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Research Article

Integrated Production and Distribution Planning for the Iron Ore Concentrate

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This paper studies the production and distribution planning problem faced by the iron ore mining companies, which aims to minimize the total costs for the whole production and distribution system of the iron ore concentrate. The ores are first mined from multiple ore locations, and then sent to the corresponding dressing plant to produce ore concentrate, which will be sent to distribution centers and finally fulfill the customers' demands. This paper also tackles the difficulty of variable cut-off grade when making mining production planning decisions. A mixed-integer programming model is developed and then solved by a Lagrange relaxation (LR) procedure. Computational results indicate that the proposed solution method is more efficient than the standard solution software CPLEX.

1. Introduction

The iron ore mining industry is a typical key resource industry providing basic raw materials for the steel industry which is one of the fundamental industries of the national economy. It has long suffered from the pressure of market fluctuations and high production and distribution costs, mainly due to the lack of coordination and flexibility in decision making. To face the challenge, it is essential to make economic coordinated production and distribution planning decisions in the environment of supply chain.

This paper studies the integrated production and distribution planning problem in the iron ore concentrate supply chain which is derived from the industrial practice. There are mainly three stages in the iron ore concentrate supply chain, i.e., the mining stage, the ore concentrate production stage, and the distribution stage. In the mining stage, the crude ores are mined from multiple ore locations, satisfying the mining capacity limits and the quality requirements (e.g., the minimum ore grade requirement). Then, in the ore concentrate production stage, these crude ores are blended, sent to the dressing plant for beneficiation due to their low ore grade, and finally refined into the final product of iron ore concentrates (or pellets, etc.) with higher iron grade. In the distribution stage, the ore concentrates are transported from the dressing plants to the distribution centers, usually through railway or sea transportation. Customers' demands are then fulfilled by the distribution centers. Due to the mass-transit transportation mode which is common in bulk material logistics, centralized inventory and transportation of multiple types of ore concentrates are maintained.

It is worth emphasizing that the iron ore grade is a significant factor throughout the iron ore concentrate supply chain, since the production costs, distribution costs, and revenue are ore grade dependent. This is unique compared to other supply chains. For example, in the mining stage, the ores are mined in a proper cut-off grade, which determines whether the ore is dumped as waste or sent to the dressing plant for further processing according to its natural

iron grade. Under the premise that the mining capacity is limited, if the cut-off grade in one mining location is too high, then more ores are dumped as waste and the ores finally sent to the dressing plant are less in amount but with higher average ore grade. Since the dressing plant has minimum requirement on the grades of the blended crude ores, the coordinated determination of the cut-off grades in multiple ore locations will significantly affect the production of the iron ore concentrates, as well as the corresponding inventory and transportation. In this paper, to promote the flexibility of the supply chain, we consider the variable cut-off grade which varies in different time periods.

This paper aims to coordinate the production and distribution of iron ore products under the consideration of practical process constraints, so as to minimize the total costs for the production and distribution system of the iron ore concentrates. The rest of the paper is organized as follows. Section 2 provides the literature review, Section 3 describes the problem studied in this paper and then a mixed-integer programming model is established. A Lagrange relaxation (LR) approach is developed in Section 4 to solve the problem. Computational tests in Section 5 illustrate the efficiency of the algorithm proposed. Finally, conclusion is drawn in Section 6.

2. Literature Review

Most related research addressed a single component of the overall production-distribution system, such as mining, production and scheduling, inventory, warehousing, or transportation. Newman et al. [1] provided a significant review of operations research in mine planning, including long-term and short-term production planning. The authors pointed out that it was promising to consider variable cut-off grade rather than fixed cut-off grade which depended on an arbitrary delineation between ore and waste. Lagos et al. [2] considered an open-pit mining problem involving extraction and processing decisions under capacity constraints and ore grade uncertainty. Berton et al. [3] proposed a mine-to-mill simulation method to determine the limitations of existing ore storage facilities and help their resizing in a context of production expansion. Luo et al. [4] provided a new low carbon iron-making supply chain planning problem in the steel industry under the carbon cap and trade mechanism, such as optimal carbon trade, raw material purchasing, raw materials and sinters inventory levels, and sintering and iron-making production schemes to be determined, aiming to minimize the total cost.

Integrated production and distribution planning has attracted much attention from the researchers in the related area. Sarmiento et al. [5] understand analysis performed on models that integrate decisions of different production and distribution functions for a simultaneous optimization by integrated analysis. Cohen et al. [6] presented a comprehensive model framework for linking decisions and performance throughout the material-production-distribution supply chain. They restrict their attention to discrete batch manufacturing operations that can be organized into multistage processing lines and to arborescent distribution networks. These systems make extensive use of intermediate buffer storage facilities. Pirkul et al. [7] provided a mixed-integer programming model for the plant and warehouse location problem where the objective is to minimize the total transportation and distribution costs and the fixed costs for opening and operating plants and warehouses. Erengüç et al. [8] presented a review on integrated production/distribution planning in supply chains. Fumero et al. [9] provided an integrated optimization model for production and distribution planning with the aim of optimally coordinating important and interrelated logistic decisions. Sun et al. [10] proposed a new risk-averse model aiming to improve production scheduling reliability integrated with shipment assignments in terms of total operating cost. Khalifehzadeh et al. [11] studied a multiobjective production-distribution system and the objectives are to minimize total costs and maximize the reliability of transportations system. Karaoğlan et al. [12] studied a variant of the problem, in which single product with limited shelf life is produced at single facility and the goal is to determine the minimum time required to produce and deliver all customer demands. Wei et al. [13] considered a two-stage production process, where, in the first stage, raw materials are transformed into continuous resources that feed the discrete production of end products in the second stage, and production and distribution decisions are considered simultaneously. Lacomme et al. [14] provided an integrated production and transportation scheduling problem by considering multiple vehicles for optimization of supply chains. In order to obtain the optimal solution, the production and transportation are considered together. Miranda et al. [15] proposed a mixed-integer programming model to integrate production, inventory, distribution, and routing decisions in a single framework.

Some scholars have also conducted research on production-distribution planning in the aspect of supply chain modeling and optimization. Nishi et al. [16] proposed a distributed supply chain planning model, which is characterized by sharing some nonconfidential information and is applied for medium-term planning in many enterprises in the oil supply chain. The model is solved using the augmented Lagrangian relaxation algorithm. Steinrücke [17] studied an approach to integrate production-transportation planning and scheduling in an aluminum supply chain. The mixed-integer model based on continuous time considering the lowest total cost of production and transportation in the supply chain with early rewards is established. This model can coordinate the quantity and time among members in the supply chain and is solved by a heuristic variable relaxation algorithm. Chen et al. [18] established the optimal production planning model in the supply chain, wherein the suppliers of raw materials or semifinished products in different geographical locations, the production of finished products, and distribution to different customers are included. The modeling method of integrated plan and its application can significantly improve economic efficiency.

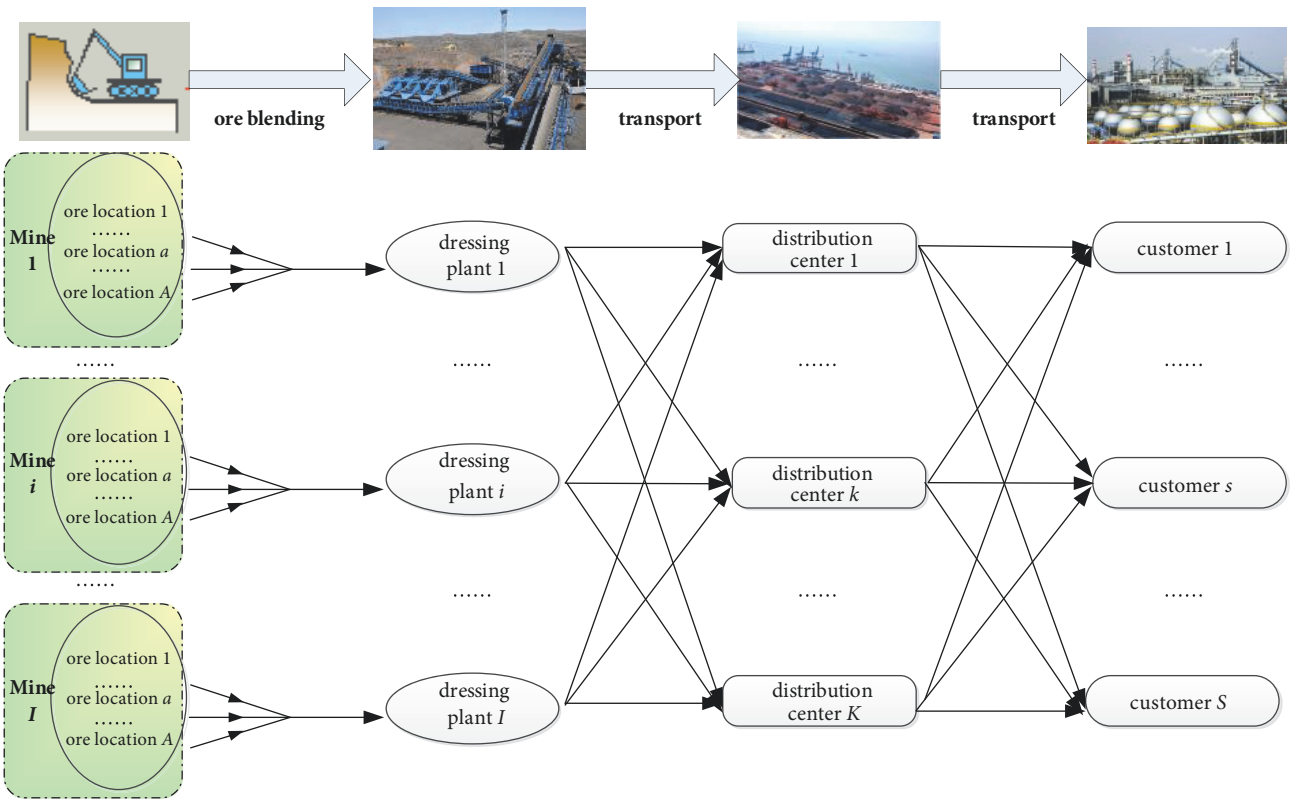

Figure 1: Open-pit mine supply chain diagram.

3. Problem Description and Model Formulation

3.1. Problem Description. The iron ore concentrate production and distribution system includes the processes of mining production, ore dressing production, warehouse stocking, and distribution. The goal is to minimize the total costs of the whole system, such as mining costs, mineral processing/dressing, inventory, and transportation. As indicated above, unique and interesting characteristics could be found in the system, especially in the production stage. For the mining production, as to each mining location, it is inevitable that the ore grade is not consistent along with the production horizon. These inevitable fluctuations of ore grade in different time periods for each mining location have significant impact on the production amount and quality of the mined crude ores and will further influence the following ore dressing production. Therefore, under the premise of production capacity limits, the consideration of variable cut-off grades is necessary in the coordinated production planning, allowing the flexibility in the mining stage. The ore dressing production process is to convert the mined iron ore material into the iron ore concentrate. Also, in order to meet the demand of different ore concentrate products from back-end customers in the supply chain, the front-end mining production and mineral processing/dressing production should be coordinated. It is different from the traditional single (multi-) product and single (multi-) cycle supply chain network model.

Figure 1 is the diagram of the iron ore concentrate production and distribution system. Iron ores are mined from multiple mining locations and then blended to meet the input ore grade requirement of ore dressing production. The ore concentrates produced in the ore dressing plants are then transported to the distribution center in discrete lot size. One distribution center might hold the inventory of limited types of ore concentrates according to the customers it serves. The ore concentrates are finally transported from the distribution center to fulfill the demand of each customer.

We have the following practical assumptions:

(1) In each mine, there are multiple ore locations which can produce crude ore of different ore grades, but the total mining capacity of the mine is limited.

(2) In each mine, all the crude ores mined from different ore locations are transported to and processed in one dressing plant corresponding to the given mine, and only one type of ore concentrate (i.e., the final product of each mine) is produced.

(3) Each type of ore concentrate is transported in large lot sizes from the dressing plant of each mine to each distribution center, while different types of ore concentrates can be transported in the same lot from the distribution center to the customer.

In the next section, we propose a mixed-integer programming model to address the integrated production and distribution planning problem for the iron ore concentrate.

3.2. Model

Sets and Parameters

I: The number of mines and dressing plants (one-to-one correspondence), $i = 1, 2, \ldots, I$.

A_i: The number of ore locations of mine $i, a = 1, 2, \ldots, A_i$.

B_i: The number of cutting off grade options in mine $i, b = 1, 2, \ldots, B_i$.

K: The number of distribution centers, $k = 1, 2, \ldots, K$.

S: The number of customers, $s = 1, 2, \ldots, S$.

T: The number of time periods in the planning horizon, $t = 1, 2, \ldots, T$.

θ_{ia}: The fixed resource consumption of ore locations a of mine i.

C_{iab}: The unitary mining cost of ore locations a in mine i when employing the cut-off grade option b.

CX_i: The unitary production cost of dressing plant of mine i.

CH_i: The unitary inventory cost of ore dressing plant of mine i.

CY_{ik}: The unitary transportation cost of ore concentrates from dressing plant of mine i to distribution center k.

CW_{ik}: The unitary inventory cost at distribution center k for the ore concentrate product of mine i.

CR_{iks}: The unitary transportation cost of ore concentrates from dressing plant of mine i to customer s.

CR_{ks}: The unitary transportation cost of ore concentrates from distribution center k to customer s.

CU_{ikt}: The setup cost for ore concentrates from dressing plant of mine i at distribution center k in period t.

CQ_{kst}: The setup cost for ore concentrates when transported from distribution center k to customer s in period t.

V_{it}^{max}: The upper limit of the stock of ore concentrates at dressing plant of mine i in period t.

V_{it}^{min}: The lower limit of the stock of ore concentrates at dressing plant of mine i in period t.

H_{kt}^{max}: The upper limit of the stock of ore concentrates from all dressing plants at distribution center k in period t.

D_{ist}: The demand of the ore concentrate product of mine i by customer s in period t.

g_{iabt}: The grade of the crude ore when employing the cut-off grade option b at the ore location a of mine i in period t.

G_i: The minimum values of grade required for the dressing plant of mine i.

l_i: The production rate of concentrate of dressing plant of mine i.

v_{iabt}: The setup cost of ore locations a of mine i when employing the cut-off grade option b in period t.

U_{it}: The mining capacity of mine i in period t.

Q_{it}: The production capacity of dressing plant of mine i in period t.

w_{iabt}: The unitary resource consumption of ore locations a of mine i when employing the cut-off grade option b in period t.

$\overline{C}_{ikt}$: The maximum transport capacity of ore concentrates from mine i to distribution center k in period t.

M: A very large positive number.

Variables

x_{iabt}: The amount of ore mined with cut-off grade option b in mining location a of mine i during period t.

y_{it}: The amount of ore concentrate produced in the dressing plant of mine i in period t.

IM_{it}: The inventory of ore concentrates produced by dressing plant of mine i in period t.

z_{ikt}: The amount of ore concentrate transported from dressing plant i to distribution center k in period t.

IC_{ikt}: The inventory of the ore concentrate product of mine i at distribution center k in period t.

e_{ikst}: The amount of the ore concentrate product of mine i transported from distribution center k to customer s in period t.

φ_{iabt}: 1, if cut-off grade option b is utilized by the mining production of location a of mine i in period t; 0, otherwise.

α_{ikt}: 1, if the ore concentrates i was transported to distribution center k in period t; 0, otherwise.

β_{kst}: 1, if there are concentrates transported from distribution center k to customer s in period t; 0, otherwise.

Using the above parameters and decision variables, the established 0-1 integer programming model is formulated as follows.

$$\begin{aligned}\min \quad TC &= \sum_{i=1}^{I}\sum_{a=1}^{A_i}\sum_{b=1}^{B_i}\sum_{t=1}^{T} v_{iabt}\varphi_{iabt} \\ &+ \sum_{i=1}^{I}\sum_{a=1}^{A_i}\sum_{b=1}^{B_i}\sum_{t=1}^{T} C_{iab}x_{iabt} \\ &+ \sum_{i=1}^{I}\sum_{t=1}^{T} CX_i y_{it} + \sum_{i=1}^{I}\sum_{t=1}^{T} CH_i IM_{it} \\ &+ \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} CU_{ikt}\alpha_{ikt} \\ &+ \sum_{k=1}^{K}\sum_{s=1}^{S}\sum_{t=1}^{T} CQ_{kst}\beta_{kst} \\ &+ \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} CY_{ik}z_{ikt} \\ &+ \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} CW_{ik}IC_{ikt} \\ &+ \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{s=1}^{S}\sum_{t=1}^{T} CR_{ks}e_{ikst}\end{aligned} \tag{1}$$

Subject to:

$$\sum_{a\in A_i}\left(\theta_{ia}\sum_{b\in B_i}\varphi_{iabt} + \sum_{b\in B_i} w_{iabt}x_{iabt}\right) \le U_{it}, \quad \forall i\in I,\ t\in T \tag{2}$$

$$\sum_{a\in A_i}\sum_{b\in B_i} x_{iabt} \le Q_{it}, \quad \forall i\in I,\ t\in T \tag{3}$$

$$\sum_{a\in A_i}\sum_{b\in B_i} g_{iabt}x_{iabt} \ge G_i \sum_{a\in A_i}\sum_{b\in B_i} x_{iabt}, \quad \forall i\in I,\ t\in T \tag{4}$$

$$\sum_{a\in A_i}\sum_{b\in B_i} x_{iabt} \ge l_i y_{it}, \quad \forall i\in I,\ t\in T \tag{5}$$

$$x_{iabt} \le M\varphi_{iabt}, \quad \forall i\in I,\ a\in A_i,\ b\in B_i,\ t\in T \tag{6}$$

$$\sum_{b\in B_i}\varphi_{iabt} \le 1, \quad \forall i\in I,\ a\in A_i,\ t\in T \tag{7}$$

$$y_{it} + IM_{i,t-1} - \sum_{k\in K} z_{ikt} = IM_{it}, \quad \forall i\in I,\ t\in T \tag{8}$$

$$z_{ikt} + IC_{i,k,t-1} - \sum_{s\in S} e_{ikst} = IC_{ikt}, \quad \forall i\in I,\ k\in K,\ t\in T \tag{9}$$

$$V_{it}^{min} \le IM_{it} \le V_{it}^{max}, \quad \forall i\in I,\ t\in T \tag{10}$$

$$\sum_{i\in I} IC_{ikt} \le H_{kt}^{max}, \quad \forall k\in K,\ t\in T \tag{11}$$

$$z_{ikt} \le \overline{C}_{ikt}\alpha_{ikt}, \quad \forall i\in I,\ k\in K,\ t\in T \tag{12}$$

$$\sum_{i\in I} e_{ikst} \le M\beta_{kst}, \quad \forall k\in K,\ s\in S,\ t\in T \tag{13}$$

$$\sum_{k\in K} e_{ikst} \ge D_{ist}, \quad \forall i\in I,\ s\in S,\ t\in T \tag{14}$$

$$x_{iabt}, y_{it}, IM_{it}, z_{ikt}, IC_{ikt}, e_{ikst} \ge 0, \quad \forall i\in I,\ a\in A_i,\ b\in B_i,\ k\in K,\ s\in S,\ t\in T \tag{15}$$

$$\varphi_{iabt}, \alpha_{ikt}, \beta_{kst} \in \{0,1\}, \quad \forall i\in I,\ a\in A_i,\ b\in B_i,\ k\in K,\ s\in S,\ t\in T \tag{16}$$

The objective function (1) minimizes the total production and distribution costs including setup cost of mine mining, mining costs, mineral processing costs, inventory cost of the dressing plant, setup costs and transportation costs from mine (dressing plant) to distribution center, transportation setup costs, inventory costs of distribution center, and transportation cost of distribution center to customer. Constraints (2) indicate the limitation of mining production capacity; constraints (3) indicate the limitation of ore dressing capacity; constraints (4) indicate that the minimum input grade requirement of each dressing plant is satisfied; constraints (5) show the converting rational relationship between the iron ore and the ore concentrate; constraints (6) ensure the logic correctness between related binary and continuous variables; constraints (7) indicate that, for the mining location, only one option of cut-off grade can be chosen at each period; constraints (8) indicate the ore concentrate inventory balance constraints in each dressing plant; constraints (9) indicate the inventory balance constraints in the distribution center; constraints (10) indicate the upper and lower limits of the inventory of ore concentrate in each concentrate in each period; constraints (11) indicate the upper limit of the inventory at the distribution center k in period t; constraints (12) indicate the logical constraints on whether or not the ore concentrate produced in dressing plant i is transported to distribution center k at time t; constraints (13) indicate the logical constraints on whether or not the ore concentrate is transported from distribution center k to customer s at time t; constraints (14) are the demand fulfillment constraints; constraints (15) and (16) are the range of variables.

4. Lagrange Relaxation Approach

Lagrange relaxation algorithm is a widely used optimization algorithm. The basic principle of Lagrange relaxation

algorithm is to use the Lagrange multiplier to relax the constraints that are difficult to handle in the original problem. Thus, the problem is transformed into a more easily solved Lagrange relaxation subproblem, and the optimal solution for obtaining the original problem is gradually approximated by solving the Lagrange dual problem.

The Lagrange relaxation algorithm is well applied to solve various production-distribution problems. Jayaraman et al. [19] used a heuristic procedure with the aid of Lagrangian relaxation to solve the production-distribution, facility location-allocation problem. Shen [20] proposed a Lagrangian relaxation solution algorithm to deal with the multicommodity supply chain design problem. Altiparmak et al. [21] developed a steady-state GA to solve the multiproduct, multistage SCN design problem. The results were compared using CPLEX, Lagrangian heuristic, hybrid genetic, and simulated annealing algorithms. An et al. [22] used Lagrange relaxation algorithm to solve a mixed-integer programming model which jointly considers production planning and detailed scheduling. In the following part, we propose a Lagrange relaxation procedure to solve the problem.

The first step of the Lagrange approach is to determine which constraints should be relaxed. The big-M constraints such as constraints (6), (12), and (13) are not considered because the default CPLEX solver performs bad when solving models of similar structure. In this paper, we considered two candidate constraints, i.e., constraints (2) and (7), to be relaxed. We first introduce the Lagrange approach procedure based on the relaxation of constraints (2) and then give the details of differentiation when applying the approach based on the relaxation of constraints (7).

4.1. The First Relaxation Strategy. To develop Lagrange relaxation algorithm to solve the problem, there are three key steps, i.e., solving the Lagrange dual subproblem, constructing the feasible solution, and updating the Lagrange multiplier.

4.1.1. Solving the Subproblem. This paper relaxes the constraints (2) first. By Introducing Lagrange multiplier vectors $\lambda_{it} > 0$, we get the following relaxation problem (LR^1):

$$\begin{aligned} L^1 = \min \Bigg\{ & \sum_{i=1}^{I}\sum_{a=1}^{A_i}\sum_{b=1}^{B_i}\sum_{t=1}^{T} v_{iabt}\varphi_{iabt} + \sum_{i=1}^{I}\sum_{a=1}^{A_i}\sum_{b=1}^{B_i}\sum_{t=1}^{T} C_{iab}x_{iabt} + \sum_{i=1}^{I}\sum_{t=1}^{T} CX_i y_{it} + \sum_{i=1}^{I}\sum_{t=1}^{T} CH_i IM_{it} + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} CU_{ikt}\alpha_{ikt} \\ & + \sum_{k=1}^{K}\sum_{s=1}^{S}\sum_{t=1}^{T} CQ_{kst}\beta_{kst} + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} CY_{ik}z_{ikt} + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} CW_{ik}IC_{ikt} \\ & + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{s=1}^{S}\sum_{t=1}^{T} CR_{iks}e_{ikst} + \sum_{i=1}^{I}\sum_{t=1}^{T}\lambda_{it}\left[\sum_{a=1}^{A_i}\left(\theta_{ia}\sum_{b=1}^{B_i}\varphi_{iabt} + \sum_{b=1}^{B_i} w_{iabt}x_{iabt}\right) - U_{it}\right]\Bigg\} \end{aligned} \tag{17}$$

subject to constraints(3)~ (16).

Then, the corresponding Lagrange dual problem (LD^1) can be obtained.

$$LD^1 = \max_{\{\lambda\}} \left\{\min L^1\right\} \tag{18}$$

4.1.2. Heuristic Algorithm to Construct the Upper Bound. The optimal solution of LD^1 is probably not the feasible solution of the original problem due to the relaxation of constraints (2). In order to obtain the upper bound of the problem, we develop a heuristic procedure to construct the feasible solution of the original problem from the optimal solution of LD^1.

For easy description, we first define a parameter EX_{it} to denote the amount of the exceeding capacity for mine i in time period t, i.e.,

$$EX_{it} = \sum_{a=1}^{A_i}\left(\theta_{ia}\sum_{b=1}^{B_i}\varphi_{iabt} + \sum_{b=1}^{B_i} w_{iabt}x_{iabt}\right) - U_{it} \quad \forall i, t \tag{19}$$

and the detailed steps of the heuristic algorithm are described below.

Step 1. For each mine i and time period t, calculate the value EX_{it} of each mine i at each time period t; if $EX_{it} > 0$, set binary parameter $BF_{it}=1$ and turn to Step 2; otherwise, stop.

Step 2. For each x_{iabt}, if $x_{iabt} > 0$, adjust the value of x_{iabt} proportionately according to the following expression.

$$x_{iabt} = x_{iabt} - \left(\frac{EX_{it}}{w_{iabt}}\right) * \left(\frac{g_{iabt}}{\sum_{a=1}^{A_i}\sum_{b=1}^{B_i} g_{iabt}}\right) \tag{20}$$

Step 3. If $x_{iabt} < 0$, add the value of $\theta_{ia} - x_{iabt} * w_{iabt}$ into R_{it}, and then set $x_{iabt} = 0$, $\varphi_{iabt} = 0$. If $R_{it} > 0$, turn to Step 4; otherwise, turn to Step 5.

Step 4. For the current mine i and time period t, select the x_{iabt}, which satisfies $x_{iabt} > 0$, with the highest g_{iabt} for secondary adjustment.

$$x_{iabt} = x_{iabt} - \frac{R_{it}}{w_{iabt}} \tag{21}$$

Step 5. Up to the present, variable x_{iabt} can be determined in the current mine i and time period t; from constraints (5) we can get the value of the corresponding variable y_{it}; turn Step 6.

Step 6. Adjust variables IM_{it} according to constraints (8), and let z_{ikt} remain unchanged.

4.1.3. Updating the Lagrange Multiplier. In this paper, the subgradient algorithm is used to update the Lagrange multiplier.

In the m iteration, after solving the subproblem, we calculate the dual function value through the following expression:

$$g(\lambda^m) = \sum_{a=1}^{A_i}\left(\theta_{ia}\sum_{b=1}^{B_i}\varphi_{iabt} + \sum_{b=1}^{B_i} w_{iabt}x_{iabt}\right) - U_{it} \tag{22}$$

where λ^m denotes the multiplier vector composed by all $\{\lambda_{it}\}$ in the m iteration.

Then, update the multiplier. Calculate the multiplier to be used in the next iteration through the following formula:

$$\lambda^{m+1} = \lambda^m + s^m g(\lambda^m) \tag{23}$$

where s^m is the step size in the m iteration, being calculated as follows:

$$s^m = \omega\frac{L^U - L^m}{\|g(\lambda^m)\|^2}, \quad 0 < \omega < 2 \tag{24}$$

where L^U is the current best upper bound obtained by the heuristic given in Section 4.1.2, L^m is the current optimal solution of the subproblem in the m iteration, and the initial value of the parameter ω is set to 0.5.

4.1.4. The Lagrange Approach. The overall steps of the approach are described as follows:

Step 1. Initialization. Let m=0, UB=∞, LB=0, $\lambda^{(0)}$ =0, where m is the index of iterations, $\lambda^{(m)}$ is the Lagrange multiplier in the m generation, and LB is the largest lower bound of the current problem.

Step 2. Solving the subproblem. Given the Lagrange multiplier $\lambda^{(m)}$, the subproblem is solved by CPLEX software to get the optimal solution of the problem LD and the corresponding objective function value $g_{LR}^{(m)}$. If $g_{LR}^{(m)} > LB$, update LB.

Step 3. Construction of feasible solution. Implement the heuristic procedure in Section 4.1.2 to construct the feasible solution of the original problem. Denote the feasible solutions that are constructed as $X^{(m)}$ and the corresponding objective function value as $g_{LD}^{(m)}$, if $g_{LD}^{(m)} < UB$, and then update UB.

Step 4. If one of the following stopping criteria is satisfied, the algorithm stops, and the current solution $X^{(m)}$ is the final best solution obtained for the original problem (denoted as $X*$).

(1) Dual gap gap=|(UB-LB)/LB| < ε, where ε is a smaller positive number.

(2) m is greater than the upper limit of the number of iterations given in advance.

Step 5. Update the Lagrange multiplier as described in Section 4.1.3, let m=m+1, and go to Step 2.

4.2. The Second Relaxation Strategy. In the following part, instead of relaxing constraints (2), we relax constraints (7). The framework and the multiplier updating is similar to the first strategy, except the formulation of the subproblem and the heuristic procedure to construct feasible solution.

Introducing Lagrange multiplier vectors $\mu_{iat} > 0$, after the relaxation constraint (7), we get the following relaxation problem (LR^2):

$$\begin{aligned} L^2 = \min \Bigg\{ & \sum_{i=1}^{I}\sum_{a=1}^{A_i}\sum_{b=1}^{B_i}\sum_{t=1}^{T} v_{iabt}\varphi_{iabt} \\ & + \sum_{i=1}^{I}\sum_{a=1}^{A_i}\sum_{b=1}^{B_i}\sum_{t=1}^{T} C_{iab}x_{iabt} + \sum_{i=1}^{I}\sum_{t=1}^{T} CX_i y_{it} \\ & + \sum_{i=1}^{I}\sum_{t=1}^{T} CH_i IM_{it} + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} CU_{ikt}\alpha_{ikt} \\ & + \sum_{k=1}^{K}\sum_{s=1}^{S}\sum_{t=1}^{T} CQ_{kst}\beta_{kst} + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} CY_{ik}z_{ikt} \\ & + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{t=1}^{T} CW_{ik}IC_{ikt} + \sum_{i=1}^{I}\sum_{k=1}^{K}\sum_{s=1}^{S}\sum_{t=1}^{T} CR_{iks}e_{ikst} \\ & + \sum_{i=1}^{I}\sum_{a=1}^{A_i}\sum_{t=1}^{T}\mu_{iat}\left(\sum_{b=1}^{B_i}\varphi_{iabt} - 1\right)\Bigg\} \end{aligned} \tag{25}$$

constraints (2)~(6), (8)~(16).

Further, the corresponding Lagrange dual problem (LD^2) is obtained.

$$LD^2 = \max_{\{\mu\}}\left\{\min L^2\right\} \tag{26}$$

The corresponding steps of the heuristic procedure are described as follows.

For ease of description, the following parameters are defined.

(1) exc_iat. It . means the sum of the optional projects of mining i and mining location a in a period of time t.

$$exc\_iat = \sum_{b=1}^{B_i}\varphi_{iabt} \quad \forall i, a, t \tag{27}$$

(2) f_iat. It . indicates whether the sum of the optional projects a mining i and mining location a in a period of time t is beyond mark value; f_iat =0 means not beyond; f_iat =1 means beyond.

(3) Sum_cs_{iabt}. It i. ndicates the sum of the starting cost and mining cost of a mining i and mining location a in a period of time t, that is

$$Sum_{cs_{iabt}} = v_{iabt}\varphi_{iabt} + C_{iab}x_{iabt}. \tag{28}$$

Table 1: Parameter settings and symbol definitions used in numerical analysis.

Parameter	symbol definitions
UB^C	The optimal solution obtained by CPLEX software
UB^1	The upper bound obtained by the Lagrangian approach based on the first relaxation strategy
UB^2	The upper bound obtained by the Lagrangian approach based on the second relaxation strategy
MT^C	The maximum computing time by CPLEX software (in seconds)
AT^C	The average computing time by CPLEX software
MT^1	The maximum computing time of the Lagrangian approach based on the first relaxation strategy
AT^1	The average computing time of the Lagrangian approach based on the first relaxation strategy
MT^2	The maximum computing time of the Lagrangian approach based on the second relaxation strategy
AT^2	The average computing time of the Lagrangian approach based on the second relaxation strategy
GAP^1	$(UB^1\text{-}UB^C)/\ UB^C\times100\%$
GAP^2	$(UB^2\text{-}UB^C)/\ UB^C\times100\%$

(4) pro_it. It i. ndicates the total amount of mining at all points of mine i in a period of time t.

$$pro\_it = \sum_{a=1}^{A_i}\sum_{b=1}^{B_i} x_{iabt} \quad \forall i, t \tag{29}$$

The detailed steps of the heuristic algorithm are described below.

Step 1. Calculate the value exc_iat of mining location a of each mine i at each time period t; if exc_iat >0, make f_iat =1, and turn Step 2; otherwise, stop.

Step 2. Calculate the value Sum_cs_{iabt} of 1 for all φ_{iabt} of current mine i and mining location a at each time period t, to sort it from low to high; calculate the value pro_it of optional project b in turn; if $pro\_it \geq l_i y_{it}$, the value x_{iabt} and φ_{iabt} of corresponding b are the same; otherwise, turn Step 3.

Step 3. Choose the second smallest Sum_cs_{iabt} according to the order of the Sum_cs_{iabt}; follow the above principles to judge in turn, until the end of the loop.

5. Computational Tests

To evaluate the computational efficiency of the proposed solution approach, we generate test instances that reflect real application. To benchmark the performance of the approach, the MILP models for the instances are solved using CPLEX 12.6. CPLEX 12.6 is also employed to solve the Lagrangian subproblem. All the computational experiments are performed on a computer with Intel Core i3-2350 2.30 GHz CPU and 4GB RAM.

We have generated eight groups of test instances based on the real-world data, but of different scales. We consider different levels of mines, transit centers, steel enterprises, and time periods, and therefore the scale of instance groups can be denoted as "I-K-S-T". The scales of the eight groups are "3-2-2-3", "4-3-3-5", "6-5-6-8", "8-5-12-10", "10-5-12-10", "15-5-15-10", "25-5-15-10", and "30-5-20-10". Five cases are set for each group according to different combinations of mining locations (Ai) and different cutting off grade options (Bi), i.e., Case I: Ai={3}, Bi={3}; Case II: Ai={5}, Bi={3}; Case III: Ai={10}, Bi={5}; Case IV: Ai={15}, Bi={5}; Case V: Ai={20}, Bi={5}.

The parameter values are randomly generated within a given interval and uniformly distributed, as shown below. It should be noted that, due to the different scale of the calculation example, the parameter value setting related to mining capacity and mineral processing capacity will be adjusted in proportion according to the actual situation; otherwise there will be no feasible solution obtained. Table 1 shows the parameter settings and symbol definitions involved in the numerical experiment.

Table 2 shows the average computation time and the maximum computation time of the first four instance groups using CPLEX software and two Lagrange relaxation strategies, respectively. In the first four instance groups, the optimal or feasible solution can be found within the acceptable time (3600 seconds) in most cases. Observe that the proposed Lagrangian approach based on the relaxation strategy is superior to that of the second relaxation strategy, while they both performs much better than the standard solution software CPLEX.

The results of the last four instance groups are shown in Table 3. The maximum numbers of iterations of the last four instance groups are set to 100, 500, 1000, and 3000, respectively. On average, the approach based on the second relaxation strategy is 40.01% faster than the CPLEX, and the approach based on the first relaxation strategy is 66.31% faster than the CPLEX. Note that the approach based on the second relaxation strategy is superior to that of the first strategy for some cases.

6. Conclusion

(1) Taking open-pit mines as background, this paper conducted an overall analysis and description of the ore concentrate supply chain constructed by a number of mining spots, dressing plants, distribution centers and steel enterprises, and established. The variable cut-off grades is considered to achieve flexible mining production in the environment of supply chain.

Table 2: Results of the first four instance groups.

Case number	A_i&B_i	MT^C	AT^C	MT^1	AT^1	MT^2	AT^2
3-2-2-3	I	1.332	1.217	1.294	1.273	1.351	1.301
	II	1.887	1.369	1.306	1.241	1.308	1.234
	III	1.383	1.283	1.275	1.220	1.467	1.279
	IV	1.671	1.525	1.76	1.669	1.544	1.486
	V	1.791	1.660	2.088	2.051	1.956	1.695
4-3-3-5	I	1.610	1.578	1.614	1.594	1.592	1.554
	II	3.868	2.338	2.034	2.022	3.689	2.207
	III	9.912	5.112	2.659	2.488	10.439	4.806
	IV	52.71	11.714	2.507	2.446	15.556	7.043
	V	23.746	15.655	13.424	13.313	19.135	14.377
6-5-6-8	I	8.482	6.519	7.957	7.899	10.607	7.499
	II	147.982	63.910	136.342	136.151	511.334	123.349
	III	>3600	—	31.995	21.839	600.279	359.695
	IV	607.37	365.091	37.015	31.516	452.278	382.634
	V	2205.71	1064.77	38.352	37.469	2966.05	1144.306
8-5-12-10	I	300.800	137.948	22.876	18.345	600.37	171.952
	II	3544.42	1326.38	73.235	54.387	3539.15	1195.01
	III	1896.12	1119.792	67.342	56.363	1743.23	989.64
	IV	2679.91	712.124	62.588	57.048	432.834	239.303
	V	3014.41	942.568	41.334	38.106	532.223	341.817
Average			289.128		24.422		249.609

Table 3: Results of the last four instance groups.

Case number	A_i&B_i	MT^C	AT^C	MT^1	AT^1	MT^2	AT^2	GAP^1	GAP^2
10-5-12-10	I	83.213	54.882	41.934	38.282	50.983	45.1848	0.29%	2.7%
	II	1898.77	890.269	131.231	129.849	980.321	780.26	0.18%	0.09%
	III	2449.73	1669.95	571.933	483.982	267.965	114.68	0.11%	0.05%
	IV	2572.42	855.199	223.206	139.909	376.99	274.807	0.01%	0.07%
	V	2092.78	881.135	583.234	556.611	321.343	172.623	0.01%	0.02%
15-5-15-10	I	1397.86	840.568	210.432	173.903	1022.83	816.482	0.3%	0.26%
	II	1162.39	1159.03	1623.35	693.934	1093.34	613.926	0.34%	0.31%
	III	839.346	598.342	421.345	210.467	498.472	290.313	0.4%	0.45%
	IV	902.342	478.321	390.848	201.234	480.532	276.644	0.36%	0.35%
	V	1304.35	890.325	909.322	309.587	1092.64	492.933	0.51%	0.55%
25-5-15-10	I	2932.82	736.915	857.55	757.098	526.33	245.879	0.22%	0.24%
	II	2264.27	1493.34	2059.31	1476.29	86.258	82.557	0.26%	0.25%
	III	1345.35	873.325	1024.35	640.352	230.345	98.343	0.25%	0.28%
	IV	130.322	128.276	356.321	186.561	113.537	110.617	0.28%	0.30%
	V	1932.34	903.345	983.342	672.346	302.345	178.342	0.27%	0.29%
30-5-20-10	I	1890.23	1422.83	997.321	885.355	412.345	208.241	0.5%	0.51%
	II	2023.22	1893.35	1872.34	1592.23	902.242	390.342	0.48%	0.52%
	III	2582.34	2012.34	1904.53	1356.32	1073.23	980.123	0.61%	0.65%
	IV	890.355	683.225	793.234	583.562	493.344	201.354	1.0%	0.9%
	V	1790.34	1340.35	903.342	793.125	563.233	298.342	1.5%	1.6%
Average			990.266		594.049		333.599	0.394%	0.519%

(2) A mixed integer linear programming model is proposed to optimize the integration of the mine production planning and distribution planning in the supply chain. The model considered coordination and balance between quantity and quality indicators when making the planning decisions.

(3) Calculation experiments on random instances generated from practical data shows the superior performance of the proposed Lagrange relaxation algorithm compared with CPLEX software.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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