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Research Article

Formation Tracking via Iterative Learning Control for Multiagent Systems with Diverse Communication Time-Delays

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In this paper, we consider the formation tracking problem for multiagent systems with diverse communication time-delays by using iterative learning control (ILC) method based on the frequency domain analysis. A first-order ILC law for multiagent systems with diverse communication time-delays is first proposed and its convergence conditions are given by the general Nyquist stability criterion and Gershgorin's disk theorem. Then, in order for the system to track accurately, a second-order ILC law is presented. The conditions for system tracking with zero error are established. Numerical simulations show that the proposed ILC laws for multiagent systems with diverse communication time-delays are able to achieve effectively formation tracking. And the convergence speed remains the same as the learning control algorithm without communication delay.

1. Introduction

Multiagent systems can make it more efficient to complete some parallel and complicated tasks than a single agent and more and more scholars have begun to pay attention to multiagent systems cooperation [1–3]. As well known, ILC has been widely utilized for dynamic systems that operate repetitively on the finite time interval, because of its low complexity and high precision. Therefore, the multiagent systems that can operate repetitively using ILC method are a wise choice to achieve flexible task. In 2009, Ahn et al. firstly used ILC to implement circular formation for multiagent systems [4]. Afterward, ILC for multiagent systems was widely used in practical applications. For example, in [5], multiple satellites cooperatively kept the trajectory around the earth; in [6], multiple robots formation moved in a shape; in [7], multiple high-speed trains cooperatively controlled driving speed; in [8, 9], multiple UAVs flew for trajectory tracking. And all the above-mentioned papers were based on ILC method.

However, note that all the above-mentioned results were based on an ideal communication environment, in which data transmitted between multiple agents will not be mistaken. Due to the limited bandwidth of communication channels, noise interference, and signal fading, multiagents inevitably have communication time-delays during signal transmission [10]. Since communication time-delays extensively exit, the performance of systems may be greatly reduced. For example, in [11–14], multiagent systems with communication time-delays have attracted great attention. Since ILC is introduced into multiagent systems, more and more scholars have paid attention to ILC for multiagent systems with communication time-delays. An ILC tracking strategy was proposed for nonlinear multiagent systems with communication time-delays in [15]. Using the 2D analysis, [16] obtained convergence conditions in terms of linear matrix inequality and addressed both exponential convergence and monotonic convergence problems associated with the ILC process corresponding to the proposed consensus learning protocol. In [17], a distributed ILC algorithm was proposed for multiagent systems subject to 2D switching topologies and varying communication time-delays. In [18], an adaptive iterative learning control method, which includes a P-type feedback term and an iterative learning term along the iteration axis, was designed for the consensus problem of homogeneous multiagent systems with state time-delays. In [19], an ILC scheme for multiagent systems with one-step random time-delay was proposed. All the above-mentioned papers have proposed suitable learning law and convergence conditions to guarantee convergence of system errors. However, the bounds of tracking errors were subject to the accuracy of the estimated delay in [15]. The works [16, 18] discussed consensus problem of multiagent systems not formation tracking. And [19] showed that the convergence speed got slower as the increase of the time-delays rate. Therefore, time-delays compensation method for formation tracking of multiagent systems with diverse time-delays is a more challenging issue.

Motivated by the above observations, this paper proposes an ILC time-delays compensation method for multiagent systems. And we can keep convergence speed as multiagent systems without time-delays. A frequency domain ILC model is established for multiagent systems with linear SISO discrete time dynamics. Since ILC is a two-dimensional system with evolution along two independent axes, frequency domain and iteration domain, we discuss the convergence of arbitrary frequency in iteration dimension by exchange variable frequency and parameter iteration. Then we investigate eigenvalues of the characteristic equation to analyze the condition of convergence by taking z-transformation in iteration domain. Because the characteristic equations are in matrix form, it is difficult to analyze their eigenvalues. To this end, we use the generalized Nyquist criteria and Gershgorin disc theorem to analyze the range of eigenvalues and obtain the conditions for system convergence. The results show that the convergence condition of the system has nothing to do with the communication time-delays. The system errors cannot converge to zero but a certain value. In order to compensate for the influence of time-delays, this paper proposes a method using second-order ILC to make the system error converge to zero. The same frequency domain analysis method as first-order ILC was used to analyze the convergence conditions of the second-order ILC for multiagent systems with communication time-delays. Simulations demonstrate that second-order ILC can effectively compensate for the effects of communication delay.

The remainder of the paper is organized as follows. In Section 2, an ILC model is established for multiagent systems with linear SISO discrete time dynamics based on frequency domain. Moreover, we proposed the control objective which is formation tracking for multiagent systems. The conditions of system convergence are obtained in Section 3 when using first-order ILC and second-order ILC, respectively. In Section 4, numerical simulations show the effectiveness of the proposed protocol.

Notations. 1 and 0 denote the column vectors of appropriate dimensions whose elements are all ones and all zeros, respectively; I denotes an identity matrix with the required dimensions.

2. Problem Formulation

2.1. System. Consider the multiagent systems consisting of n agents labeled from 1 through n, whose communication topology graph is denoted by $\mathcal{G}$. And the dynamic of each agent is a linear SISO discrete time system:

$$\begin{aligned} x_{i,k}(t+1) &= Ax_{i,k}(t) + Bu_{i,k}(t) \\ y_{i,k}(t) &= Cx_{i,k}(t) \end{aligned} \tag{1}$$

where $i = 1, 2, \ldots, n$ represents the ith agent, $t = 1, 2, \ldots, T$ is the discrete time index, $k = 1, 2 \ldots$ is iteration index, $x_{i,k}(t) \in \mathbb{R}$ is the ith agent state of the kth iteration in time t, $u_{i,k}(t) \in \mathbb{R}$ is the control input signal, and $y_{i,k}(t) \in \mathbb{R}$ is the output. Multiple agents have the same dynamics A, B, and C. Without loss of generality, let system (1) relative degree be 1; that is, $CB \neq 0$. And the dynamics for each agent is stable; that is, $|A| < 1$. Taking the z-transformation of system (1), we get

$$Y_{i,k}(z) = \frac{BC}{z-A}U_{i,k}(z) + \frac{C}{z-A}x_{i,k}(0) \tag{2}$$

where $z = e^{j\omega}$, $\omega \in (-\infty, \infty)$ with $j = \sqrt{-1}$, ω is frequency, and $x_{i,k}(0)$ is the ith agent initial state of the kth iteration; let $G(z) = BC(z-A)^{-1}$, $\gamma_{i,k}(z) = C(z-A)^{-1}x_{i,k}(0)$, then (2) can be rewritten as

$$Y_{i,k}(z) = G(z)U_{i,k}(z) + \gamma_{i,k}(z) \tag{3}$$

Assumption 1. The initial resetting condition for all agents and all the iteration satisfied desired input. Then, $x_{i,k}(0)$ can be abbreviated as $x_i(0)$.

Remark 2. Assumption 1 is common in ILC for multiagent systems such as [16, 19]. If the condition is not met, we can think of it as a robust problem against initial shifts. In our future work, the ILC with initial-state learning of formation tracking control will be one of our objectives to consider.

2.2. Control Objective. For system (1), the objective to realize in this paper is that multiagent systems can move in the desired formation tracking reference trajectory as the increase of iteration. That is, for $i = 1, 2, \ldots, n$, $t = 1, 2, \ldots, T$, we have $\lim_{k\to\infty} y_{i,k}(t) = y_i^d(t)$, and $y_i^d(t)$ is desired output of the ith agent and denoted as

$$y_i^d(t) = r(t) + d_i(t) \tag{4}$$

where $r(t)$, $t = 1, 2, \ldots, T$ is desired reference trajectory of multiagent systems. In practice, not all agents can obtain $r(t)$, but a portion of agents can. To this end, we denote the reference-accessibility matrix $\Phi = \mathrm{diag}\{\varphi_1, \varphi_2, \cdots \varphi_n\}$, which is a diagonal, nonnegative, real matrix. If the ith agent has direct information about $r(t)$, then $\varphi_i > 0$; otherwise, $\varphi_i = 0$. $d_i(t)$ is the desired output deviation from desired reference trajectory of the ith agent. If multiple agents have the same desired output deviation, then the agents will have the same output. So $d_i(t) \neq d_j(t)$, $i \neq j$. $d_{ij}(t) = d_i(t) - d_j(t)$ represents the desired relative formation between the ith agent and the jth agent. It is worth pointing out that each agent can obtain its own output $y_{i,k}(t)$ and its own desired output deviation $d_i(t)$. For convenience, we denote

$$\delta_{i,k}(t) = d_i(t) - y_{i,k}(t) \tag{5}$$

We denote the output error of the ith agent in the kth iteration:

$$e_{i,k}(t) = r(t) + d_i(t) - y_{i,k}(t) \tag{6}$$

Taking z-transformation for (6), we get

$$E_{i,k}(z) = R(z) + D_i(z) - Y_{i,k}(z) \tag{7}$$

When it is satisfied $\lim_{k\longrightarrow\infty} E_{i,k}(z) = 0$, that is, $\lim_{k\longrightarrow\infty} Y_{i,k}(z) = Y_i^d(z)$, multiagent systems realize formation tracking control.

2.3. Preliminaries in Graph Theory. Let $\mathscr{G} = (\mathscr{V}, \mathscr{E}, \mathscr{A})$ be directed graph, where $\mathscr{V} = \{v_1, \ldots, v_n\}$ is the set of vertices, $\mathscr{E} \subseteq \mathscr{V} \times \mathscr{V}$ is the set of edge, and $\mathscr{A} = [a_{ij}]$ $(a_{ij} \geq 0)$ is the weighted adjacency matrix of the graph $\mathscr{G}$. If there is an edge from the ith agent to the jth agent, that is, $(v_i, v_j) \in \mathscr{E}$, then $a_{ji} > 0$, representing the fact that information is transmitted from ith agent to jth agent. Otherwise, $a_{ji} = 0$. Moreover, we assume that $a_{ii} = 0$. The index set of neighbors of node v_i is $\mathscr{N}_i = \{j : (v_i, v_j) \in \mathscr{E}\}$. The Laplacian matrix $\mathscr{L} = [l_{ij}] \in \mathbb{R}^{n\times n}$ of graph $\mathscr{G}$ is denoted as $\mathscr{L} = \mathscr{D} - \mathscr{A}$, where $\mathscr{D} \triangleq \operatorname{diag}\{d_1, \ldots, d_n\}$ with $d_i = \sum_{j=1}^{n} a_{ij}$. A path in the directed graph $\mathscr{G}$ is a finite sequence $(v_{i_1}, v_{i_2}, \ldots, v_{i_j})$, and $(v_{i_l}, v_{i_{l+1}}) \in \mathscr{E}$, $l = 1, 2, \cdots j-1$. If there is a vertex that can be connected to all other vertices through paths, then $\mathscr{G}$ is said to have a spanning tree, and this vertex is called the root vertex.

Assumption 3. The graph of multiagent system (1) is a directed graph. And its graph is connected; that is, each agent has a path to the other agents so that they can exchange information.

Lemma 4. *Vector* $\mathbf{1}$ *is a right eigenvector of the Laplacian* $\mathscr{L}$ *with eigenvalue 0; i.e.,*

$$\mathscr{L}\mathbf{1} = 0\mathbf{1} \tag{8}$$

Lemma 5 (see [20]). *If an irreducible matrix* $M = (a_{i,j}) \in \mathbb{C}^{n\times n}$, $n \geq 1$ *is weakly generalized diagonally dominant and at least one of the rows is strictly diagonally dominant, A is nonsingular.*

Remark 6. Assumption 3 implies $\mathscr{L} + \Phi$ is a matrix that satisfies the conditions of *Lemma 5*. In $\mathscr{L} + \Phi$, the magnitude of the diagonal entry is $\varphi_i + \sum_{j\in\mathscr{N}_i} a_{i,j}$, $i = 1, 2, \ldots, n$, and the sum of the magnitudes of all nondiagonal entries is $\sum_{j\in\mathscr{N}_i} a_{i,j}$, $i = 1, 2, \ldots, n$. Since $\varphi_i + \sum_{j\in\mathscr{N}_i} a_{i,j} \geq \sum_{j\in\mathscr{N}_i} a_{i,j}$, $i = 1, 2, \ldots, n$ and at least one of the rows is satisfied $\varphi_i + \sum_{j\in\mathscr{N}_i} a_{i,j} > \sum_{j\in\mathscr{N}_i} a_{i,j}$, matrix $\mathscr{L} + \Phi$ is nonsingular.

2.4. Communication Time-Delays. When multiagent systems cooperate through the network, there are time-delays in transmission due to the limited bandwidth of communication channels, noise interference, and signal fading. Therefore, we introduce communication time-delays into multiagent systems to research more close to the practical systems. A block diagram of multiagent systems with communication time-delays is illustrated in Figure 2 if the communication graph of multiagent systems is shown in Figure 1. When the ith agent can obtain $\delta_{j,k}(t)$ from the jth agent, we denote $\dot{\delta}_{ji,k}(t)$ as the signal which the ith agent received from the jth agent:

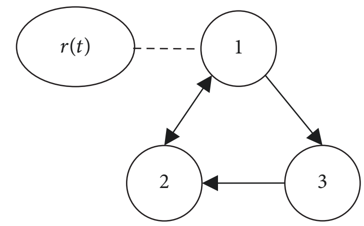

Figure 1: Communication graph of the multiagent system.

$$\dot{\delta}_{ji,k}(t) = \delta_{j,k}\left(t - \tau_{i,j}\right) \tag{9}$$

where $\tau_{i,j}$ is communication time-delay when the jth agent sends signal to the ith agent. $\tau_{i,j} \in \{0, 1, 2 \cdots, \tau_{max}\}$, $i \neq j$, $\tau_{max} < T$.

3. Compensation to ILC for Multiagent System with Communication Time-Delays

In order to make the multiagent systems more close to the practical engineering applications, we analyze the ILC for multiagent systems with communication time-delays first. The result shows the system error of first-order ILC with communication time-delays cannot converge to zero. Second, the method to compensate communication time-delay is given. And the convergence analysis is obtained in Section 3.2.

3.1. First-Order ILC for Multiagent Systems with Communication Time-Delays. For multiagent systems with communication time-delays, a first-order ILC law is proposed as

$$\begin{aligned} u_{i,k+1}(t) &= u_{i,k}(t) \\ &\quad + \Gamma \left\{ \sum_{j\in\mathscr{N}_i} a_{i,j} \left[\delta_{i,k}(t+1) - \delta_{j,k}\left(t + 1 - \tau_{i,j}\right)\right] \right. \\ &\quad \left. + \varphi_i e_{i,k}(t+1) \right\} \end{aligned} \tag{10}$$

where $\Gamma \in \mathbb{R}$ and $\Gamma \neq 0$ is a gain to be determined. This ILC law is multiple agents joint learning law. The control input of the ith agent needs previous iterative control input, self-error, and adjacent agent information. In (10), $\tau_{i,j}$ is communication time-delay mentioned above.

Taking the z-transformation of (10), we get

$$U_{i,k+1}(z) = U_{i,k}(z)$$

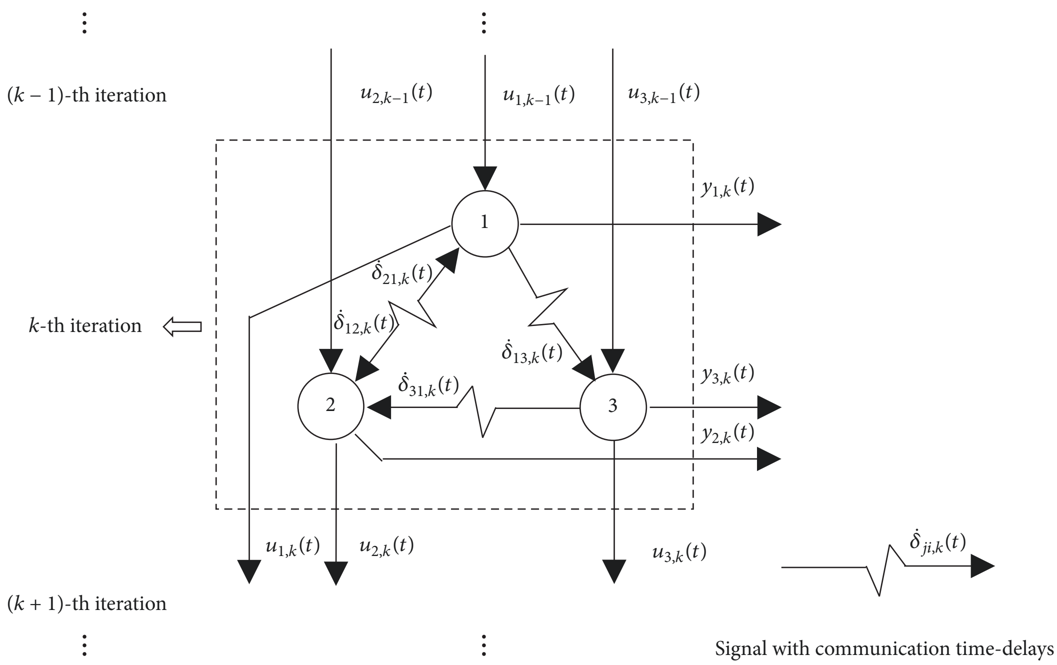

Figure 2: The structure of multiagent systems with communication time-delays.

$$+ z\Gamma \left\{ \sum_{j\epsilon\mathcal{N}_i} a_{i,j} \left[\Delta_{i,k}(z) - z^{-\tau_{i,j}} \Delta_{j,k}(z) \right] + \varphi_i E_{i,k}(z) \right\} \tag{11}$$

By (7), (11) and *Assumption 1*, we have that

$$\begin{aligned} E_{i,k+1}(z) - E_{i,k}(z) &= G(z)\left[U_{i,k}(z) - U_{i,k+1}(z)\right] \\ &= -z\Gamma G(z) \\ &\cdot \left\{ \sum_{j\epsilon\mathcal{N}_i} a_{i,j} \left[\Delta_{i,k}(z) - z^{-\tau_{i,j}} \Delta_{j,k}(z) \right] + \varphi_i E_{i,k}(z) \right\} \end{aligned} \tag{12}$$

Let $E_k(z) = [E_{1,k}(z), E_{2,k}(z), \ldots, E_{n,k}(z)]^T$; then (12) can be rewritten as

$$\begin{aligned} E_{k+1}(z) - E_k(z) &= -z\Gamma G(z)\left[\mathscr{L}(z) + \Phi\right]\left[E_k(z) - R(z)\mathbf{1}\right] \\ &\quad - z\Gamma G(z)\Phi R(z)\mathbf{1} \end{aligned} \tag{13}$$

Further, we can show that

$$\begin{aligned} E_{k+1}(z) &= \{I - z\Gamma G(z)\left[\mathscr{L}(z) + \Phi\right]\} E_k(z) \\ &\quad + z\Gamma G(z) R(z) \mathscr{L}(z)\mathbf{1} \end{aligned} \tag{14}$$

where $\mathscr{L}(z)$ is

$$\mathscr{L}(z) = \begin{cases} -a_{i,j} z^{-\tau_{i,j}}, & j\epsilon\mathcal{N}_i \\ \sum_{j\epsilon\mathcal{N}_i} a_{i,j}, & i = j \\ 0, & \text{others} \end{cases} \tag{15}$$

Theorem 7. *Consider multiagent system (1); let Assumptions 1 and 3 hold and the learning law (10) be applied. Then the system error of multiagent systems can converge, given that any one of the following conditions is satisfied for all* $i = 1, 2, \ldots, n$:

$$\begin{aligned} &(1)\ (1 > A > 0) \wedge (P_{i,1} > 0) \wedge (P_{i,2} > 0) \wedge (P_{i,3} > 0) \\ &\quad \wedge (P_{i,4} > 0) \wedge (P_{i,5} > 0) \\ &(2)\ (-1 < A < 0) \wedge (P_{i,1} > 0) \wedge (P_{i,2} < 0) \wedge (P_{i,3} > 0) \\ &\quad \wedge (P_{i,4} < 0) \wedge (P_{i,5} > 0) \end{aligned} \tag{16}$$

where

$$\begin{aligned} P_{i,1} &= 2 - Q_i \\ P_{i,2} &= -16(1-A)^2 + 4(1-A)(3-A)Q_i \\ &\quad + (A-2)Q_i^2 \\ P_{i,3} &= 2(1-A)^2 - (1-A)Q_i \\ P_{i,4} &= -4(1-A)^2 + 2(1-A)(3-A)Q_i \\ &\quad + (A-2)Q_i^2 \\ P_{i,5} &= 4(1-A)^2 - 4(1-A)Q_i + S_i \\ Q_i &= \Gamma BC(K_i + \varphi_i) \end{aligned}$$

$$S_i = (\Gamma BC)^2 \left(2K_i\varphi_i + {\varphi_i}^2\right)$$
$$K_i = \sum_{j\in\mathcal{N}_i} a_{i,j} \tag{17}$$

Proof. As we know, ILC is employed for dealing with the repeated tracking control. For this kind of machines, let them learn as people by using previous errors correct this control input. Thus, by repeating the learning process, the tracking performance can be achieved with high accuracy. Since ILC is a two-dimensional system with evolution along two independent axes, time domain and iteration domain, the scholars Fang et al. proposed an ILC 2D model [21, 22]. This paper considers a two-dimensional system of frequency domain and iteration domain after taking z-transformation. Considering $E_k(z)$ as two-dimensional function $E(k, z)$, we discuss the convergence of arbitrary frequency in iteration dimension by exchange variable frequency and parameter iteration. Thus, we can exchange variable z and parameter k. And $E_k(z)$ can be rewritten as $E_z(k)$; that is, $E_z(k) = [E_{1,z}(k), E_{2,z}(k), \ldots, E_{n,z}(k)]^T$. Then, (14) can be rewritten as

$$\begin{aligned} E_z(k+1) &= \{I - z\Gamma G(z)[\mathscr{L}(z) + \Phi]\} E_z(k) \\ &\quad + z\Gamma G(z) R(z) \mathscr{L}(z) \mathbf{1} \end{aligned} \tag{18}$$

Taking the z-transformation of (18), we get

$$\begin{aligned} \mathbb{z}\mathbb{E}_z(\mathbb{z}) &= \{I - z\Gamma G(z)[\mathscr{L}(z) + \Phi]\} \mathbb{E}_z(\mathbb{z}) \\ &\quad + \frac{\mathbb{z}}{\mathbb{z}-1} z\Gamma G(z) R(z) \mathscr{L}(z) \mathbf{1} \end{aligned} \tag{19}$$

Further, we get $\{(\mathbb{z} - 1)I + z\Gamma G(z)[\mathscr{L}(z) + \Omega]\}\mathbb{E}_z(\mathbb{z}) - \mathbb{z}(\mathbb{z}-1)^{-1} z\Gamma G(z)R(z)\mathscr{L}(z)\mathbf{1} = 0$. The characteristic equation is

$$\det\{(\mathbb{z}-1)I + z\Gamma G(z)[\mathscr{L}(z) + \Phi]\} = 0 \tag{20}$$

When $\mathbb{z} = 1$, $\det\{(\mathbb{z} - 1)I + z\Gamma G(z)[\mathscr{L}(z) + \Phi]\} = \det\{\Gamma G(z)[\mathscr{L}(z)+\Phi]\}$. From *Remark 6*, we can get that $\mathscr{L}(z)+\Omega$ is nonsingular; that is, $\det[\mathscr{L}(z)+\Phi] \neq 0$. Since $\Gamma G(z) \neq 0$, $\mathbb{z} = 1$ is not the root of characteristic equation.

When $\mathbb{z} \neq 1$, both sides of (20) are divided by $\mathbb{z} - 1$:

$$\det\left\{I + \frac{z\Gamma G(z)[\mathscr{L}(z) + \Phi]}{\mathbb{z}-1}\right\} = 0 \tag{21}$$

Next, we need to prove that all roots of (21) have modulus less than unity. Let $F(\mathbb{z}) = z\Gamma G(z)[\mathscr{L}(z)+\Phi](\mathbb{z}-1)^{-1}$. Based on the general Nyquist stability criterion [23], the roots of (21) have modulus less than unity, if the eigenloci $\lambda[F(\omega,\beta)]$ of

$$F(\omega,\beta) = \frac{e^{j\beta}\Gamma G\left(e^{j\beta}\right)\left[\mathscr{L}\left(e^{j\beta}\right) + \Phi\right]}{e^{j\omega}-1} \tag{22}$$

do not enclose the point $(-1, j0)$ for $\beta \in [-\pi,\pi)$, $\omega \in [-\pi,\pi)$, and $\omega \neq 0$. By Gershgorin's disk theorem, we have $\lambda[F(\omega,\beta)] \in \bigcup_{i\in n} F_i$ for all $\beta \in [-\pi,\pi)$, $\omega \in [-\pi,\pi)$, and $\omega \neq 0$, where

$$\begin{aligned} F_i = \Bigg\{\varsigma : \varsigma \in C, &\left|\varsigma - \frac{e^{j\beta}\Gamma BC(K_i + \varphi_i)}{(e^{j\omega}-1)(e^{j\beta}-A)}\right| \\ &\leq \sum_{j\in\mathcal{N}_i}\left|\frac{K_i\Gamma BC}{(e^{j\omega}-1)(e^{j\beta}-A)}\right|\Bigg\} \end{aligned} \tag{23}$$

So the eigenloci $\lambda[F(\omega,\beta)]$ do not enclose the point $(-1, j0)$ for $\beta \in [-\pi,\pi)$, $\omega \in [-\pi,\pi)$, and $\omega \neq 0$ when the point $(-a, j0)$ with $a \geq 1$ is not in the disc F_i for all $i = 1, 2, \ldots, n$, $\beta \in [-\pi,\pi]$, $\omega \in [-\pi,\pi]$, and $\omega \neq 0$. That is, $|a + e^{j\beta}\Gamma BC(K_i + \varphi_i)/[(e^{j\omega}-1)(e^{j\beta}-A)]| > |K_i\Gamma BC/[(e^{j\omega}-1)(e^{j\beta}-A)]|$ for all $i = 1, 2, \ldots, n$, $\beta \in [-\pi,\pi)$, $\omega \in [-\pi,\pi)$ and $\omega \neq 0$, when $a \geq 1$. We denote $f_i(a)$ as

$$\begin{aligned} f_i(a) &= \left|a\left(e^{j\omega}-1\right)\left(e^{j\beta}-A\right) + e^{j\beta}\Gamma BC(K_i+\varphi_i)\right|^2 \\ &\quad - \left|K_i\Gamma BC\right|^2 \end{aligned} \tag{24}$$

Note that $|a + e^{j\beta}\Gamma BC(K_i + \varphi_i)/(e^{j\omega}-1)(e^{j\beta}-A)|^2 - |K_i\Gamma BC/(e^{j\omega}-1)(e^{j\beta}-A)|^2 = |1/(e^{j\omega}-1)(e^{j\beta}-A)|^2 f_i(a)$ for all $a \geq 1$, $\beta \in [-\pi,\pi)$, $\omega \in [-\pi,\pi)$ and $\omega \neq 0$. So, $|a+e^{j\beta}\Gamma BC(K_i+\varphi_i)/(e^{j\omega}-1)(e^{j\beta}-A)|^2 - |K_i\Gamma BC/(e^{j\omega}-1)(e^{j\beta}-A)|^2 > 0$ as long as $f_i(a) > 0$ for all $a \geq 1$, $\beta \in [-\pi,\pi)$, $\omega \in [-\pi,\pi)$, and $\omega \neq 0$. Further, we can show that

$$\begin{aligned} f_i(a) &= 2(1-\cos\omega)\left(1 + A^2 - 2A\cos\beta\right)a^2 \\ &\quad + 2a\Gamma BC(K_i + \varphi_i) \\ &\quad \cdot \left[\cos\omega - 1 - A\cos(\omega-\beta) + A\cos(\beta)\right] \\ &\quad + (\Gamma BC)^2\left(2K_i\varphi_i + \varphi_i^2\right) \end{aligned} \tag{25}$$

Using the conditions of *Theorem 7*, we can prove $f_i(a) > 0$ for all $a \geq 1$, $\beta \in [-\pi,\pi)$, $\omega \in [-\pi,\pi)$, and $\omega \neq 0$ in Appendix. Then eigenloci of $\lambda[F(\omega,\beta)]$ for all $\beta \in [-\pi,\pi)$, $\omega \in [-\pi,\pi)$, and $\omega \neq 0$ do not enclose the point $(-1, j0)$. Thus the roots of (21) have modulus less than unity. That is, the system achieves a formation tracking asymptotically.

In addition, let the final value of $E_z(k)$ be $E_z(\infty)$. Multiply $\mathbb{z}-1$ on both sides of $\{(\mathbb{z}-1)I + z\Gamma G(z)[\mathscr{L}(z)+\Phi]\}\mathbb{E}_z(\mathbb{z}) = (\mathbb{z}/(\mathbb{z}-1))z\Gamma G(z)R(z)\mathscr{L}(z)\mathbf{1}$ and take the limit:

$$\begin{aligned} &\lim_{\mathbb{z}\to 1}\{(\mathbb{z}-1)I + z\Gamma G(z)[\mathscr{L}(z)+\Phi]\}(\mathbb{z}-1)\mathbb{E}_z(\mathbb{z}) \\ &\quad = \lim_{\mathbb{z}\to 1}\mathbb{z}z\Gamma G(z)R(z)\mathscr{L}(z)\mathbf{1} \end{aligned} \tag{26}$$

By final value theorem, we get

$$z\Gamma G(z)[\mathscr{L}(z)+\Phi]E_z(\infty) = z\Gamma G(z)R(z)\mathscr{L}(z)\mathbf{1} \tag{27}$$

Since $rank[\mathscr{L}(z) + \Phi] = n$, $\mathscr{L}(z) + \Phi$ is invertible. And $z\Gamma G(z) \neq 0$ is satisfied; thus $E_z(\infty) = R(z)[\mathscr{L}(z) + \Phi]^{-1}\mathscr{L}(z)\mathbf{1}$. This completes the proof. □

When there is not communication time-delay in multi-agent systems, we can get system error converging to zero combining equation (27) and *Lemma 4*. Otherwise, system error cannot converge to zero and converges to a certain value about desired reference trajectory, graph, and time-delays as shown in (27).

3.2. Second-Order ILC for Multiagent Systems with Time-Delays. In this subsection, we analyze convergence condition of second-order ILC for multiagent systems. Using previous two errors to correct the control input can compensate the influence of communication time-delays. Therefore, we propose the second-order ILC law:

$$\begin{aligned} u_{i,k+1}(t) &= u_{i,k}(t) \\ &\quad + \Gamma\left\{\sum_{j\in\mathcal{N}_i} a_{i,j}\left[\delta_{i,k}(t+1)-\delta_{j,k}\left(t+1-\tau_{i,j}\right)\right]\right. \\ &\quad \left.+\phi_i e_{i,k}(t+1)\right\} \\ &\quad + \Lambda\left\{\sum_{j\in\mathcal{N}_i} a_{i,j}\left[\delta_{i,k-1}(t+1)-\delta_{j,k-1}\left(t+1-\tau_{i,j}\right)\right]\right. \\ &\quad \left.+\phi_i e_{i,k-1}(t+1)\right\} \end{aligned} \tag{28}$$

where Γ is first-order learning gain, Λ is second-order learning gain, $\Gamma, \Lambda \in \mathbb{R}$, and $\Gamma \neq 0, \Lambda \neq 0$.

Theorem 8. *Consider multiagent system (1); let Assumptions 1 and 3 hold and the learning law (28) be applied. Then the formation tracking objective* $\lim_{k\longrightarrow\infty} Y_{i,k}(z) = Y_i^d(z)$ *is achieved, given that* $\Lambda = -\Gamma$, $Q_i > 0$, *and any one of the following conditions are satisfied for all* $i = 1, 2, \ldots, n$:

$$\begin{aligned} &(1)\ \ (1 > A > 0) \wedge (P_{i,6} > 0) \wedge (P_{i,7} > 0) \\ &(2)\ \ (-1 < A < 0) \wedge (P_{i,7} > 0) \wedge (P_{i,8} < 0) \\ &(3)\ \ (1 > A > 0) \wedge (P_{i,9} > 0) \wedge (P_{i,10} > 0) \end{aligned} \tag{29}$$

where

$$\begin{aligned} P_{i,6} &= Q_i - 2(1+A) \\ P_{i,7} &= (1+A)^2 + 2(1+A)Q_i + S \\ P_{i,8} &= Q_i - 1 - A \\ P_{i,9} &= 1 - A - Q_i \\ P_{i,10} &= (1-A)^2 - 2(1-A)Q_i + S_i \\ Q_i &= \Gamma BC(K_i + \varphi_i) \\ S_i &= (\Gamma BC)^2\left(2K_i\varphi_i + {\varphi_i}^2\right) \\ K_i &= \sum_{j\in\mathcal{N}_i} a_{i,j} \end{aligned} \tag{30}$$

Proof. Taking the z-transformation of (28), we get

$$\begin{aligned} U_{i,k+1}(z) &= U_{i,k}(z) \\ &\quad + z\Gamma\left\{\sum_{j\in\mathcal{N}_i} a_{i,j}\left[\Delta_{i,k}(z) - z^{-\tau_{i,j}}\Delta_{j,k}(z)\right]\right. \\ &\quad \left.+\phi_i E_{i,k}(z)\right\} \\ &\quad + z\Lambda\left\{\sum_{j\in\mathcal{N}_i} a_{i,j}\left[\Delta_{i,k-1}(z) - z^{-\tau_{i,j}}\Delta_{j,k-1}(z)\right]\right. \\ &\quad \left.+\phi_i E_{i,k-1}(z)\right\} \end{aligned} \tag{31}$$

By (7), (31), and *Assumption 1*, we have that

$$\begin{aligned} E_{i,k+1}(z) - E_{i,k}(z) &= -z\Gamma G(z) \\ &\quad \cdot \left\{\sum_{j\in\mathcal{N}_i} a_{i,j}\left[\Delta_{i,k}(z) - z^{-\tau_{i,j}}\Delta_{j,k}(z)\right] + \phi_i E_{i,k}(z)\right\} \\ &\quad - z\Lambda G(z)\left\{\sum_{j\in\mathcal{N}_i} a_{i,j}\left[\Delta_{i,k-1}(z) - z^{-\tau_{i,j}}\Delta_{j,k-1}(z)\right]\right. \\ &\quad \left.+\phi_i E_{i,k-1}(z)\right\} \end{aligned} \tag{32}$$

Let $E_k(z) = [E_{1,k}(z), E_{2,k}(z), \ldots, E_{n,k}(z)]^T$; (32) can be rewritten as

$$\begin{aligned} E_{k+1}(z) - E_k(z) &= -z\Gamma G(z)\left[\mathcal{L}(z) + \Phi\right]\left[E_k(z) - R(z)\mathbf{1}\right] \\ &\quad - z\Gamma G(z)\Phi R(z)\mathbf{1} \\ &\quad - z\Lambda G(z)\left[\mathcal{L}(z) + \Phi\right]\left[E_{k-1}(z) - R(z)\mathbf{1}\right] \\ &\quad - z\Lambda G(z)\Phi R(z)\mathbf{1} \\ &= -z\Gamma G(z)\left[\mathcal{L}(z) + \Phi\right]E_k(z) \\ &\quad - z\Lambda G(z)\left[\mathcal{L}(z) + \Phi\right]E_{k-1}(z) \\ &\quad + zG(z)R(z)(\Lambda + \Gamma)\mathcal{L}(z)\mathbf{1} \end{aligned} \tag{33}$$

Exchanging variable frequency z and parameter iteration k, $E_k(z)$ can be rewritten as $E_z(k)$ in (33):

$$\begin{aligned} E_z(k+1) - E_z(k) &= -z\Gamma G(z)\left[\mathcal{L}(z) + \Phi\right]E_z(k) \\ &\quad - z\Lambda G(z)\left[\mathcal{L}(z) + \Phi\right]E_z(k-1) \\ &\quad + zG(z)R(z)(\Lambda + \Gamma)\mathcal{L}(z)\mathbf{1} \end{aligned} \tag{34}$$

Taking z-transformation of (34), we get

$$\begin{aligned} \mathbb{z}\mathbb{E}_z(\mathbb{z}) - \mathbb{E}_z(\mathbb{z}) &= -z\Gamma G(z)\left[\mathscr{L}(z) + \Phi\right]\mathbb{E}_z(\mathbb{z}) \\ &\quad - \mathbb{z}^{-1} z\Lambda G(z)\left[\mathscr{L}(z) + \Phi\right]\mathbb{E}_z(\mathbb{z}) \\ &\quad + \frac{\mathbb{z}}{\mathbb{z}-1} zG(z)R(z)(\Lambda + \Gamma)\mathscr{L}(z)\mathbf{1} \end{aligned} \tag{35}$$

From *Theorem 8*, $\Lambda = -\Gamma$. Further, we have that

$$(\mathbb{z}-1)\left\{I + z\mathbb{z}^{-1}\Gamma G(z)\left[\mathscr{L}(z) + \Phi\right]\right\}\mathbb{E}_z(\mathbb{z}) = 0 \tag{36}$$

The characteristic equation of system is

$$\det(\mathbb{z}-1)\left\{I + z\mathbb{z}^{-1}\Gamma G(z)\left[\mathscr{L}(z) + \Phi\right]\right\} = 0 \tag{37}$$

Let $p(\mathbb{z}) = (\mathbb{z}-1)\{I + z\mathbb{z}^{-1}\Gamma G(z)[\mathscr{L}(z) + \Phi]\}$. Since $p(1) = 0$ and $\det\{I + z\Gamma G(z)[\mathscr{L}(z) + \Phi]\} \neq 0$, $\mathbb{z} = 1$ is the only root of (37).

When $\mathbb{z} \neq 1$, both sides of (37) are divided by $\mathbb{z} - 1$:

$$\det\left\{I + \frac{z\Gamma G(z)\left[\mathscr{L}(z) + \Phi\right]}{\mathbb{z}}\right\} = 0 \tag{38}$$

Next, we need to prove that all roots of (38) have modulus less than unity. Similar to the proof of *Theorem 7*, the conditions of *Theorem 8* are easily verified.

Taking the limit of (36) yields

$$\lim_{\mathbb{z}\longrightarrow 1}(\mathbb{z}-1)\left\{I + z\mathbb{z}^{-1}\Gamma G(z)\left[\mathscr{L}(z) + \Phi\right]\right\}\mathbb{E}_z(\mathbb{z}) = 0 \tag{39}$$

By final value theorem, we get

$$\{I + z\Gamma G(z)\left[\mathscr{L}(z) + \Phi\right]\}E_z(\infty) = 0 \tag{40}$$

Since $\det\{I+z\Gamma G(z)[\mathscr{L}(z)+\Phi]\} \neq 0$ as proved above, $rank\{I+z\Gamma G(z)[\mathscr{L}(z)+\Phi]\} = n$. Based on Sylvester inequality, we get $rank\{I + z\Gamma G(z)[\mathscr{L}(z) + \Phi]\} + rank(E_z(\infty)) \leq n$. Therefore, $rank[E_z(\infty)] = 0$; that is $E_z(\infty) = 0$. *Theorem 8* is thus proved. □

The result of *Theorem 7* shows that the error of multiagent system cannot converge to zero considering diverse communication time-delays. In *Theorem 8*, using second-order ILC and the fact that the second-order learning gain is opposite of the first-order learning gain, the error of multiagent system can converge to zero considering diverse communication time-delays. In addition, the convergence speed can be kept as convergence speed of multiagent systems without time-delays.

4. Simulation

In this section, numerical simulations are presented to illustrate the first-order and second-order ILC for multiagent systems. Consider a multiagent system consisting of 4 agents in the directed graph as shown in Figure 3. It can be seen that the graph is connected graph. In addition, only the 1st agent can obtain the desired reference trajectory.

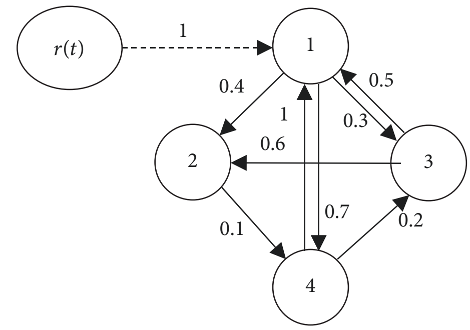

FIGURE 3: Directed graph of multiagent system.

The Laplacian matrix $\mathscr{L}$ of graph $\mathscr{G}$ and the reference-accessibility matrix Φ are

$$\begin{aligned} \mathscr{L} &= \begin{pmatrix} 1.5 & 0 & -0.5 & -1 \\ -0.4 & 1 & -0.6 & 0 \\ -0.3 & 0 & 0.5 & -0.2 \\ -0.7 & -0.1 & 0 & 0.8 \end{pmatrix}, \\ \Phi &= \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix} \end{aligned} \tag{41}$$

From $\mathscr{L}$ and Φ, we get $\max_{i=1,2,3,4}(2K_i + \phi_i) = 4$. Let desired reference trajectory be $r(t) = 150 + 80 \times \sin t$. The desired output deviation of each agent is chosen as

$$\begin{aligned} d_1(t) &= 200 - t \\ d_2(t) &= 80 \\ d_3(t) &= -80 \\ d_4(t) &= -200 + t \end{aligned} \tag{42}$$

Set the initial state of 4 agents as 875, 575, 175, and -125. It satisfies *Assumption 1* that the initial resetting conditions for all agents and all the iterations meet desired input. When dynamics of system is Case 1, $A = 0.5, B = -0.8, C = -0.7$, we set $\Gamma = 0.6$. It can be seen that above learning gains satisfy condition 1 of *Theorem 7*. When dynamics of system is Case 2, $A = -0.5, B = 0.8, C = 0.7$, we set $\Gamma = 0.6$. It can be seen that above learning gains satisfy condition 2 of *Theorem 7*. In Case 1 or Case 2, when $t = 1, 2, \ldots, 100$ and iteration is 200, the trajectory of the multiagent system without communication time-delays is shown as Figure 4. From this figure, it can be seen that the desired formation is well achieved to track desired reference trajectory.

In order to measure the formation accuracy quantitatively, the disagreement among all agents on their output errors is defined as $error(k) = (1/nT)\sum_{i=1}^{n}\sum_{t=1}^{T}\|e_{i,k}(t)\|$. When $\lim_{k\longrightarrow\infty} error(k) = 0$, multiagent system achieves formation tacking. Based on this, Figure 5 shows formation

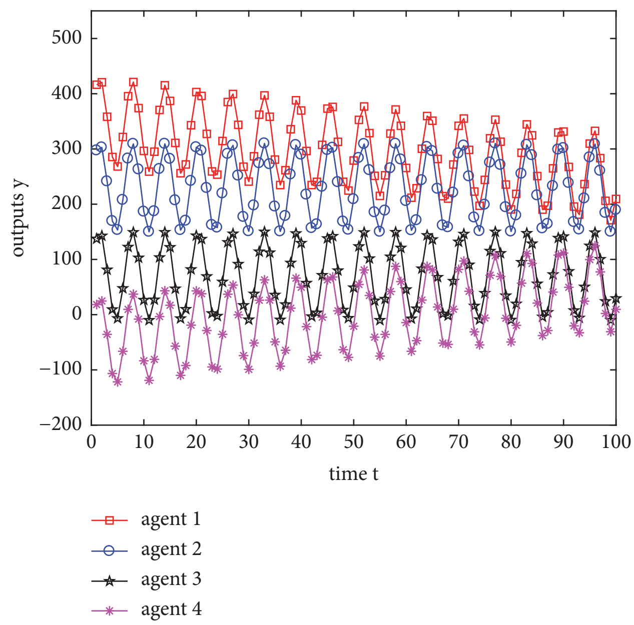

Figure 4: Trajectory of multiagent system without communication time-delays at iteration 200.

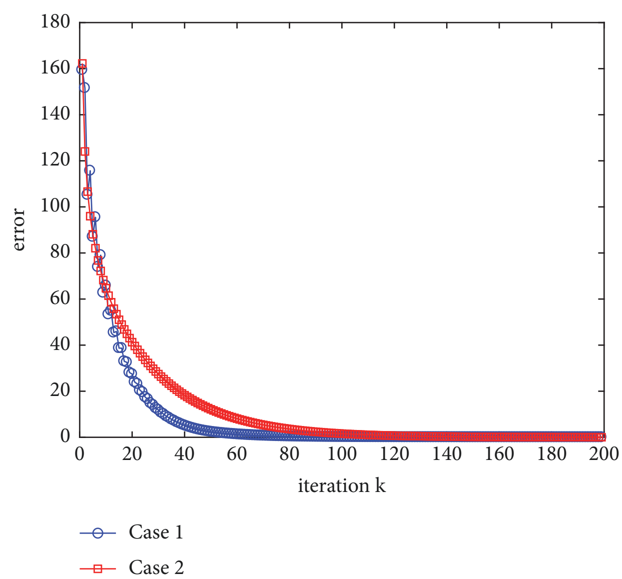

Figure 5: Convergence of formation tracking errors for multiagent system without communication time-delays.

performance of the system at the first 200 iterations in Cases 1 and 2. From Figure 5, when iteration is around 120, the multiagent systems error in Case 1 can converge to zero and when iteration is around 150, the multiagent systems error in Case 2 can converge to zero.

When there are communication time-delays in the multiagent system, we set 3 time-delays graphs of the multiagent system as shown in Figure 6.

From Figure 7, the system errors cannot converge to zero but a certain value as the increase of the iteration when there are time-delays. In addition, the bigger the communication time-delays are, the bigger the bounds of tracking errors are. Multiagent system cannot achieve formation tracking and the trajectory of multiagent system is as shown in Figure 8 when time-delays graph 1 holds.

By *Theorem 8*, we use second-order ILC and set $\Lambda = -\Gamma$ when the time-delays graphs of multiagent systems are as shown in Figure 6. It can be seen that Case 1, $A = 0.5, B = -0.8, C = -0.7, \Lambda = -0.6$, satisfies conditions 1 and 3 of *Theorem 8* and Case 2, $A = -0.5, B = 0.8, C = 0.7, \Lambda = -0.6$, satisfies condition 2 of *Theorem 8*. The system errors are as shown in Figure 9. From Figure 9, when iteration is around 120 in Case 1, all the multiagent systems errors with communication time-delays and without communication time-delays can converge to zero. When iteration is around 150 in Case 2, all the multiagent systems errors with communication time-delays and without communication time-delays can converge to zero. Therefore, second-order ILC formula can keep the same learning error convergence speed, no matter what time-delays are. In Case 1 or Case 2, multiagent systems can achieve formation tracking and the trajectory of multiagent systems is shown as Figure 10.

5. Conclusion

When using first-order ILC to implement the formation tracking for multiagent system with communication time-delays, the system error cannot converge to zero but a certain value. Using second-order ILC and making second-order learning gain be opposite of first-order learning gain can compensate influence of communication time-delays. Based on the frequency domain ILC model for multiagent systems, we analyze the convergence condition by considering the convergence of arbitrary frequency in iteration dimension. We use the generalized Nyquist criteria and Gershgorin disc theorem to analyze the range of eigenvalues of the system characteristic equation and obtain the conditions for system convergence. The second-order ILC proposed in this paper for compensating for the influence of time-delays not only can make the system error converge to zero, but also can ensure that the convergence speed is the same as the convergence speed for system without time-delays. The simulation results also verify the effectiveness of the conclusion.

Appendix

$f_i(a)$ is a quadratic function. Since $z \neq 1$ and with *Assumption 1*, $1-\cos\omega > 0$ and $1 + A^2 - 2A\cos\beta > 0$. That is, $2(1-\cos\omega)(1 + A^2 - 2A\cos\beta) > 0$ and $f_i(a)$ has minimum value for $a \in \mathbb{R}$. The minimum value of $f_i(a)$ for $a \in \mathbb{R}$ is

$$\begin{aligned}\min_{a\in\mathbb{R}} f_i(a) = {} & \frac{(\Gamma BC)^2}{2(1-\cos\omega)(1+A^2-2A\cos\beta)}\Big[2(1 \\ & -\cos\omega)\left(1+A^2-2A\cos\beta\right)\left(2K_i\varphi_i+\varphi_i^2\right) \\ & -\left[\cos\omega-1-A\cos(\omega-\beta)+A\cos(\beta)\right]^2(K_i \\ & +\varphi_i)^2\Big]\end{aligned} \qquad (A.1)$$

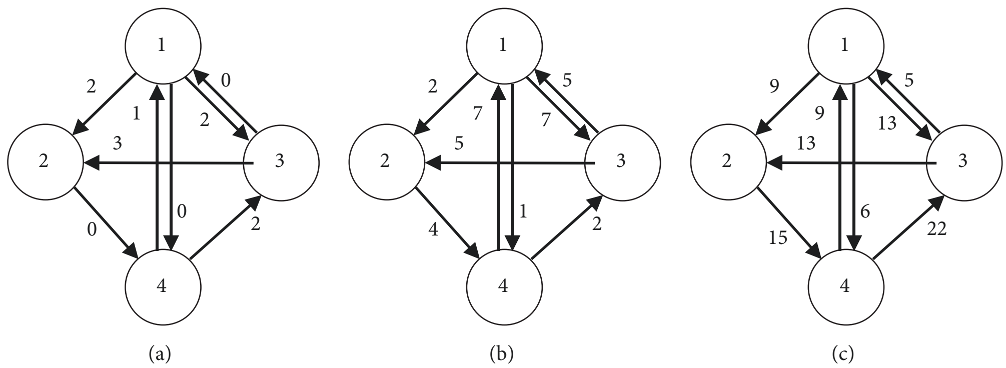

FIGURE 6: The time-delays graphs of multiagent systems. (a) Time-delays graph 1. (b) Time-delays graph 2. (c) Time-delays graph 3.

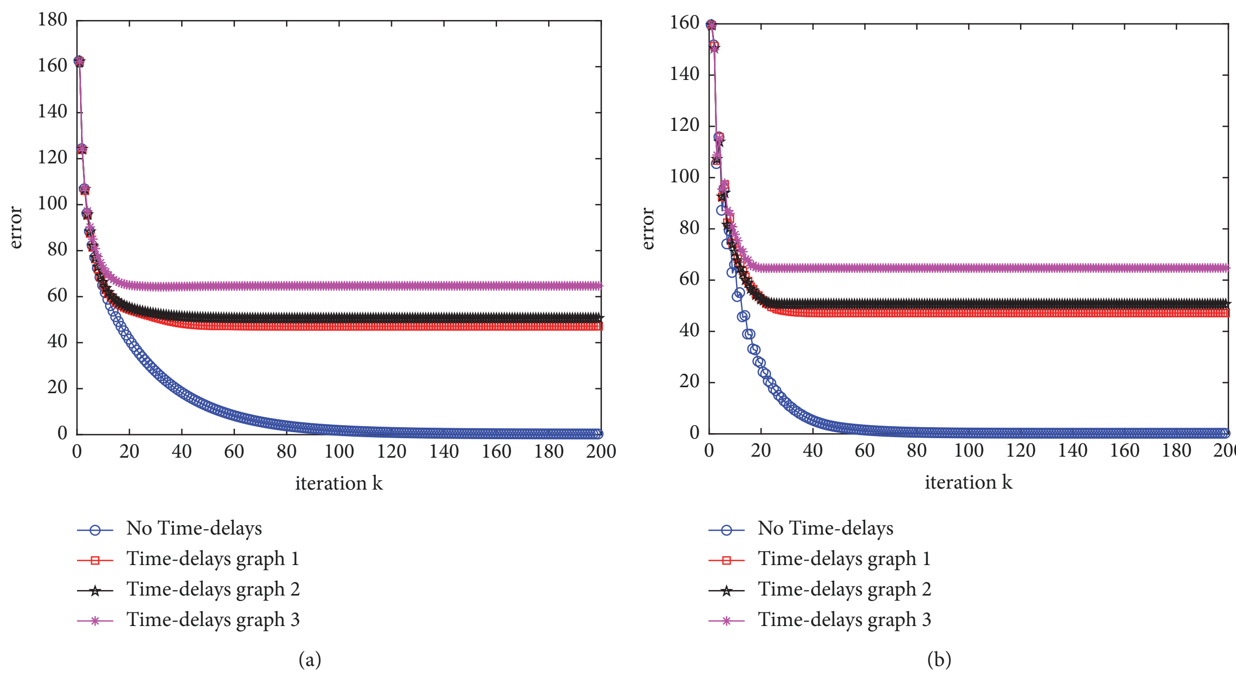

FIGURE 7: Convergence of formation tracking errors for multiagent system with communication time-delays. (a) Case 1. (b) Case 2.

Taking $\omega = -\pi$ and $\beta = 0$ for $\min_{a\in\mathbb{R}} f_i(a)$, we get $\min_{a\in\mathbb{R}} f_i(a) = -(K_i \Gamma BC)^2/2(1-\cos\omega)(1+A^2-2A\cos\beta)$. This example can prove that $\min_{a\in\mathbb{R}} f_i(a) > 0$ is not valid for all $\beta \in [-\pi,\pi)$, $\omega \in [-\pi,\pi)$ and $\omega \neq 0$. Thus, $f_i(a) > 0$ for all $a \geq 1$ as long as

$$\begin{gathered} \arg\min_a f_i(a) < 1 \\ f_i(1) > 0 \end{gathered} \tag{A.2}$$

where

$$\begin{aligned} &\arg\min_a f_i(a) \\ &= \frac{-\Gamma BC(K_i+\varphi_i)\left[\cos\omega - 1 - A\cos(\omega-\beta) + A\cos(\beta)\right]}{2(1-\cos\omega)\left(1+A^2-2A\cos\beta\right)} \end{aligned} \tag{A.3}$$

We denote

$$\begin{aligned} g(\omega,\beta) &= 2(1-\cos\omega)\left(1+A^2-2A\cos\beta\right) \\ &\quad + \Gamma BC(K_i+\varphi_i) \\ &\quad \cdot\left[\cos\omega - 1 - A\cos(\omega-\beta) + A\cos(\beta)\right] \end{aligned} \tag{A.4}$$

$g(\omega,\beta)$ is function of two variables. We need to obtain the extremum of $g(\omega,\beta)$. Taking the partial derivative with respect to ω, we get

$$\begin{aligned} &g'_\omega(\omega,\beta) \\ &\quad = 2\sin\omega\left(1+A^2-2A\cos\beta\right) \\ &\qquad + \Gamma BC(K_i+\varphi_i)\left[-\sin\omega + A\sin(\omega-\beta)\right] \end{aligned} \tag{A.5}$$

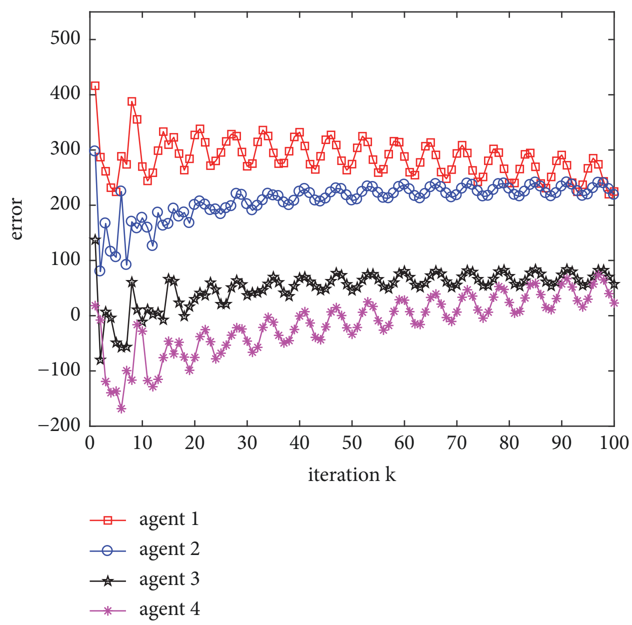

Figure 8: Trajectory of multiagent system with communication time-delays graph 1 at iteration 200.

Taking the partial derivative with respect to β, we get

$$\begin{aligned} g'_{\beta}(\omega,\beta) &= 4A\sin\beta(1-\cos\omega) \\ &\quad + \Gamma ABC(K_i+\varphi_i)[-\sin(\omega-\beta)-\sin(\beta)] \end{aligned} \quad (A.6)$$

Then we take second derivative of $g(\omega,\beta)$.

$$\begin{aligned} g''_{\beta\beta}(\omega,\beta) &= 4A\cos\beta(1-\cos\omega) \\ &\quad + \Gamma ABC(K_i+\varphi_i)[\cos(\omega-\beta)-\cos(\beta)] \\ g''_{\beta\omega}(\omega,\beta) &= 4A\sin\omega\sin\beta \\ &\quad + \Gamma ABC(K_i+\varphi_i)[-\cos(\omega-\beta)] \\ g''_{\omega\omega}(\omega,\beta) &= 2\cos\omega\left(1+A^2-2A\cos\beta\right) \\ &\quad + \Gamma BC(K_i+\varphi_i)[-\cos\omega+A\cos(\omega-\beta)] \end{aligned} \quad (A.7)$$

Next we denote $h(\omega,\beta) = f_i(1)$:

$$\begin{aligned} h(\omega,\beta) &= 2(1-\cos\omega)\left(1+A^2-2A\cos\beta\right) \\ &\quad + 2\Gamma BC(K_i+\varphi_i) \\ &\quad \cdot [\cos\omega - 1 - A\cos(\omega-\beta) + A\cos(\beta)] \\ &\quad + (\Gamma BC)^2\left(2K_i\varphi_i+\varphi_i^2\right) \end{aligned} \quad (A.8)$$

$h(\omega,\beta)$ is function of two variables. Similarly, we do the same operation of $h(\omega,\beta)$ as $g(\omega,\beta)$. Taking the partial derivative with respect to ω, we get

$$\begin{aligned} h'_{\omega}(\omega,\beta) &= 2\sin\omega\left(1+A^2-2A\cos\beta\right) \\ &\quad + 2\Gamma BC(K_i+\varphi_i)[-\sin\omega+A\sin(\omega-\beta)] \end{aligned} \quad (A.9)$$

Taking the partial derivative with respect to β, we get

$$\begin{aligned} h'_{\beta}(\omega,\beta) &= 4A\sin\beta(1-\cos\omega) \\ &\quad + 2\Gamma ABC(K_i+\varphi_i)[-\sin(\omega-\beta)-\sin(\beta)] \end{aligned} \quad (A.10)$$

Then we take second derivative of $h(\omega,\beta)$.

$$\begin{aligned} h''_{\beta\beta}(\omega,\beta) &= 4A\cos\beta(1-\cos\omega) \\ &\quad + \Gamma ABC(K_i+\varphi_i)[\cos(\omega-\beta)-\cos(\beta)] \\ h''_{\beta\omega}(\omega,\beta) &= 4A\sin\omega\sin\beta \\ &\quad + 2\Gamma ABC(K_i+\varphi_i)[-\cos(\omega-\beta)] \\ h''_{\omega\omega}(\omega,\beta) &= 2\cos\omega\left(1+A^2-2A\cos\beta\right) \\ &\quad + 2\Gamma BC(K_i+\varphi_i)[-\cos\omega+A\cos(\omega-\beta)] \end{aligned} \quad (A.11)$$

From

$$\begin{aligned} g'_{\omega}(\omega,\beta) &= 0 \\ g'_{\beta}(\omega,\beta) &= 0 \end{aligned} \quad (A.12)$$

and

$$\begin{aligned} h'_{\omega}(\omega,\beta) &= 0 \\ h'_{\beta}(\omega,\beta) &= 0, \end{aligned} \quad (A.13)$$

we get $(-\pi,-\pi)$ and $(-\pi,0)$ which are the stationary points of both $h(\omega,\beta)$ and $g(\omega,\beta)$ for all $\beta \in [-\pi,\pi)$, $\omega \in [-\pi,\pi)$, and $\omega \neq 0$.

When $(\omega,\beta) = (-\pi,-\pi)$, second derivative of $g(\omega,\beta)$ is

$$\begin{aligned} g''_{\beta\beta}(\omega,\beta) &= -8A + 2\Gamma ABC(K_i+\varphi_i) \\ g(-\pi,-\pi) &= 2(1+A)^2 - (1+A)Q_i \end{aligned} \quad (A.14)$$

There is no such A and $\Gamma BC(K_i+\varphi_i)$ satisfied $g(-\pi,-\pi) > 0$ and $g''_{\beta\beta}(\omega,\beta) > 0$. Then (A.2) is not satisfied.

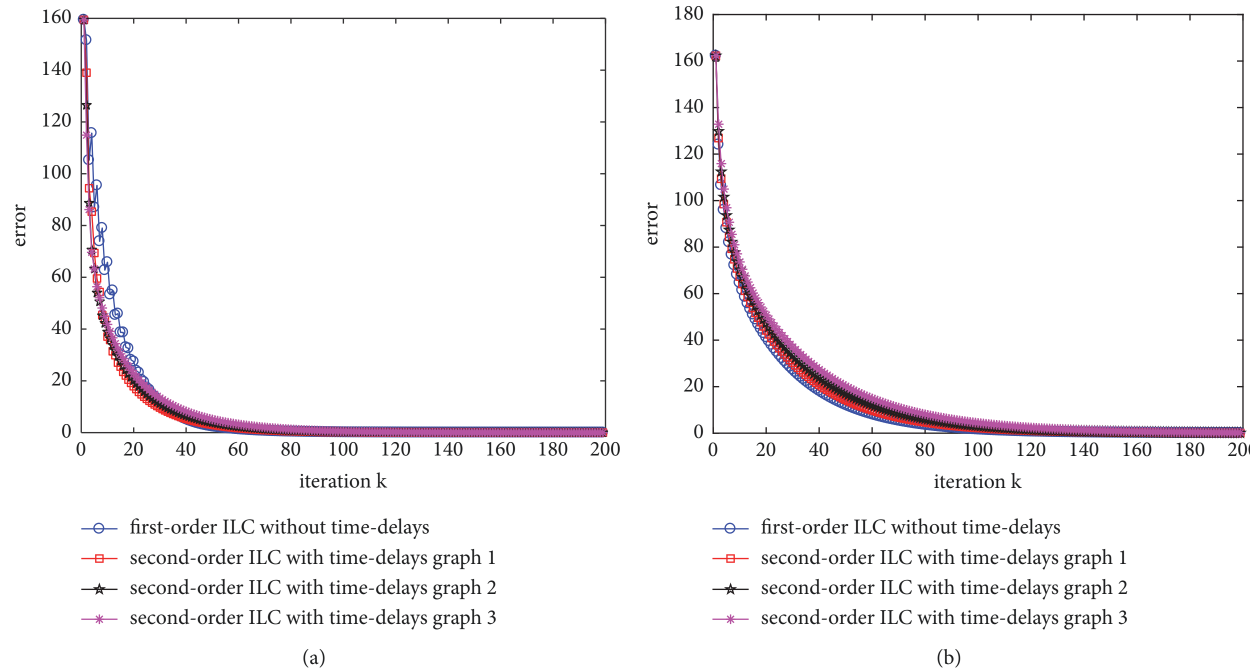

FIGURE 9: Convergence of formation tracking errors via second-order ILC for multiagent system with communication time-delays and via first-order ILC for multiagent system without communication time-delays. (a) Case 1. (b) Case 2.

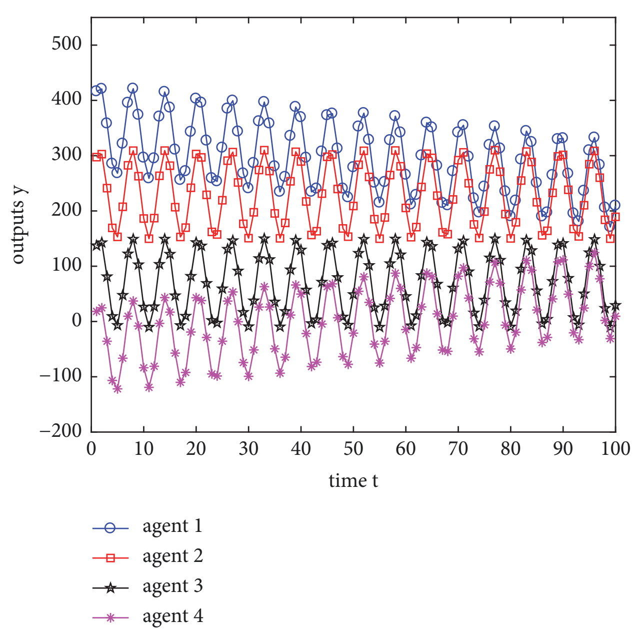

FIGURE 10: Trajectory of multiagent systems with communication time-delays via second-order ILC at iteration 200.

When $(\omega, \beta) = (-\pi, 0)$, the second derivative of $g(\omega, \beta)$ is

$$\begin{aligned} g''_{\beta\beta}(-\pi, 0) &= 8A - 2\Gamma ABC (K_i + \varphi_i) \\ g''_{\beta\omega}(-\pi, 0) &= \Gamma ABC (K_i + \varphi_i) \\ g''_{\omega\omega}(-\pi, 0) &= -2(1 - A)^2 + \Gamma BC (1 - A)(K_i + \varphi_i) \end{aligned} \tag{A.15}$$

and the second derivative of $h(\omega, \beta)$ is

$$\begin{aligned} h''_{\beta\beta}(-\pi, 0) &= 8A - 4\Gamma BC (K_i + \varphi_i) \\ h''_{\beta\omega}(-\pi, 0) &= 2\Gamma ABC (K_i + \varphi_i) \\ h''_{\omega\omega}(-\pi, 0) &= -2(1 - A)^2 \\ &\quad + 2\Gamma BC (1 - A)(K_i + \varphi_i) \end{aligned} \tag{A.16}$$

If condition 1 and condition 2 of *Theorem 7* are satisfied, then

$$\begin{aligned} g''_{\beta\beta}(-\pi, 0)\, g''_{\omega\omega}(-\pi, 0) - \left[g''_{\beta\omega}(-\pi, 0)\right]^2 &> 0 \\ g''_{\beta\beta}(-\pi, 0) &> 0 \\ g(-\pi, 0) &> 0 \\ h''_{\beta\beta}(-\pi, 0)\, h''_{\omega\omega}(-\pi, 0) - \left[h''_{\omega\beta}(-\pi, 0)\right]^2 &> 0 \\ h''_{\beta\beta}(-\pi, 0) &> 0 \\ h(-\pi, 0) &> 0 \end{aligned} \tag{A.17}$$

is satisfied and $g''_{\beta\beta}(-\pi, -\pi) < 0, h''_{\beta\beta}(-\pi, -\pi) < 0$ are satisfied. That is, only $g(-\pi, 0)$ and $h(-\pi, 0)$ are minimum values of $g(\omega, \beta)$ and $h(\omega, \beta)$, respectively. Overall, if any one of the two conditions in Theorem holds, then (A.2) is satisfied. That is, $f_i(a) > 0$ for all $a \geq 1$, $\beta \in [-\pi, \pi)$, $\omega \in [-\pi, \pi)$, and $\omega \neq 0$ is proved.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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