**Cosine-Trapezoidal Soft-Starting Control Strategy for a Belt Conveyor**

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Abstract

To improve the soft-starting performance of a belt conveyor, a cosine-trapezoidal soft-starting control strategy is proposed. With this strategy, the speed curve for soft-starting is S-shaped, continuous, and smooth. The acceleration and jerk are also continuous and smooth without discontinuities. The time-interval ratio of the horizontal and cosine curves can be changed via a single parameter setting, so that the strategy can be modified to meet different practical needs. In this paper, the soft-starting control strategy of cosine trapezoid is analyzed and simulated based on current international and domestic standards. The results show that compared with the classical soft-starting control strategy, the cosine-trapezoidal soft-starting control strategy enables the belt conveyor to start at its highest efficiency. Moreover, the energy savings are obvious, the safety performance is good, and the controllable performance is excellent.

1. Introduction

Because the conveyor belt has a certain viscoelasticity, the belt conveyor belongs to the class of flexible bodies. Therefore, the characteristics should be taken in full account in designing a soft-starting control strategy for the belt conveyor, so as to avoid or reduce as far as possible any impact damage of the motor on the belt conveyor. In addition, it is necessary to pay attention to ensure that the starting-up stage of the belt conveyor maintains a high starting efficiency and low energy consumption.

The ideal starting process of the belt conveyor should ensure a smooth continuous acceleration and its maximum value is small to reduce the jolting impact of the motor on the belt conveyor. That is, the maximum jerk must be kept within a reasonable range to ensure a soft impact of motor on the belt conveyor [1–6]. Hence the speed curve should be S-shaped and the output smooth [7–10]. At the same time, it is necessary to ensure high efficiency and low energy consumption at startup. Although some reports have analyzed the operation efficiency and energy consumption of the belt conveyor [11–14], analyzing the efficiency and energy at startup is seldom performed.

The simplest starting mode of belt conveyor is equal acceleration soft-starting control strategy, which is not often used because of its many shortcomings.

\[ v(t) = \frac{V}{T} t, \quad 0 \leq t \leq T \]  

(1)

\[ a(t) = \frac{V}{T}, \quad 0 \leq t \leq T \]  

(2)

\[ j(t) = \begin{cases} 0, & 0 < t < T \\ \infty, & t = 0 \\ -\infty, & t = T \end{cases} \]  

(3)

In 1983, for the purpose of minimizing the transient stress in the belt, Dr. A. Harrison [15] analyzed the circulating elastic waves and long period mass-spring oscillations of the whole belt conveyor system which disturbed the belt speed and belt tension, obtained the wave equation and deduced sinusoidal acceleration control strategy of the belt conveyor.

\[ v(t) = \frac{V_c}{2} \left( 1 - \cos \frac{\pi t}{T} \right), \quad 0 \leq t \leq T \]  

(4)
In 1987, Nordell [16] put forward the triangular acceleration control strategy of belt conveyor.

\[
\begin{align*}
    a(t) &= \frac{\pi V_e}{2T} \sin \frac{\pi t}{T} = \frac{1.57 V_e}{2T} \sin \frac{\pi t}{T}, \quad 0 \leq t \leq T \\
    j(t) &= \frac{4.93 V_e}{T^2} \cos \frac{\pi t}{T}, \quad 0 \leq t \leq T
\end{align*}
\]

In 1987, Nordell [16] put forward the triangular acceleration control strategy of belt conveyor.

\[
\begin{align*}
    v(t) &= \begin{cases} 
        \frac{2V_e t^2}{T^2}, & 0 \leq t \leq \frac{T}{2} \\
        V_e \left(1 - \frac{4t^2}{T^2} - \frac{t^2}{2} \right), & T \leq t \leq T
    \end{cases} \\
    a(t) &= \begin{cases} 
        \frac{4V_e t}{T^2}, & 0 \leq t \leq \frac{T}{2} \\
        \frac{4V_e}{T} \left(1 - \frac{t}{T}\right), & T \leq t \leq T
    \end{cases} \\
    j(t) &= \begin{cases} 
        \frac{4V_e}{T^2}, & 0 \leq t \leq \frac{T}{2} \\
        \frac{-4V_e}{T}, & \frac{T}{2} \leq t \leq T
    \end{cases}
\end{align*}
\]

In addition, parabolic acceleration soft-starting control strategy is often used [17].

\[
\begin{align*}
    v(t) &= V_e \left(\frac{3t^2}{T^2} - \frac{2t^3}{T^3}\right), \quad 0 \leq t \leq T \\
    a(t) &= \frac{1.5V_e}{T} \left(4t - \frac{4t^2}{T^2}\right), \quad 0 \leq t \leq T \\
    j(t) &= \frac{6V_e}{T^2} \left(1 - \frac{2t}{T}\right), \quad 0 \leq t \leq T
\end{align*}
\]

In all expressions, \(v(t)\) represents the startup speed, \(a(t)\) is the startup acceleration, \(j(t)\) denotes the startup acceleration, \(T\) denotes the startup time, and \(V_e\) is the rated speed.

From (2) and (3), the acceleration of the equal acceleration control strategy is discontinuous (mutation) at the start and end of the startup and the jerk is infinite. Therefore, the impact is greater on starting up the belt conveyor, and so its application is little used. The other three soft-starting control strategies are however widely used. Therefore, these three strategies are compared with our proposed soft-starting control strategy presented below.

Referred to here as the cosine-trapezoidal control strategy, the theoretical analysis and simulation experiments that were performed and present comparison results with the above-mentioned classic control strategies are described in this paper. It is also concluded that this cosine-trapezoidal soft-starting control strategy is better than the others regarding soft-starting time-consuming start time, startup energy consumption, safety and control.

### 2. Cosine-Trapezoidal Soft-Starting Control Strategy

With \(T\) the startup time of the belt conveyor the cosine-trapezoidal soft-starting control strategy (Figure 1) is characterized by a maximum acceleration value \(a_{\text{max}}\) in the fixed time interval \((T_1, T_2)\) and cosine curves in time intervals \((0, T_1)\) and \((T_2, T)\); the fixed times satisfy

\[
T_1 = T - T_2, \\
T_2 - T_1 = 2NT_1, \\
N = 0, 1, 2, 3, \ldots
\]

In the formula above, \(N\) denotes the number of half cycles of the cosine curve. That is,

\[
T_1 = \frac{T}{2N + 2}, \\
T_2 = \frac{(2N + 1)T}{2N + 2},
\]

\[N = 0, 1, 2, 3, \ldots\]

Therefore, the expression for acceleration is

\[
a(t) = \begin{cases} 
    a_{\text{max}} \left[1 - \cos \left(\frac{\pi t}{T_1}\right)\right], & 0 < t < T_1 \\
    2a_{\text{max}}, & T_1 < t < T_2 \\
    a_{\text{max}} \left[1 - \cos \left(\frac{\pi t}{T_1}\right)\right], & T_2 < t < T
\end{cases}
\]

Integrating these expressions with respect to \(t\) yields the velocity expressions

\[
v(t) = \begin{cases} 
    a_{\text{max}} \left[t - \frac{T_1}{\pi} \sin \left(\frac{\pi t}{T_1}\right)\right], & 0 < t < T_1 \\
    a_{\text{max}} (2t - T_1), & T_1 < t < T_2 \\
    a_{\text{max}} \left[T_2 - T_1 + t - \frac{T_1}{\pi} \sin \left(\frac{\pi t}{T_1}\right)\right], & T_2 < t < T
\end{cases}
\]

Differentiating (15) with respect to \(t\) yields the jerk expressions

\[
j(t) = \begin{cases} 
    a_{\text{max}} \frac{\pi}{T_1} \sin \left(\frac{\pi t}{T_1}\right), & 0 < t < T_1 \\
    0, & T_1 < t < T_2 \\
    a_{\text{max}} \frac{\pi}{T_1} \sin \left(\frac{\pi t}{T_1}\right), & T_2 < t < T
\end{cases}
\]
The running speed of the belt conveyor should reach the rated speed of $V_e$ when $t=T$. Hence, from (16), the condition is obtained:

$$a_{\text{max}} \left[ T_2 - T_1 + T - \frac{T}{\pi} \sin \left( \frac{\pi}{T_1} T \right) \right] = V_e \quad (18)$$

which, on substituting (14), becomes

$$a_{\text{max}} = \frac{V_e}{T}, \quad \frac{N+1}{2N+1} = \frac{V_e}{T} \left( \frac{1}{2} + \frac{1}{4N+2} \right) \quad (19)$$

The substitution of (14) and (19) into (15)–(17) provides expressions parametrized by $N$ and $T$ for position, speed, acceleration, and jerk of the soft-starting control strategy:

$$d(t) = \left\{ \begin{array}{ll}
\frac{(N+1)V_e}{2N+1}, & 0 < t < T_1 \\
\frac{T}{T} - \frac{1}{2N+2} + \frac{2(2N+2)^2 - (2N+2)^2 \pi^2}{(2N+2)^2 \pi^2}, & T_1 < t < T_2 \\
\frac{NT}{2N+1} + \frac{T}{2N+2} - \frac{1}{2N+2}, & T_2 < t < T
\end{array} \right. \quad (20)$$

$$v(t) = \left\{ \begin{array}{ll}
\frac{(N+1)V_e}{2N+1}, & 0 < t < T_1 \\
\frac{T}{T} - \frac{1}{2N+2} + \frac{2(2N+2)^2 - (2N+2)^2 \pi^2}{(2N+2)^2 \pi^2}, & T_1 < t < T_2 \\
\frac{NT}{2N+1} + \frac{T}{2N+2} - \frac{1}{2N+2}, & T_2 < t < T
\end{array} \right. \quad (21)$$

$$a(t) = \left\{ \begin{array}{ll}
\frac{N+1}{2N+1} \frac{V_e}{T}, & 0 < t < T_1 \\
\frac{T}{T} - \frac{1}{2N+2} + \frac{2(2N+2)^2 - (2N+2)^2 \pi^2}{(2N+2)^2 \pi^2}, & T_1 < t < T_2 \\
\frac{NT}{2N+1} + \frac{T}{2N+2} - \frac{1}{2N+2}, & T_2 < t < T
\end{array} \right. \quad (22)$$

$$j(t) = \left\{ \begin{array}{ll}
\frac{(N+1)(2N+2) \pi V_e}{2N+1} \frac{V_e}{T^2}, & 0 < t < T_1 \\
\frac{T}{T} - \frac{1}{2N+2} + \frac{2(2N+2)^2 - (2N+2)^2 \pi^2}{(2N+2)^2 \pi^2}, & T_1 < t < T_2 \\
\frac{NT}{2N+1} + \frac{T}{2N+2} - \frac{1}{2N+2}, & T_2 < t < T
\end{array} \right. \quad (23)$$

Setting $T=50$ s, $V_e=5$ m/s, and $N=2$, simulation curves for the position, speed, acceleration, and jerk using our control strategy were plotted (see Figures 2–5).

### 3. Simulation and Performance Analysis

The rated speed, maximum acceleration, and power of the belt conveyor all conform with the domestic and international standards. Limited by space, this study only uses a set of recommended values for analysis; the procedures and results of other numerical analyses are similar to those given below.

#### 3.1. Startup Time and Jerk Analysis

In terms of the maximum acceleration and rated speed, the startup times and jerks of the four soft-starting control strategies can be given (Tables 1 and 2). Setting the rated speed $V_e=5$ m/s and maximum acceleration $a_{\text{max}}=0.2$ m/s$^2$ for the belt conveyor, plots for the four soft-starting control strategies provide a base for comparison (Figure 6) and in particular a comparison of our cosine-trapezoidal control strategy for various values of $N$ (Figure 7).

Given the same maximum acceleration, the startup time is much shorter using the cosine-trapezoidal control strategy than for the other three classical strategies. Moreover, its startup efficiency is the highest.

Comparing cosine-trapezoidal strategies with different $N$ (Figure 7), the startup time shortened and the startup efficiency increases gradually as $N$ increases. The shortest startup time of the cosine-trapezoidal soft-starting control strategy is $V_e/a_{\text{max}}$ (i.e., $N$ large).

From Table 2 and Figure 5, it can be seen that the jerk of the other three classical soft-starting strategies has a mutation, so their maximum ramp rate of jerk can be infinite at mutation. While the jerk of the cosine-trapezoidal soft-starting control strategy has no mutation, so its ramp rate of jerk is continuous and smooth. In addition, the jerk of the other three classical soft-starting control strategies is always nonzero during the startup process. But the jerk of the cosine-trapezoidal soft-starting control strategy is zero in $[T_1,T_2]$. Therefore, the cosine-trapezoidal soft-starting control strategy is conducive to reducing the damage of the soft impact on the belt conveyor.
### Table 1: Startup times comparison.

<table>
<thead>
<tr>
<th>Control strategy</th>
<th>Sinusoidal</th>
<th>Triangle</th>
<th>Parabolic</th>
<th>Cosine-trapezoidal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Startup time</td>
<td>$\frac{1.5V_e}{a_{\text{max}}}$</td>
<td>$\frac{2V_e}{a_{\text{max}}}$</td>
<td>$\frac{1.5V_e}{a_{\text{max}}}$</td>
<td>$\frac{2N + 2V_e}{2N + 1a_{\text{max}}}$</td>
</tr>
</tbody>
</table>

### Table 2: Jerks comparison.

<table>
<thead>
<tr>
<th>Control strategy</th>
<th>Sinusoidal</th>
<th>Triangle</th>
<th>Parabolic</th>
<th>Cosine-trapezoidal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum value</td>
<td>$\frac{2a_{\text{max}}}{V_e}$</td>
<td>$\frac{a_{\text{max}}}{V_e}$</td>
<td>$\frac{2.67a_{\text{max}}}{V_e}$</td>
<td>$\frac{1.57(2N + 1)a_{\text{max}}}{V_e}$</td>
</tr>
<tr>
<td>Maximum ramp rate</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\infty$</td>
<td>$\frac{4.93(2N + 1)^2a_{\text{max}}}{V_e}$</td>
</tr>
<tr>
<td>Mutation</td>
<td>$t = 0, T$</td>
<td>$t = 0, \frac{T}{2}, T$</td>
<td>$t = 0, T$</td>
<td>None</td>
</tr>
</tbody>
</table>

3.2. **Acceleration Analysis.** Given a fixed startup time and rated speed, the maximum acceleration for the different soft-starting control strategies is listed in Table 3. Setting $V_e = 5 \text{ m/s}$ and $T = 50 \text{ s}$, values of the maximum acceleration were then plotted for comparison (Figure 8). The maximum change in acceleration of the cosine-trapezoidal soft-starting control strategy for different values of $N$ was also plotted (Figure 9).

Table 3 and Figure 8 show that the startup times are the same and the maximum acceleration of the belt conveyor
### Table 3: Maximum acceleration comparison.

<table>
<thead>
<tr>
<th>Control strategy</th>
<th>Sinusoidal</th>
<th>Triangle</th>
<th>Parabolic</th>
<th>Cosine-trapezoidal</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum acceleration</td>
<td>$\frac{1.57 V}{T}$</td>
<td>$\frac{2V}{T}$</td>
<td>$\frac{1.5V}{T}$</td>
<td>$\frac{2(N+1)V}{T}$</td>
</tr>
</tbody>
</table>

![Figure 6: Startup times for the different soft-starting control strategies.]

![Figure 8: Acceleration curves for the different soft-starting control strategies.]

![Figure 7: Change in startup times for the cosine-trapezoidal soft-starting control strategy with $N$.](image)

![Figure 9: Change in acceleration for the cosine-trapezoidal soft-starting control strategy with $N$.](image)

Using the cosine-trapezoidal soft-starting control strategy is much lower than the three classical strategies. That is, the belt conveyor experiences by far a softer impact under a cosine-trapezoidal strategy than under the other three strategies.

From Figure 9, with a cosine-trapezoidal strategy, as $N$ increases, the maximum acceleration diminishes gradually. In the limit of $N$ large, the maximum acceleration tends to $V_c/T$ (see Table 2).

### 3.3 Energy Saving Analysis

In the startup process, the energy saving criterion for the belt conveyor relates to the energy consumption rather than power. When the belt conveyor is moving at a constant speed, the circular force of the conveyor roller of the belt conveyor is given as

$$F_U = CF_H + F_{S1} + F_{S2} + F_{St}$$  \hspace{1cm} (24)

where $C$ is an added coefficient of resistance, $F_H$ the main resistance ($N$), $F_{S1}$ the main special resistance ($N$), $F_{S2}$ an added special resistance ($N$), and $F_{St}$ the inclined resistance ($N$).

During startup, the force of inertia of the belt conveyor is

$$F_A = (m_L + m_D) a(t)$$  \hspace{1cm} (25)

where $m_L$ is the equivalent mass (kg) of the moving parts (conveyor belt, material, and roller) performing linear motion and $m_D$ is the equivalent mass (kg) of the rotating parts (without the roller) undergoing linear motion.

During the startup period, the power of the belt conveyor (kW) is
Table 4: Energy consumption and power of the different soft-starting control strategies.

| Control strategy       | Energy consumption/kJ | Comparison content | Power/kW |   |
|------------------------|------------------------|--------------------|---------|
|                        | No load                | Full load          | No load | Full load |
| Sinusoidal             | 5453.1                 | 6078.1             | 138.9   | 154.9     |
| Triangle               | 7125                   | 8125               | 142.5   | 162.5     |
| Parabolic (N=2)        | 5562.5                 | 6562.5             | 148.3   | 175       |
| Cosine-trapezoidal     | 4637.2                 | 5636.7             | 154.6   | 187.9     |

Table 5: Energy consumption and power of the cosine-trapezoidal soft-starting control strategy for small $N$.

| $N$   | Energy consumption/kJ | Comparison content | Power/kW |   |
|-------|------------------------|--------------------|---------|
|       | No load                | Full load          | No load | Full load |
| 0     | 7430.9                 | 8418.2             | 148.6   | 168.4     |
| 1     | 5075.6                 | 6074.2             | 152.3   | 182.2     |
| 2     | 4637.2                 | 5636.7             | 154.6   | 187.9     |
| 3     | 4452.8                 | 5452.4             | 155.9   | 190.8     |
| 4     | 4351.0                 | 5350.8             | 156.6   | 192.6     |

\[ p(t) = \frac{\left[ F_U + (m_L + m_D) a(t) \right] v(t)}{1000} \]  
\[ (26) \]

whereas its energy consumption (kJ) is

\[ w(t) = \int_0^t p(x) \, dx \]  
\[ (27) \]

Set the rated speed of the belt conveyor to $V_e=5$ m/s, the maximum acceleration to $a_{\text{max}}=0.2$ m/s$^2$, the circumferential driving force of the drive roller to $F_U=50000$ N, the equivalent mass of the moving body of the unloaded belt conveyor converted into linear motion of the conveyor belt $m_{L,K}=40000$ kg, the full-loaded belt conveyor to $m_{L,m}=120000$ kg, and $m_D=30000$ kg. The energy consumption and power of the four soft-starting control strategies for these settings are listed in Table 4, and the change in the energy consumption and power of the cosine-trapezoidal soft-starting control strategy for small values of $N$ are listed in Table 5.

From Table 4, it can be found that when the maximum acceleration is fixed, the energy consumption of the cosine-trapezoidal strategy is the lowest among the four strategies: the no-load energy consumption is 85% for the sinusoidal, 65.1% for the triangle, and 83.4% for the parabola, whereas the full-loaded energy consumption is 92.7% for the sinusoidal, 69.4% for the triangle, and 85.9% for the parabola. The power is slightly higher for the cosine-trapezoidal strategy than for the other three strategies.

From Table 5, using the cosine-trapezoidal strategy, with different $N$ values, the energy consumption diminishes with $N$ whereas the startup power increases slightly. The energy saving with the cosine-trapezoidal strategy is evident.

3.4. Parameter $N$ Analysis

3.4.1. Influence of the Change in $N$ on the Startup Time. Setting $V_e=5$ m/s and $a_{\text{max}}=0.2$ m/s$^2$ into the startup time expression of the cosine-trapezoidal soft-starting control strategy gives

\[ T = \frac{2N + 2}{2N + 1} \frac{V_e}{a_{\text{max}}} = \frac{50(N + 1)}{2N + 1} \text{(s)} \]  
\[ (28) \]

The startup time changes with $N$. From (28) and the plot (Figure 10), the startup time decreases with $N$. Hence, from the perspective of a quick startup and improved efficiency, $N$ should be large.

3.4.2. Influence of the Change in $N$ on the Energy Consumption. Setting $V_e=5$ m/s and $a_{\text{max}}=0.2$ m/s$^2$, the circumferential driving force of the drive roller is $F_U=50000$ N. The equivalent mass of the moving body of the no-load belt conveyor converted into linear motion of the conveyor belt is $m_{L,K}=40000$ kg and of the full-load belt conveyor is $m_{L,m}=120000$ kg. The
4. Conclusion

For the belt conveyor, this paper proposed a different soft-starting control strategy providing high efficiency and low equivalent mass of the rotating parts of the belt conveyor converted into linear motion of the conveyor belt is $m_D = 30000$ kg. The energy consumptions under no-load and full-load are then

$$\begin{align*}
W_{\text{No-Load}} & = \frac{21375N^2 + 28500N + 9189.96}{(2N + 1)^2} + \frac{7125N + 4000}{2N + 1} + 2218.75 \text{ (kJ)} \\
W_{\text{Full-Load}} & = \frac{24375N^2 + 32500N + 10414.559}{(2N + 1)^2} - \frac{8125N + 5000}{2N + 1} + 2968.75 \text{ (kJ)}
\end{align*}$$

During startup, the energy consumption of the belt conveyor under no-load and full-load changes with $N$ as evident in Figures 11 and 12.

Using (29) and (30), the plots of the energy consumption of the belt conveyor (Figures 11 and 12) show a decrease with increasing $N$. Hence, to reduce the energy consumption requires $N$ to be large.
energy consumption, as well as offering stable, controllable, and safe operating conditions. The speed curve of the cosine-trapezoidal soft-starting control strategy is S-shaped allowing for continuous and smooth transitions without mutation as for the acceleration curve. Compared with the classic soft-starting control strategies, the advantages of the cosine-trapezoidal soft-starting control strategy are as follows:

(1) Enhance startup efficiency: the startup time is short, and it is further shortened as $N$ increases.

(2) Provide energy savings: the energy consumption is lowest of the control strategies and can be further reduced by increasing $N$.

(3) Improve controllable performance: the startup time, acceleration, jerk, and energy consumption can be changed via the load adjustment parameter $N$.

(4) Raise safety performance: the hard/soft impacts imparted by the motor to the belt conveyor are suppressed and controlled.

**Data Availability**

The data used to support the findings of this study are available from the corresponding author upon request.

**Conflicts of Interest**

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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