Research Article

Solution and Verification of Cutter Position for Machining Split Equal-Base Circle Bevel Gear

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Received 3 April 2019; Revised 17 August 2019; Accepted 5 September 2019; Published 17 October 2019

Academic Editor: Ramon Sancibrian

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The purpose of this study is to achieve machining of a split equal-base circle bevel gear. Based on the equal-base circle bevel gear theory, a cutting coordinate system for the separate piece is developed, and the tooth surface processing path of the separate piece is analysed and planned. According to the working principle of the equal-base circle bevel gear, by analysing the cutter position and posture, the calculation method for the angle between the wheel blank coordinate system and the fixed space coordinate system is derived when machining the separate piece for different spiral angles as well as various concavity and convexity properties. The explicit function expressions for the cutter centre coordinates and axis vectors for machining the separate piece are obtained. Using MATLAB, the tool position model is verified by means of calculation, and the tooth cutting simulation of the separate piece is carried out using VERICUT software. By machining experiment and tooth surface measurement analysis, the feasibility of machining the separate piece and the correctness of the tool position mathematical model are verified.

1. Introduction

Extralarge bevel gears are core components of large-scale key equipment, which are used extensively in important fields such as power generation, ships, and mines. To date, the extralarge bevel gears used in key equipment have all been straight bevel gears. When the diameter of the wheel in the gear pair is more than 3000 mm, the wheel blank rigidity is poor owing to its overall structure, which is presented as a ring and thin wall shape, and this particular structure can be deformed easily during processing, transportation, and assembling. Therefore, the split structure has often been adopted in existing extralarge straight bevel gears.

Compared to the curved bevel gear, the straight bevel gear exhibits certain disadvantages, such as weak carrying capacity, poor transmission stability, large impact, and high noise, which impose serious restrictions on the improvement of major equipment and industrial upgrades. Using the split curved bevel gear to replace the existing split straight bevel gear has become an important trend in industrial development and upgrades and is also a key problem that urgently needs to be solved. Moreover, with the continuous improvement in the bevel gear machining theory, equipment, and technology, this approach provides a powerful guarantee for the processing and manufacturing of split extralarge curved bevel gears [1–4].

General curved tooth bevel gears that are split along the tooth space, such as Gleason and Oerlikon, will have a greater impact on the strength of the teeth on both sides of the splitting position, owing to their larger helix angle. When stretching, bending, and other deformations occur in the process of machining split gears, it is difficult to improve the machining precision by means of active modification to the machining path. Furthermore, the machining theories for the general curved bevel gear are complex and are all based on the entire wheel blank; therefore, the machining theory for the split wheel blank is more complicated and it is difficult to realise machining to split the general curved bevel gear.
The equal-base circle gear is a new type of curved tooth bevel gear; its wheel and pinion are formed by finger-shaped milling cutters with a general machine tool, and the machine tool structure is simple, while the cost of the cutter applied in the machining is low [5–7]. Furthermore, tooth surface machining of the equal-base circle gear can be achieved by a two-axis linkage machine, making it easy to apply tooth machining following wheel blank splitting. Thus, the extralarge equal-base circle bevel gear is an ideal curved bevel gear for replacing the existing extralarge straight bevel gear [8–11].

The existing theory of machining the equal-base circle bevel gear is based on its integral machining. In this case, the gear rotary centre certainly coincides with the machine turning axis, which is a necessary condition for two-axis linkage machining of the equal-base circle bevel gear. For the extralarge equal-base circle bevel gear, particularly when the diameter reaches approximately 7000 mm, few devices can meet this requirement. Therefore, when machining a separate block of the split extralarge equal-base circle bevel gear on a small-size machine tool, the rotary centre of the wheel blank is located outside the machine, the original motion control theory of the cutting gear is no longer applicable, and indexing during the machining process cannot be realised with ease. Thus, a systematic theoretical study of nonintegral machining and its realisation method offers scientific significance and engineering application value for the manufacturing of extralarge split curved bevel gears.

The remainder of this paper is organised as follows: Section 2 describes the calculation principle and method and derives the calculation formulae for the cutter centre and tool axis vector for different orders, rotation directions, and concavity and convexity of each tooth surface. Section 3 verifies the accuracy of the calculation method of the tool position, using the calculation and drawing function of the MATLAB software. Section 4 discusses the verification process for the mathematical model deduced earlier and the simulation result analysis. Conclusions and further developments are presented in Section 5.

2. Cutter Position Calculation

To achieve machining of the tooth surface on each individual block, firstly, the cutting coordinate system for the split block is established, and processing path planning is carried out. Thereafter, the instantaneous cutter position and posture corresponding to each position of the tooth surface are analysed. Finally, the tool axis vector and cutter centre coordinates are solved.

2.1. Machining Path Planning. Figure 1 illustrates the coordinate system of the entire wheel blank cutting, in the process of machining the equal-base circle bevel gear; when the finger milling cutter moves at a uniform speed from the outer to the inner along the reference cone element, the wheel blank rotates at a specific speed, and the tool surface envelops the tooth surface. After machining a tooth surface, the wheel blank rotates around the axis to complete indexing, and the milling cutter returns to the previous starting point for milling the next tooth surface. In this manner, all concave surfaces have the same cutter trajectory and rule, and the same is true for all convex surfaces.

However, during the process of tooth cutting on the split block of a superlarge wheel blank, the wheel blank cannot rotate around its true geometric central axis because the indexing rotation centre of the split block often exceeds the machine tool specification. However, in order to produce equal-base circle bevel gears, assuming that the wheel blank does not move, the cutter must move along the relative motion trajectory with the wheel blank as a reference frame. The cutter centre and tool axis vectors change instantaneously along the tooth line of each tooth surface (Figure 2), to ensure an instantaneous conjugate relationship between the cutter and tooth surface. Moreover, the tool centre trajectory and posture of all concave tooth surfaces differ, as do those of all convex tooth surfaces.

In Figures 1 and 2, $S_i(O - i, j, k_i)$ is the coordinate system fixed on the wheel blank; $S_j(O - l_i, j, k_j)$ is the coordinate system fixed on the tool, and $S(O - i, j, k)$ is the fixed space coordinate system. The geometric space relationship can be found in the relevant references and is not described here.

The relative positional relationship between the finger-shaped milling cutter and tooth line on the crown gear plane is illustrated in Figure 3. When the cutter moves along the offset curve $L'$ of the theoretical tooth line, the theoretical
A split wheel blank often contains more than one tooth, and the cutting path of each tooth is different, so these need to be calculated separately. In order to enable convenient processing, calculation, and programming, it is necessary to plan the cutting path as a whole. According to the characteristics of equal-base circle bevel gears, there are primarily two feasible processing paths. In the first path, after processing a convex tooth surface, the cutter moves to the next feeding point of the adjacent convex tooth surface through an appropriate motion path and continues to process the next one. Once all the convex surfaces have been processed, the tool returns to the feeding point of the nearest concave surface, and the remaining concave surfaces are processed sequentially. The processing path is illustrated in Figure 4.

In the second path, after processing the first convex tooth surface, the tool moves directly to the feeding point of the adjacent concave tooth surface and alternately processes the convex and concave tooth surfaces. The processing path is illustrated in Figure 5.

According to the above machining path, the tooth surface forming theory of the equal-base circle bevel gear, and the gear cutting coordinate system, as illustrated in Figure 2, the first cutting point coordinates of all convex and concave surfaces can be obtained. According to these first cutting points, the tool can move to the starting points of any machining path of the tooth surface; thus, the problem whereby the split wheel blank cannot complete indexing rotation is solved. Moreover, the processing order of the tooth surface can be planned arbitrarily according to the processing requirements to adapt to different processing schemes. Further planning schemes of processing routes are not described here.

2.2. Cutter Centre Coordinate Calculation. In Figure 2, the distance between the origin $O_i$ and the origin $O$ in the wheel blank coordinate system is expressed as $R_c$, and the radius vector of the origin $O_i$ in the fixed space coordinate system $S(O-i,j,k)$ is

$$\mathbf{O}_i \mathbf{O} = R_c (\sin \delta \mathbf{i} + \cos \delta \mathbf{k}),$$

(1)

where

$$R_c = \sqrt{R^2 + (r_0 - s)^2} \pm 2R(r_0 - s)\sin \beta,$$

(2)

where “+” indicates a concave tooth surface and “−” indicates a convex tooth surface; $\delta$ is the reference cone angle (°), where $i = 1, 2$ correspond to the pinion and wheel, respectively; $r_0$ is the distance between the theoretical tooth line and its offset curve, determined by the cutter size (mm); $s$ is the modification in the normal direction of the theoretical tooth line (mm); $R$ is the cone distance at any point (mm); and $\beta$ is the helix angle (°).

Thereafter, the transformation matrix between the wheel blank coordinate system and fixed coordinate system can be determined, according to the indexing relationship among different tooth flanks. The relative positional relationship between the finger-shaped milling cutter and tooth line on the crown wheel plane is illustrated in Figure 6.

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**Figure 2: Tooth surface machining coordinate system of split block.**

**Figure 3: Relationship between cutter path and tooth trace.**

tooth line $L$ is the enveloping curve formed by the cutter surface. The tooth line modification of the equal-base circle bevel gear can be realised by moving the cutter by a distance $s$ along the normal direction of the theoretical tooth line at different cone distances $R$. When the tool moves along the modified actual tooth line $L''_0$, the modified tooth line $L_0$ can be created, and the tool centre trajectory is $L'_0$. The tool movement in numerical control machining is driven by its reference point, and the reference point of the finger milling cutter is at the centre point $O_{\alpha}$. Therefore, the machining path in the split wheel blank is formed by a series of cutter centres in the tool movement process.
As indicated in Figure 6, according to the machining principle of the equal-base circle bevel gear, at the outer starting point, the angle between the tool centre track of the concave tooth flank and theoretical tooth line is $\theta_d^1$ when machining concave tooth surfaces; when machining convex tooth surfaces, the angle between the tool centre track and theoretical tooth line is $\theta_d^2$. Here, $\theta_d^1$ and $\theta_d^2$ are the angles between the polar radius for the theoretical tooth trace and that of the tool centre path, where the cone distance is $R_e$.

$$\theta_d^1 = \theta_d^2 = \arcsin \left( \frac{r_0 - s}{R_e} \cos \beta_e \right).$$  \hfill (3)

where $R_e$ is the outer cone distance (mm) and $\beta_e$ is the helix angle corresponding to the cone distance $R_e$ (°). The relationship between the cutter location and tooth line of the reference cone is illustrated in Figure 7.

In Figure 7, $e_1$ is the angle between the cutter centre of machining the concave tooth surface and the starting point of the reference cone tooth line, while $e_2$ is the angle for machining the convex tooth surface. According to the rolling relationship between the crown wheel and wheel blank, we can obtain

$$e_1 = \frac{\theta_d^1}{\sin \delta_i}, \quad e_2 = \frac{\theta_d^2}{\sin \delta_i}.$$  \hfill (4)

After machining a concave tooth surface, the wheel blank requires indexing, and the index angle is $\theta_m$ in Figure 7. After processing all concave tooth surfaces, the wheel blank should be turned at an angle of $\theta_m$. Then, once the first convex tooth surface is processed, during the next convex tooth surface processing, the index angle is obviously $\theta_n$ in Figure 7, and

$$\theta_m = \theta_n = \frac{360^\circ}{z_i},$$  \hfill (5)

where $z_i$ is the tooth number of the gear. The index angle between the concave and convex tooth flanks is $\theta_x$ (Figure 7), and $\theta_1 = \theta_2 = (\theta_x/2)$, so it is easy to determine that
The wheel blank coordinate system $S_i(O_i - j_i, k_i)$ can rotate around the axis $k_i$ of the fixed coordinate system and the outer of the theoretical tooth line is always in the plane $j_i, k_i$. When the angle $e$ between the axis $j_i$ of the wheel blank coordinate system and axis $i$ of the fixed coordinate system is fixed, the processed tooth surface position in the fixed coordinate system is also determined. Therefore, the value of $e$ determines the relative position of each convex and concave tooth surface in the fixed coordinate system.

The value of $e$ differs according to the convexity and concavity of the tooth surface. The angle is set as $e_0 = e_m$, that is,

$$e_m = \frac{\theta_c}{\sin \delta_i + \theta_i \pm (n-1) \theta_m}. \quad (7)$$

For a convex tooth surface, the angle is set as $e_0 = e_n$, that is,

$$e_n = \frac{\theta_c}{\sin \delta_i + \theta_i \pm (n-1) \theta_n}. \quad (8)$$

In equations (7) and (8), “+” indicates machining one by one along the counterclockwise direction, while “-” indicates processing along the clockwise direction. Moreover, $m$ and $n$ represent the $m$-th concave tooth surface or $n$-th convex tooth surface being processed, respectively, where $m, n = 1, 2, 3, \ldots$

According to the concavity and convexity of the tooth surface, relevant parameters are substituted into equations (7) and (8), respectively. Corresponding to the different rotation directions, and the concavity and convexity of the tooth surface, the calculation formulae for the angle $e$ between axis $j_i$ and axis $i$ are obtained, in which the lower corner marks $m$ and $n$ correspond to concave and convex tooth surfaces, respectively, while the lower corner marks $l$ and $r$ correspond to the left-turning and right-turning gears, respectively.

When processing the $m$-th left-handed concave tooth surface, $e_0 = e_{ml}$, where $e_{ml}$ is

$$e_{ml} = -2(\tan \beta - \beta) + \arcsin\left(\frac{r_0 - s/R_c}{\cos \beta}\right) \sin \delta_i \pm (m - 1) \theta_m. \quad (9)$$

When processing the $m$-th right-handed concave tooth surface, $e_0 = e_{mr}$, where $e_{mr}$ is

$$e_{mr} = \frac{2(\tan \beta - \beta) + \arcsin\left(\frac{r_0 - s/R_c}{\cos \beta}\right)}{\sin \delta_i} \pm (m - 1) \theta_m. \quad (10)$$

When processing the $n$-th left-handed convex tooth surface, $e_0 = e_{nl}$, where $e_{nl}$ is

$$e_{nl} = -2(\tan \beta - \beta) - \arcsin\left(\frac{r_0 - s/R_c}{\cos \beta}\right) \sin \delta_i \pm (n - 1) \theta_n. \quad (11)$$

When processing the $n$-th right-handed convex tooth surface, $e_0 = e_{nr}$, where $e_{nr}$ is

$$e_{nr} = \frac{2(\tan \beta - \beta) - \arcsin\left(\frac{r_0 - s/R_c}{\cos \beta}\right)}{\sin \delta_i} \sin \delta_i \sin e_{ml}, \quad (12)$$

As can be observed in Figure 2, the vector $R_0$ in the fixed coordinate system points to the origin $O_0$ of the cutter coordinate system from its origin $O$, and the length $R_0$ of $R_0$ is determined by using equation (2). As the machining proceeds, that is, with the change in the origin $O_0$, $R_0$ will have different values, with a series of coordinate values of the vector serving as the cutter centre coordinates in the entire machining process. The cutter axis vector in the fixed coordinate system can be defined as $R_0 = [x_0, y_0, z_0]$, which can be obtained by combination with the vector projection relation in Figure 2:

$$\begin{align*}
x_0 &= R_c \sin \delta_i, \\
y_0 &= 0, \\
z_0 &= R_c \cos \delta_i. \quad (13)
\end{align*}$$

According to the coordinate transformation matrix from the fixed space coordinate system to the wheel blank coordinate system,

$$M_{0i} = \begin{bmatrix} \sin e & -\cos e & 0 \\ \cos e & \sin e & 0 \\ 0 & 0 & 1 \end{bmatrix}. \quad (14)$$

The value of parameter $e$ in equation (14) is based on equations (9)–(12).

By converting all of the vectors $R_0$ into the wheel blank coordinate system, a series of values $R_i$ that wrap around the wheel blank can be obtained. The cutter centre vector can be set as $R_i = [x_i, y_i, z_i]$ in the wheel blank coordinate system, according to the gear rotation as well as the concave and convex characteristics of the tooth surface, and the calculation formula for the cutter centre coordinates can be categorised into four situations as follows.

For the left-handed concave tooth surface, the cutter centre coordinates are

$$\begin{align*}
x_i &= \sqrt{R_i^2 + (r_0 - s)^2} - 2R_i (r_0 - s) \sin \beta \sin \delta_i \sin e_{ml}, \\
y_i &= \sqrt{R_i^2 + (r_0 - s)^2} + 2R_i (r_0 - s) \sin \beta \sin \delta_i \cos e_{ml}, \\
z_i &= \sqrt{R_i^2 + (r_0 - s)^2} - 2R_i (r_0 - s) \sin \beta \cos \delta_i. \quad (15)
\end{align*}$$

For the left-handed convex tooth surface, the cutter centre coordinates are
\[
\begin{align*}
    x_i &= \sqrt{R^2 + (r_0 - s)^2} - 2R(r_0 - s) \sin \beta \sin \delta_i \sin e_{ml}, \\
    y_i &= \sqrt{R^2 + (r_0 - s)^2} - 2R(r_0 - s) \sin \beta \sin \delta_i \cos e_{ml}, \\
    z_i &= \sqrt{R^2 + (r_0 - s)^2} - 2R(r_0 - s) \sin \beta \cos \delta_i.
\end{align*}
\]

For the right-handed concave tooth surface, the cutter centre coordinates are

\[
\begin{align*}
    x_i &= \sqrt{R^2 + (r_0 - s)^2} + 2R(r_0 - s) \sin \beta \sin \delta_i \sin e_{mr}, \\
    y_i &= \sqrt{R^2 + (r_0 - s)^2} + 2R(r_0 - s) \sin \beta \sin \delta_i \cos e_{mr}, \\
    z_i &= \sqrt{R^2 + (r_0 - s)^2} + 2R(r_0 - s) \sin \beta \cos \delta_i.
\end{align*}
\]

For the right-handed convex tooth surface, the cutter centre coordinates are

\[
\begin{align*}
    x_i &= \sqrt{R^2 + (r_0 - s)^2} - 2R(r_0 - s) \sin \beta \sin \delta_i \sin e_{mr}, \\
    y_i &= \sqrt{R^2 + (r_0 - s)^2} - 2R(r_0 - s) \sin \beta \sin \delta_i \cos e_{mr}, \\
    z_i &= \sqrt{R^2 + (r_0 - s)^2} - 2R(r_0 - s) \sin \beta \cos \delta_i.
\end{align*}
\]

2.3. Tool Axis Vector Calculation. The solution of the cutter axis vector is provided for solving the tool rotation angle in postprocessing, and the tool position is controlled by calculated cutter centre coordinates. Therefore, the following cutter axis vectors are treated as free vectors, and in all coordinate transformations, we are only concerned with rotation, rather than translation.

In the tool coordinate system \( S_c(O_c, i_1, j_1, k_1) \), the finger milling cutter always rotates around the axis \( i_1 \) during processing, so the initial tool axis vector can be defined as \( a_{c_0} = [1, 0, 0] \).

When converting \( a_{c_0} \) from the tool coordinate system \( S_c \) to the fixed coordinate system \( S \), the vector in this case can be defined as \( a = [a_x, a_y, a_z] \). Through coordinate transformation, we obtain

\[
\begin{align*}
    a_x &= \cos \delta_i \sin e_{ml}, \\
    a_y &= \cos \delta_i \cos e_{ml}, \\
    a_z &= -\sin \delta_i.
\end{align*}
\]

For the left-handed concave tooth surface, the tool axis vector is

\[
\begin{align*}
    a_{xi} &= \cos \delta_i \sin e_{ml}, \\
    a_{yi} &= \cos \delta_i \cos e_{ml}, \\
    a_{zi} &= -\sin \delta_i.
\end{align*}
\]

At this point, a series of tool axis vectors can be obtained, with a difference in the origin of the wheel blank coordinate system. Thereafter, through postprocessing of the tool axis vectors, numerical control machining of the split extralarge curved bevel gear can be realised.

3. Calculation Verification of Cutter Position Solution

In order to verify the correctness of the tool position calculation method according to its mathematical model, MATLAB is used for programming and verification. Firstly, a 3D model of the tooth surface is constructed, and the tooth surface is fixed in the wheel blank coordinate system. Then, a 3D model of the tooth surface is constructed in the same coordinate system, and the tool spatial attitude is adjusted according to the obtained tool axis vector. Finally, the cutter centre point \( O_n \) on the 3D model is moved parallel to the calculated instantaneous cutter centre. According to the spatial positional relationship between the tooth and cutter surfaces, the accuracy of the above calculation method is evaluated.

Next, an example is provided to validate the mathematical model of the cutter position solution, the design parameters of which are presented in Table 1. Furthermore, the wheel blank is divided into six equal sections in this example.

Firstly, according to the design parameters of a pair of equal-base circular curved bevel gears, the 3D model for the
finger milling cutter is programmed and drawn in the wheel blank coordinate system using MATLAB, as illustrated in Figure 8.

Secondly, the discrete points of the tooth surface are obtained according to the tooth surface equation. In the wheel blank coordinate system, the left and right tooth surfaces of a tooth space on the split block are programmed and drawn using MATLAB, as illustrated in Figure 9.

Thereafter, according to equations (21)–(24), the instantaneous tool axis vector corresponding to the tooth space is calculated, and the initial cutter axis vector is rotated on this basis. Furthermore, according to equations (15)–(18), the instantaneous tool centre coordinate corresponding to the tooth space is calculated, and the tool is moved parallel based on this. At this stage, the relative positional relationship between the tool axis vector, tool centre trajectory, finger milling cutter, and tooth surface is established, as illustrated in Figure 10.

When it is necessary to verify the other tooth space on the split block, the corresponding shaft including angle $\epsilon$ can be calculated by using equations (9)–(12), and the above steps are repeated. It is necessary to change the cone distance parameters $R$ when verifying the different instantaneous tool positions, which move along the tool centre trajectory curve. In the same coordinate system, once the 3D drawing of the instantaneous tooth and tool surfaces has been completed, the contact line between the tool and tooth surfaces is created, and the conjugate curve is clearly visible. After changing the instantaneous tool position several times, no separation or interference exists between the two surfaces, and no deviation of the instantaneous tool centre from the tool centre trajectory exists. This demonstrates that the cutter position calculation models are correct.

In order to further verify, in the MATLAB environment, the cutter moves according to the calculation results of the above cutter position and simulates the machining motion, and the tooth surfaces of gear and pinion are enveloped separately. Thereafter, TCA (tooth contact analysis) is carried out on the obtained tooth surface, and the contact trajectory, contact area, and transmission error curve are obtained, respectively [12–15], and the results are shown in Figure 11. In this example, the modifications on the tooth line and tooth profile are, respectively, as follows: $s_0 = 0.09$ and $s_{50} = 0.009$. The contact trace of the tooth surface is almost a straight line, the contact area of the tooth surface becomes wider, and the transmission error is larger, whose value is 0.0013 rad/s.

### Table 1: Basic geometric parameters of a gear pair.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tooth number of pinion $z_1$</td>
<td>12</td>
</tr>
<tr>
<td>Tooth number of wheel $z_2$</td>
<td>36</td>
</tr>
<tr>
<td>Outer transverse module $M_{te}$</td>
<td>24</td>
</tr>
<tr>
<td>Face width $B$</td>
<td>130</td>
</tr>
<tr>
<td>Outer spiral angle $\beta_e$</td>
<td>$0^\circ$</td>
</tr>
<tr>
<td>Normal pressure angle</td>
<td>$20^\circ$</td>
</tr>
<tr>
<td>Shaft angle $\gamma$</td>
<td>$90^\circ$</td>
</tr>
</tbody>
</table>

#### 4. Verification of Simulation Machining

In order to verify the accuracy of the cutter position mathematical model and feasibility of the machining split gear in practice further, according to the actual situation, machine tool modelling, cutter modelling, and split wheel blank modelling are carried out. For the gear indicated in Table 1, the machining trajectory calculation and numerical control programming are conducted according to the above mathematical model. With the aid of VERICUT software, the cutting simulation is completed.

##### 4.1. Machine Tool and Cutter Modelling

The machine tool requires three moving motions ($X/Y/Z$) and two rotary motions ($A/B$) according to the solution of the tool axis vectors presented above. Moreover, because the machine tool motion is complex during the cutting process, it is necessary to construct the machine tool model by means of third-party software. Thus, all components of the machine tools are modelled in the INVENTOR environment. Then, by importing each component module of the machine tool into the VERICUT software in turn, adjusting their assembly relationship, and defining all the coordinate systems, construction of the final model of the machine tool is achieved, as illustrated in Figure 12.

Thereafter, the tool profile is calculated according to the gear design parameters, and the profile parameters are input in the tool management of VERICUT, as indicated in Figure 13. The generated 3D tool model is illustrated in Figure 14.

##### 4.2. Split Wheel Blank Modelling

A split wheel blank model of the equal-base circle bevel gear, which constitutes the simulated cutting, is established in the UG software, saved as a .stl format file, and imported into the VERICUT software. The model of the split wheel blank to be processed is presented in Figure 15. Moreover, in order to verify whether the simulated tooth surface meets the processing requirements, we need to compare and analyse the errors between the machined and theoretical tooth surfaces. Therefore, it is necessary to establish a split gear blank model with the standard tooth surface and import it in the same manner. The established standard model is illustrated in Figure 16.

##### 4.3. Simulation and Result Analysis

For the above example presented in Table 1, the cutter positions are calculated by the formulae deduced previously, following which the relevant calculations of NC machining are completed by postprocessing. As the next step, the subroutines of different convex and concave surfaces can be established, and the execution sequences of the subroutines are adjusted according to different cutting path plans. Thereafter, the main program calls the subroutine, and the simulation process can be completed. Figure 17 presents the overall simulation interface, which includes the machine tool, wheel blank, and tool.
Figures 18 and 19 illustrate the cutting interfaces for the convex and concave gear surfaces, respectively.

Following the simulation cutting, the obtained tooth surface is compared with the standard tooth surface model, through the automatic comparison and analysis function of the software. The detection accuracy of overcut and undercut is set to 0.01 mm. The comparison results are presented in Figure 20.

From the results in Figure 19, it can be observed that the inner of the convex tooth surface machined by the finger milling cutter is undercut, while the outer of the concave surface is slightly overcut, and the remainder is basically coincident with the theoretical tooth surface. If the detection accuracy is reduced to 0.5 mm, the overcut and undercut in Figure 20 will be almost invisible. The specific effect is illustrated in Figure 21.

Several discrepancies are identified between the simulation cutting and theoretical tooth surfaces, according to the above comparison results.

Based on a comprehensive analysis of all the factors, the occurrence of a small amount of overcut or undercut is mainly caused by the theoretical tooth surface modelling accuracy, cutting tool modelling accuracy, calculation errors in the NC machining calculation, and error accumulation.

4.4. Machining Experiment. On the basis of the above simulation, machining experiment is carried out on a five-axis CNC machine tool. According to the gear parameters in Table 1, the split gear billet is designed and processed, which material is aluminium alloy. In order to reduce the cutting force and the tool wear in finishing, rough slotting was carried out with standard tools before finishing. A fine milling cutter is designed according to the gear parameters in Table 1, and a special cutter bar is designed and manufactured. The cutter is connected with the spindle of the machine tool through the cutter bar and a spring chuck. The
processed photographs and the finished product photograph are shown in Figure 22.

Because of the particularity of the equal-base circle bevel gear, the existing gear measuring centre cannot detect and analyse the tooth surface. Therefore, the tooth surface measurement after machining is carried out on the coordinate measuring machine. Fifty-five measuring points are taken on the whole tooth surface. The measured values are compared with the corresponding points on the theoretical tooth surface by C language programming [16, 17]. The photograph of the measurement site is shown in Figure 23, and the analysis result of tooth surface is shown in Figure 24.

In Figure 24, the fine solid line represents the theoretical tooth surface, the blue solid line represents the machined tooth surface, the positive number represents the machined tooth surface overcutting, and the negative number

Figure 11: Analysis results of TCA. (a) Contact trace. (b) Convex-side contact area of gear. (c) Transmission error.

Figure 12: Model of machine tool.
Figure 13: Input interface of cutter parameters.

Figure 14: 3D model of finger milling cutter.
represents the machined tooth surface undercutting. As can be seen from the graph, convex surface is overcut more, concave surface is undercut more, and the error at outer is greater than that at inner. The errors in root and top parts of teeth are larger, while the errors in the middle part of teeth depth are smaller. For a large size split gear, it can fully meet the engineering requirements.

It is noteworthy that the results of this tooth surface machining and measurement are somewhat different from those of simulation processing, but the general trend of errors
Figure 18: Simulation cutting of convex surface.

Figure 19: Simulation cutting of concave surface.

Figure 20: Contrast result within precision of 0.01 mm.
is similar, and this may be due to the fact that simulation processing is carried out in an ideal state, while the actual machining includes many errors such as tool manufacturing, workpiece positioning, and machine tool movement. However, these do not affect the demonstration of the feasibility of machining and the correctness of tool position calculation.
5. Conclusions

Based on the research conducted, the following conclusions can be drawn:

(1) A new approach for machining a tooth surface on the separate block of the split curve bevel gear blank is proposed. Particularly for superlarge bevel gears, the concept of machining the tooth surface after splitting the wheel blank is feasible.

(2) The cutter axis vector and cutter centre coordinate for the split equal-base circle bevel gear are solved correctly.

(3) This research offers certain scientific significance and engineering application value, especially for machining superlarge bevel gears on small-size machine tools.

In addition, the manufacture of superlarge assembled bevel gears includes many technical links, such as the design and manufacture of the mounting base of the split gear blank, the assembly of all the split gear blanks on it, and the adjustment of the tooth surface contact area. The research in this paper only completes the theoretical research and feasibility verification of the tooth cutting on split wheel blank, and other related researches will be carried out in the subsequent papers.

Data Availability

The simulation process program and image file data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This research was supported by the Natural Science Foundation of Henan Province of China (nos. 182300410190 and 162300410090) and the Key Scientific and Technological Project of Henan Province of China (nos. 182102210284 and 182102210042).

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