

Research Article

Dynamic Currency Futures and Options Hedging Model

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In this paper, we consider a risk averse competitive firm that adopts currency futures and options for hedging purpose. Based on the assumption of unbiased markets of currency futures and options, we propose the optimal hedging model in dynamic setting. By using two-stage optimization method, we prove that it is desirable for the prudent enterprise to buy exchange rate options to hedge currency risk. Furthermore, we derive the closed-form solutions of the multiperiod hedging problem with the quadratic utility function. We investigate an empirical study incorporated into GARCH-t prediction on the efficiency of hedging with currency futures and options. The empirical results demonstrate that hedging with currency futures and options can reduce the silver export firm's risk exposure. Profits and the effective boundaries are compared in three cases: hedging with futures and options synchronously, only with futures and without any hedge. The results of multiple comparisons among different hedging strategies show that hedging with linear and nonlinear derivatives is advisable for the export firm.

1. Introduction

Since 2005, China has begun to carry out exchange rate reform, from a single fixed exchange rate system pegged to the US dollar to a floating exchange rate system with reference to a package of currencies. Since the reform of the exchange rate, the pressure of RMB exchange rate appreciation in fluctuation has been constantly increasing, resulting in increasing exchange rate risk faced by foreign-related enterprises and bringing tremendous impacts on China's import and export enterprises. Under this background, what strategies should be used for the import or export enterprises to cope with the exchange rate risk is a practical problem that has to be solved urgently.

With the exchange rate fluctuation increasing, many enterprises begin to transfer the exchange rate risk through financial derivatives. As we know, foreign exchange futures and options are two commonly used derivatives in international trade for exchange rate risk management. Numerous scholars have verified the influences of financial derivatives hedging on production by empirical research. Allayannis and Weston [1] used data from 720 large nonfinancial firms in the United States from 1990 to 1995 and found that firms using financial hedging tools increased their corporate value by 4.87

percent. Clark and Meffeh [2] also found that using financial hedging tools could improve enterprises' profits. But different financial hedging tools have different hedge performances. They concluded that, in short term, tools such as options and forwards can significantly help enterprises ease exchange rate risk. While in long run, only the combination of forward, option, and swap can help enterprises avoid exchange rate risk. Barjaktarovi et al. (2011) explained how currency option contracts were used to speculate or hedge based on anticipated foreign exchange rate movement. Although the positive role of derivatives in the exchange rate risk management for import and export enterprises has been verified to a certain extent by empirical researches, it lacks theoretical support to the generality and universal applicability of the conclusion.

Many scholars applied the foreign exchange futures and options to reduce the exchange rate risk enterprises faced. In terms of foreign exchange futures hedging, Wong [3] studied the decisions on production operation and foreign currency futures hedging of the export enterprises. Wong [4] examined the behaviors of competitive exporters under price and exchange rate uncertainties and found that the key to the optimal production and hedging decisions depended on the degree of imperfection of the forward market (either

commodity futures or foreign exchange futures exist), the dependence structure of price, and exchange rate risk. Broll and Wong [5] investigated the exchange rate hedging problem in that a competitive exporter exported products to two countries. They established a hedging model of exchange rate futures to study the impact of exchange rate changes on the export decisions. Exchange rate futures hedging related researches can refer to Lence [6], Lien and Wong [7], Broll et al. [8], Wong [9], and so on. As for exchange rate options hedging, Machnes [10] studied the decision-making problems of the competitive companies using commodity options to hedge price uncertainty. Wong [11] made a contrastive study for an export firm with regard to the risk and found that the variance of options hedging was lower than that of no options hedging. Lien and Wong [12] derived an optimal hedging strategy for the risk averse hedgers under delivery risk through futures options hedging. Their study thus provides a theoretical basis for options hedging with multiple delivery prices. Since both exchange rate futures and options can be used as risk hedging tools, some scholars have conducted comparative studies on the hedging effects between the two derivatives. Battermann et al. [13] assumed that the export price was known, enterprises only needed to deal with the random exchange rate risk. They compared the hedging effects of exchange rate futures and exchange rate options and showed that exchange rate futures have more advantages than the exchange rate options. But it does not mean to say that options are useless. Some scholars point out that futures have an advantage in hedging linear risk, and options are more suitable to hedge nonlinear risk (Bajo [14, 15] and Wong [16]). As we all know, linear risk and nonlinear risk are generally prevailing in the real investment. Lapan et al. [17] established the classical LMH (Lapan, Moschini, and Hanson) model and pointed out that options played a role due to the convexity utility function. Frechette [18] presented various optimal hedging portfolio models including futures and options when the marginal cost of hedging was nonzero. Wong [19] studied the optimal hedging and decision-making problem of the competitive export enterprises that faced the exchange rate risk. They proved that there were two different sources of nonlinear risk, wherein one was the multiplicative property of the price and exchange rate, and another was the nonlinear marginal utility function. Wong [20] examined the problem wherein state-dependent preference and commodity futures and options hedging were considered for the competitive firms. There was only a linear risk stemming from stochastic price, but a nonlinear relationship between preference and the state made the enterprises face nonlinear risk. For the studies of joint hedge with exchange rate futures and options, see Sakong et al. [21]; Moschini and Lapan [22]. Despite literatures mentioned above have studied futures hedging, options hedging and futures and options joint hedging, the situation they assume is static. While in actual hedging practice, investors have to adjust the positions of futures and options dynamically to minimize risks or maximize returns.

Dynamic hedge is what needs to adjust the strategies as the price or other characteristics of the portfolio or security

changes. Moreover, risks of some securities cannot be hedged with static positions. For example, option price is not linear with the underlying asset's price, which means that risk of option can only be hedged dynamically. There are many researches on dynamic futures hedging. The used methods mainly involve time-varying GARCH models and dynamic programming method. Kotkatvuorinberg [23] used Copula DCC-EGARCH model to estimate bivariate error correction and study the effectiveness of currency futures hedging. Zhang et al. [24] employed GARCH-Copula models to examine the options hedging problem. Dynamic programming method is another widely used method to solve dynamic investment decisions. Change and Wong [25] developed a quadratic utility model of a multinational firm that faced exchange rate risk exposure to a foreign currency cash flow. Chi et al. [26] analyzed the hedger position's value alteration and used the dynamic programming method to set up the multiperiod futures dynamic hedging optimal model. Li et al. [27] constructed the minimum variance model for the estimation of the optimal hedge ratio based on the stochastic differential equation. However, the application of the dynamic programming method in risk hedging, especially in options hedging, is rare.

To sum up, this paper takes account of the following three aspects: First, with the development of economic globalization, trade across countries is increasingly frequent. How to use financial derivatives to manage exchange rate risk is a practical problem. Previous studies have shown that futures and options have advantages in hedging linear and nonlinear risks, respectively, and in the actual hedging practice, linear risk and nonlinear risk coexist generally. Therefore, this paper establishes a hedging model of exchange rate futures and options to hedge the exchange rate risk. Second, a large number of literatures have studied the dynamic hedging of futures, but few study the dynamic hedging model with options. In this paper, a dynamic hedging model of exchange rate futures and options is constructed by combining GARCH model with dynamic programming method. Third, we verify the effects of futures hedging, option hedging and futures and option combined hedging and then further to provide risk management ways for the import and export enterprises to avoid foreign exchange risk.

The rest of this paper is organized as follows. In Section 2, assumptions and notations are presented. Section 3 provides the one-stage hedging model with exchange rate options and futures. We show that, for the prudent firm, it is necessary to buy options for hedging. Under the quadratic utility, we demonstrate the explicit positions of the exchange rate futures and options. Section 4 studies the multistage hedging problem. We deduce the optimal positions of the exchange rate futures and options by using dynamic programming method. An empirical analysis illustrated the hedging effectiveness in Section 4. The performance of the derived hedging strategy is compared in three cases of hedging with futures and options, only with futures and without hedging. The comparison is performed in terms of the terminal wealth, the terminal wealth based on utility, and also the variance of wealth accumulation path. Section 5 concludes the paper.

2. Assumptions and Notations

Suppose a competitive export enterprise exports a single product to the foreign market. It may obtain a predetermined foreign order in advance. That is, demand or output is given as Q . We consider the price risk and exchange rate risk faced by the enterprise. The price of the product sold abroad is \bar{P} in foreign currency. The exchange rate is \bar{S} . Referring to the study of Lapan and Moschini [22], we suppose that

$$\bar{P} = \alpha + \beta\bar{S} + \tilde{\varepsilon}, \quad \beta < 0 \quad (1)$$

where $\tilde{\varepsilon}$ is a zero mean and independent random variable with \bar{S} . Furthermore, according to Chang and Wong [25], we assume that

$$\tilde{S} = \bar{S} + \tilde{\theta} \quad (2)$$

Here, $E(\tilde{S}) = \bar{S}$.

The imported or exported products in China involve a wide range, such as clothing, toys, and electromechanical and pharmaceutical products, while the derivative markets corresponding to these small commodities usually do not exist. Generally speaking, exchange rate derivatives are common. Since the relationship between commodity price and exchange rate is assumed to be shown in (1), i.e. hedging exchange rate risk also eliminates some commodity price risk synchronously, we assume that the exporter uses divisible and tradable exchange rate rather than commodity futures and option for hedging. Suppose that the futures price is F at the current time. For the sake of simplicity, we also assume that the strike price of the exchange rate option is K , and the corresponding premium is recorded as V . Since Wong [16] points out that the assumption of futures and options markets which are unbiased is because firms aim to hedge rather than arbitrage, futures and options markets are assumed to be unbiased for studying the pure hedging roles of exchange rate futures and options, i.e., $E(\tilde{S}) = F$ and $E(K - \tilde{S})^+ = V$.

3. One-Stage Hedging Model with Exchange Rate Options and Futures

To begin with, we consider one-stage static hedging problem with exchange rate options and futures. The firm determines the exchange rate futures position X and the exchange rate option position Y at 0 (current time) to maximize its utility. According to the assumptions above, the enterprise's profit at time 1 is

$$\bar{\Pi} = \bar{P}\tilde{S}Q + (F - \tilde{S})X + [V - (K - \tilde{S})^+]Y \quad (3)$$

where X and Y are the exchange rate futures and options positions that are sold (negative when be bought) and $(K - \tilde{S})^+ = \max(K - \tilde{S}, 0)$.

The firm's utility function is $U(\bar{\Pi})$. Assume that the decision-maker of the firm is risk averse, then $U'(\bar{\Pi}) > 0$ and $U''(\bar{\Pi}) < 0$. Based on the discussion above, the problem faced by the firm can be written as (P_1) :

$$\max_{X,Y} E[U(\bar{\Pi})] \quad (4)$$

where $\bar{\Pi}$ is described in (3).

Proposition 1. *Suppose the exchange rate futures market and option market are unbiased. When the risk aversion export enterprise's utility function satisfies $U'''(\bar{\Pi}) \geq 0$, then it is optimal for the enterprise to buy options for hedging.*

Proof. Since the enterprise is risk aversion, then its utility function satisfies $U'(\bar{\Pi}) > 0$ and $U''(\bar{\Pi}) < 0$. Therefore, the relation of the optimal positions of futures and options X^*, Y^* is map1-to-1, i.e., $X^* = X(Y^*)$. We apply the two-stage optimization method to prove Proposition 1. In the first stage, let

$$H(Y) = \arg \max_X E[U(\bar{\Pi})] \quad (5)$$

When $Y = 0$, based on (5), we have

$$E[U'(\bar{\Pi}_0)(F - \bar{S})] = 0 \quad (6)$$

According to the assumption of unbiased market, (6) can be written as follows:

$$-\text{cov}\{E[U'(\bar{\Pi}_0) | S], S\} = 0 \quad (7)$$

Since

$$\begin{aligned} & \frac{\partial E[U'(\bar{\Pi}_0) | S]}{\partial S} \\ &= E\{U''(\bar{\Pi}_0)[(\alpha + 2\beta S + \varepsilon)Q - H(0)] | S\} \\ &= E\{U''(\bar{\Pi}_0 | S)[pQ + s\beta Q - H(0)]\} \end{aligned} \quad (8)$$

and

$$\begin{aligned} & \frac{\partial^2 E[U'(\bar{\Pi}_0) | S]}{\partial S^2} \\ &= E\{[U'''(\bar{\Pi}_0)(pQ + s\beta Q - H(0))^2] | S\} \\ & \quad + E[U''(\bar{\Pi}_0 | S)]2\beta Q \end{aligned} \quad (9)$$

then due to the firm's utility, we have

$$\frac{\partial^2 E[U'(\bar{\Pi}_0) | S]}{\partial S^2} > 0 \quad (10)$$

That is, $E[U'(\bar{\Pi}_0) | S]$ is concave with regard to S . On the other hand, since $E\{E[U'(\bar{\Pi}_0) | S]\} = E[U'(\bar{\Pi}_0)]$, then the equation of

$$E[U'(\bar{\Pi}_0) | S] = E[U'(\bar{\Pi}_0)] \quad (11)$$

has at least one solution and at most two solutions.

When (11) has only one solution $S_1 \in (\bar{S}, \tilde{S})$, then (7) can be expressed by

$$E\{U'(\bar{\Pi}_0 | S) - E[U'(\bar{\Pi}_0)]\} [E(\tilde{S}) - S] = 0 \quad (12)$$

Equation (12) is further expressed in an integral form

$$\int_{\underline{S}}^{\bar{S}} \{E[U'(\bar{\Pi}_0) | S] - E[U'(\bar{\Pi}_0)]\} (f - S) f(s) ds \quad (13)$$

$$= 0$$

If $S \in (\underline{S}, S_1)$, the first term of the integrand in (13) is negative, and the second is positive. When $S \in (S_1, \bar{S})$, the first term of the integrand in (13) is positive, and the second is negative. Therefore, the left hand of (13) is negative, which leads a contradictory to (13).

If (11) has two solutions, i.e., $S_1, S_2: \underline{S} < S_1 < S_2 < \bar{S}$, then when $S = S_1$ and $S = S_2$ we have

$$E[U'(\bar{\Pi}_0) | S] = E[U'(\bar{\Pi}_0)] \quad (14)$$

So, it yields

$$\begin{aligned} \frac{\partial E[U(\bar{\Pi})]}{\partial Y} \Big|_{Y=0} &= E\{U'(\bar{\Pi}_0) [V - (K - \bar{S})^+]\} \\ &= \text{cov}(E[U'(\bar{\Pi}_0) | S], V - (K - \bar{S})^+) \\ &= E\{[EU'(\bar{\Pi}_0) - EU'(\bar{\Pi}_0) | S] [(K - \bar{S})^+ - V]\} \\ &= \int_{\underline{S}}^{\bar{S}} [EU'(\bar{\Pi}_0) - EU'(\bar{\Pi}_0) | S] [(K - S)^+ - V] \\ &\quad \cdot f(S) dS = \int_{\underline{S}}^K [EU'(\bar{\Pi}_0) - EU'(\bar{\Pi}_0) | S] \\ &\quad \cdot (K - S) f(S) dS \end{aligned} \quad (15)$$

Let $R(K) = \int_{\underline{S}}^K [EU'(\bar{\Pi}_0) - EU'(\bar{\Pi}_0) | S] (K - S) f(S) dS$. We then proof that $R(K) < 0$ according to the functional features of $R(K)$. In fact, it is evident that $R(\underline{S}) = 0$. From (6), we have $R(\bar{S}) = 0$. Since

$$R'(K) = \int_{\underline{S}}^K [EU'(\bar{\Pi}_0) - EU'(\bar{\Pi}_0) | S] f(S) dS \quad (16)$$

and

$$R''(K) = [EU'(\bar{\Pi}_0) - EU'(\bar{\Pi}_0) | K] f(K) \quad (17)$$

then when $S \in (S_1, S_2)$, we have $R''(K) > 0$. Similarly, we obtain that when $S \in (\underline{S}, S_1) \cup (S_2, \bar{S})$, we have $R''(K) < 0$. Based on the discussion above, we have $R(K) < 0$. That is, $(\partial E[U(\bar{\Pi})]/\partial Y)|_{Y=0} < 0$ and $Y^* < 0$. \square

Referring to the definition in Kim [28], the risk aversion investor whose utility function satisfies $U'''(\bar{\Pi}) \geq 0$ is called a prudent investor. The results in Proposition 1 show that it is wise for the prudent investor to buy unbiased options for hedging, which is consistent with the real intention of enterprises who adopt hedging with the purpose of appreciation and preservation. We know that although selling options

can obtain option premiums, investors have to face the daily marking risk caused by the additional margin, which may lead to greater liquidity risk for investors. Jorion [29] divided liquidity risk into asset liquidity risk and capital liquidity risk. He pointed out that the capital liquidity risk is the potentially fatal risks faced by investors. For option sellers, the liquidity risk they face mainly includes capital liquidity risk. As a failure case of hedging, China Southern Airlines desire to lock in the cost of raw materials by buying call options, which can be regarded as a hedge, but selling put options based on bull market judgment creates a new margin risk exposure. Therefore, the conclusion of Proposition 1 coincides with the original intention of options hedging to preserve and increase appreciation. We note the utility function of the prudent enterprise in Proposition 1 without a specific form. No matter what the utility function of the enterprise is, as long as it satisfies $U'(\bar{\Pi}) > 0$, $U''(\bar{\Pi}) < 0$, and $U'''(\bar{\Pi}) \geq 0$, it is optimal to use options for hedging. We can find many utility functions that satisfy $U'(\bar{\Pi}) > 0$, $U''(\bar{\Pi}) < 0$, and $U'''(\bar{\Pi}) \geq 0$, such as the negative exponential utility function and the quadratic utility function. Many scholars have established portfolio or futures hedging model assuming that the investor's utility is a quadratic utility function. Steil [30] obtained the hedging position of exchange rate equity with quadratic utility. Lien [31] studied futures hedging under the negative exponential utility and the quadratic utility. Bodnar [32] gave the strategies of multiperiod portfolio under negative exponential utility and the quadratic utility. Because of the complexity of the multistage dynamic programming method under the negative exponential utility function, it is difficult for us to obtain the explicit solutions. We then study the dynamic hedging problem in the framework of the quadratic utility.

Proposition 2. Assume that exchange rate futures and exchange rate option markets are unbiased. If the export enterprise's utility function is the quadratic utility function $U(\bar{\Pi}) = \bar{\Pi} - b\bar{\Pi}^2$, ($b > 0$), then the optimal positions of exchange rate futures and options are

$$\begin{aligned} X^* &= \frac{A_{13}A_{21} - A_{11}A_{23}}{A_{13}^2 - A_{12}A_{23}} Q \\ Y^* &= \frac{A_{11}A_{13} - A_{12}A_{21}}{A_{13}^2 - A_{12}A_{23}} Q \end{aligned} \quad (18)$$

where $A_{11} = (\bar{P} + \beta\bar{S}) \text{cov}(\theta, \theta) + \beta \text{cov}(\theta^2, \theta)$, $A_{12} = \text{cov}(\theta, \theta)$, $A_{13} = \text{cov}((-\theta)^+, \theta)$, $A_{21} = (\bar{P} + \beta\bar{S}) \text{cov}(\theta, (-\theta)^+) + \beta \text{cov}(\theta^2, (-\theta)^+)$, $A_{22} = A_{13} = \text{cov}(\theta, (-\theta)^+)$, and $A_{23} = \text{cov}((-\theta)^+, (-\theta)^+)$.

Proof. The first-order conditions of the objective function are

$$\text{cov}(\bar{\Pi}^*, \bar{\theta}) = 0 \quad (19)$$

$$\text{cov}(\bar{\Pi}^*, (-\theta)^+) = 0 \quad (20)$$

Due to the operational rules of covariance, (19) and (20) can be rewritten as

$$Q(\bar{P} + \beta\bar{S}) \text{cov}(\theta, \theta) + \beta Q \text{cov}(\theta^2, \theta) - X^* \text{cov}((-\theta)^+, \theta) - Y^* \text{cov}((-\theta)^+, \theta) = 0 \quad (21)$$

$$Q(\bar{P} + \beta\bar{S}) \text{cov}(\theta, (-\theta)^+) + \beta Q \text{cov}(\theta^2, (-\theta)^+) - X^* \text{cov}(\theta, (-\theta)^+) - Y^* \text{cov}((-\theta)^+, (-\theta)^+) = 0 \quad (22)$$

Let $A_{11} = (\bar{P} + \beta\bar{S}) \text{cov}(\theta, \theta) + \beta \text{cov}(\theta^2, \theta)$, $A_{12} = \text{cov}(\theta, \theta)$, $A_{13} = \text{cov}((-\theta)^+, \theta)$, $A_{21} = (\bar{P} + \beta\bar{S}) \text{cov}(\theta, (-\theta)^+) + \beta \text{cov}(\theta^2, (-\theta)^+)$, $A_{22} = A_{13} = \text{cov}(\theta, (-\theta)^+)$, and $A_{23} = \text{cov}((-\theta)^+, (-\theta)^+)$, then we have

$$\begin{aligned} A_{11} - A_{12}X^* - A_{13}Y^* &= 0 \\ A_{21} - A_{13}X^* - A_{23}Y^* &= 0 \end{aligned} \quad (23)$$

By solving equations, we can obtain the optimal positions of the exchange rate futures and options presented in Proposition 1. \square

Corollary 3. Suppose that θ_t has a symmetric distribution function of $G(\theta_t)$, then $dG(\theta_t) = dG(-\theta_t)$. Let $E(\theta_t) = 0$, $E(\theta_t^2) = \sigma^2$, $V = \int_0^{+\infty} \theta_t dG(\theta_t)$, and $\varphi = \int_0^{+\infty} \theta_t^3 dG(\theta_t)$. In one-stage hedging, the optimal positions of exchange rate futures and options are

$$\begin{aligned} X^* &= \left[\bar{P} + \beta\bar{S} + \beta \frac{\varphi - V\sigma^2}{\sigma^2/2 - 2V^2} \right] Q \\ Y^* &= 2\beta Q \frac{\varphi - V\sigma^2}{\sigma^2/2 - 2V^2} \end{aligned} \quad (24)$$

The study above is to establish the optimal hedging model of exchange rate futures and options in one-stage, while, in actual hedging practice, the enterprise has to adjust the positions dynamically according to the market conditions so as to maximize its utility based on the final wealth. We then extend the one-stage hedging problem to a multiperiod dynamic case and derive the dynamic positions of exchange rate futures and options.

4. Multistage Hedging Model with Futures and Options

Proposition 4. Suppose that the markets of exchange rate futures and option are unbiased. If the firm's utility function is quadratic of $U(\bar{\Pi}) = \bar{\Pi} - b\bar{\Pi}^2$, ($b > 0$), then the optimal positions of the exchange rate futures and options at stage t are

$$\begin{aligned} X_t^* &= \frac{A_{13,t}A_{21,t} - A_{11,t}A_{23,t}}{A_{13,t}^2 - A_{12,t}A_{23,t}} \sum_{\tau=t}^T Q_\tau \\ Y_t^* &= \frac{A_{11,t}A_{13,t} - A_{12,t}A_{21,t}}{A_{13,t}^2 - A_{12,t}A_{23,t}} \sum_{\tau=t}^T Q_\tau \end{aligned} \quad (25)$$

where $A_{11,t} = (P_{t-1} + \beta_t S_{t-1}) \text{cov}(\theta_t, \theta_t) + \beta_t \text{cov}(\theta_t^2, \theta_t)$, $A_{12,t} = \text{cov}(\theta_t, \theta_t)$, $A_{13,t} = \text{cov}((-\theta_t)^+, \theta_t)$, $A_{21,t} = (P_{t-1} + \beta_t S_{t-1}) \text{cov}(\theta_t, (-\theta_t)^+) + \beta_t \text{cov}(\theta_t^2, (-\theta_t)^+)$, $A_{22} = A_{13} = \text{cov}(\theta_t^2, (-\theta_t)^+)$, and $A_{23,t} = \text{cov}((-\theta_t)^+, (-\theta_t)^+)$.

Proof. Let $\bar{S}_t = S_{1,t-1} + \bar{\theta}_t$, $\bar{P}_t = P_{1,t-1} + \beta_t \bar{\theta}_t + \bar{\varepsilon}_t$, and $F_t = S_{t-1}$. Then, we have the firm's profit at stage t is

$$\begin{aligned} \bar{\Pi}_t &= \bar{P}_t \bar{S}_t Q_T + (S_{t-1} - \bar{S}_t) X_t + [V_t - (S_{t-1} - \bar{S}_t)^+] Y_t \\ &= (P_{t-1} + \beta_t \bar{\theta}_t + \bar{\varepsilon}_t) (S_{t-1} + \bar{\theta}_t) Q_t - \bar{\theta}_t X_t \\ &\quad + [V_t - (-\bar{\theta}_t)^+] Y_t \end{aligned} \quad (26)$$

At the beginning of stage t , the firm decides the optimal positions X_t^*, Y_t^* to maximize its terminal utility. Let $\bar{W}_T = \sum_{\tau=1}^T \bar{\Pi}_\tau$. Then, the optimal hedging model can be described as follows:

$$(P_2) \max_{X_t, Y_t} E[U(\bar{W}_T)] \quad (27)$$

We use the dynamic programming method to solve the problem (P_2) . To begin with, at stage T , the firm decides the optimal position X_T^*, Y_T^* to maximize its utility. Let

$$\begin{aligned} V_T(W_{T-1} | \Omega_T) &= \max_{H_T, Z_T} E[U(W_{T-1} + \bar{\Pi}_T) | \Omega_T] \\ &= E[U(W_{T-1} + \bar{\Pi}_T^*) | \Omega_T] \end{aligned} \quad (28)$$

where $W_{T-1} = \sum_{\tau=1}^T$. Then, the first-order conditions of (29) satisfy

$$\text{cov}(\bar{\Pi}_T^*, \bar{\theta}_T) = 0 \quad (29)$$

$$\text{cov}(\bar{\Pi}_T^*, (-\bar{\theta}_T)^+) = 0 \quad (30)$$

Let $A_{11,t} = (P_{t-1} + \beta_t S_{t-1}) \text{cov}(\theta_t, \theta_t) + \beta_t \text{cov}(\theta_t^2, \theta_t)$, $A_{12,t} = \text{cov}(\theta_t, \theta_t)$, $A_{13,t} = \text{cov}((-\theta_t)^+, \theta_t)$, $A_{21,t} = (P_{t-1} + \beta_t S_{t-1}) \text{cov}(\theta_t, (-\theta_t)^+) + \beta_t \text{cov}(\theta_t^2, (-\theta_t)^+)$, $A_{22} = A_{13} = \text{cov}(\theta_t^2, (-\theta_t)^+)$, and $A_{23,t} = \text{cov}((-\theta_t)^+, (-\theta_t)^+)$. Comparing (30) and (32) with (19) and (20), we can obtain the optimal positions at stage T are

$$\begin{aligned} X_T^* &= \frac{A_{13,T}A_{21,T} - A_{11,T}A_{23,T}}{A_{13,T}^2 - A_{12,T}A_{23,T}} Q_T \\ Y_T^* &= \frac{A_{11,T}A_{13,T} - A_{12,T}A_{21,T}}{A_{13,T}^2 - A_{12,T}A_{23,T}} Q_T \end{aligned} \quad (31)$$

At stage $T-1$, let

$$\begin{aligned} V_{T-1}(W_{T-2} | \Omega_{T-1}) &= \max_{H_{T-1}, Z_{T-1}} E\{[V(W_{T-2} + \bar{\Pi}_{T-1}) | \Omega_T] | \Omega_{T-1}\} \\ &= \max_{H_{T-1}, Z_{T-1}} E[U(W_{T-2} + \bar{\Pi}_{T-1} + \bar{\Pi}_T^*) | \Omega_{T-1}] \end{aligned} \quad (32)$$

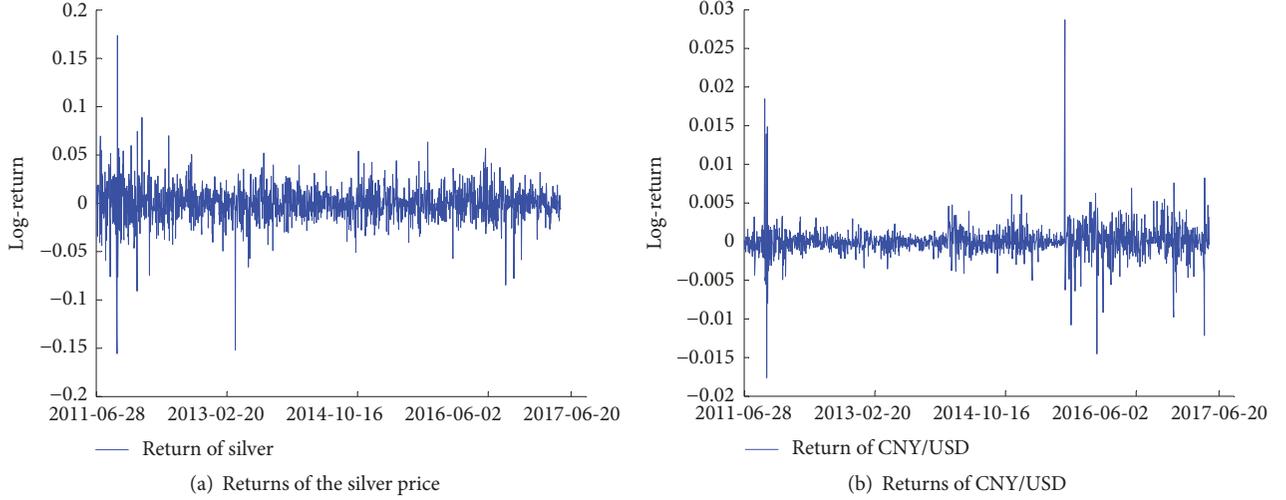


FIGURE 1: Returns of the silver price and the exchange rate.

then the first-order conditions of (33) can be expressed by

$$\text{cov}(\tilde{\Pi}_{T-1} + \tilde{\Pi}_T^*, \tilde{\theta}_{T-1}) = 0 \quad (33)$$

$$\text{cov}(\tilde{\Pi}_{T-1} + \tilde{\Pi}_T^*, (-\tilde{\theta}_{T-1})^+) = 0 \quad (34)$$

Since

$$\begin{aligned} \tilde{\Pi}_{T-1} + \tilde{\Pi}_T^* &= P_{T-1}S_{T-1}(Q_{T-1} + Q_T) - \theta_{T-1}X_{T-1} \\ &\quad - (-\theta_{T-1})^+ Y_{T-1} + V_{T-1}Y_{T-1} \\ &\quad + (P_{T-1} + \beta_T S_{T-1})\theta_T Q_T + \beta_T \theta_T^2 Q_T \\ &\quad + \varepsilon_T Q_T (S_{T-1} + \theta_{T-1}) - \theta_T X_T^* + V_T Y_T^* \\ &\quad - (-\theta_T)^+ Y_T^* \end{aligned} \quad (35)$$

and θ_{T-1} is independent on θ_T and ε_T , we have

$$\begin{aligned} \text{cov}[P_{T-1}S_{T-1}(Q_{T-1} + Q_T) - \theta_{T-1}X_{T-1} \\ - (-\theta_{T-1})^+ Y_{T-1}, \tilde{\theta}_{T-1}] = 0 \end{aligned} \quad (36)$$

$$\begin{aligned} \text{cov}[P_{T-1}S_{T-1}(Q_{T-1} + Q_T) - \theta_{T-1}X_{T-1} \\ - (-\theta_{T-1})^+ Y_{T-1}, (-\tilde{\theta}_{T-1})^+] = 0 \end{aligned} \quad (37)$$

According to (37) and (40), we obtain the optimal positions at stage $T - 1$ as

$$\begin{aligned} X_{T-1}^* &= \frac{A_{13,T-1}A_{21,T-1} - A_{11,T-1}A_{23,T-1}}{A_{13,T-1}^2 - A_{12,T-1}A_{23,T-1}} (Q_{T-1} + Q_T) \\ Y_{T-1}^* &= \frac{A_{11,T-1}A_{13,T-1} - A_{12,T-1}A_{21,T-1}}{A_{13,T-1}^2 - A_{12,T-1}A_{23,T-1}} (Q_{T-1} + Q_T) \end{aligned} \quad (38)$$

In this way, we can deduce the optimal positions of futures and options on the basis of mathematical induction at stage t are

$$X_t^* = \frac{A_{13,t}A_{21,t} - A_{11,t}A_{23,t}}{A_{13,t}^2 - A_{12,t}A_{23,t}} \sum_{\tau=t}^T Q_\tau \quad (39)$$

$$Y_t^* = \frac{A_{11,t}A_{13,t} - A_{12,t}A_{21,t}}{A_{13,t}^2 - A_{12,t}A_{23,t}} \sum_{\tau=t}^T Q_\tau$$

□

Corollary 5. Suppose that θ_t has a symmetric distribution function $G(\theta_t)$, then $dG(\theta_t) = dG(-\theta_t)$. Assume the firm export products to the foreign market at the terminal time. That is, $Q_t = 0$, ($t = 1, 2, \dots, T - 1$), $Q_T = Q$. In the case of multistage, the optimal positions of exchange rate futures and options are

$$\begin{aligned} X_t^* &= \left[P_{t-1} + \beta_t S_{t-1} + \beta_t \frac{\varphi_t - V_t \sigma_t^2}{\sigma_t^2/2 - 2V_t^2} \right] Q \\ Y_t^* &= 2\beta_t Q \frac{\varphi_t - V_t \sigma_t^2}{\sigma_t^2/2 - 2V_t^2} \end{aligned} \quad (40)$$

where $E(\theta_t) = 0$, $E(\theta_t^2) = \sigma_t^2$, $V_t = \int_0^{+\infty} \sigma_t dG(\theta_t)$, and $\varphi_t = \int_0^{+\infty} \sigma_t^3 dG(\theta_t)$.

5. Empirical Analysis

China has become a major producer and consumer of silver, also a big importer and exporter. According to the accessible data, we assume there is a firm in China exports silver to the US dollar region countries. The firm intends to buy the exchange rate futures and options for hedging. Data resources are from Wind database. Figure 1 shows the daily yield of the silver exported price and the exchange rate (RMB/US dollar).

TABLE 1: Basic statistical characteristics of returns.

Objective	Maximum	Minimum	Mean	Median	Standard Deviation	Skewness	Kurtosis
Silver price	0.1736	-0.1556	-0.0005	-0.0006	0.0199	-0.4517	14.6842
CNY/USD	0.0287	-0.0176	0.0000	0	0.0021	1.8943	40.6029

TABLE 2: Normality test of returns.

Objective	P-value	J-B statistics	Critical value
Silver price	1.0000e-03	8.1429e+03	5.9490
CNY/USD	1.0000e-03	8.4688e+04	5.9490

Note. $ae \pm c$ denote that $a \times e^{\pm c}$, the same after.

From Figure 1, we can observe that the returns of the silver price and the exchange rate show cluster effect (that is, large fluctuations are often accompanied by large fluctuations, and small fluctuations are often accompanied by small fluctuations). We further test the ARCH effects, wherein the statistics of the returns are presented in Table 1.

Table 1 gives the statistical description of ARCH effects test results with regard to the silver price and the exchange rate. We find that the returns of the silver price and the exchange rate CNY/USD have nonzero skewness of -0.4517 and 1.8943 , respectively. The returns also have peak thick tails. Based on LM statistic, the ARCH effects corresponding to the first 20 lags of the two exchange rate returns are further confirmed (In order to save space, the specific numerical value is omitted here). The main objective of this paper is to examine the hedging roles of the currency futures and options. We further to test the nonnormal of the returns.

Table 2 shows that the J-B statistic is larger than the critical value at the 5% significance level and the P-value is smaller than the significant level. That is, the hypothesis of normal distribution under J-B test is rejected. Six typical models of GARCH-n, GARCH-t, GIR-n, GIR-t, EGARCH-n, and EGARCH-t are tested to depict the marginal distributions of the silver price and the exchange rate of CNY/USD. Three fitting criteria measured by LLF (optimized log-likelihood objective function value), AIC (Akaike information criteria), and BIC (Bayesian information criteria) are used to test how the outlined models fit the in-sample data. The values of LLF, AIC, and BIC for different models are in Tables 3 and 4:

Since the bigger the LLF, the better is and the smaller the AIC and BIC, the better is, from Tables 3 and 4, we find that models of GARCH-t model capture the fatness tails of the unconditional distributions both silver price and CNY/USD better. Then, we use GARCH-t model to fit the return series for the returns of silver price and the exchange rate. The detail description of GARCH-t model can be referred to Huang et al. [33], if the yield sequence r_t has the forms

$$\begin{aligned}
 r_t &= C + ARr_{t-1} + \varepsilon_t \\
 \varepsilon_t &= \sigma_t z_t \\
 \sigma_t^2 &= K + GARCH\sigma_{t-1}^2 + ARCH\varepsilon_{t-1}^2 \\
 z_t &\sim t(d)
 \end{aligned} \tag{41}$$

where $t(d)$ obeys Student-t distribution with degree of freedom d . K is the conditional variance constant. GARCH means the coefficients related to lagged conditional variances. ARCH is the coefficients related to lagged innovations (residuals), and C denotes the conditional mean constant. The method to estimate for the parameters could be artificial algorithm. In this paper, we use MLE method to estimate the parameters in GARCH model. We let the information set $\Omega_{t-1} = \{a_0, a_1, \dots, a_{t-1}\}$. The joint density function can then be written as $f(a_0, a_1, \dots, a_t) = f(a_t | \Omega_{t-1})f(a_{t-1} | \Omega_{t-2}) \cdots f(a_1 | \Omega_0)f(a_0)$. Given data a_1, \dots, a_t the log-likelihood is as follows:

$$LLF = \sum_{k=0}^{n-1} f(a_{n-k} | \Omega_{n-k-1}) \tag{42}$$

This can be evaluated using the model volatility equation for any assumed distribution for the error term. Here, LLF can be maximized numerically to obtain MLE . The estimation results can be seen in Table 5.

Table 5 shows the maximum likelihood estimation of parameters, the validity of ARCH and Ljungbox tests for GARCH-t models, and the fitted values of AIC (Akaike information criterion) and BIC (Bayesian information criterion). It can be seen from Table 5 that the Ljung-Box test for GARCH-t model residuals at 5% confidence level cannot reject the original hypothesis of order 1, 3, 5, and 7 autocorrelation, and the original hypothesis of ARCH effect in orders 4, 6, 8, and 10 cannot be rejected. The values of AIC and BIC are smaller, but the LLF is larger. According to the P-value of parameter estimation, it shows that GARCH-t model can fit the residual sequence of silver price and the exchange rate well. We then give the expression of GARCH-t distributions for the silver price and the exchange rate of CNY/USD as follows:

$$\begin{aligned}
 r_t &= -5.2123e - 04 - 9.2936e - 02r_{t-1} + \varepsilon_t \\
 \varepsilon_t &= \sigma_t z_t
 \end{aligned} \tag{43}$$

$$\begin{aligned}
 \sigma_t^2 &= 3.2615e - 06 + 9.6126\sigma_{t-1}^2 + 2.7763e - 02\varepsilon_{t-1}^2 \\
 z_t &\sim t(4.9082)
 \end{aligned}$$

$$\begin{aligned}
 r_t &= -3.7917e - 05 + 1.8163e - 02r_{t-1} + \varepsilon_t \\
 \varepsilon_t &= \sigma_t z_t \\
 \sigma_t^2 &= 2.0000e - 07 + 7.3552e - 01\sigma_{t-1}^2 + 2.6448e
 \end{aligned} \tag{44}$$

$$- 01e - 02\varepsilon_{t-1}^2$$

$$z_t \sim t(3.4282)$$

where (43) and (44) are the expressions for the silver price and the exchange rate, respectively.

TABLE 3: Fitting criteria of silver price.

LLF	3.6748e+03	3.7755 e +03	3.6805 e +03	3.7747 e +03	3.6813 e +03	3.7749 e +03
AIC	-7.3415 e +03	-7.5390 e +03	-7.3490 e +03	-7.5374 e +03	-7.3506 e +03	-7.5378 e +03
BIC	-7.3223 e +03	-7.5102 e +03	-7.3201 e +03	-7.4990 e +03	-7.3218e+03	-7.5042 e +03

TABLE 4: Fitting criteria of CNY/USD.

LLF	7.0932e+03	7.3090e+03	7.1016e+03	7.2108e+03	7.0781e+03	7.2247e+03
AIC	-1.4178e+04	-1.4606e+04	-1.4191e+04	-1.4606e+04	-1.4144e+04	-1.4605e+04
BIC	-1.4159e+04	-1.4577e+04	-1.4162e+04	-1.4567e+04	-1.4115e+04	-1.4502e+04

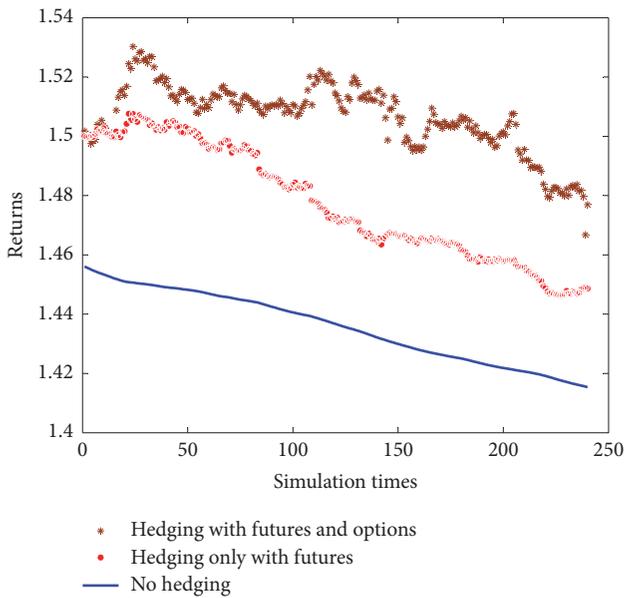


FIGURE 2: Gains under three circumstances.

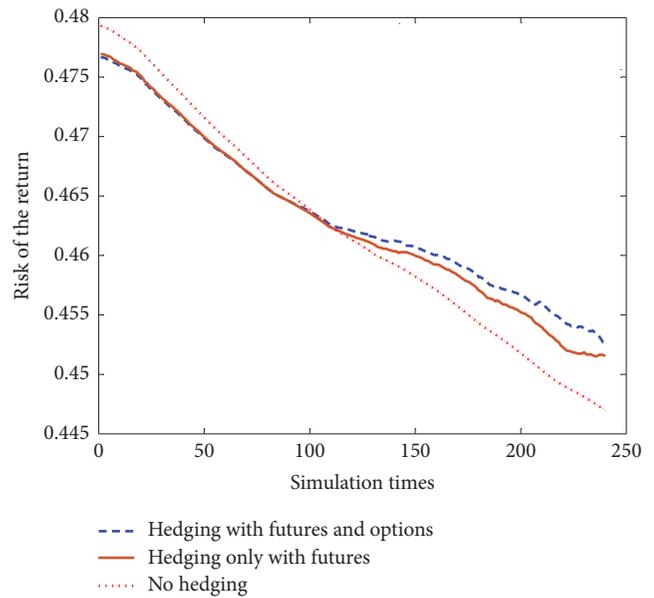


FIGURE 3: Volatility in the process of wealth accumulation.

The current time is June 1, 2016. We suppose that a year later, i.e., at June 1, 2017. The enterprise exports 10,000 ounces of silver to the country settled in US dollars. By solving the proposed model, we obtain the returns in three cases of options and futures hedging, only futures hedging, and no hedging shown in Figure 2.

In order to avoid the silver export risk the enterprise faces with, the return of futures and options joint hedging is greater than that of only futures hedging. The result shows that the combination of options and futures is better than using futures hedging alone. Without any hedging, the profit is minimal, which reflects the advantage of using derivatives for hedging. In fact, when the enterprise uses futures or options for hedging, futures and options themselves are regarded as financial assets to participate in the investment. That is to say, in the process of hedging, the enterprise can obtain additional income by trading derivatives. At the same time, linear instrument futures and nonlinear instrument options have their own advantages in hedging against linear and nonlinear risks. The combination of futures and options can bring different hedging experiences to hedgers and obtain

better hedging effect. The original intention of investors to use derivatives hedging is to keep assets to preserve and increase appreciation. We say that hedging is the primary goal, followed by appreciation. Therefore, for hedgers, in pursuit of appropriate returns, they will also pay attention to the volatility in the process of wealth accumulation. Even if the return is high, if the hedger is exposed to excessive fluctuation process, the psychological cost of the hedger is relatively high, and the hedger may have to bear the psychological impact of ups and downs. This paper compares and analyzes the stability of wealth accumulation process when using derivatives hedging. We compare the standard deviation of the simulated wealth in three cases as shown in Figure 3.

From Figure 3, it is not difficult to find that, in the prehedging stage, the volatility of wealth accumulation under the joint hedging of futures and options is relatively small, followed by only using futures hedging and no hedging. In the latter stage, the opposite is true, but in general, there is not much difference in the volatility of wealth paths. Moreover,

TABLE 5: Parameters estimations of GARCH-t model.

GARCH-t	CNY/USD		Silver price	
Parameters	Estimated values	Std	Estimated values	Std
C	-3.7917e-05	2.9301e-05	-5.2123e-04	3.9959e-04
K	2.0000e-07	4.8079e-08	3.2615e-06	1.5352e-06
GARCH	0.7355	0.0327	0.9613	0.0097
ARCH	0.2645	0.0494	0.0278	0.0074
AR	0.0182	0.0274	-0.0929	0.0234
d	3.4282	0.3276	4.9082	0.5513
LLF	7.3090e+03		3.7755e+03	
AIC	-1.4606e+04		-7.5390e+03	
BIC	-1.4577e+04		-7.5102e+03	
Lags	P-values	Q- statistics	P-values	Q- statistics
Ljung-Box test				
QW(1)	0.1684	1.8970	0.0552	7.8133
QW(3)	0.5638	2.0418	0.0554	8.5825
QW(5)	0.8301	2.1356	0.1024	9.1720
QW(7)	0.9453	2.414	0.2165	9.5341
Engles test				
LM(4)	0.7190	2.0910	0.0718	8.6029
LM(6)	0.9029	2.1748	0.1810	8.8711
LM(8)	0.9721	2.2566	0.3425	8.9979
LM(10)	0.9937	2.2799	0.4114	10.3380

TABLE 6: Kruskal-Wallis ANOVA.

Source	SS	df	MS	Chi-sq	Prob>Chi-sq
Columns	2.48687e+07	2	12434328.16	574.86	1.47798e-125
Error	6.23528e+06	717	8696.35		
Total	3.11039e+07	719			

TABLE 7: Comparison of three effective boundary values.

Group name	Confidence interval			
1	2	158.65	203.15	247.64
1	3	409.88	454.38	498.88
2	3	206.73	251.23	295.73

the empirical part of this paper assumes that the utility function of silver export enterprises is a quadratic utility function, we can not only study profits or volatility. Therefore, we study the effective boundary (revenue/standard deviation) in three cases. In order to avoid using ANOVA multiple comparisons when comparing multiple groups when the groups do not satisfy the normal distribution, Kruskal-Wallis test is used to compare the effective boundaries under different strategies. The results of statistical analysis of Kruskal-Wallis test are shown in Table 6.

As can be seen from Table 6, because the mean of the three groups are unequal, the effective boundaries are different in the three cases of using options and futures, using only futures and no hedging. Table 7 further compares the three effective boundaries:

From the comparison matrix in Table 7, we can see the three effective boundaries from large to small corresponding to options and futures hedging, futures hedging, and no hedging. Generally speaking, if the exporters do not adopt any hedge, their effective boundaries are small. From the confidence intervals of the mean between different groups, we can see that the effective boundary increases by nearly 454.38 on average, which is larger than the average value-added of 251.23 of futures hedging. The results show that hedging with futures or options can effectively avoid the risk faced by the export enterprise.

6. Conclusion

Since the inception of floating exchange rates, firms engaged in international operations have been highly interested in developing ways and means to protect themselves against exchange rate risk. Price or exchange rate uncertainty results in a reduction of production and exports. Therefore, the major role of financial markets enabling firms to reduce price or currency risks is their impact on production and export levels.

This paper studies the problem of exchange rate futures and options hedging for a risk averse export enterprise. Firstly, under unbiased markets of exchange rate futures and options, we prove that the prudent exporter is profitable to buy exchange rate options for hedging. The result validates the necessity of options hedging when the risk exposure is nonlinear. Considering that, in actual hedging practice, investors will adjust their investment strategies dynamically according to the market conditions; this paper generalizes the

existing static futures and options hedging to the dynamic situation. We establish the dynamic hedging model of exchange rate futures and options and present the explicit positions under the quadratic utility function by using the dynamic programming method. Finally, through empirical analysis, it is found that the export enterprise's profit and effective boundary are the largest by using both exchange rate futures and option hedging, followed by only using exchange rate futures hedging, and then without any hedging strategy. We demonstrate the role of exchange rate futures and options hedging in the exchange rate risk management of the export enterprise. Therefore, it is suggested that exporter should adopt hedging strategies of exchange rate futures and options.

This paper focuses on the application of exchange rate futures and options in hedging. Therefore, the transaction cost is neglected in this study and the firm is assumed to have sufficient futures margin. In the future study, the daily peering risk of futures hedging and the transaction cost of futures and options can be considered.

Data Availability

The data are the export price of silver and the exchange rate of CNY/USD. The data can be downloaded from Wind database, whose website is <http://www.wind.com.cn/newsite/data.html>. However, downloading data is payable. Fortunately, our school (CCNU) bought the database. So we have a barrier-free access to download the data.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References

- [1] G. Allayannis and J. P. Weston, "The use of foreign currency derivatives and firm market value," *Review of Financial Studies*, vol. 14, no. 1, pp. 243–276, 2001.
- [2] E. Clarka and S. Mefteh, "Foreign currency derivatives use firm value and the effect of the exposure profile: evidence from france," *International Journal of Business*, vol. 15, no. 2, pp. 183–196, 2010.
- [3] K. P. Wong, "Operational and financial hedging for exporting firms," *International Review of Economics & Finance*, vol. 16, no. 4, pp. 459–470, 2007.
- [4] K. P. Wong, "International trade and hedging under joint price and exchange rate uncertainty," *International Review of Economics & Finance*, vol. 27, no. 2, pp. 160–170, 2013.
- [5] U. Broll and K. P. Wong, "Trade and cross hedging exchange rate risk," *International Economics and Economic Policy*, vol. 12, no. 4, pp. 509–520, 2015.
- [6] S. H. Lence, "On the optimal hedge under unbiased futures prices," *Economics Letters*, vol. 47, no. 3–4, pp. 385–388, 1995.
- [7] D. Lien and K. P. Wong, "Multinationals and futures hedging under liquidity constraints," *Global Finance Journal*, vol. 16, no. 2, pp. 210–220, 2005.
- [8] U. Broll, P. Welzel, and K. P. Wong, "Export and strategic currency hedging," *Open Economies Review*, vol. 20, no. 5, pp. 717–732, 2009.
- [9] K. P. Wong, "Production and hedging in futures markets with multiple delivery specifications," *Decisions in Economics and Finance*, vol. 37, no. 2, pp. 413–421, 2014.
- [10] Y. Machnes, "Further results on comparative statics under uncertainty," *European Journal of Political Economy*, vol. 9, no. 1, pp. 141–146, 1993.
- [11] K. P. Wong, "Production decisions in the presence of options. a note," *International Review of Economics & Finance*, vol. 11, no. 1, pp. 17–25, 2002.
- [12] D. Lien and K. P. Wong, "Delivery risk and the hedging role of options," *Journal of Futures Markets*, vol. 22, no. 4, pp. 339–354, 2002.
- [13] H. L. Battermann and U. Broll, "Inflation risk, hedging, and exports," *Review of Development Economics*, vol. 5, no. 3, pp. 355–362, 2001.
- [14] E. Bajo, M. Barbi, and S. Romagnoli, "Optimal corporate hedging using options with basis and production risk," *The North American Journal of Economics and Finance*, vol. 30, pp. 56–71, 2014.
- [15] E. Bajo, M. Barbi, and S. Romagnoli, "A generalized approach to optimal hedging with option contracts," *European Journal of Finance*, vol. 21, no. 9, pp. 714–733, 2015.
- [16] K. P. Wong, "Cross-hedging ambiguous exchange rate risk," *Journal of Futures Markets*, vol. 37, no. 2, pp. 132–147, 2017.
- [17] H. Lapan, G. Moschini, and S. Hanson, "Production, hedging, and speculative decisions with options and futures markets," *American Journal of Agricultural Economics*, vol. 75, no. 3, pp. 748–750, 1993.
- [18] D. L. Frechette, "The demand for hedging with futures and options," *Journal of Futures Markets*, vol. 21, no. 8, pp. 693–712, 2001.
- [19] K. P. Wong, "Currency hedging with options and futures," *European Economic Review*, vol. 47, no. 5, pp. 833–839, 2003.
- [20] K. P. Wong, "Production and hedging under state-dependent preferences," *Journal of Futures Markets*, vol. 32, no. 10, pp. 945–963, 2012.
- [21] Y. Sakong, D. J. Hayes, and A. Hallam, "Hedging production risk with options," *American Journal of Agricultural Economics*, vol. 75, no. 2, p. 408, 1993.
- [22] G. Moschini and H. Lapan, "The Hedging Role of Options and Futures Under Joint Price, Basis, and Production Risk," *International Economic Review*, vol. 36, no. 4, p. 1025, 1995.
- [23] J. Kotkatvuori-Örnberg, "Dynamic conditional copula correlation and optimal hedge ratios with currency futures," *International Review of Financial Analysis*, vol. 47, pp. 60–69, 2016.
- [24] W. G. Zhang, X. Yu, and Y. J. Liu, "Trade and currency options hedging model," *Journal of Computational and Applied Mathematics*, vol. 343, pp. 328–340, 2018.

- [25] E. C. Chang and K. P. Wong, "Cross-hedging with currency options and futures," *Journal of Financial and Quantitative Analysis*, vol. 38, no. 3, pp. 555–574, 2003.
- [26] T. G. Chi, P. F. YU, and Y. G. Wang, "Research on multi-period futures dynamic hedging model," *Chinese Journal of Management Science*, vol. 18, no. 3, pp. 17–24, 2010.
- [27] Q. Li, Y. Zhou, X. Zhao, and X. Ge, "Dynamic hedging based on fractional order stochastic model with memory effect," *Mathematical Problems in Engineering*, vol. 2016, Article ID 6817483, 8 pages, 2016.
- [28] M. S. Kimball, "Standard risk aversion," *Econometrica*, vol. 61, no. 3, pp. 589–611, 1993.
- [29] P. Jorion, *Value at Risk: The New Benchmark for Managing Financial Risk*, McGraw-Hill, New York, NY, USA, 2nd edition, 2001.
- [30] B. Steil, "Currency options and the optimal hedging of contingent foreign exchange exposure," *Economica*, vol. 60, no. 240, pp. 413–431, 1993.
- [31] D. Lien, "The effect of liquidity constraints on futures hedging," *Journal of Futures Markets*, vol. 23, no. 6, pp. 603–613, 2003.
- [32] T. Bodnar, N. Parolya, and W. Schmid, "A closed-form solution of the multi-period portfolio choice problem for a quadratic utility function," *Annals of Operations Research*, vol. 229, no. 1, pp. 121–158, 2015.
- [33] J.-J. Huang, K.-J. Lee, H. Liang, and W.-F. Lin, "Estimating value at risk of portfolio by conditional copula-GARCH method," *Insurance: Mathematics & Economics*, vol. 45, no. 3, pp. 315–324, 2009.

