Research Article

A New Type of Combination Synchronization among Multiple Chaotic Systems

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Based on former combination synchronization studies, a new type of combination synchronization approach is developed in this research, with the consideration of parallel combination of drive systems. This new synchronization approach is referred to as combination synchronization-II. As a representative case, the combination synchronization-II between three drive systems and one response system is studied. Applying Lyapunov stability theorem and active backstepping design, sufficient conditions for the proposed combination synchronization approach are derived. Numerical simulations are performed to show the feasibility and effectiveness of the proposed approach. Based on the investigation in this research, the proposed approach provides an applicable method for designing universal combination synchronization among multiple chaotic systems.

1. Introduction

Chaos phenomena have been widely observed in many systems and chaos synchronization plays an important role in the research on chaos. Great efforts were devoted to chaos synchronization of chaotic systems, a very important subfield of nonlinear science [1]. Since the pioneering work of Pecora and Carroll [2], chaos synchronization has been investigated in a variety of fields, such as ecological systems, chemical reactions, engineering science, and secure communication [3–5].

Many research works have been made to achieve various types of chaos synchronization, for example, complete synchronization [6, 7], phase synchronization [8, 9], and generalized synchronization [10, 11]. Nevertheless, most of the research works of chaos synchronization are limited to the synchronization of two chaotic systems, i.e., one drive system and one response system. In 2012, Luo et al. [12] developed a new type of synchronization, combination synchronization, which was applied to investigate the synchronization of two drive systems and one response system. From then on, the synchronization among multiple chaotic systems was extensively studied by many researchers [13–18].

The combination synchronization in recent literature develops in various types. Xi et al. proposed the function projective combination synchronization and determined the stability criterion for such combination synchronization of fractional-order chaotic systems [19]. Wang et al. investigated a novel adaptive generalized combination complex synchronization for different real and complex nonlinear systems with unknown parameters [20]. Vincent et al. developed a multiswitching combination synchronization of chaotic systems [18], and this synchronization type is further developed by Ahmad et al. as globally exponential multi-switching-combination synchronization control for chaotic systems in the field of secure communications [21]. Based on the combination synchronization, with the consideration of four or more chaotic systems, the researchers further proposed and explored combination-combination synchronization in the cases where the numbers of drive systems and response systems are both larger than one [13, 22–25].

The application of combination synchronization has been widespread for different chaotic systems as recorded in literature. The chaos and combination synchronization of a new fractional-order Lorenz-like system with two stable node-foci was thoroughly studied by Alam et al. [26]. Khan and Shikha

FIGURE 1: The parallel combination of two and three drive systems. DS: drive system; RS: response system.

used robust adaptive sliding mode control to investigate the combination synchronization of identical Genesio time-delay chaotic system [27]. Moreover, the combination synchronization scheme has also been introduced for complex-variable chaotic systems [17]. Sun et al. launched investigations on the finite-time combination synchronization of complex-variable chaotic systems with unknown parameters via sliding mode control [28, 29]. Singh et al. proposed a novel scheme for the dual combination synchronization among four master systems and two slave systems for the fractional-order complex chaotic systems with stability analysis [30].

As so far, the combination synchronization approaches focus on series combination of drive systems; i.e., the drive systems can be regarded as one combined supersystem to synchronize with the response system. Actually, there are two basic ways for combination of three systems, series

combination ($\bullet - \bullet - \bullet$) and parallel combination ($\begin{smallmatrix} \bullet & \bullet \\ \diagdown & \diagup \\ & \bullet \end{smallmatrix}$). Based on the two combination ways, many complex systems can be composed, such as electric circuit and ecological food web [31, 32]. The combination synchronization based on parallel combination of drive systems with one response system is described as the following: (1) the drive systems are divided into at least two groups, and each group exists as one of the parallel branches in the system combination; (2) the drive systems on the same branch jointly drive the response system; (3) the drive systems on different branches synchronize with the response system separately; and (4) the response system takes merely one controller in the combination synchronization. In this research, we refer the combination synchronization based on such parallel combination of drive systems as combination synchronization-II, whereas we refer to the former Runzi's combination synchronization as combination synchronization-I [12].

In comparing with former research works on the combination synchronization, the combination synchronization-II designed in this research may be a similar but extended type to combination complex synchronization or dual combination synchronization. The combination complex synchronization, for example, as described in the research of Sun et al. [17, 28, 29], suggests that the real part and the imaginary part of the drive systems and the response system, which are all complex systems, achieve combination synchronization, respectively. In this sense, the combination synchronization-II also considers that the synchronization of the chaotic systems can be divided into a few parallel branches or parts, similar to the division of real and imaginary parts in combination complex synchronization.

However, the combination synchronization-II exhibits three aspects of difference from the combination complex synchronization. Firstly, the response system commonly exists in every parallel branch divided for the synchronization. Secondly, the division of parallel branches for synchronization relies on the relationship between the chaotic systems in reality, and this is very different from the relation of real and imaginary parts in complex systems. Thirdly, according to the number of systems in each branch, synchronization between two chaotic systems or combination synchronization among three or more chaotic systems is achieved, implying that the synchronization in different branches can be unidentical. Based on the comparison with previous approaches in literature, the combination synchronization-II may be more general than the combination complex synchronization or the combination synchronization-I and it shows advantages in practical applications for more system synchronization modes. Therefore, the research on the combination synchronization-II still remains open and deserves investigation.

The organization of this research is organized as follows. Section 2 shows the scheme of combination synchronization-II with three drive systems and one response system, which shows two cases. In Section 3, we study the first case of the combination synchronization-II among four chaotic systems. Section 4 focuses on the achievement of the second case of the combination synchronization-II. At last, conclusions are described in Section 5.

2. The Scheme of Combination Synchronization-II

The combination synchronization-II is a drive-response type synchronization. Consider multiple drive systems and one response system, the parallel combination for drive systems may have various cases. It is easy to identify that the parallel combination of two drive systems has a simple structure as Figure 1(a). For three drive systems, there exist two cases for the parallel combination of drive systems, as shown in Figures 1(b) and 1(c). The case of Figure 1(b) describes three parallel branches, each one of which has merely one drive system whereas the case of Figure 1(c) demonstrates two branches, one of which has two serial drive systems. It should be noticed that the parallel combination of two drive systems can be simplified from the two cases of Figure 1(b) or Figure 1(c). Moreover, more complex cases of parallel combinations can be extended similarly from these two cases.

Based on the above description, the two cases of Figures 1(b) and 1(c) can be regarded as representative cases. Therefore, we focus on the combination synchronization-II of three drive systems and one response system. In this section, the combination synchronization-II scheme for three drive systems and one response system is designed. The three drive systems are given as follows:

$$\dot{x}^{(i)} = f^{(i)}(x^{(i)}) \quad (i = 1, 2, 3), \quad (1)$$

and the response system is given by

$$\dot{z} = h(z) + u(x^{(1)}, x^{(2)}, x^{(3)}, z). \quad (2)$$

In the above equations, the symbols have the following meaning: $x^{(i)} = (x_1^{(i)}, x_2^{(i)}, \dots, x_n^{(i)})^T$ ($i = 1, 2, 3$), $z = (z_1, z_2, \dots, z_n)^T \in R^n$ are the state vectors of systems (1) and (2), respectively; $f^{(i)}$ ($i = 1, 2, 3$), $h: R^n \rightarrow R^n$ are four continuous vector functions; $u: R^n \times R^n \times R^n \times R^n \rightarrow R^n$ is a controller to be designed.

Definition 1. Combination synchronization-II between the drive systems (1) and the response system (2) is achieved if there exist constant matrixes $A_i, B_j \in R^n$ ($i = 1, 2, 3$; $j = 1, 2, \dots, l$) and $B_j \neq 0$ such that

$$\lim_{l \rightarrow +\infty} \left\| \sum_{j=1}^l \sum_{i=k_{j-1}+1}^{k_j} A_i x^{(i)} - B_j z \right\| = 0, \quad (3)$$

in which $\|\cdot\|$ represents the matrix norm; l and k_j ($j = 0, 1, 2, \dots, l$) are integers and satisfy $1 < l \leq 3$ and $0 = k_0 < k_1 < \dots < k_l = 3$.

Remark 2. The constant matrixes A_i ($i = 1, 2, 3$), B_j ($j = 1, 2, \dots, l$) are called scaling matrixes. Moreover, these matrixes can be extended to functional matrixes of the state variables $x^{(i)}$ ($i = 1, 2, 3$) and z .

Remark 3. If (1) $l = 1$; or (2) $l = 2, k_1 = 1, A_1 = B_1 = 0$; or (3) $l = 2, k_1 = 2, A_3 = B_2 = 0$, the combination synchronization-II problem will be turned into the combination synchronization-I problem.

Remark 4. If (1) $l = 2, k_1 = 1$, (li) $A_1 = 0$ or (lii) $A_2 = 0$; or (2) $l = 2, k_1 = 2$, (2i) $A_2 = 0$ or (2ii) $A_3 = 0$; or (3) $l = 3, k_1 = 1, k_2 = 2$, (3i) $A_1 = B_1 = 0$ or (3ii) $A_2 = B_2 = 0$ or (3iii) $A_3 = B_3 = 0$, the combination synchronization-II problem between three drive systems and one response system will be reduced to the combination synchronization-II problem between two drive systems and one response system.

Remark 5. If (1) $l = 1, B_1 = I$, (li) $A_1 = A_2 = 0$ or (lii) $A_1 = A_3 = 0$ or (liii) $A_2 = A_3 = 0$; or (2) $l = 2, k_1 = 1$, (2i) $B_1 = I$ and $A_2 = A_3 = B_2 = 0$ or (2ii) $B_2 = I$ and $A_1 = A_2 = B_1 = 0$ or (2iii) $B_2 = I$ and $A_1 = A_3 = B_1 = 0$; or (3) $l = 2, k_1 = 2$, (3i) $B_1 = I$ and $A_1 = A_3 = B_2 = 0$ or (3ii) $B_1 = I$ and $A_2 = A_3 = B_2 = 0$ or (3iii) $B_2 = I$ and $A_1 = A_2 = B_1 = 0$; or (4) $l = 3, k_1 = 1, k_2 = 2$, (4i) $B_1 = I$

and $A_2 = A_3 = B_2 = B_3 = 0$ or (4ii) $B_2 = I$ and $A_1 = A_3 = B_1 = B_3 = 0$ or (4iii) $B_3 = I$ and $A_1 = A_2 = B_1 = B_2 = 0$, the combination synchronization-II problem will be reduced to the projective synchronization, where I is an $n \times n$ identity matrix.

Remark 6. If (1) $l = 1, A_1 = A_2 = A_3 = 0$; or (2) $l = 2, k_1 = 1$ or $2, A_1 = A_2 = A_3 = 0$, (2i) $B_1 = 0$ or (2ii) $B_2 = 0$; or (3) $l = 3, k_1 = 1, k_2 = 2, A_1 = A_2 = A_3 = 0$, (3i) $B_1 = B_2 = 0$ or (3ii) $B_1 = B_3 = 0$ or (3iii) $B_2 = B_3 = 0$, the combination synchronization-II problem will be turned into a chaos control problem.

Remark 7. The combination synchronization-II scheme as described in Definition 1 can be extended to the cases which have more than three drive systems.

According to Definition 1, l can have two values, 2 and 3, for the combination synchronization-II between three drive systems and one response system. In this research, we define the case of $l = 3$ as Case 1, and the case of $l = 2$ as Case 2, corresponding to the two cases of Figures 1(b) and 1(c). The applications of the combination synchronization-II scheme for the two cases are provided in the following.

3. Combination Synchronization-II of Case 1

The combination synchronization-II scheme is applied to four chaotic systems. Such application can adapt to a variety of chaotic systems in engineering science, secure communication, etc. As widely known, Lorenz system, Chen system, Rössler system, and Lü system are four basic and classical chaotic systems, which have been studied by many researchers [33–35] and can be extended for designing combination synchronization-II scheme. Also, the memristor chaotic systems, for example, are also suitable cases for the investigation of combination synchronization-II. Recently, autonomous memristor systems of infinite chaotic attractors were proposed and electronic circuits of such memristor chaotic systems have been designed [36, 37]. With circuitry realization where memristor chaotic systems show parallel combination, the combination synchronization-II scheme can be extended for investigating multiple memristor systems of infinite chaotic attractors [36, 37].

In this section, the combination synchronization-II of Case 1 is realized among the four classic chaotic systems, Lorenz system, Chen system, Rössler system, and Lü system. We consider the first three systems as drive systems and Lü system as response system. The Lorenz system, Chen system, and Rössler system are described by the following systems (4), (5), and (6), respectively,

$$\begin{aligned} \dot{x}_1 &= a_1(x_2 - x_1), \\ \dot{x}_2 &= b_1 x_1 - x_2 - x_1 x_3, \\ \dot{x}_3 &= x_1 x_2 - c_1 x_3, \end{aligned} \quad (4)$$

$$\begin{aligned}
\dot{y}_1 &= a_2 (y_2 - y_1), \\
\dot{y}_2 &= (c_2 - a_2) y_1 + c_2 y_2 - y_1 y_3, \\
\dot{y}_3 &= y_1 y_2 - b_2 y_3. \\
\dot{w}_1 &= -w_2 - w_3, \\
\dot{w}_2 &= w_1 + a_3 w_2, \\
\dot{w}_3 &= b_3 + w_3 (w_1 - c_3),
\end{aligned} \tag{5}$$

As described in literature [12, 13], Lorenz system shows chaotic behavior when $a_1 = 10$, $b_1 = 28$, and $c_1 = 8/3$; Chen system exhibits chaotic behavior when $a_2 = 35$, $b_2 = 3$, and $c_2 = 28$; Rössler system exhibits chaotic behavior when $a_3 = 0.2$, $b_3 = 0.2$, and $c_3 = 5.7$. On the other hand, the controlled Lü system is given by

$$\begin{aligned}
\dot{z}_1 &= a_4 (z_2 - z_1) + u_1, \\
\dot{z}_2 &= c_4 z_2 - z_1 z_3 + u_2, \\
\dot{z}_3 &= z_1 z_2 - b_4 z_3 + u_3.
\end{aligned} \tag{6}$$

Lü system exhibits chaotic behavior when $a_4 = 36$, $b_4 = 3$, and $c_4 = 20$. In (7), u_1 , u_2 , and u_3 are controllers to be designed.

Notice that to avoid confusions in the calculations, the state variables of the three drive systems in Definition 1 are redenoted as $x = (x_1, x_2, x_3)^T$, $y = (y_1, y_2, y_3)^T$ and $w = (w_1, w_2, w_3)^T$ in (4), (5), and (6). Simultaneously, the constant matrixes in Definition 1 are also redenoted as A , B , C , D , E , and H . According to Definition 1, the combination synchronization-II of Case 1 can be redescribed by

$$\begin{aligned}
&\lim_{t \rightarrow +\infty} \|Ax - Bz\| + \lim_{t \rightarrow +\infty} \|Cy - Dz\| \\
&+ \lim_{t \rightarrow +\infty} \|Ew - Hz\| = 0.
\end{aligned} \tag{8}$$

For convenience of the calculations, we write the constant matrixes A , B , C , D , E , and H as $A = \text{diag}(\alpha_1, \alpha_2, \alpha_3)$, $B = \text{diag}(\beta_1, \beta_2, \beta_3)$, $C = \text{diag}(\gamma_1, \gamma_2, \gamma_3)$, $D = \text{diag}(\delta_1, \delta_2, \delta_3)$, $E = \text{diag}(\varepsilon_1, \varepsilon_2, \varepsilon_3)$, and $H = \text{diag}(\eta_1, \eta_2, \eta_3)$, and let

$$\begin{aligned}
e_1 &= Bz - Ax, \\
e_2 &= Dz - Cy, \\
e_3 &= Hz - Ew,
\end{aligned} \tag{9}$$

in which $e_1 = (e_{11}, e_{12}, e_{13})^T$, $e_2 = (e_{21}, e_{22}, e_{23})^T$, and $e_3 = (e_{31}, e_{32}, e_{33})^T$. Hence, the error system can be written as the following:

$$\begin{aligned}
\dot{e}_{11} &= \frac{a_4 \beta_1}{\beta_2} e_{12} - a_4 e_{11} + f_1 + \beta_1 u_1, \\
\dot{e}_{12} &= c_4 e_{12} - \frac{\beta_2}{\beta_1 \beta_3} e_{11} e_{13} - \frac{\alpha_3 \beta_2}{\beta_1 \beta_3} x_3 e_{11} - \frac{\alpha_1 \beta_2}{\beta_1 \beta_3} x_1 e_{13} \\
&+ g_1 + \beta_2 u_2,
\end{aligned}$$

$$\begin{aligned}
\dot{e}_{13} &= \frac{\beta_3}{\beta_1 \beta_2} e_{11} e_{12} + \frac{\alpha_2 \beta_3}{\beta_1 \beta_2} x_2 e_{11} + \frac{\alpha_1 \beta_3}{\beta_1 \beta_2} x_1 e_{12} - b_4 e_{13} \\
&+ h_1 + \beta_3 u_3, \\
\dot{e}_{21} &= \frac{a_4 \delta_1}{\delta_2} e_{22} - a_4 e_{21} + f_2 + \delta_1 u_1, \\
\dot{e}_{22} &= c_4 e_{22} - \frac{\delta_2}{\delta_1 \delta_3} e_{21} e_{23} - \frac{\gamma_3 \delta_2}{\delta_1 \delta_3} y_3 e_{21} - \frac{\gamma_1 \delta_2}{\delta_1 \delta_3} y_1 e_{23} \\
&+ g_2 + \delta_2 u_2, \\
\dot{e}_{23} &= \frac{\delta_3}{\delta_1 \delta_2} e_{21} e_{22} + \frac{\gamma_2 \delta_3}{\delta_1 \delta_2} y_2 e_{21} + \frac{\gamma_1 \delta_3}{\delta_1 \delta_2} y_1 e_{22} - b_4 e_{23} \\
&+ h_2 + \delta_3 u_3, \\
\dot{e}_{31} &= \frac{a_4 \eta_1}{\eta_2} e_{32} - a_4 e_{31} + f_3 + \eta_1 u_1, \\
\dot{e}_{32} &= c_4 e_{32} - \frac{\eta_2}{\eta_1 \eta_3} e_{31} e_{33} - \frac{\varepsilon_3 \eta_2}{\eta_1 \eta_3} w_3 e_{31} - \frac{\varepsilon_1 \eta_2}{\eta_1 \eta_3} w_1 e_{33} \\
&+ g_3 + \eta_2 u_2, \\
\dot{e}_{33} &= \frac{\eta_3}{\eta_1 \eta_2} e_{31} e_{32} + \frac{\varepsilon_2 \eta_3}{\eta_1 \eta_2} w_2 e_{31} + \frac{\varepsilon_1 \eta_3}{\eta_1 \eta_2} w_1 e_{32} - b_4 e_{33} \\
&+ h_3 + \eta_3 u_3,
\end{aligned} \tag{10}$$

in which

$$\begin{aligned}
f_1 &= \frac{a_4 \alpha_2 \beta_1}{\beta_2} x_2 - a_4 \alpha_1 x_1 - a_1 \alpha_1 (x_2 - x_1), \\
g_1 &= (c_4 + 1) \alpha_2 x_2 - b_1 \alpha_2 x_1 + \left(\alpha_2 - \frac{\alpha_1 \alpha_3 \beta_2}{\beta_1 \beta_3} \right) x_1 x_3, \\
h_1 &= \left(\frac{\alpha_1 \alpha_2 \beta_3}{\beta_1 \beta_2} - \alpha_3 \right) x_1 x_2 + (c_1 - b_4) \alpha_3 x_3, \\
f_2 &= \frac{a_4 \gamma_2 \delta_1}{\delta_2} y_2 - a_4 \gamma_1 y_1 - a_2 \gamma_1 (y_2 - y_1), \\
g_2 &= (c_4 - c_2) \gamma_2 y_2 - (c_2 - a_2) \gamma_2 y_1 \\
&+ \left(\gamma_2 - \frac{\gamma_1 \gamma_3 \delta_2}{\delta_1 \delta_3} \right) y_1 y_3, \\
h_2 &= \left(\frac{\gamma_1 \gamma_2 \delta_3}{\delta_1 \delta_2} - \gamma_3 \right) y_1 y_2 + (b_2 - b_4) \gamma_3 y_3, \\
f_3 &= -a_4 \varepsilon_1 w_1 + \left(\frac{a_4 \varepsilon_2 \eta_1}{\eta_2} + \varepsilon_1 \right) w_2 + \varepsilon_1 w_3, \\
g_3 &= -\frac{\varepsilon_1 \varepsilon_3 \eta_2}{\eta_1 \eta_3} w_1 w_3 - \varepsilon_2 w_1 + (c_4 - a_3) \varepsilon_2 w_2, \\
h_3 &= \frac{\varepsilon_1 \varepsilon_2 \eta_3}{\eta_1 \eta_2} w_1 w_2 - \varepsilon_3 w_1 w_3 + (c_3 - b_4) \varepsilon_3 w_3 - b_3 \varepsilon_3.
\end{aligned} \tag{11}$$

Based on the active backstepping design method, the controllers u_1 , u_2 , and u_3 can be designed in the following

for realizing the combination synchronization-II of Case 1 among Lorenz system, Chen system, Rössler system, and Lü system. For the details of the active backstepping design method, one can refer to Luo et al. [12].

Theorem 8. *If the controllers are chosen as*

$$\begin{aligned}
 u_1 &= -\frac{s_1}{\beta_1} f_1 - \frac{s_2}{\delta_1} f_2 - \frac{s_3}{\eta_1} f_3, \\
 u_2 &= -s_1 \left(\frac{(a_4 - 1)(c_4 + 1)}{a_4 \beta_1} + \frac{a_4 \beta_1}{\beta_2^2} - \frac{\alpha_3 x_3}{\beta_1 \beta_3} \right) v_{11} \\
 &\quad - s_2 \left(\frac{(a_4 - 1)(c_4 + 1)}{a_4 \delta_1} + \frac{a_4 \delta_1}{\delta_2^2} - \frac{\gamma_3 y_3}{\delta_1 \delta_3} \right) v_{21} \\
 &\quad - s_3 \left(\frac{(a_4 - 1)(c_4 + 1)}{a_4 \eta_1} + \frac{a_4 \eta_1}{\eta_2^2} - \frac{\varepsilon_3 w_3}{\eta_1 \eta_3} \right) v_{31} - (c_4 \\
 &\quad - a_4 + 2) \left(\frac{s_1}{\beta_2} v_{12} + \frac{s_2}{\delta_2} v_{22} + \frac{s_3}{\eta_2} v_{32} \right) - \frac{s_1}{\beta_2} g_1 - \frac{s_2}{\delta_2} \\
 &\quad \cdot g_2 - \frac{s_3}{\eta_2} g_3, \\
 u_3 &= -s_1 \left[\left(\frac{a_4 - 1}{a_4 \beta_1^2} v_{11} + \frac{1}{\beta_1} \left(\frac{1}{\beta_2} - \frac{\beta_2}{\beta_3^2} \right) v_{12} \right) \right. \\
 &\quad \cdot (v_{11} + \alpha_1 x_1) + \frac{\alpha_2}{\beta_1 \beta_2} x_2 v_{11} \Big] \\
 &\quad - s_2 \left[\left(\frac{a_4 - 1}{a_4 \delta_1^2} v_{21} + \frac{1}{\delta_1} \left(\frac{1}{\delta_2} - \frac{\delta_2}{\delta_3^2} \right) v_{22} \right) \right. \\
 &\quad \cdot (v_{21} + \gamma_1 y_1) + \frac{\gamma_2}{\delta_1 \delta_2} y_2 v_{21} \Big] \\
 &\quad - s_3 \left[\left(\frac{a_4 - 1}{a_4 \eta_1^2} v_{31} + \frac{1}{\eta_1} \left(\frac{1}{\eta_2} - \frac{\eta_2}{\eta_3^2} \right) v_{32} \right) \right. \\
 &\quad \cdot (v_{31} + \varepsilon_1 w_1) + \frac{\varepsilon_2}{\eta_1 \eta_2} w_2 v_{31} \Big] + (b_4 - 1) \left(\frac{s_1}{\beta_3} v_{13} \right. \\
 &\quad \left. + \frac{s_2}{\delta_3} v_{23} + \frac{s_3}{\eta_3} v_{33} \right) - \frac{s_1}{\beta_3} h_1 - \frac{s_2}{\delta_3} h_2 - \frac{s_3}{\eta_3} h_3,
 \end{aligned} \tag{12}$$

in which

$$\begin{aligned}
 s_1 &= \operatorname{sgn}(\|x\|) (1 - \operatorname{sgn}(\|y\|)) (1 - \operatorname{sgn}(\|w\|)), \\
 s_2 &= \operatorname{sgn}(\|y\|) (1 - \operatorname{sgn}(\|x\|)) (1 - \operatorname{sgn}(\|w\|)), \\
 s_3 &= \operatorname{sgn}(\|w\|) (1 - \operatorname{sgn}(\|x\|)) (1 - \operatorname{sgn}(\|y\|)),
 \end{aligned} \tag{13}$$

$$\begin{pmatrix} v_{11} \\ v_{12} \\ v_{13} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{(1 - a_4) \beta_2}{a_4 \beta_1} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_{11} \\ e_{12} \\ e_{13} \end{pmatrix},$$

$$\begin{pmatrix} v_{21} \\ v_{22} \\ v_{23} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{(1 - a_4) \delta_2}{a_4 \delta_1} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_{21} \\ e_{22} \\ e_{23} \end{pmatrix},$$

$$\begin{pmatrix} v_{31} \\ v_{32} \\ v_{33} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{(1 - a_4) \eta_2}{a_4 \eta_1} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_{31} \\ e_{32} \\ e_{33} \end{pmatrix}, \tag{14}$$

the drive systems (4), (5), and (6) will achieve combination synchronization-II (Case 1) with the response system (7).

Proof. Based on the linear transformation described in (14), we construct (v_{11}, v_{12}, v_{13}) -subsystem. According to the definition of combination synchronization-II, we have $\|y\| = 0$ and $\|w\| = 0$ for this subsystem. Then direct calculations lead to

$$\begin{aligned}
 \dot{v}_{11} &= \frac{a_4 \beta_1}{\beta_2} \left(v_{12} - \frac{(1 - a_4) \beta_2}{a_4 \beta_1} v_{11} \right) - a_4 v_{11} + f_1 \\
 &\quad + \beta_1 u_1|_{\|x\|>0, \|y\|=0, \|w\|=0} = -v_{11} + \frac{a_4 \beta_1}{\beta_2} v_{12}, \\
 \dot{v}_{12} &= \frac{(1 - a_4) \beta_2}{a_4 \beta_1} \left(-v_{11} + \frac{a_4 \beta_1}{\beta_2} v_{12} \right) \\
 &\quad + c_4 \left(v_{12} - \frac{(1 - a_4) \beta_2}{a_4 \beta_1} v_{11} \right) - \frac{\beta_2}{\beta_1 \beta_3} v_{11} v_{13} \\
 &\quad - \frac{\alpha_3 \beta_2}{\beta_1 \beta_3} x_3 v_{11} - \frac{\alpha_1 \beta_2}{\beta_1 \beta_3} x_1 v_{13} + g_1 \\
 &\quad + \beta_2 u_2|_{\|x\|>0, \|y\|=0, \|w\|=0} \\
 &= -\frac{a_4 \beta_1}{\beta_2} v_{11} - v_{12} - \frac{\beta_2}{\beta_1 \beta_3} (v_{11} + \alpha_1 x_1) v_{13}, \\
 \dot{v}_{13} &= \left(\frac{\beta_3}{\beta_1 \beta_2} v_{11} + \frac{\alpha_1 \beta_3}{\beta_1 \beta_2} x_1 \right) \left(v_{12} - \frac{(1 - a_4) \beta_2}{a_4 \beta_1} v_{11} \right) \\
 &\quad + \frac{\alpha_2 \beta_3}{\beta_1 \beta_2} x_2 v_{11} - b_4 v_{13} + h_1 \\
 &\quad + \beta_3 u_3|_{\|x\|>0, \|y\|=0, \|w\|=0} \\
 &= \frac{\beta_2}{\beta_1 \beta_3} (v_{11} + \alpha_1 x_1) v_{12} - v_{13}.
 \end{aligned} \tag{15}$$

We can take a Lyapunov function as

$$V_1 = \frac{1}{2} v_{11}^2 + \frac{1}{2} v_{12}^2 + \frac{1}{2} v_{13}^2 \tag{16}$$

for the (v_{11}, v_{12}, v_{13}) -subsystem. The derivative of V_1 is given by

$$\begin{aligned}\dot{V}_1 &= v_{11}\dot{v}_{11} + v_{12}\dot{v}_{12} + v_{13}\dot{v}_{13} \\ &= v_{11}\left(-v_{11} + \frac{a_4\beta_1}{\beta_2}v_{12}\right) \\ &\quad + v_{12}\left(-\frac{a_4\beta_1}{\beta_2}v_{11} - v_{12} - \frac{\beta_2}{\beta_1\beta_3}(v_{11} + \alpha_1x_1)v_{13}\right) \quad (17) \\ &\quad + v_{13}\left(\frac{\beta_2}{\beta_1\beta_3}(v_{11} + \alpha_1x_1)v_{12} - v_{13}\right) \\ &= -v_{11}^2 - v_{12}^2 - v_{13}^2 \leq 0.\end{aligned}$$

According to the Lyapunov stability theorem, $\dot{V}_1 \leq 0$ as shown in (17) suggests that the equilibrium $(0, 0, 0)$ is globally asymptotically stable for the (v_{11}, v_{12}, v_{13}) -subsystem. Likewise, we construct (v_{21}, v_{22}, v_{23}) -subsystem and we have $\|x\| = 0$ and $\|w\| = 0$ for this subsystem. Meanwhile, the following can be obtained:

$$\begin{aligned}\dot{v}_{21} &= \frac{a_4\delta_1}{\delta_2}\left(v_{22} - \frac{(1-a_4)\delta_2}{a_4\delta_1}v_{21}\right) - a_4v_{21} + f_2 \\ &\quad + \delta_1u_1|_{\|x\|=0, \|y\|>0, \|w\|=0} = -v_{21} + \frac{a_4\delta_1}{\delta_2}v_{22}, \\ \dot{v}_{22} &= \frac{(1-a_4)\delta_2}{a_4\delta_1}\left(-v_{21} + \frac{a_4\delta_1}{\delta_2}v_{22}\right) \\ &\quad + c_4\left(v_{22} - \frac{(1-a_4)\delta_2}{a_4\delta_1}v_{21}\right) - \frac{\delta_2}{\delta_1\delta_3}v_{21}v_{23} \\ &\quad - \frac{\gamma_3\delta_2}{\delta_1\delta_3}v_{21} - \frac{\gamma_1\delta_2}{\delta_1\delta_3}v_{23} + g_2 \\ &\quad + \delta_2u_2|_{\|x\|=0, \|y\|>0, \|w\|=0} \quad (18) \\ &= -\frac{a_4\delta_1}{\delta_2}v_{21} - v_{22} - \frac{\delta_2}{\delta_1\delta_3}(v_{21} + \gamma_1\gamma_1)v_{23}, \\ \dot{v}_{23} &= \left(\frac{\delta_3}{\delta_1\delta_2}v_{21} + \frac{\gamma_1\delta_3}{\delta_1\delta_2}v_{21}\right)\left(v_{22} - \frac{(1-a_4)\delta_2}{a_4\delta_1}v_{21}\right) \\ &\quad + \frac{\gamma_2\delta_3}{\delta_1\delta_2}v_{21} - b_4v_{23} + h_2 \\ &\quad + \delta_3u_3|_{\|x\|=0, \|y\|>0, \|w\|=0} \\ &= \frac{\delta_2}{\delta_1\delta_3}(v_{21} + \gamma_1\gamma_1)v_{22} - v_{23},\end{aligned}$$

Choosing a candidate Lyapunov function,

$$V_2 = \frac{1}{2}v_{21}^2 + \frac{1}{2}v_{22}^2 + \frac{1}{2}v_{23}^2, \quad (19)$$

for the (v_{21}, v_{22}, v_{23}) -subsystem. The derivative of V_2 is given by

$$\begin{aligned}\dot{V}_2 &= v_{21}\dot{v}_{21} + v_{22}\dot{v}_{22} + v_{23}\dot{v}_{23} \\ &= v_{21}\left(-v_{21} + \frac{a_4\delta_1}{\delta_2}v_{22}\right) \\ &\quad + v_{22}\left(-\frac{a_4\delta_1}{\delta_2}v_{21} - v_{22} - \frac{\delta_2}{\delta_1\delta_3}(v_{21} + \gamma_1\gamma_1)v_{23}\right) \quad (20) \\ &\quad + v_{23}\left(\frac{\delta_2}{\delta_1\delta_3}(v_{21} + \gamma_1\gamma_1)v_{22} - v_{23}\right) \\ &= -v_{21}^2 - v_{22}^2 - v_{23}^2 \leq 0,\end{aligned}$$

which suggests that (v_{21}, v_{22}, v_{23}) asymptotically approaches $(0, 0, 0)$ as time increases to infinite.

Construct (v_{31}, v_{32}, v_{33}) -subsystem which satisfies $\|x\| = 0$ and $\|y\| = 0$. Then we have

$$\begin{aligned}\dot{v}_{31} &= \frac{a_4\eta_1}{\eta_2}\left(v_{32} + \frac{(a_4-1)\eta_2}{a_4\eta_1}v_{31}\right) - a_4v_{31} + f_3 \\ &\quad + \eta_1u_1|_{\|x\|=0, \|y\|=0, \|w\|>0} = -v_{31} + \frac{a_4\eta_1}{\eta_2}v_{32}, \\ \dot{v}_{32} &= \frac{(1-a_4)\eta_2}{a_4\eta_1}\left(-v_{31} + \frac{a_4\eta_1}{\eta_2}v_{32}\right) \\ &\quad + c_4\left(v_{32} + \frac{(a_4-1)\eta_2}{a_4\eta_1}v_{31}\right) - \frac{\eta_2}{\eta_1\eta_3}v_{31}v_{33} \\ &\quad - \frac{\varepsilon_3\eta_2}{\eta_1\eta_3}w_3v_{31} - \frac{\varepsilon_1\eta_2}{\eta_1\eta_3}w_1v_{33} + g_3 \\ &\quad + \eta_2u_2|_{\|x\|=0, \|y\|=0, \|w\|>0} \quad (21) \\ &= -\frac{a_4\eta_1}{\eta_2}v_{31} - v_{32} - \frac{\eta_2}{\eta_1\eta_3}(v_{31} + \varepsilon_1w_1)v_{33}, \\ \dot{v}_{33} &= \left(\frac{\eta_3}{\eta_1\eta_2}v_{31} + \frac{\varepsilon_1\eta_3}{\eta_1\eta_2}w_1\right)\left(v_{32} + \frac{(a_4-1)\eta_2}{a_4\eta_1}v_{31}\right) \\ &\quad + \frac{\varepsilon_2\eta_3}{\eta_1\eta_2}w_2v_{31} - b_4v_{33} + h_3 \\ &\quad + \eta_3u_3|_{\|x\|=0, \|y\|=0, \|w\|>0} \\ &= \frac{\eta_2}{\eta_1\eta_3}(v_{31} + \varepsilon_1w_1)v_{32} - v_{33}.\end{aligned}$$

Choosing a candidate Lyapunov function,

$$V_3 = \frac{1}{2}v_{31}^2 + \frac{1}{2}v_{32}^2 + \frac{1}{2}v_{33}^2, \quad (22)$$

for the (v_{31}, v_{32}, v_{33}) -subsystem. The derivative of V_3 is given by

$$\begin{aligned}\dot{V}_3 &= v_{31}\dot{v}_{31} + v_{32}\dot{v}_{32} + v_{33}\dot{v}_{33} \\ &= v_{31}\left(-v_{31} + \frac{a_4\eta_1}{\eta_2}v_{32}\right)\end{aligned}$$

$$\begin{aligned}
& + v_{32} \left(-\frac{a_4 \eta_1}{\eta_2} v_{31} - v_{32} - \frac{\eta_2}{\eta_1 \eta_3} (v_{31} + \varepsilon_1 w_1) v_{33} \right) \\
& + v_{33} \left(\frac{\eta_2}{\eta_1 \eta_3} (v_{31} + \varepsilon_1 w_1) v_{32} - v_{33} \right) \\
& = -v_{31}^2 - v_{32}^2 - v_{33}^2 \leq 0,
\end{aligned} \tag{23}$$

which suggests that (v_{31}, v_{32}, v_{33}) asymptotically approaches $(0, 0, 0)$.

Taking the above results together, we have $(v_{11}, v_{12}, v_{13}) \rightarrow (0, 0, 0)$, $(v_{21}, v_{22}, v_{23}) \rightarrow (0, 0, 0)$, and $(v_{31}, v_{32}, v_{33}) \rightarrow (0, 0, 0)$.

$(0, 0, 0)$ as $t \rightarrow +\infty$. Notice the linear transformation described by (14), we know $e_{ij} \rightarrow 0$ ($i = 1, 2, 3; j = 1, 2, 3$). It means that the drive systems (4), (5), and (6) achieve combination synchronization-II with the response system (7). $\square$

From Theorem 8, the following corollaries can be deduced. Since the proofs of the corollaries are similar to Theorem 8, we omit the corollary proofs for the sake of simplification.

Corollary 9. *If the controllers are chosen as*

$$u = \begin{pmatrix} \frac{(\beta_1 u_1|_{\|x\|>0, \|y\|=0, \|w\|=0} + \delta_1 u_1|_{\|x\|=0, \|y\|>0, \|w\|=0} + \eta_1 u_1|_{\|x\|=0, \|y\|=0, \|w\|>0})}{(\beta_1 + \delta_1 + \eta_1)} \\ \frac{(\beta_2 u_2|_{\|x\|>0, \|y\|=0, \|w\|=0} + \delta_2 u_2|_{\|x\|=0, \|y\|>0, \|w\|=0} + \eta_2 u_2|_{\|x\|=0, \|y\|=0, \|w\|>0})}{(\beta_2 + \delta_2 + \eta_2)} \\ \frac{(\beta_3 u_3|_{\|x\|>0, \|y\|=0, \|w\|=0} + \delta_3 u_3|_{\|x\|=0, \|y\|>0, \|w\|=0} + \eta_3 u_3|_{\|x\|=0, \|y\|=0, \|w\|>0})}{(\beta_3 + \delta_3 + \eta_3)} \end{pmatrix}, \tag{24}$$

in which u_1, u_2 , and u_3 are described by (10), we will have

$$\lim_{t \rightarrow +\infty} \|Ax + Cy + Ew - (B + D + H)z\| = 0. \tag{25}$$

Hence, the drive systems (4), (5), and (6) can achieve combination synchronization-I with the response system (7).

Corollary 10. (i) Let $\alpha_1 = \alpha_2 = \alpha_3 = 0$ and $\beta_1 = \beta_2 = \beta_3 = 0$; if the controllers are chosen as (10) or (24) with removing all the terms related to v_{11}, v_{12}, v_{13} , the drive systems (5) and (6) can achieve combination synchronization-II or combination synchronization-I with the response system (7).

(ii) Let $\gamma_1 = \gamma_2 = \gamma_3 = 0$ and $\delta_1 = \delta_2 = \delta_3 = 0$; if the controllers are chosen as (10) or (24) with removing all the terms related to v_{21}, v_{22}, v_{23} , the drive systems (4) and (6) can achieve combination synchronization-II or combination synchronization-I with the response system (7).

(iii) Let $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0$ and $\eta_1 = \eta_2 = \eta_3 = 0$; if the controllers are chosen as (10) or (24) with removing all the terms related to v_{31}, v_{32}, v_{33} , the drive systems (4) and (5) can achieve combination synchronization-II or combination synchronization-I with the response system (7).

Corollary 11. (i) Let $\beta_1 = \beta_2 = \beta_3 = 1, \gamma_1 = \gamma_2 = \gamma_3 = 0, \delta_1 = \delta_2 = \delta_3 = 0, \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0$, and $\eta_1 = \eta_2 = \eta_3 = 0$; if the controllers are chosen as

$$u = \begin{pmatrix} -f_1 \\ -\left(\frac{(a_4 - 1)(c_4 + 1)}{a_4} + a_4 - \alpha_3 x_3 \right) v_{11} - (c_4 - a_4 + 2) v_{12} - g_1 \\ \frac{1 - a_4}{a_4} v_{11} (v_{11} + \alpha_1 x_1) - \alpha_2 x_2 v_{11} + (b_4 - 1) v_{13} - h_1 \end{pmatrix}, \tag{26}$$

the drive system (4) will achieve projective synchronization with the response system (7).

(ii) Let $\alpha_1 = \alpha_2 = \alpha_3 = 0, \beta_1 = \beta_2 = \beta_3 = 0, \delta_1 = \delta_2 = \delta_3 = 1, \varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0$, and $\eta_1 = \eta_2 = \eta_3 = 0$; if the controllers are chosen as

$$u = \begin{pmatrix} -f_2 \\ -\left(\frac{(a_4 - 1)(c_4 + 1)}{a_4} + a_4 - \gamma_3 y_3 \right) v_{21} - (c_4 - a_4 + 2) v_{22} - g_2 \\ \frac{1 - a_4}{a_4} v_{21} (v_{21} + \gamma_1 y_1) - \gamma_2 y_2 v_{21} + (b_4 - 1) v_{23} - h_2 \end{pmatrix}, \tag{27}$$

the drive system (5) will achieve projective synchronization with the response system (7).

(iii) Let $\alpha_1 = \alpha_2 = \alpha_3 = 0$, $\beta_1 = \beta_2 = \beta_3 = 0$, $\gamma_1 = \gamma_2 = \gamma_3 = 0$, $\delta_1 = \delta_2 = \delta_3 = 0$, and $\eta_1 = \eta_2 = \eta_3 = 1$; if the controllers are chosen as

$$u = \begin{pmatrix} -f_3 \\ -\left(\frac{(a_4 - 1)(c_4 + 1)}{a_4} + a_4 - \varepsilon_3 w_3\right) v_{31} - (c_4 - a_4 + 2) v_{32} - g_3 \\ \frac{1 - a_4}{a_4} v_{31} (v_{31} + \varepsilon_1 w_1) - \varepsilon_2 w_2 v_{31} + (b_4 - 1) v_{33} - h_3 \end{pmatrix}, \quad (28)$$

the drive system (6) will achieve projective synchronization with the response system (7).

Corollary 12. Let $\alpha_1 = \alpha_2 = \alpha_3 = 0$, $\gamma_1 = \gamma_2 = \gamma_3 = 0$, $\varepsilon_1 = \varepsilon_2 = \varepsilon_3 = 0$, and (i) $\beta_1 = \beta_2 = \beta_3 = 1$, $\delta_1 = \delta_2 = \delta_3 = 0$, $\eta_1 = \eta_2 = \eta_3 = 0$, or (ii) $\beta_1 = \beta_2 = \beta_3 = 0$, $\delta_1 = \delta_2 = \delta_3 = 1$, $\eta_1 = \eta_2 = \eta_3 = 0$, or (iii) $\beta_1 = \beta_2 = \beta_3 = 0$, $\delta_1 = \delta_2 = \delta_3 = 0$, $\eta_1 = \eta_2 = \eta_3 = 1$; if the controllers are chosen as

$$u = \begin{pmatrix} 0 \\ -\left(\frac{(a_4 - 1)(c_4 + 1)}{a_4} + a_4\right) v_1 - (c_4 - a_4 + 2) v_2 \\ \frac{1 - a_4}{a_4} v_1^2 + (b_4 - 1) v_3 \end{pmatrix}, \quad (29)$$

in which $v_1 = z_1$, $v_2 = z_2 - (a_4 - 1)z_1/a_4$, $v_3 = z_3$, the equilibrium $(0, 0, 0)$ of the response system (7) is globally asymptotically stable.

Numerical simulations are performed to illustrate the above combination synchronization-II among systems (4), (5), (6), and (7). Fourth-order Runge-Kutta method is applied with time step equal to 0.001, and the initial values of variables x , y , w , and z are provided randomly. We choose $\alpha_i = \beta_i = \gamma_i = \delta_i = \eta_i = 1$, $\varepsilon_i = 3$ ($i = 1, 2, 3$). The parameter values of systems (4), (5), (6), and (7) are the same as described along with the systems, i.e., $a_1 = 10$, $b_1 = 28$, $c_1 = 8/3$, $a_2 = 35$, $b_2 = 3$, $c_2 = 28$, $a_3 = 0.2$, $b_3 = 0.2$, $c_3 = 5.7$, $a_4 = 36$, $b_4 = 3$, and $c_4 = 20$. With these given conditions, the controller can be calculated as

$$\begin{aligned} u_1 &= 26s_1(x_1 - x_2) + s_2(y_1 - y_2) \\ &\quad + s_3(108w_1 - 111w_2 - 3w_3), \\ u_2 &= -s_1\left(56 + \frac{5}{12} - x_3\right)e_{11} - s_2\left(56 + \frac{5}{12} - y_3\right)e_{21} \\ &\quad - s_3\left(56 + \frac{5}{12} - 3w_3\right)e_{31} \\ &\quad + 14s_1\left(e_{12} - \frac{35}{36}e_{11}\right) + 14s_2\left(e_{22} - \frac{35}{36}e_{21}\right) \\ &\quad + 14s_3\left(e_{32} - \frac{35}{36}e_{31}\right) - 7s_1(3x_2 - 4x_1) \end{aligned}$$

$$\begin{aligned} &-s_2(7y_1 + 12y_2) \\ &-3s_3(19.8w_2 - 3w_1w_3 - w_1), \\ u_3 &= -s_1e_{11}\left(\frac{35}{36}(e_{11} + x_1) + x_2\right) \\ &-s_2e_{21}\left(\frac{35}{36}(e_{21} + y_1) + y_2\right) \\ &-s_3e_{31}\left(\frac{35}{36}(e_{31} + 3w_1) + 3w_2\right) \\ &\quad + 2(s_1e_{13} + s_2e_{23} + s_3e_{33}) + \frac{1}{3}s_1x_3 \\ &\quad - 3s_3(3w_1w_2 - w_1w_3 + 2.7w_3 - 0.2). \end{aligned} \quad (30)$$

Substituting the equations of controller into the error system, we then have (notice that $s_1 = 1$, $s_2 = 0$, and $s_3 = 0$ for the dynamics of e_{11} , e_{12} , and e_{13} , $s_1 = 0$, $s_2 = 1$, and $s_3 = 0$ for the dynamics of e_{21} , e_{22} , and e_{23} , $s_1 = 0$, $s_2 = 0$, and $s_3 = 1$ for the dynamics of e_{31} , e_{32} , and e_{33}),

$$\begin{aligned} \dot{e}_{11} &= 36(e_{12} - e_{11}), \\ \dot{e}_{12} &= 34e_{12} - \left(70 + \frac{1}{36}\right)e_{11} - e_{11}e_{13} - x_1e_{13}, \\ \dot{e}_{13} &= e_{11}e_{12} + x_1e_{12} - e_{13} - \frac{35}{36}e_{11}(e_{11} + x_1), \\ \dot{e}_{21} &= 36(e_{22} - e_{21}), \\ \dot{e}_{22} &= 34e_{22} - \left(70 + \frac{1}{36}\right)e_{21} - e_{21}e_{23} - y_1e_{23}, \\ \dot{e}_{23} &= e_{21}e_{22} + y_1e_{22} - e_{23} - \frac{35}{36}e_{21}(e_{21} + y_1), \\ \dot{e}_{31} &= 36(e_{32} - e_{31}), \\ \dot{e}_{32} &= 34e_{32} - \left(70 + \frac{1}{36}\right)e_{31} - e_{31}e_{33} - 3w_1e_{33}, \\ \dot{e}_{33} &= e_{31}e_{32} + 3w_1e_{32} - e_{33} - \frac{35}{36}e_{31}(e_{31} + 3w_1). \end{aligned} \quad (31)$$

Via numerical calculations on the above error system, Figure 2 demonstrates the time response of the synchronization errors e_{ij} ($i = 1, 2, 3$ and $j = 1, 2, 3$), which converges

FIGURE 2: Time response of synchronization errors e_{ij} , $i=1, 2, 3$ and $j=1, 2, 3$, for the combination synchronization-II of Case 1 among systems (4), (5), (6), and (7).

to zero, suggesting that the combination synchronization-II among systems (4), (5), (6), and (7) indeed achieves. Figure 3 depicts time response of the states, x_i , y_i , w_i , and z_i , showing the synchronization between the drive systems and the response system.

4. Combination Synchronization-II of Case 2

In this section, the combination synchronization-II of Case 2 among the four chaotic systems, Lorenz system, Chen system, Rössler system, and Lü system, is realized. Lorenz system, Chen system, and Rössler system are also taken as drive systems, in which Lorenz system and Chen system are on the same branch as shown in Figure 1(c); Lü system is taken as the response system.

According to Definition 1, the combination synchronization of Case 2 among the four chaotic systems can be described by

$$\lim_{t \rightarrow +\infty} \|Ax + By - Cz\| + \lim_{t \rightarrow +\infty} \|Dw - Ez\| = 0. \quad (32)$$

Let

$$\begin{aligned} e_1 &= Cz - Ax - By, \\ e_2 &= Ez - Dw, \end{aligned} \quad (33)$$

and then the error system can be described by

$$\dot{e}_{11} = \frac{a_4 \gamma_1}{\gamma_2} e_{12} - a_4 e_{11} + f_1 + \gamma_1 u_1,$$

$$\begin{aligned} \dot{e}_{12} &= c_4 e_{12} - \frac{\gamma_2}{\gamma_1 \gamma_3} (e_{11} e_{13} + (\alpha_3 x_3 + \beta_3 \gamma_3) e_{11} \\ &\quad + (\alpha_1 x_1 + \beta_1 \gamma_1) e_{13}) + g_1 + \gamma_2 u_2, \\ \dot{e}_{13} &= \frac{\gamma_3}{\gamma_1 \gamma_2} (e_{11} e_{12} + (\alpha_2 x_2 + \beta_2 \gamma_2) e_{11} \\ &\quad + (\alpha_1 x_1 + \beta_1 \gamma_1) e_{12}) - b_4 e_{13} + h_1 + \gamma_3 u_3, \\ \dot{e}_{21} &= \frac{a_4 \varepsilon_1}{\varepsilon_2} e_{22} - a_4 e_{21} + f_2 + \varepsilon_1 u_1, \\ \dot{e}_{22} &= c_4 e_{22} - \frac{\varepsilon_2}{\varepsilon_1 \varepsilon_3} e_{21} e_{23} - \frac{\delta_3 \varepsilon_2}{\varepsilon_1 \varepsilon_3} w_3 e_{21} - \frac{\delta_1 \varepsilon_2}{\varepsilon_1 \varepsilon_3} w_1 e_{23} \\ &\quad + g_2 + \varepsilon_2 u_2, \\ \dot{e}_{23} &= \frac{\varepsilon_3}{\varepsilon_1 \varepsilon_2} e_{21} e_{22} + \frac{\delta_2 \varepsilon_3}{\varepsilon_1 \varepsilon_2} w_2 e_{21} + \frac{\delta_1 \varepsilon_3}{\varepsilon_1 \varepsilon_2} w_1 e_{22} - b_4 e_{23} \\ &\quad + h_2 + \varepsilon_3 u_3, \end{aligned} \quad (34)$$

in which

$$\begin{aligned} f_1 &= \frac{a_4 \alpha_2 \gamma_1}{\gamma_2} x_2 + \frac{a_4 \beta_2 \gamma_1}{\gamma_2} y_2 - a_4 (\alpha_1 x_1 + \beta_1 \gamma_1) \\ &\quad - a_1 \alpha_1 (x_2 - x_1) - a_2 \beta_1 (y_2 - y_1), \\ g_1 &= -b_1 \alpha_2 x_1 + (c_4 + 1) \alpha_2 x_2 + (a_2 - c_2) \beta_2 y_1 \\ &\quad + (c_4 - c_2) \beta_2 y_2 + \alpha_2 x_1 x_3 + \beta_2 y_1 y_3 \\ &\quad - \frac{\gamma_2}{\gamma_1 \gamma_3} (\alpha_1 x_1 + \beta_1 \gamma_1) (\alpha_3 x_3 + \beta_3 \gamma_3), \\ h_1 &= (c_1 - b_4) \alpha_3 x_3 + (b_2 - b_4) \beta_2 y_3 \\ &\quad + \frac{\gamma_3}{\gamma_1 \gamma_2} (\alpha_1 x_1 + \beta_1 \gamma_1) (\alpha_2 x_2 + \beta_2 y_2) \\ &\quad - \alpha_3 x_1 x_2 - \beta_3 y_1 y_2, \end{aligned} \quad (35)$$

$$f_2 = -a_4 \delta_1 w_1 + \left(\frac{a_4 \delta_2 \varepsilon_1}{\varepsilon_2} + \delta_1 \right) w_2 + \delta_1 w_3,$$

$$g_2 = -\frac{\delta_1 \delta_3 \varepsilon_2}{\varepsilon_1 \varepsilon_3} w_1 w_3 - \delta_2 w_1 + (c_4 - a_3) \delta_2 w_2,$$

$$h_2 = \frac{\delta_1 \delta_2 \varepsilon_3}{\varepsilon_1 \varepsilon_2} w_1 w_2 - \delta_3 w_1 w_3 + (c_3 - b_4) \delta_3 w_3 - b_3 \delta_3.$$

The active backstepping design method is also applied for designing the controllers u_1 , u_2 , and u_3 for the error system (34).

Theorem 13. If the controllers u_1 , u_2 , and u_3 are designed as

$$u_1 = -\frac{s_1}{\gamma_1} f_1 - \frac{s_2}{\varepsilon_1} f_2,$$

$$u_2 = -s_1 \left(\frac{(a_4 - 1)(c_4 + 1)}{a_4 \gamma_1} + \frac{a_4 \gamma_1}{\gamma_2^2} - \frac{\alpha_3 x_3 + \beta_3 \gamma_3}{\gamma_1 \gamma_3} \right)$$

FIGURE 3: Time response of states x_i , y_i , w_i , and z_i , $i = 1, 2, 3$, and the comparison of states between the drive systems and the response system.

$$\begin{aligned}
 & \cdot v_{11} - (c_4 - a_4 + 2) \left(\frac{s_1}{\gamma_2} v_{12} + \frac{s_2}{\varepsilon_2} v_{22} \right) \\
 & - s_2 \left(\frac{(a_4 - 1)(c_4 + 1)}{a_4 \varepsilon_1} + \frac{a_4 \varepsilon_1}{\varepsilon_2^2} - \frac{\delta_3 w_3}{\varepsilon_1 \varepsilon_3} \right) v_{21} - \frac{s_1}{\gamma_2} \\
 & \cdot g_1 - \frac{s_2}{\varepsilon_2} g_2, \\
 u_3 = & -s_1 \left[\frac{a_4 - 1}{a_4 \gamma_1^2} v_{11}^2 \right. \\
 & + \frac{1}{\gamma_1} \left(\frac{\alpha_2 x_2 + \beta_2 y_2}{\gamma_2} + \frac{(a_4 - 1)(\alpha_1 x_1 + \beta_1 y_1)}{a_4 \gamma_1} \right) \\
 & \cdot v_{11} \left. \right] - s_2 \left[\left(\frac{a_4 - 1}{a_4 \varepsilon_1^2} v_{21} + \frac{1}{\varepsilon_1} \left(\frac{1}{\varepsilon_2} - \frac{\varepsilon_2}{\varepsilon_3^2} \right) v_{22} \right) \right. \\
 & \cdot (v_{21} + \delta_1 w_1) + \frac{\delta_2}{\varepsilon_1 \varepsilon_2} w_2 v_{21} \left. \right] - \frac{s_1}{\gamma_1} \left(\frac{1}{\gamma_2} - \frac{\gamma_2}{\gamma_3^2} \right) (v_{11} \\
 & + \alpha_1 x_1 + \beta_1 y_1) v_{12} + (b_4 - 1) \left(\frac{s_1}{\gamma_3} v_{13} + \frac{s_2}{\varepsilon_3} v_{23} \right) \\
 & - \frac{s_1}{\gamma_3} h_1 - \frac{s_2}{\varepsilon_3} h_2,
 \end{aligned} \tag{36}$$

in which f_1 , g_1 , h_1 , f_2 , g_2 , and h_2 are described by (35) and

$$\begin{aligned}
 s_1 &= \operatorname{sgn}(\|x\| + \|y\|) (1 - \operatorname{sgn}(\|w\|)), \\
 s_2 &= \operatorname{sgn}(\|w\|) (1 - \operatorname{sgn}(\|x\| + \|y\|)),
 \end{aligned} \tag{37}$$

$$\begin{pmatrix} v_{11} \\ v_{12} \\ v_{13} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{(1 - a_4) \gamma_2}{a_4 \gamma_1} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_{11} \\ e_{12} \\ e_{13} \end{pmatrix},$$

$$\begin{pmatrix} v_{21} \\ v_{22} \\ v_{23} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ \frac{(1-a_4)\varepsilon_2}{a_4\varepsilon_1} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} e_{21} \\ e_{22} \\ e_{23} \end{pmatrix}, \quad (38)$$

the combination synchronization-II of Case 2 will be achieved among systems (4), (5), (6), and (7).

Proof. Based on the linear transformation described in (38), we construct (v_{11}, v_{12}, v_{13}) -subsystem. For this subsystem, we have $\|w\| = 0$. Then direct calculations lead to

$$\begin{aligned} \dot{v}_{11} &= \frac{a_4\gamma_1}{\gamma_2} \left(v_{12} - \frac{(1-a_4)\gamma_2}{a_4\gamma_1} v_{11} \right) - a_4 v_{11} + f_1 \\ &+ \gamma_1 u_1|_{\|x\|+\|y\|>0, \|w\|=0} = -v_{11} + \frac{a_4\gamma_1}{\gamma_2} v_{12}, \\ \dot{v}_{12} &= \frac{(1-a_4)\gamma_2}{a_4\gamma_1} \left(-v_{11} + \frac{a_4\gamma_1}{\gamma_2} v_{12} \right) + c_4 \left(v_{12} \right. \\ &\quad \left. - \frac{(1-a_4)\gamma_2}{a_4\gamma_1} v_{11} \right) - \frac{\gamma_2}{\gamma_1\gamma_3} (v_{11}v_{13} \\ &\quad + (\alpha_3x_3 + \beta_3y_3)v_{11} + (\alpha_1x_1 + \beta_1y_1)v_{13}) + g_1 \\ &\quad + \gamma_2 u_2|_{\|x\|+\|y\|>0, \|w\|=0} = -\frac{\gamma_2}{\gamma_1\gamma_3} (v_{11} + \alpha_1x_1 \\ &\quad + \beta_1y_1)v_{13} - \frac{a_4\gamma_1}{\gamma_2} v_{11} - v_{12}, \\ \dot{v}_{13} &= \frac{\gamma_3}{\gamma_1\gamma_2} \left((\alpha_2x_2 + \beta_2y_2)v_{11} \right. \\ &\quad \left. + (v_{11} + \alpha_1x_1 + \beta_1y_1) \left(v_{12} - \frac{(1-a_4)\gamma_2}{a_4\gamma_1} v_{11} \right) \right) \\ &\quad - b_4v_{13} + h_1 + \gamma_3 u_3|_{\|x\|+\|y\|>0, \|w\|=0} = \frac{\gamma_2}{\gamma_1\gamma_3} (v_{11} \\ &\quad + \alpha_1x_1 + \beta_1y_1)v_{12} - v_{13}. \end{aligned} \quad (39)$$

Take a Lyapunov function as

$$V_1 = \frac{1}{2}v_{11}^2 + \frac{1}{2}v_{12}^2 + \frac{1}{2}v_{13}^2 \quad (40)$$

for the (v_{11}, v_{12}, v_{13}) -subsystem. The derivative of V_1 is given by

$$\begin{aligned} \dot{V}_1 &= v_{11}\dot{v}_{11} + v_{12}\dot{v}_{12} + v_{13}\dot{v}_{13} = v_{11} \left(-v_{11} + \frac{a_4\gamma_1}{\gamma_2} v_{12} \right) \\ &\quad + v_{12} \left(-\frac{\gamma_2}{\gamma_1\gamma_3} (v_{11} + \alpha_1x_1 + \beta_1y_1)v_{13} - \frac{a_4\gamma_1}{\gamma_2} v_{11} \right. \\ &\quad \left. - v_{12} \right) + v_{13} \left(\frac{\gamma_2}{\gamma_1\gamma_3} (v_{11} + \alpha_1x_1 + \beta_1y_1)v_{12} - v_{13} \right) \\ &= -v_{11}^2 - v_{12}^2 - v_{13}^2 \leq 0, \end{aligned} \quad (41)$$

which suggests that the equilibrium $(0, 0, 0)$ is globally asymptotically stable for the (v_{11}, v_{12}, v_{13}) -subsystem. Likewise, we construct (v_{21}, v_{22}, v_{23}) -subsystem which satisfies $\|x\| + \|y\| = 0$. Then we have

$$\begin{aligned} \dot{v}_{21} &= \frac{a_4\varepsilon_1}{\varepsilon_2} \left(v_{22} + \frac{(a_4-1)\varepsilon_2}{a_4\varepsilon_1} v_{21} \right) - a_4v_{21} + f_2 \\ &\quad + \varepsilon_1 u_1|_{\|x\|+\|y\|=0, \|w\|>0} = -v_{21} + \frac{a_4\varepsilon_1}{\varepsilon_2} v_{22}, \\ \dot{v}_{22} &= \frac{(1-a_4)\varepsilon_2}{a_4\varepsilon_1} \left(-v_{21} + \frac{a_4\varepsilon_1}{\varepsilon_2} v_{22} \right) \\ &\quad + c_4 \left(v_{22} + \frac{(a_4-1)\varepsilon_2}{a_4\varepsilon_1} v_{21} \right) - \frac{\varepsilon_2}{\varepsilon_1\varepsilon_3} v_{21}v_{23} \\ &\quad - \frac{\delta_3\varepsilon_2}{\varepsilon_1\varepsilon_3} w_3v_{21} - \frac{\delta_1\varepsilon_2}{\varepsilon_1\varepsilon_3} w_1v_{23} + g_2 \\ &\quad + \varepsilon_2 u_2|_{\|x\|+\|y\|=0, \|w\|>0} \\ &= -\frac{a_4\varepsilon_1}{\varepsilon_2} v_{21} - v_{22} - \frac{\varepsilon_2}{\varepsilon_1\varepsilon_3} (v_{21} + \delta_1w_1)v_{23}, \\ \dot{v}_{23} &= \left(\frac{\varepsilon_3}{\varepsilon_1\varepsilon_2} v_{21} + \frac{\delta_1\varepsilon_3}{\varepsilon_1\varepsilon_2} w_1 \right) \left(v_{22} + \frac{(a_4-1)\varepsilon_2}{a_4\varepsilon_1} v_{21} \right) \\ &\quad + \frac{\delta_2\varepsilon_3}{\varepsilon_1\varepsilon_2} w_2v_{21} - b_4v_{23} + h_2 + \varepsilon_3 u_3|_{\|x\|+\|y\|=0, \|w\|>0} \\ &= \frac{\varepsilon_2}{\varepsilon_1\varepsilon_3} (v_{21} + \delta_1w_1)v_{22} - v_{23}. \end{aligned} \quad (42)$$

Choosing a candidate Lyapunov function,

$$V_2 = \frac{1}{2}v_{21}^2 + \frac{1}{2}v_{22}^2 + \frac{1}{2}v_{23}^2, \quad (43)$$

for the (v_{21}, v_{22}, v_{23}) -subsystem. The derivative of V_2 is given by

$$\begin{aligned} \dot{V}_2 &= v_{21}\dot{v}_{21} + v_{22}\dot{v}_{22} + v_{23}\dot{v}_{23} \\ &= v_{21} \left(-v_{21} + \frac{a_4\varepsilon_1}{\varepsilon_2} v_{22} \right) \\ &\quad + v_{22} \left(-\frac{a_4\varepsilon_1}{\varepsilon_2} v_{21} - v_{22} - \frac{\varepsilon_2}{\varepsilon_1\varepsilon_3} (v_{21} + \delta_1w_1)v_{23} \right) \\ &\quad + v_{23} \left(\frac{\varepsilon_2}{\varepsilon_1\varepsilon_3} (v_{21} + \delta_1w_1)v_{22} - v_{23} \right) \\ &= -v_{21}^2 - v_{22}^2 - v_{23}^2 \leq 0, \end{aligned} \quad (44)$$

which suggests that (v_{21}, v_{22}, v_{23}) asymptotically approaches $(0, 0, 0)$.

Considering the above results together, we have $(v_{11}, v_{12}, v_{13}) \rightarrow (0, 0, 0)$ and $(v_{21}, v_{22}, v_{23}) \rightarrow (0, 0, 0)$ as $t \rightarrow +\infty$. Via the linear transformation described by (38), $e_{ij} \rightarrow 0$ ($i = 1, 2$; $j = 1, 2, 3$) is obtained. It means the establishment of (32). $\square$

The following corollary of combination synchronization-I between three drive systems and one response system can be directly obtained from Theorem 13. For the other cases, one can refer to Corollaries 9~12.

Corollary 14. *If the controllers are chosen as*

$$u = \begin{pmatrix} \frac{(\gamma_1 u_1 |_{\|x\|+\|y\|>0, \|w\|=0} + \varepsilon_1 u_1 |_{\|x\|+\|y\|=0, \|w\|>0})}{(\gamma_1 + \varepsilon_1)} \\ \frac{(\gamma_2 u_2 |_{\|x\|+\|y\|>0, \|w\|=0} + \varepsilon_2 u_2 |_{\|x\|+\|y\|=0, \|w\|>0})}{(\gamma_2 + \varepsilon_2)} \\ \frac{(\gamma_3 u_3 |_{\|x\|+\|y\|>0, \|w\|=0} + \varepsilon_3 u_3 |_{\|x\|+\|y\|=0, \|w\|>0})}{(\gamma_3 + \varepsilon_3)} \end{pmatrix}, \quad (45)$$

in which u_1 , u_2 , and u_3 are described by (36), we will have

$$\lim_{t \rightarrow +\infty} \|Ax + By + Dw - (C + E)z\| = 0. \quad (46)$$

Hence, the drive systems (4), (5), and (6) can achieve combination synchronization-I with the response system (7).

For demonstrating the second case of combination synchronization-II, an example of numerical simulation is provided in the following. The parameter values for the combination synchronization-II and the dynamics of the four chaotic systems are given as $\alpha_i = \beta_i = \gamma_i = \delta_i = \eta_i = \varepsilon_i = 1$ ($i = 1, 2, 3$), $a_1 = 10$, $b_1 = 28$, $c_1 = 8/3$, $a_2 = 35$, $b_2 = 3$, $c_2 = 28$, $a_3 = 0.2$, $b_3 = 0.2$, $c_3 = 5.7$, $a_4 = 36$, $b_4 = 3$, and $c_4 = 20$. Then based on the above theoretical calculations, the controller can be determined as

$$\begin{aligned} u_1 &= s_1 (26x_1 - 26x_2 + y_1 - y_2) \\ &\quad + s_2 (36w_1 - 37w_2 - w_3), \\ u_2 &= -s_1 \left(56 + \frac{5}{12} - x_3 - y_3 \right) e_{11} \\ &\quad + 14 \left(s_1 \left(e_{12} - \frac{35}{36} e_{11} \right) + s_2 \left(e_{22} - \frac{35}{36} e_{21} \right) \right) \\ &\quad - s_2 \left(56 + \frac{5}{12} - w_3 \right) e_{21} \\ &\quad + s_1 (28x_1 - 21x_2 - 7y_1 + 8y_2 + x_1 y_3 + x_3 y_1) \\ &\quad + s_2 (w_1 w_3 + w_1 - 19.8w_2), \\ u_3 &= -s_1 \left[\frac{35}{36} e_{11}^2 + \left(x_2 + y_2 + \frac{35}{36} (x_1 + y_1) \right) e_{11} \right] \\ &\quad - s_2 \left[\frac{35}{36} e_{21} (e_{21} + w_1) + w_2 e_{21} \right] \\ &\quad + 2 (s_1 e_{13} + s_2 e_{23}) \\ &\quad + s_1 \left(\frac{1}{3} x_3 - x_1 y_2 - x_2 y_1 \right) \\ &\quad - s_2 (w_1 w_2 - w_1 w_3 + 2.7w_3 - 0.2). \end{aligned} \quad (47)$$

FIGURE 4: Time response of synchronization errors e_{ij} , $i=1, 2$ and $j=1, 2, 3$, for the combination synchronization-II of Case 2 among systems (4), (5), (6), and (7).

With this controller, the error system can be changed as the following (notice that $s_1 = 1$ and $s_2 = 0$ for the dynamics of e_{11} , e_{12} , and e_{13} , $s_1 = 0$ and $s_2 = 1$ for the dynamics of e_{21} , e_{22} , and e_{23}):

$$\begin{aligned} \dot{e}_{11} &= 36(e_{12} - e_{11}), \\ \dot{e}_{12} &= 34e_{12} - \left(70 + \frac{1}{36} \right) e_{11} - e_{11}e_{13} - (x_1 + y_1)e_{13}, \\ \dot{e}_{13} &= e_{11}e_{12} - e_{13} - \frac{35}{36}e_{11}^2 + (x_1 + y_1)e_{12} \\ &\quad - \frac{35}{36}(x_1 + y_1)e_{11}, \\ \dot{e}_{21} &= 36(e_{22} - e_{21}), \\ \dot{e}_{22} &= 34e_{22} + \left(43 - \frac{7}{36} \right) e_{21} - e_{21}e_{23} - w_1e_{23}, \\ \dot{e}_{23} &= e_{21}e_{22} - \frac{35}{36}e_{21}e_{21} + w_1e_{22} - \frac{35}{36}w_1e_{21} - e_{23}. \end{aligned} \quad (48)$$

By numerically solving the above error system, Figure 4 is plotted. Figure 4 shows that the synchronization errors e_{ij} ($i = 1, 2$ and $j = 1, 2, 3$) approach zero as time increases. This suggests that the combination synchronization-II of Case 2 as described above is realized. The time response of states $x_i + y_i$, w_i , and z_i as shown in Figure 5 and their comparison further prove that.

FIGURE 5: Comparison of states between $x_i + y_i$ and z_i , and w_i and z_i , $i = 1, 2, 3$.

5. Discussion and Conclusions

This research develops a new type of combination synchronization, which is referred as combination synchronization-II. Based on the active backstepping design method, the combination synchronization-II between three drive systems and one response system is achieved. Numerical simulations are provided to show the feasibility and effectiveness of the proposed combination synchronization approach, with applications in four classical chaotic systems. Moreover, it should be noticed that such application can be extended to other chaotic systems. For example, when multiple memristor systems of infinite chaotic attractors stay at different chaotic states, the combination synchronization-II can take these memristor chaotic systems into the same oscillation.

The proposed combination synchronization approach develops the synchronization techniques among multiple chaotic systems. On one hand, the proposed approach potentially has wider applications than former combination synchronizations. On the other hand, the proposed approach

can be applied for more complex system combinations. Via comparison with previous approaches, the following advantages of combination synchronization-II can be emphasized:

- (1) The combination synchronization-II is more general than the combination synchronization-I or combination complex synchronization. Based on the norm properties and the conditions of combination synchronization-II, the sufficient conditions for the combination synchronization-I or the combination complex synchronization can be deduced.
- (2) The combination synchronization-II can be applied for a variety of system combination modes than the combination synchronization-I or the combination complex synchronization. This shows advantages in practical applications. For example, in secure communication when several signals are transmitted, diversified modes of system combinations can effectively increase antitranslated capability.

- (3) Based on the proposed approach, an applicable universal combination synchronization scheme can be designed for multiple chaotic systems which feature series and/or parallel system combinations. This enables wider potential applications of the combination synchronization approach in many fields.

Data Availability

The data of numerical results are generated during the study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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