Hindawi
Mathematical Problems in Engineering
Volume 2019, Article ID 7989768, 6 pages
https://doi.org/10.1155/2019/7989768

Letter to the Editor

Comment on "Corrigendum to 'On Interval-Valued Hesitant Fuzzy Soft Sets'" and "Generalized Trapezoidal Fuzzy Soft Set and Its Application in Medical Diagnosis"

Ahmed Mostafa Khalil,[1,2] **Sheng-Gang Li**,[1] **Hong-Xia Li**,[3] **and Hai-Dong Zhang**[4]

[1] *College of Mathematics and Information Science, Shaanxi Normal University, Xi'an 710062, China*
[2] *Department of Mathematics, Faculty of Science, Al-Azhar University, Assiut 71524, Egypt*
[3] *School of mathematics and statistics, Long Dong University, Qingyang 745000, China*
[4] *School of Mathematics and Computer Science, Northwest University for Nationalities, Lanzhou 730030, China*

Correspondence should be addressed to Sheng-Gang Li; shengganglinew@126.com

Received 10 December 2017; Accepted 11 December 2018; Published 26 March 2019

Academic Editor: Erik Cuevas

1. Introduction and Preliminaries

The aim of this article is to correct assertions (3) and (4) of Theorem 17 proposed by Khalil et al. [1] and assertions (3) and (4) of Theorem 29 proposed by Zhang et al. [2]. We first generalize (in an unambiguous manner) the fitting or remedy of the involved preliminary notions concerning interval-valued hesitant fuzzy soft set and generalized trapezoidal fuzzy soft sets and then use these notions to remedy the flaw in those assertions.

The concept of soft set was introduced by Molodtsov [3] in 1999. Maji et al. [4] defined some operations on soft sets and showed that the distributive law on soft sets is varied. Later on, Ali et al. [5] pointed out that the distributive laws of soft sets are not true in general. It is necessary to present the theory in a mathematical (at least errorless) way. So, in this paper, we will show how to remedy flows in soft set theory by correcting assertions (3) and (4) of Theorem 17 in [1] and assertions (3) and (4) of Theorem 29 in [2] (including also remedy preliminaries they involved).

In this section we present (in a clear, rigorous, and nonburdensome manner) some notions concerning interval-valued hesitant fuzzy soft set and generalized trapezoidal fuzzy soft sets, most of which are generalizations of the fitting or remedy of the original notions in the references. (All key notions involved in this note are defined for arbitrary sets; this is convenient for subsequent study (including categorical approach to soft sets, study process relating to soft sets since soft sets can be used in decision-making and study approximation and control relating to soft sets since soft sets can be looked to be L-sets).)

First, we present hesitant fuzzy set, soft set, fuzzy soft set, and interval-valued hesitant fuzzy soft set. We will use Y^X to denote the set of all mappings from X to Y, use 2^X to denote the set of all subsets of X (it is a completely distributive complete lattice with the order $\subseteq$), and write $\downarrow E = \{p \in P \mid \exists e \in E, p \leq e\}$ for each subset E of a poset $(P, \leq)$. For a mapping $\Phi : X \longrightarrow L$ (called L-subset or L-set and write also $\Phi \in L^X$, where L is a lattice with the least element 0), we call $\operatorname{supp} \Phi = \{x \in X \mid \Phi(x) \neq 0\}$ the support of Φ; sometimes we identify Φ with its restriction to $\operatorname{supp} \Phi$ for convenience.

Definition 1. (1) (cf. [6]) Each closed interval $a = [a^L, a^U] = \{x \mid a^L \leq x \leq a^U\}$ of R (the set of all real numbers) is called an interval number (we will identify a with the corresponding real number a if $a^L = a^U = a$). The set of all interval numbers (resp., all interval numbers included in $[0, 1]$) is denoted by $\mathbb{R}$ (resp., by $\mathbb{I}$). Obviously, $\mathbb{I}$ is a completely distributive complete lattice with the point-wise (We can identify a closed interval

$[a,b]$ of R with a point $\langle a,b\rangle$ in R^2.) order $\leq$ which is defined by

$$\left[a^L,a^U\right] \leq \left[b^L,b^U\right] \Longleftrightarrow a^L \leq b^L \text{ and } a^U \leq b^U. \tag{1}$$

(2) (cf. [6]) For two interval numbers $a = [a^L,a^U]$ and $b = [b^L,b^U]$, let $l_a = a^U - a^L$ and $l_b = b^U - b^L$, and define the degree $p(a \geq b)$ (i.e., $p(b \leq a)$) of possibility of $a \geq b$ (i.e., $b \leq a$) by

$$p(a \geq b) = \begin{cases} 0, & l_a + l_b = 0,\ a < b, \\ \dfrac{1}{2}, & l_a + l_b = 0,\ a = b, \\ 1, & l_a + l_b = 0,\ a > b, \\ \max\left\{1 - \max\left(\dfrac{b^+ - a^-}{l_a + l_b}, 0\right), 0\right\}, & l_a + l_b \neq 0. \end{cases} \tag{2}$$

The conclusions of Theorems 2 and 3 (proofs of which will be given in our other paper) will be used in the sequel.

Theorem 2. *The relation $\leqslant$ on $\mathbb{I}$, defined by*

$$a \leqslant b \Longleftrightarrow p(a \leq b) \geq p(a \geq b), \tag{3}$$

is a total preorder on $\mathbb{I}$ whose restriction to R is the same as the restriction of the point-wise order $\leq$ to R.

Theorem 3. *For any two elements $\mathscr{A},\mathscr{B} \in 2^{(\mathbb{I})}$ (the set of all nonempty finite subsets of $\mathbb{I}$), where $\mathscr{A} = \{[a_1^L,a_1^U], [a_2^L,a_2^U],\cdots,[a_m^L,a_m^U]\}$ is an m-element set satisfying $[a_1^L,a_1^U] \leqslant [a_2^L,a_2^U] \leqslant \cdots \leqslant [a_m^L,a_m^U]$, and $\mathscr{B} = \{[b_1^L,b_1^U],[b_2^L,b_2^U],\ldots,[b_n^L,b_n^U]\}$ is an n-element set satisfying $[b_1^L,b_1^U] \leqslant [b_2^L,b_2^U] \leqslant \cdots \leqslant [b_n^L,b_n^U]$ (m and n are nature numbers). Define $\mathscr{A} \preceq \mathscr{B}$ iff we have the following.*

(1) $[a_k^L,a_k^U] \leq [b_k^L,b_k^U]$ (for $k = 1,2,\ldots,m-1$) and $[a_m^L,a_m^U] \leq [b_k^L,b_k^U]$ (for $k = m,m+1,\ldots,n$) hold in $(\mathbb{I},\leq)$ when $m \leq n$.

(2) $[a_k^L,a_k^U] \leq [b_k^L,b_k^U]$ (for $k = 1,2,\ldots,n-1$) and $[a_k^L,a_k^U] \leq [b_n^L,b_n^U]$ (for $k = n,n+1,\ldots,m$) hold in $(\mathbb{I},\leq)$ when $m \leq n$.

Then $(2^{(\mathbb{I})},\preceq)$ is a completely distributive complete lattice. Moreover, the following hold (without loss of generality we assume $m \leq n$):

$$\begin{aligned} \mathscr{A} \vee \mathscr{B} &= \left\{\left[a_1^L \vee b_1^L, a_1^U \vee b_1^U\right], \left[a_2^L \vee b_2^L, a_2^U \vee b_2^U\right], \ldots, \right. \\ &\quad \left.\left[a_m^L \vee b_m^L, a_m^U \vee b_m^U\right], \left[a_m^L \vee b_n^L, a_m^U \vee b_n^U\right]\right\}, \\ \mathscr{A} \wedge \mathscr{B} &= \left\{\left[a_1^L \wedge b_1^L, a_1^U \wedge b_1^U\right], \left[a_2^L \wedge b_2^L, a_2^U \wedge b_2^U\right], \ldots, \right. \\ &\quad \left.\left[a_m^L \wedge b_m^L, a_m^U \wedge b_m^U\right], \left[a_m^L \wedge b_n^L, a_m^U \wedge b_n^U\right]\right\}. \end{aligned} \tag{4}$$

Definition 4 (cf. [7, 8]). Each element (Write also $\Gamma = \{\Gamma(x_1)/x_1,\Gamma(x_2)/x_2,\ldots,\Gamma(x_n)/x_n\}$ if $X = \{x_1,x_2,\ldots,x_n\}$ is a finite set.) $\Gamma \in (2^{(\mathbb{I})})^X$ (i.e., a family $\{\Gamma(x)\}_{x\in X}$ consisting of nonempty finite subsets of $\mathbb{I}$) is called an interval-valued hesitant fuzzy set on X, where $\Gamma(x)$ denotes all possible interval valued membership degrees of the element $x \in X$ to Γ, and $2^{(\mathbb{I})}$ has the order $\preceq$ as defined in Theorem 3.

Definition 5 (cf. [3]). Each element $\Gamma \in (2^X)^I$ (i.e., a family $\{\Gamma(i)\}_{i\in I}$ of subsets of X) is called a soft set on X, where X is called an initial universe set and I is called a set of parameters.

Definition 6 (cf. [9, 10]). Each element $\Gamma \in ([0,1]^X)^I$ (i.e., a family $\{\Gamma(i)\}_{i\in I}$ of fuzzy sets on X) is called a fuzzy soft set on X, and each element $\Phi \in ((2^{(\mathbb{I})})^X)^I$ is called an interval-valued hesitant fuzzy soft set on X; we will use $\gamma_{i,x}^{(k)}$ to denote the k-th largest element (it exists by Theorem 3) in $(\Phi(i)(x),\leqslant)$ ($i \in I, x \in X, k \leq |\Phi(i)(x)|$, and $|\Phi(i)(x)|$ denotes the cardinality of $\Phi(i)(x)$).

Definition 7 (cf. [1]). Let $\Phi,\Psi \in ((2^{(\mathbb{I})})^X)^I$ with $\operatorname{supp}\Phi = U$ and $\operatorname{supp}\Psi = V$. If, for each $i \in U$, there exists $j \in V$ such that $\Phi(i) \preceq \Psi(j)$ (i.e., $\Phi(i)(x) \preceq \Psi(j)(x)$ for each $x \in X$), then we say that Φ is a generalized interval-valued hesitant fuzzy soft subset of Ψ (written as $\Phi \preceq \Psi$ or $\Phi \Subset \Psi$). If both $\Phi \preceq \Psi$ and $\Psi \preceq \Phi$ hold, then we write $\Phi \cong \Psi$.

Proposition 8 holds.

Proposition 8. $\Phi \cong \Psi \Longleftrightarrow \downarrow\Phi = \downarrow\Psi$.

Definition 9 (cf. [10]). For a family $\{\Phi_1,\Phi_2\} \subseteq ((2^{(\mathbb{I})})^X)^I$, we call $\Phi_1 \otimes \Phi_2 \in ((2^{(\mathbb{I})})^X)^{I\times I}$, defined by

$$(\Phi_1 \otimes \Phi_2)(i_1,i_2) = \Phi_1(i_1) \vee \Phi_2(i_2) \quad (\forall (i_1,i_2) \in I \times I), \tag{5}$$

the hyper-product of $\{\Phi_1,\Phi_2\}$ in $((2^{(\mathbb{I})})^X)^I$ and call $\Phi_1 \times \Phi_2 \in ((2^{(\mathbb{I})})^X)^{I\times I}$, defined by

$$(\Phi_1 \times \Phi_2)(i_1,i_2) = \Phi_1(i_1) \wedge \Phi_2(i_2) \quad (\forall (i_1,i_2) \in I \times I), \tag{6}$$

the product of $\{\Phi_1,\Phi_2\}$ in $((2^{(\mathbb{I})})^X)^I$.

Next, we present trapezoidal fuzzy set, trapezoidal fuzzy soft set, and generalized trapezoidal fuzzy soft set.

Definition 10 (cf. [2, 11, 12]). An element $n \in [0,1]^R$, defined by

$$n(x) = (n_1,n_2,n_3,n_4)(x) = \begin{cases} \dfrac{x-n_1}{n_2-n_1}, & n_1 \leq x < n_2, \\ 1, & n_2 \leq x < n_3, \\ \dfrac{x-n_4}{n_3-n_4}, & n_3 \leq x \leq n_4, \\ 0, & x \in R - [n_1,n_4], \end{cases} \tag{7}$$

is called trapezoidal fuzzy number. The set of all trapezoidal fuzzy numbers satisfying $0 \le n_1 \le n_2 \le n_3 \le n_4 \le 1$ is denoted by $\mathbb{J}$; it is a Hutton algebra (i.e., a completely distributive complete lattice with an order-reversion involution) with the point-wise order $\le$. Each element $\xi \in \mathbb{J}^X$ is called a trapezoidal fuzzy set on X, each element $\Gamma \in (\mathbb{J}^X)^I$ (i.e., a family $\{\Gamma(i)\}_{i\in I}$ of trapezoidal fuzzy sets on X) is called a trapezoidal fuzzy soft set on X, and each element $\Phi \in (\mathbb{J}^X \times \mathbb{J})^I$ is called a generalized trapezoidal fuzzy soft set on X.

Definition 11 (cf. [2]). Let $\mathscr{A} = \{\Phi_k\}_{k\in K} \subseteq (\mathbb{J}^X \times \mathbb{J})^I$.

(1) The supremum (Each element (as a family) of $((2^{(\mathbb{I})})^X)^I$ is just a poset with the point-wise order, but $(\mathbb{J}^X \times \mathbb{J})^I$ is a Hutton algebra with respect to the point-wise order (see Theorem 17).) $\bigvee_{k\in K}\Phi_k$ (write as also $\underset{\cdot}{\bigcup}_{k\in K}\Phi_k$) and the infimum $\bigwedge_{k\in K}\Phi_k$ (write as also $\underset{\cdot}{\bigcap}_{k\in K}\Phi_k$) of $\{\Phi_k\}_{k\in K}$ in $(\mathbb{J}^X \times \mathbb{J})^I$ is called the generalized trapezoidal point-wise union and the generalized trapezoidal point-wise intersection of $\mathscr{A}$ in $(\mathbb{J}^X \times \mathbb{J})^I$, respectively.

(2) We call $\bigotimes \mathscr{A} = \bigotimes_{k\in K}\Phi_k \in (\mathbb{J}^X \times \mathbb{J})^{(I^K)}$, defined by

$$\left(\bigotimes_{k\in K}\Phi_k\right)\left(\{i_k\}_{k\in K}\right) = \bigvee_{k\in K}\Phi_k(i_k) \quad \left(\forall \{i_k\}_{k\in K} \in I^K\right), \tag{8}$$

the hyper-product of $\mathscr{A}$ and $\prod \mathscr{A} = \prod_{k\in K}\Phi_k \in (\mathbb{J}^X \times \mathbb{J})^{(I^K)}$, defined by

$$\left(\prod_{k\in K}\Phi_k\right)\left(\{i_k\}_{k\in K}\right) = \bigwedge_{k\in K}\Phi_k(i_k) \quad \left(\forall \{i_k\}_{k\in K} \in I^K\right), \tag{9}$$

the product of $\mathscr{A}$.

2. Correction to Paper [1]

Khalil et al. [1] proposed the following Theorem 12 (and hoped it to be a corrected version of assertions (3) and (4) of Theorem 36 in Zhang et al. [10]):

Theorem 12 (see [1, Theorem 17, p.3]). *Let* $\{\Phi, \Psi, \Gamma\} \subseteq ((2^{(\mathbb{I})})^X)^I$ *with* $\operatorname{supp}\Phi = U$, $\operatorname{supp}\Psi = V$, *and* $\operatorname{supp}\Gamma = W$. *Then*

(3) $\Phi \times (\Psi \otimes \Gamma) \cong (\Phi \times \Psi) \otimes (\Phi \times \Gamma)$.

(4) $\Phi \otimes (\Psi \times \Gamma) \cong (\Phi \otimes \Psi) \times (\Phi \otimes \Gamma)$.

However, neither (3) nor (4) is correct (see Example 13).

Example 13. Let $X = \{u, v\}$ be a set of two cars and $I = \{i, j, k, l\}$ be a set of parameters, where i (resp., j, k, l) stands for the parameter "cheap" (resp., "equipment", "fuel consumption", and expensive). Again let $\{\Phi, \Psi, \Gamma\} \subseteq ((2^{(\mathbb{I})})^X)^I$, defined by

$$\begin{aligned}
&\Phi(i) = \left\{\frac{\{[0.3, 0.5], [0.4, 0.8]\}}{u}, \frac{\{[0.1, 0.2], [0.4, 0.5]\}}{v}\right\},\\
&\Phi(j) = \left\{\frac{\{[0.2, 0.3], [0.4, 0.5]\}}{u}, \frac{\{[0.7, 0.9], [0.5, 0.6]\}}{v}\right\},\\
&\Psi(k) = \left\{\frac{\{[0.2, 0.6], [0.6, 0.8]\}}{u}, \frac{\{[0.4, 0.5], [0.7, 0.8]\}}{v}\right\},\\
&\Gamma(l) = \left\{\frac{\{[0.1, 0.4], [0.2, 0.7]\}}{u}, \frac{\{[0.3, 0.7], [0.4, 0.9]\}}{v}\right\},
\end{aligned} \tag{10}$$

where $\operatorname{supp}\Phi = U = \{i, j\}$, $\operatorname{supp}\Psi = V = \{k\}$, and $\operatorname{supp}\Gamma = W = \{l\}$.

(1) Let $\Theta = \Phi \times (\Psi \otimes \Gamma)$ and $\Xi = (\Phi \times \Psi) \otimes (\Phi \times \Gamma)$. As

$$\begin{aligned}
&\Psi(k) \vee \Gamma(l) = \left\{\frac{\{[0.2, 0.6], [0.6, 0.8]\}}{u}, \frac{\{[0.4, 0.7], [0.7, 0.9]\}}{v}\right\},\\
&\Theta(i, k, l) = \Phi(i) \wedge [\Psi(k) \vee \Gamma(l)]\\
&\quad = \left\{\frac{\{[0.2, 0.5], [0.4, 0.8]\}}{u}, \frac{\{[0.1, 0.2], [0.4, 0.5]\}}{v}\right\}
\end{aligned} \tag{11}$$

and

$$\begin{aligned}
&\Theta(j, k, l) = \Phi(j) \wedge [\Psi(k) \vee \Gamma(l)]\\
&\quad = \left\{\frac{\{[0.2, 0.3], [0.4, 0.5]\}}{u}, \frac{\{[0.4, 0.7], [0.5, 0.6]\}}{v}\right\},
\end{aligned} \tag{12}$$

similarly,

$$\begin{aligned}
&\Phi(i) \wedge \Psi(k) = \left\{\frac{\{[0.2, 0.5], [0.4, 0.8]\}}{u}, \frac{\{[0.1, 0.2], [0.4, 0.5]\}}{v}\right\},\\
&\Phi(j) \wedge \Psi(k) = \left\{\frac{\{[0.2, 0.3], [0.4, 0.5]\}}{u}, \frac{\{[0.4, 0.5], [0.7, 0.8]\}}{v}\right\},\\
&\Phi(i) \wedge \Gamma(l) = \left\{\frac{\{[0.1, 0.4], [0.2, 0.7]\}}{u}, \frac{\{[0.1, 0.2], [0.4, 0.5]\}}{v}\right\},
\end{aligned} \tag{13}$$

and

$$\Phi(j) \wedge \Gamma(l) = \left\{\frac{\{[0.1, 0.3], [0.2, 0.5]\}}{u}, \frac{\{[0.3, 0.6], [0.4, 0.9]\}}{v}\right\}. \tag{14}$$

Thus

$$\begin{aligned}
&\Xi((i,k),(i,l)) = [\Phi(i)\wedge\Psi(k)]\vee[\Phi(i)\wedge\Gamma(l)] \\
&\quad = \left\{\frac{\{[0.2,0.5],[0.4,0.8]\}}{u},\frac{\{[0.1,0.2],[0.4,0.5]\}}{v}\right\}, \\
&\Xi((j,k),(j,l)) = [\Phi(j)\wedge\Psi(k)]\vee[\Phi(j)\wedge\Gamma(l)] \\
&\quad = \left\{\frac{\{[0.2,0.3],[0.4,0.5]\}}{u},\frac{\{[0.4,0.6],[0.7,0.9]\}}{v}\right\}, \\
&\Xi((i,k),(j,l)) = [\Phi(i)\wedge\Psi(k)]\vee[\Phi(j)\wedge\Gamma(l)] \\
&\quad = \left\{\frac{\{[0.2,0.5],[0.4,0.8]\}}{u},\frac{\{[0.3,0.6],[0.4,0.9]\}}{v}\right\}, \\
&\Xi((j,k),(i,l)) = [\Phi(j)\wedge\Psi(k)]\vee[\Phi(i)\wedge\Gamma(l)] \\
&\quad = \left\{\frac{\{[0.2,0.4],[0.4,0.7]\}}{u},\frac{\{[0.4,0.5],[0.7,0.8]\}}{v}\right\}.
\end{aligned} \tag{15}$$

As $\Xi((i,k),(j,l)) \not\leq \Theta(i,k,l)$ and $\Xi((i,k),(j,l)) \not\leq \Theta(j,k,l)$, $(\Phi\times\Psi)\otimes(\Phi\times\Gamma) \not\leq \Phi\times(\Psi\otimes\Gamma)$. Therefore, $\Phi\times(\Psi\otimes\Gamma)\cong(\Phi\times\Psi)\otimes(\Phi\times\Gamma)$ does not hold.

(2) Now we show $\Phi\otimes(\Psi\times\Gamma)\cong(\Phi\otimes\Psi)\times(\Phi\otimes\Gamma)$ does not hold. Let $\Delta=\Phi\otimes(\Psi\times\Gamma)$ and $\Omega=(\Phi\otimes\Psi)\times(\Phi\otimes\Gamma)$. As

$$\begin{aligned}
&\Psi(k)\wedge\Gamma(l) \\
&\quad = \left\{\frac{\{[0.1,0.4],[0.2,0.7]\}}{u},\frac{\{[0.3,0.5],[0.4,0.8]\}}{v}\right\}, \\
&\Delta(i,k,l) = \Phi(i)\vee[\Psi(k)\wedge\Gamma(l)] \\
&\quad = \left\{\frac{\{[0.3,0.5],[0.4,0.8]\}}{u},\frac{\{[0.3,0.5],[0.4,0.8]\}}{v}\right\}
\end{aligned} \tag{16}$$

and

$$\begin{aligned}
&\Delta(j,k,l) = \Phi(j)\vee[\Psi(k)\wedge\Gamma(l)] \\
&\quad = \left\{\frac{\{[0.2,0.4],[0.4,0.7]\}}{u},\frac{\{[0.5,0.6],[0.7,0.9]\}}{v}\right\},
\end{aligned} \tag{17}$$

similarly,

$$\begin{aligned}
&\Phi(i)\vee\Psi(k) \\
&\quad = \left\{\frac{\{[0.3,0.6],[0.6,0.8]\}}{u},\frac{\{[0.4,0.5],[0.7,0.8]\}}{v}\right\}, \\
&\Phi(j)\vee\Psi(k) \\
&\quad = \left\{\frac{\{[0.2,0.6],[0.6,0.8]\}}{u},\frac{\{[0.5,0.6],[0.7,0.9]\}}{v}\right\}, \\
&\Phi(i)\vee\Gamma(l) \\
&\quad = \left\{\frac{\{[0.3,0.5],[0.4,0.8]\}}{u},\frac{\{[0.3,0.7],[0.4,0.9]\}}{v}\right\},
\end{aligned} \tag{18}$$

and

$$\begin{aligned}
&\Phi(j)\vee\Gamma(l) \\
&\quad = \left\{\frac{\{[0.2,0.4],[0.4,0.7]\}}{u},\frac{\{[0.5,0.7],[0.7,0.9]\}}{v}\right\}.
\end{aligned} \tag{19}$$

Thus

$$\begin{aligned}
&\Omega((i,k),(i,l)) = [\Phi(i)\vee\Psi(k)]\wedge[\Phi(i)\vee\Gamma(l)] \\
&\quad = \left\{\frac{\{[0.3,0.5],[0.4,0.8\}}{u},\frac{\{[0.3,0.5],[0.4,0.8\}}{v}\right\}, \\
&\Omega((j,k),(j,l)) = [\Phi(j)\vee\Psi(k)]\wedge[\Phi(j)\vee\Gamma(l)] \\
&\quad = \left\{\frac{\{[0.2,0.4],[0.4,0.7]\}}{u},\frac{\{[0.5,0.6],[0.7,0.9]\}}{v}\right\}, \\
&\Omega((i,k),(j,l)) = [\Phi(i)\vee\Psi(k)]\wedge[\Phi(j)\vee\Gamma(l)] \\
&\quad = \left\{\frac{\{[0.2,0.4],[0.4,0.7]\}}{u},\frac{\{[0.4,0.7],[0.5,0.8]\}}{v}\right\}, \\
&\Omega((j,k),(i,l)) = [\Phi(j)\vee\Psi(k)]\wedge[\Phi(i)\vee\Gamma(l)] \\
&\quad = \left\{\frac{\{[0.2,0.5],[0.4,0.8]\}}{u},\frac{\{[0.3,0.6],[0.4,0.9]\}}{v}\right\}.
\end{aligned} \tag{20}$$

As $\Omega((i,k),(j,l)) \not\leq \Delta(i,k,l)$ and $\Omega((i,k),(j,l)) \not\leq \Delta(j,k,l)$, $(\Phi\otimes\Psi)\times(\Phi\otimes\Gamma) \not\leq \Phi\otimes(\Psi\times\Gamma)$. Therefore, $\Phi\otimes(\Psi\times\Gamma)\cong(\Phi\otimes\Psi)\times(\Phi\otimes\Gamma)$ does not hold.

Correction 1 to Theorem 12. Let $\{\Phi,\Psi,\Gamma\}\subseteq((2^{(\mathbb{I})})^X)^I$ with $\operatorname{supp}\Phi=U$, $\operatorname{supp}\Psi=V$, and $\operatorname{supp}\Gamma=W$. Then

(3) $\{[\Phi(i)\wedge\Psi(j)]\vee[\Phi(i)\wedge\Gamma(k)]\}_{((i,j),(i,k))\in(U\times V)\times(U\times W)} = \{\Phi(i)\wedge[\Psi(j)\vee\Gamma(k)]\}_{(i,j,k)\in U\times V\times W}$, and thus $\Phi\times(\Psi\otimes\Gamma)\preceq(\Phi\times\Psi)\otimes(\Phi\times\Gamma)$.

(4) $\{[\Phi(i)\vee\Psi(j)]\wedge[\Phi(i)\vee\Gamma(k)]\}_{((i,j),(i,k))\in(U\times V)\times(U\times W)} = \{\Phi(i)\vee[\Psi(j)\wedge\Gamma(k)]\}_{(i,j,k)\in U\times V\times W}$, and thus $\Phi\otimes(\Psi\times\Gamma)\preceq(\Phi\otimes\Psi)\times(\Phi\otimes\Gamma)$.

Proof. (3) For each $(i,j,k)\in U\times V\times W$ and each $x\in X$,

$$\begin{aligned}
&[\Phi(i)(x)\wedge\Psi(j)(x)]\vee[\Phi(i)(x)\wedge\Gamma(k)(x)] \\
&\quad = \Phi(i)(x)\wedge[\Psi(j)(x)\vee\Gamma(k)(x)] \\
&\quad = [\Phi\times(\Psi\otimes\Gamma)](i,j,k)(x)
\end{aligned} \tag{21}$$

by Theorem 3, which means

$$\begin{aligned}
&\{[\Phi(i)\wedge\Psi(j)] \\
&\quad \vee[\Phi(i)\wedge\Gamma(k)]\}_{((i,j),(i,k))\in(U\times V)\times(U\times W)} = \{\Phi(i) \\
&\quad \wedge[\Psi(j)\vee\Gamma(k)]\}_{(i,j,k)\in U\times V\times W},
\end{aligned} \tag{22}$$

and thus $\Phi\times(\Psi\otimes\Gamma)\preceq(\Phi\times\Psi)\otimes(\Phi\times\Gamma)$.

(4) For each $(i,j,k)\in U\times V\times W$ and each $x\in X$,

$$\begin{aligned}
&[\Phi(i)(x)\vee\Psi(j)(x)]\wedge[\Phi(i)(x)\vee\Gamma(k)(x)] \\
&\quad = \Phi(i)(x)\vee[\Psi(j)(x)\wedge\Gamma(k)(x)] \\
&\quad = [\Phi\otimes(\Psi\times\Gamma)](i,j,k)(x)
\end{aligned} \tag{23}$$

by Theorem 3, which means

$$\begin{aligned}
&\{[\Phi(i)\vee\Psi(j)] \\
&\quad \wedge[\Phi(i)\vee\Gamma(k)]\}_{((i,j),(i,k))\in(U\times V)\times(U\times W)} = \{\Phi(i) \\
&\quad \vee[\Psi(j)\wedge\Gamma(k)]\}_{(i,j,k)\in U\times V\times W},
\end{aligned} \tag{24}$$

and thus $\Phi\otimes(\Psi\times\Gamma)\preceq(\Phi\otimes\Psi)\times(\Phi\otimes\Gamma)$. □

Correction 2 to Theorem 12. Let $\{\Phi, \Psi, \Gamma\} \subseteq ((2^{(\mathbb{I})})^X)^I$ with $\operatorname{supp}\Phi = U$, $\operatorname{supp}\Psi = V$, and $\operatorname{supp}\Gamma = W$. Then

(1) $\Phi \wedge (\Psi \vee \Gamma) = (\Phi \wedge \Psi) \vee (\Phi \wedge \Gamma)$.

(2) $\Phi \vee (\Psi \wedge \Gamma) = (\Phi \vee \Psi) \wedge (\Phi \vee \Gamma)$.

Proof. It follows from the fact that $(2^{(\mathbb{I})}, \preceq)$ is a completely distributive complete lattice. □

Analogously, the following (on community and associativity) hold.

Theorem 14. *Let* $\{\Phi, \Psi, \Gamma\} \subseteq ((2^{(\mathbb{I})})^X)^I$ *with* $\operatorname{supp}\Phi = U$, $\operatorname{supp}\Psi = V$, *and* $\operatorname{supp}\Gamma = W$. *Then*

(1) $\{\Phi(i) \wedge \Psi(j)\}_{(i,j)\in(U\times V)} = \{\Psi(j) \wedge \Phi(i)\}_{(i,j)\in(U\times V)}$, *and thus* $\Phi \times \Psi = \Psi \times \Phi$.

(2) $\{\Phi(i) \vee \Psi(j)\}_{(i,j)\in(U\times V)} = \{\Psi(j) \vee \Phi(i)\}_{(i,j)\in(U\times V)}$, *and thus* $\Phi \otimes \Psi = \Psi \otimes \Phi$.

(3) $\{\Phi(i) \wedge [\Psi(j) \wedge \Gamma(k)]\}_{(i,j,k)\in U\times V\times W} = \{[\Phi(i) \wedge \Psi(j)] \wedge \Gamma(k)\}_{(i,j,k)\in U\times V\times W}$, *and thus* $\Phi \times (\Psi \times \Gamma) = (\Phi \times \Psi) \times \Gamma$.

(4) $\{\Phi(i) \vee [\Psi(j) \vee \Gamma(k)]\}_{(i,j,k)\in U\times V\times W} = \{[\Phi(i) \vee \Psi(j)] \vee \Gamma(k)\}_{(i,j,k)\in U\times V\times W}$, *and thus* $\Phi \otimes (\Psi \otimes \Gamma) = (\Phi \otimes \Psi) \otimes \Gamma$.

3. Correction to Paper [2]

For the finite set case, Zhang et al. [2] defined the concept of generalized trapezoidal fuzzy soft set and showed some examples of its operations, along with an application in decision-making. In this section we first give an example to show the following Theorem 15 is incorrect. Then we consider some possible corrections or replacements of the theorem.

Theorem 15 (see [2, Theorem 29, p.7]). *Let* $\{\Phi, \Psi, \Gamma\} \subseteq (\mathbb{J}^X \times \mathbb{J})^I$ *with* $\operatorname{supp}\Phi = I_1$, $\operatorname{supp}\Psi = I_2$, *and* $\operatorname{supp}\Gamma = I_3$. *Then*

(3) $\Phi \times (\Psi \otimes \Gamma) = (\Phi \times \Psi) \otimes (\Phi \times \Gamma)$.

(4) $\Phi \otimes (\Psi \times \Gamma) = (\Phi \otimes \Psi) \times (\Phi \otimes \Gamma)$.

However, neither (3) nor (4) is correct mainly because the cardinality $|I_1\times(I_2\times I_3)|$ *of* $I_1\times(I_2\times I_3)$ *(the support of the left) is less than the cardinality* $|(I_1\times I_2)\times(I_1\times I_3)|$ *of* $(I_1\times I_2)\times(I_1\times I_3)$ *(the support of the right) in general.*

Example 16. Let $X = \{u, v\}$ be a two-element set, $I = \{i, j, k, m\}$ a four-element set, $I_1 = \{i, j\}$ a two-element set, $I_2 = \{k\}$, and $I_3 = \{m\}$. Again let $\{\Phi, \Psi, \Gamma\} \subseteq (\mathbb{J}^X \times \mathbb{J})^I$ (with supports I_1, I_2, and I_3, respectively), defined by

$$\begin{aligned}
\Phi(i) &= \left\{\frac{(0.3, 0.4, 0.6, 0.8)}{u}, \frac{(0.5, 0.7, 0.7, 0.9)}{v}; (0.1, 0.4, 0.7, 0.8)\right\},\\
\Phi(j) &= \left\{\frac{(0.2, 0.3, 0.4, 0.5)}{u}, \frac{(0.7, 0.8, 0.8, 0.9)}{v}; (0.5, 0.6, 0.7, 0.8)\right\},\\
\Psi(k) &= \left\{\frac{(0.1, 0.2, 0.2, 0.3)}{u}, \frac{(0.2, 0.3, 0.4, 0.5)}{v}; (0.4, 0.4, 0.7, 0.9)\right\},\\
\Gamma(m) &= \left\{\frac{(0.4, 0.5, 0.5, 0.8)}{u}, \frac{(0.1, 0.1, 0.4, 0.5)}{v}; (0.6, 0.8, 0.9, 0.9)\right\}.
\end{aligned} \tag{25}$$

Then $\Upsilon_1 = \Phi \times (\Psi \otimes \Gamma)$ has the following two members:

$$\begin{aligned}
\Upsilon_1(i, k, m) &= \left\{\frac{(0.3, 0.4, 0.5, 0.8)}{u}, \frac{(0.2, 0.3, 0.4, 0.5)}{v}; (0.1, 0.4, 0.7, 0.8)\right\},\\
\Upsilon_1(j, k, m) &= \left\{\frac{(0.2, 0.3, 0.4, 0.5)}{u}, \frac{(0.2, 0.3, 0.4, 0.5)}{v}; (0.5, 0.6, 0.7, 0.8)\right\}.
\end{aligned} \tag{26}$$

$\Upsilon_2 = (\Phi \times \Psi) \otimes (\Phi \times \Gamma)$ has the following four members:

$$\begin{aligned}
\Upsilon_2(i, k, i, m) &= \left\{\frac{(0.3, 0.4, 0.5, 0.8)}{u}, \frac{(0.2, 0.3, 0.4, 0.5)}{v}; (0.1, 0.4, 0.7, 0.8)\right\},\\
\Upsilon_2(j, k, i, m) &= \left\{\frac{(0.3, 0.4, 0.5, 0.8)}{u}, \frac{(0.2, 0.3, 0.4, 0.5)}{v}; (0.4, 0.4, 0.7, 0.8)\right\},\\
\Upsilon_2(i, k, j, m) &= \left\{\frac{(0.2, 0.3, 0.4, 0.5)}{u}, \frac{(0.2, 0.3, 0.4, 0.5)}{v}; (0.5, 0.6, 0.7, 0.8)\right\},\\
\Upsilon_2(j, k, j, m) &= \left\{\frac{(0.2, 0.3, 0.4, 0.5)}{u}, \frac{(0.2, 0.3, 0.4, 0.5)}{v}; (0.5, 0.6, 0.7, 0.8)\right\}.
\end{aligned} \tag{27}$$

Apparently, $\Upsilon_1 \subseteq \Upsilon_2$. However, $\downarrow \Upsilon_1 \not\supseteq \downarrow \Upsilon_2$ (because $\Upsilon_2(j, k, i, m) \not\leq \Upsilon_1(i, k, m)$ and $\Upsilon_2(j, k, i, m) \not\leq \Upsilon_1(j, k, m)$) and $|I_1 \times (I_2 \times I_3)| = 2 \neq 4 = |(I_1 \times I_2) \times (I_1 \times I_3)|$. Therefore, $\downarrow \Upsilon_1 \neq \downarrow \Upsilon_2$, especially, $\Upsilon_1 \neq \Upsilon_2$ (whether we look Υ_1 and Υ_2 to be sets or look them to be mappings).

Now we consider some possible corrections or replacements of the theorem.

A correction to Theorem 15. Let $\Phi, \Psi, \Gamma \in (\mathbb{J}^X \times \mathbb{J})^I$ with $\operatorname{supp}\Phi = I_1$, $\operatorname{supp}\Psi = I_2$, and $\operatorname{supp}\Gamma = I_3$. Then

(3) $\Phi \times (\Psi \otimes \Gamma) \subseteq (\Phi \times \Psi) \otimes (\Phi \times \Gamma)$.

(4) $\Phi \otimes (\Psi \times \Gamma) \subseteq (\Phi \otimes \Psi) \times (\Phi \otimes \Gamma)$.

Proof. We only show (3). For each $(i_1,(i_2,i_3)) \in I_1 \times (I_2 \times I_3)$,

$$\begin{aligned}[\Phi \times (\Psi \otimes \Gamma)](i_1,i_2,i_3) &= \Phi(i_1) \wedge (\Psi(i_2) \vee \Gamma(i_3)) \\ &= [\Phi(i_1) \wedge \Psi(i_2)] \vee [\Psi(i_2) \wedge \Gamma(i_3)] \\ &= [(\Phi \times \Psi) \otimes (\Phi \times \Gamma)]((i_1,i_2)(i_2,i_3))\end{aligned} \tag{28}$$

since $\mathbb{J}^X \times \mathbb{J}$ is a distributive lattice. Therefore,

$$\begin{aligned}&\{[\Phi \times (\Psi \otimes \Gamma)](i_1,i_2,i_3) \mid (i_1,i_2,i_3) \in I_1 \times (I_2 \times I_3)\} \\ &\quad \subseteq \{[(\Phi \times \Psi) \otimes (\Phi \times \Gamma)] \\ &\quad \cdot ((i_1,i_2)(i_2,i_3)) \mid ((i_1,i_2)(i_2,i_3)) \in (I_1 \times I_2) \\ &\quad \times (I_2 \times I_3)\},\end{aligned} \tag{29}$$

which implies $\Phi \times (\Psi \otimes \Gamma) \subseteq (\Phi \times \Psi) \otimes (\Phi \times \Gamma)$. □

Theorem 17. *$(\mathbb{J}^X \times \mathbb{J})^I$ is a Hutton algebra (with the pointwise order), and thus the following hold for a family $\{\Phi_{k,j} \mid j \in J_k, k \in K\} \subseteq (\mathbb{J}^X \times \mathbb{J})^K$:*

$$\begin{aligned}\left(\bigvee_{j\in J_j,k\in K} \Phi_{k,j}\right)' &= \bigwedge_{j\in J_k,k\in K} \Phi'_{k,j}, \\ \left(\bigwedge_{j\in J_k,k\in I} \Phi_{k,j}\right)' &= \bigvee_{j\in J_k,k\in K} \Phi'_{k,j}, \\ \bigvee_{k\in K}\bigwedge_{j\in J_k} \Phi_{k,j} &= \bigwedge_{f\in\mathcal{F}}\bigvee_{k\in K} \Phi_{k,f(k)}, \\ \bigwedge_{k\in K}\bigvee_{j\in J_k} \Phi_{k,j} &= \bigvee_{f\in\mathcal{F}}\bigwedge_{j\in K} \Phi_{k,f(k)},\end{aligned} \tag{30}$$

where $\mathcal{F}$ is the set of all mappings $f : K \longrightarrow \coprod_{k\in K} J_k$ (disjoint union) satisfying $f(k) \in J_k$ $(\forall k \in K)$.

Proof. It follows from the fact that $\mathbb{J}$ is a Hutton algebra. □

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported by the National Natural Science Foundations of China (Grants nos. 11771263 and 11501496), the Key Research and Development Project of Shaanxi Province of China (Grant no. 2018KW-050), the Fundamental Research Funds For the Central Universities (Grant no. 2018CBLY001), and the Fundamental for Graduate students to participate in international academic conference (Grant no. 2018CBLY001).

References

[1] A. M. Khalil, H. Zhang, L. Xiong, and W. Ma, "Corrigendum to "on interval-valued hesitant fuzzy soft sets"," *Mathematical Problems in Engineering*, vol. 2016, Article ID 5646927, 3 pages, 2016.

[2] H. Zhang, L. Shu, and S. Liao, "Generalized Trapezoidal Fuzzy Soft Set and Its Application in Medical Diagnosis," *Journal of Applied Mathematics*, vol. 2014, Article ID 312069, 12 pages, 2014.

[3] D. Molodtsov, "Soft set theory—first results," *Computers & Mathematics with Applications*, vol. 37, no. 4-5, pp. 19–31, 1999.

[4] P. K. Maji, R. Biswas, and A. R. Roy, "Soft set theory," *Computers & Mathematics with Applications*, vol. 45, no. 4-5, pp. 555–562, 2003.

[5] M. I. Ali, F. Feng, X. Liu, W. K. Min, and M. Shabir, "On some new operations in soft set theory," *Computers & Mathematics with Applications*, vol. 57, no. 9, pp. 1547–1553, 2009.

[6] Z. S. Xu and Q. L. Da, "The uncertain OWA operator," *International Journal of Intelligent Systems*, vol. 17, no. 6, pp. 569–575, 2002.

[7] V. Torra and Y. Narukawa, "On hesitant fuzzy sets and decision," in *Proceedings of the IEEE International Conference on Fuzzy Systems*, pp. 1378–1382, Jeju-do, Republic of Korea, August 2009.

[8] V. Torra, "Hesitant fuzzy sets," *International Journal of Intelligent Systems*, vol. 25, no. 6, pp. 529–539, 2010.

[9] P. K. Maji, R. Biswas, and A. R. Roy, "Fuzzy soft sets," *Journal of Fuzzy Mathematics*, vol. 9, no. 3, pp. 589–602, 2001.

[10] H. Zhang, L. Xiong, and W. Ma, "On Interval-Valued Hesitant Fuzzy Soft Sets," *Mathematical Problems in Engineering*, vol. 2015, Article ID 254764, 17 pages, 2015.

[11] A. Kaufmann and M. M. Gupta, "Introduction to fuzzy arithmetic: theory and applications: Van Nostrand Rein hold, N. York," *International Journal of Approximate Reasoning*, vol. 321, pp. 189-190, 1987.

[12] Z. Xiao, S. Xia, K. Gong, and D. Li, "The trapezoidal fuzzy soft set and its application in MCDM," *Applied Mathematical Modelling: Simulation and Computation for Engineering and Environmental Systems*, vol. 36, no. 12, pp. 5844–5855, 2012.