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*Research Article*

# Optimal Preview Control for Linear Discrete-Time Periodic Systems

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In this paper, the optimal preview tracking control problem for a class of linear discrete-time periodic systems is investigated and the method to design the optimal preview controller for such systems is given. Initially, by fully considering the characteristic that the coefficient matrices are periodic functions, the system can be converted into a time-invariant system through lifting method. Then, the original problem is also transformed into the scenario of time-invariant system. Later on, the augmented system is constructed and the preview controller of the original system is obtained with the help of existing preview control method. The controller comprises integrator, state feedback, and preview feedforward. Finally, the simulation example shows the effectiveness of the proposed preview controller in improving the tracking performance of the close-loop system.

## 1. Introduction

The basic problem of the preview control is how to utilize the future values of the reference signal or disturbance signal to improve the control performance of the system [1, 2]. In the recent era, the preview control theory has been extensively considered in many fields, such as singular systems [3, 4], time delay systems [5], stochastic systems [6, 7], and multi-agent systems [8]. Meanwhile, the application of preview control theory has also made great progress. In reference [9], the problem of multi-model adaptive preview control for discrete-time systems with large uncertainties was discussed. Reference [10] applied the preview control to the suspension system of the vehicle model, and the proposed design method of preview controller enhanced the performance of active suspension control system. Reference [11] studied the applications of preview control in the UAV landing control system, in which the preview control theory was used to design path optimization scheme to improve the mobility and rapid-reaction capability. In reference [12], the theory and method of preview control is used in the control problem of wind turbines. In addition, preview control has great potential for application in high-tech fields, such as aerocraft, electronic power system, and cruise missile [13].

The linear periodic system is one of the linear time-varying systems, which is often seen in engineering practice as described in references [14, 15]. Reference [16] revealed the application of linear periodic system in satellite attitude control. Reference [17] dealt with the vibration attenuation problem of the helicopter transmission system. Because the vibration of the helicopter transmission system was a typical periodic vibration, the problem could be described by a linear periodic system model. So far, the linear periodic systems have been widely used in hard disk drive servo systems, wind turbines, automotive engines, and other fields [18, 19]. It has also been extensively used in aerial systems, communication systems, image compression systems, speech processing, and so forth [20]. Recently, many important theoretical results of linear periodic systems have been achieved. Reference [21] presented a new lifting method which converted the standard discrete-time linear periodic system into an augmented linear time-invariant system and applied the optimal control theory to the augmented linear time-invariant system.

Although the preview control theory and periodic system theory have made many achievements, it needs to be pointed out that there is seldom a result about the preview control for periodic system. Hence, the study of optimal preview control for periodic system becomes a new issue. Certainly,

the periodic system can be treated as a time-varying system, and the preview controller can be designed by the method as mentioned in reference [22]. Nevertheless, if we take the periodicity of the coefficient matrices into full consideration, a more effective controller can be designed. This study focuses on such a problem. The core is to convert the discrete-time periodic system into a time-invariant system by the lifting method, and then a preview controller is designed to improve the tracking performance of the close-loop system.

## 2. Problem Formulation

Consider the discrete-time linear system

$$\begin{aligned} x(k+1) &= A(k)x(k) + B(k)u(k) \\ y(k) &= Cx(k) \end{aligned} \tag{1}$$

where $x(k) \in R^n$, $u(k) \in R^r$, $y(k) \in R^m$ represent the state vector, the input vector, and the output vector of the system. $A(k) \in R^{n\times n}, B(k) \in R^{n\times r}, C \in R^{m\times n}$ are the coefficient matrices of the system. Assume that $A(k)$ and $B(k)$ take $p$ as a period, namely,

$$\begin{aligned} A(k+p) &= A(k), \\ B(k+p) &= B(k) \end{aligned} \tag{2}$$

A system which satisfies the above characteristics is called the periodic system. Obviously, system (1) is a periodic system.

For convenience, denote $A_s = A(k)$, $B_s = B(k)$, where, $s \equiv k(\mathrm{mod}\, p)$ $(s = 0, 1, 2, \cdots, p-1)$.

Construct the matrices

$$\overline{A} = \begin{bmatrix} 0 & 0 & \cdots & 0 & A_0 \\ 0 & 0 & \cdots & 0 & A_1A_0 \\ 0 & 0 & \cdots & 0 & A_2A_1A_0 \\ \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & \cdots & 0 & A_{p-1}A_{p-2}\cdots A_0 \end{bmatrix} \in R^{(pn)\times(pn)} \tag{3}$$

$$\overline{B} = \begin{bmatrix} B_0 & 0 & 0 & \cdots & 0 \\ A_1B_0 & B_1 & 0 & \cdots & 0 \\ A_2A_1B_0 & A_2B_1 & B_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{p-1}A_{p-2}\cdots A_1B_0 & A_{p-1}A_{p-2}\cdots A_2B_1 & A_{p-1}A_{p-2}\cdots A_3B_2 & \cdots & B_{p-1} \end{bmatrix} \in R^{(pn)\times(pr)} \tag{4}$$

$$\overline{C} = \mathrm{diag}\left(\underbrace{C, C, \cdots, C}_{p}\right) \tag{5}$$

Now, some standard assumptions of system (1) are given below.

(A1) $(\overline{A}, \overline{B})$ is stabilizable and the matrix $\begin{bmatrix} \overline{C} & 0 \\ I-\overline{A} & \overline{B} \end{bmatrix}$ has full row rank.

(A2) $(\overline{C}, \overline{A})$ is detectable.

(A3) The reference signal $R(k)$ is previewable in the future $M_R$ period; that is, the preview length of reference signal is $pM_R$. In other words, at the current time $k$, the values of the reference signals $R(k), R(k+1), R(k+2), \cdots R(k+pM_R)$ are available. Additionally, the future values of the reference signals after $pM_R$ are assumed to be constant, namely,

$$R(j) = R(k+pM_R), \quad j \geq k + pM_R + 1 \tag{6}$$

Define the error signal by

$$e(k) = y(k) - R(k) \tag{7}$$

The objective of this article is to design a controller with preview compensation such that the output vector tracks the reference signal asymptotically.

The difference operator $\Delta$ is needed in the theoretical deduction, which is defined by

$$\Delta v(k) = v(k) - v(k-1) \tag{8}$$

for any vector $v(k)$.

## 3. The Lifting of Periodic System

The discrete-time linear periodic system (1) will be converted into a time-invariant system by employing the lifting method [21] as follows.

For any $k$, $k = pi, pi+1, \cdots, pi+(p-1)$, substituting $k$ into the state equation of system (1), then formula (9) can be obtained

$$\begin{aligned} x(pi+1) &= A_0 x(pi) + B_0 u(pi) \\ x(pi+2) &= A_1 x(pi+1) + B_1 u(pi+1) \\ x(pi+3) &= A_2 x(pi+2) + B_2 u(pi+2) \\ &\vdots \\ x(pi+p) &= A_{p-1} x(pi+p-1) + B_{p-1} u(pi+p-1) \end{aligned} \tag{9}$$

This is another description of system (1). Further, substitution of the first equation of (9) into the second one leads to

$$\begin{aligned} x(pi+2) &= A_1 [A_0 x(pi) + B_0 u(pi)] + B_1 u(pi+1) \\ &= A_1 A_0 x(pi) + A_1 B_0 u(pi) \\ &\quad + B_1 u(pi+1) \end{aligned} \tag{10}$$

Substituting the above equation into the third one of (9) gives

$$\begin{aligned} &x(pi+3) \\ &\quad = A_2 [A_1 A_0 x(pi) + A_1 B_0 u(pi) + B_1 u(pi+1)] \\ &\qquad + B_2 u(pi+2) \\ &\quad = A_2 A_1 A_0 x(pi) + A_2 A_1 B_0 u(pi) \\ &\qquad + A_2 B_1 u(pi+1) + B_2 u(pi+2) \end{aligned} \tag{11}$$

The remainder is analogous to the above procedure. We obtain

$$\begin{aligned} x(pi+p) = {} & A_{p-1} A_{p-2} \cdots A_1 A_0 x(pi) \\ & + A_{p-1} A_{p-2} \cdots A_1 B_0 u(pi) \\ & + A_{p-1} A_{p-2} \cdots A_2 B_1 u(pi+1) + \cdots \\ & + B_{p-1} u(pi+p-1) \end{aligned} \tag{12}$$

Therefore, another representation of the state equation of system (1) is

$$\overline{x}(i+1) = \overline{A}\overline{x}(i) + \overline{B}\overline{u}(i) \tag{13}$$

where

$$\overline{x}(i) = \begin{bmatrix} x(p(i-1)+1) \\ x(p(i-1)+2) \\ x(p(i-1)+3) \\ \vdots \\ x(p(i-1)+p) \end{bmatrix}, \quad \overline{u}(i) = \begin{bmatrix} u(pi) \\ u(pi+1) \\ u(pi+2) \\ \vdots \\ u(pi+p-1) \end{bmatrix} \tag{14}$$

$\overline{A}$ and $\overline{B}$ can be seen in formulas (3) and (4), respectively.

Consequently, the state equation of system (1) has been converted into (13) by operating the lifting method. For all time $i$, the coefficient matrices $\overline{A}, \overline{B}$ of system (13) are constant matrices. Thus, it is a time-invariant system.

Proceeding as in the lifting procedure of $x(k)$, $y(k)$ can be lifted as $\overline{y}(i) = \begin{bmatrix} y(p(i-1)+1) \\ y(p(i-1)+2) \\ \vdots \\ y(p(i-1)+p) \end{bmatrix}$. From the output of system (1), it is known that $\overline{y}(i)$ can be taken as the output of system (13); the output equation is

$$\overline{y}(i) = \overline{C}\overline{x}(i) \tag{15}$$

where $\overline{C}$ is given in (5). Thus, the lifted system is as follows:

$$\begin{aligned} \overline{x}(i+1) &= \overline{A}\overline{x}(i) + \overline{B}\overline{u}(i) \\ \overline{y}(i) &= \overline{C}\overline{x}(i) \end{aligned} \tag{16}$$

Now, we intend to design the preview controller for system (1) by utilizing the invariant-time system (16).

The error vector as well as reference vector can also be lifted, corresponding to the lifting method of the state and output equations. The lifting method is basically described that, from the beginning time, each $p$ components is divided into a segment and each segment is arranged into a column vector, which is taken as a value of the corresponding vector

after lifting. That is, after lifting, the reference vector and the error vector are, respectively,

$$\overline{R}(i) = \begin{bmatrix} R(p(i-1)+1) \\ R(p(i-1)+2) \\ R(p(i-1)+3) \\ \vdots \\ R(p(i-1)+p) \end{bmatrix},$$
$$\overline{e}(i) = \begin{bmatrix} e(p(i-1)+1) \\ e(p(i-1)+2) \\ e(p(i-1)+3) \\ \vdots \\ e(p(i-1)+p) \end{bmatrix} = \overline{y}(i) - \overline{R}(i) \tag{17}$$

In order to design the controller which meets our requirement, we introduce the quadratic performance index for system (16)

$$J = \sum_{i=1}^{\infty} \left[ \overline{e}^T(i) \overline{Q} \overline{e}(i) + \Delta\overline{u}^T(i) \overline{H} \Delta\overline{u}(i) \right] \tag{18}$$

where

$$\overline{Q} > 0, \quad \overline{H} > 0 \tag{19}$$

Note that

$$\Delta\overline{u}(i) = \overline{u}(i) - \overline{u}(i-1) = \begin{bmatrix} u(pi) \\ u(pi+1) \\ u(pi+2) \\ \vdots \\ u(pi+p-1) \end{bmatrix} - \begin{bmatrix} u(p(i-1)) \\ u(p(i-1)+1) \\ u(p(i-1)+2) \\ \vdots \\ u(p(i-1)+p-1) \end{bmatrix} \tag{20}$$

Using $\Delta\overline{u}(i)$ instead of $\overline{u}(i)$ in the quadratic performance index can make the close-loop system contain integrators, so that it can eliminate the static error [23].

Thus, a preview control problem of system (16) can be obtained. The quadratic performance index is given in (18), the previewable reference signal is defined by $\overline{R}(i)$, and the preview length of the reference signal is denoted by $M_R$. Namely, in the current time $i$, the current value $\overline{R}(i)$ and the $M_R$ step future values $\overline{R}(i+1), \overline{R}(i+2), \cdots \overline{R}(i+M_R)$ are available. After $M_R$, it is considered to be a constant vector.

## 4. Construction of Augmented System

By using the method in [24], the difference operator is applied to both sides of the state equation of (16) and $\overline{e}(i+1)$. Then, by combining them,

$$X_0(i+1) = \widetilde{A}_0 X_0(i) + \widetilde{B}_0 \Delta\overline{u}(i) + \widetilde{G}_{R0} \Delta\overline{R}(i+1)$$
$$\overline{e}(i) = \widetilde{C}_0 X_0(i) \tag{21}$$

can be derived, where

$$X_0(i) = \begin{bmatrix} \overline{e}(i) \\ \Delta\overline{x}(i) \end{bmatrix}$$

$$\widetilde{A}_0 = \begin{bmatrix} I_{pm} & \overline{C}\,\overline{A} \\ 0 & \overline{A} \end{bmatrix} = \left[\begin{array}{ccccc|cccc} I_m & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & I_m & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I_m & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & I_m & 0 & 0 & \cdots & 0 \\ \hline 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \end{array}\right.\left.\begin{array}{c} CA_0 \\ CA_1A_0 \\ \vdots \\ CA_{p-2}\cdots A_0 \\ CA_{p-1}A_{p-2}\cdots A_0 \\ \hline A_0 \\ A_1A_0 \\ \vdots \\ A_{p-2}\cdots A_0 \\ A_{p-1}A_{p-2}\cdots A_0 \end{array}\right] \in R^{(pm+pn)\times(pm+pn)}$$

$$\widetilde{B}_0 = \begin{bmatrix} \overline{C}\,\overline{B} \\ \overline{B} \end{bmatrix} = \left[\begin{array}{ccccc} CB_0 & 0 & 0 & \cdots & 0 \\ CA_1B_0 & CB_1 & 0 & \cdots & 0 \\ CA_2A_1B_0 & CA_2B_1 & CB_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ CA_{p-1}A_{p-2}\cdots A_1B_0 & CA_{p-1}A_{p-2}\cdots A_2B_1 & CA_{p-1}A_{p-2}\cdots A_3B_2 & \cdots & CB_{p-1} \\ \hline B_0 & 0 & 0 & \cdots & 0 \\ A_1B_0 & B_1 & 0 & \cdots & 0 \\ A_2A_1B_0 & A_2B_1 & B_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ A_{p-1}A_{p-2}\cdots A_1B_0 & A_{p-1}A_{p-2}\cdots A_2B_1 & A_{p-1}A_{p-2}\cdots A_3B_2 & \cdots & B_{p-1} \end{array}\right] \in R^{(pm+pn)\times(pr)}$$

$$\widetilde{C}_0 = \left[I_{pm} \quad 0_{(pm)\times(pn)}\right] = \left[\begin{array}{cccc|cccc} I_m & 0 & \cdots & 0 & 0 & 0 & \cdots & 0 \\ 0 & I_m & \cdots & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I_m & 0 & 0 & \cdots & 0 \end{array}\right] \in R^{(pm)\times(pm+pn)}$$

$$\widetilde{G}_{R0} = \begin{bmatrix} -I_{pm} \\ 0_{(pn)\times(pm)} \end{bmatrix} \in R^{(pm+pn)\times(pm)} \tag{22}$$

For system (21), the quadratic performance index (18) can be rewritten as

$$J = \sum_{i=1}^{\infty} \left[X_0^T(i)\,QX_0(i) + \Delta\overline{u}^T(i)\,\overline{H}\Delta\overline{u}(i)\right] \tag{23}$$

where

$$Q = \widetilde{C}_0^{\,T}\overline{Q}\widetilde{C}_0 = \begin{bmatrix} \overline{Q} & 0 \\ 0 & 0 \end{bmatrix} \in R^{(pm+pn)\times(pm+pn)} \tag{24}$$

So far, the original tracking problem is reduced to design an optimal preview controller for system (21) that minimizes the quadratic performance index (23).

## 5. Design of Optimal Preview Controller

Based on the theory of preview control, the following theorem is obtained.

**Theorem 1.** *Supposing that (A1), (A2), and (A3) hold, the preview controller for system (21) which minimizes the quadratic performance index (23) is*

$$\begin{aligned} \Delta\overline{u}(i) &= F_0X_0(i) + \sum_{j=1}^{M_R} F_R(j)\,\Delta\overline{R}(i+j) \\ &= F_e\overline{e}(i) + F_x\Delta\overline{x}(i) + \sum_{j=1}^{M_R} F_R(j)\,\Delta\overline{R}(i+j) \end{aligned} \tag{25}$$

*where*

$$\begin{aligned} F_0 &= \left[F_e \quad F_x\right] = -\left[\overline{H} + \widetilde{B}_0^T P\widetilde{B}_0\right]^{-1}\widetilde{B}_0^T P\widetilde{A}_0 \\ F_e &\in R^{(pr)\times(pm)}, \\ F_x &\in R^{(pr)\times(pn)} \\ F_R(j) &= -\left[\overline{H} + \widetilde{B}_0^T P\widetilde{B}_0\right]^{-1}\widetilde{B}_0^T\left(\xi^T\right)^{j-1}P\widetilde{G}_{R0} \\ &\in R^{(pr)\times(pm)} \quad (j = 1, \cdots, M_R) \\ \xi &= \widetilde{A}_0 + \widetilde{B}_0F_0 \end{aligned} \tag{26}$$

*P is the solution of the following Riccati equation:*

$$P = Q + \widetilde{A}_0^T P\widetilde{A}_0 - \widetilde{A}_0^T P\widetilde{B}_0\left[\overline{H} + \widetilde{B}_0^T P\widetilde{B}_0\right]^{-1}\widetilde{B}_0^T P\widetilde{A}_0 \tag{27}$$

*Proof.* First, from the references [24, 25], it can be obtained that the theorem holds provided that $(\widetilde{A}_0, \widetilde{B}_0)$ is stabilizable, $(Q^{1/2}, \widetilde{A}_0)$ is detectable, and the reference signal is previewable.

Noting the structural relationship between $\widetilde{A}_0$ and $\overline{A}$, $\widetilde{B}_0$ and $\overline{B}$, according to the results in [24], it follows that the pair $(\widetilde{A}_0, \widetilde{B}_0)$ is stabilizable if and only if $(\overline{A}, \overline{B})$ is stabilizable and the matrix $\left[\begin{smallmatrix} \overline{C} & 0 \\ I-\overline{A} & \overline{B} \end{smallmatrix}\right]$ has full row rank. Namely, when (A1) holds, $(\widetilde{A}_0, \widetilde{B}_0)$ is stabilizable. Similarly, based on the results which is proved in [23], the pair $(Q^{1/2}, \widetilde{A}_0)$ is detectable

if and only if $(\overline{C}, \overline{A})$ is detectable. That is, when A2 holds, $(Q^{1/2}, \widetilde{A}_0)$ is detectable. Therefore, if (A1), (A2), and (A3) are satisfied, then the theorem is established. This completes the proof. □

By solving $\overline{u}(i)$ from (25), Theorem 2 is concluded.

**Theorem 2.** *Supposing that (A1), (A2), and (A3) hold, the preview controller of system (1) is given by*

$$\begin{aligned} u(pi+q-1) &= u(q-1) + \sum_{s=1}^{i}\sum_{j=1}^{p} F_e(q,j)\, e(p(s-1) \\ &\quad + j) + \sum_{j=1}^{p} F_x(q,j)\left[x(p(i-1)+j) - x(-p+j)\right] \\ &\quad + \sum_{j=1}^{M_R}\sum_{s=1}^{p} F_R(q,s,j) \\ &\quad \cdot \left[R(p(i+j-1)+s) - R(p(j-1)+s)\right], \\ &\qquad q = 1, 2 \cdots p \end{aligned} \tag{28}$$

*where $u(0), u(1), \cdots, u(p-1)$ and $x(-p+1), x(-p+2), \cdots, x(0)$ are the initial values of the input signal as well as the state vector, respectively.*

*Proof.* Based on (25), the following can be derived:

$$\begin{aligned} \overline{u}(i) &= \overline{u}(i-1) + F_e \overline{e}(i) + F_x\left[\overline{x}(i) - \overline{x}(i-1)\right] \\ &\quad + \sum_{j=1}^{M_R} F_R(j)\, \Delta\overline{R}(i+j) \end{aligned} \tag{29}$$

Let $i = 1, 2, \cdots, i$ in the above equation, and adding them together, we get

$$\begin{aligned} \overline{u}(i) &= \overline{u}(0) + F_e \sum_{s=1}^{i} \overline{e}(s) + F_x\left[\overline{x}(i) - \overline{x}(0)\right] \\ &\quad + \sum_{s=1}^{i}\sum_{j=1}^{M_R} F_R(j)\, \Delta\overline{R}(s+j) \end{aligned} \tag{30}$$

Also

$$\begin{aligned} \overline{u}(i) &= \overline{u}(0) + F_e \sum_{s=1}^{i} \overline{e}(s) + F_x\left[\overline{x}(i) - \overline{x}(0)\right] \\ &\quad + \sum_{j=1}^{M_R} F_R(j) \sum_{s=1}^{i} \Delta\overline{R}(s+j) \end{aligned} \tag{31}$$

Namely,

$$\begin{aligned} \overline{u}(i) &= \overline{u}(0) + F_e \sum_{s=1}^{i} \overline{e}(s) + F_x\left[\overline{x}(i) - \overline{x}(0)\right] \\ &\quad + \sum_{j=1}^{M_R} F_R(j)\left[\overline{R}(i+j) - \overline{R}(j)\right] \end{aligned} \tag{32}$$

In order to see the structure of the controller clearly, we partition $F_e$, $F_x$, and $F_R(j)$ in Theorem 1 as

$$\begin{aligned} F_e &= \begin{bmatrix} F_e^{(1)} \\ F_e^{(2)} \\ \vdots \\ F_e^{(p)} \end{bmatrix} \\ &= \begin{bmatrix} F_e(1,1) & F_e(1,2) & \cdots & F_e(1,p) \\ F_e(2,1) & F_e(2,2) & \cdots & F_e(2,p) \\ \vdots & \vdots & & \vdots \\ F_e(p,1) & F_e(p,2) & \cdots & F_e(p,p) \end{bmatrix} \\ F_x &= \begin{bmatrix} F_x^{(1)} \\ F_x^{(2)} \\ \vdots \\ F_x^{(p)} \end{bmatrix} \\ &= \begin{bmatrix} F_x(1,1) & F_x(1,2) & \cdots & F_x(1,p) \\ F_x(2,1) & F_x(2,2) & \cdots & F_x(2,p) \\ \vdots & \vdots & & \vdots \\ F_x(p,1) & F_x(p,2) & \cdots & F_x(p,p) \end{bmatrix} \\ F_R(j) &= \begin{bmatrix} F_R^{(1)}(j) \\ F_R^{(2)}(j) \\ \vdots \\ F_R^{(p)}(j) \end{bmatrix} \\ &= \begin{bmatrix} F_R(1,1,j) & F_R(1,2,j) & \cdots & F_R(1,p,j) \\ F_R(2,1,j) & F_R(2,2,j) & \cdots & F_R(2,p,j) \\ \vdots & \vdots & & \vdots \\ F_R(p,1,j) & F_R(p,2,j) & \cdots & F_R(p,p,j) \end{bmatrix} \end{aligned} \tag{33}$$

where $F_e(i,j) \in R^{r\times m}$, $F_x(i,j) \in R^{r\times n}$ and $F_R(i,j) \in R^{r\times m}$ $(i, j = 1, 2, \cdots, p)$.

Then, the following equations are derived.

$$\begin{aligned}
&\begin{bmatrix} u(pi) \\ u(pi+1) \\ \vdots \\ u(pi+p-1) \end{bmatrix} = \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(p-1) \end{bmatrix} + \begin{bmatrix} F_e^{(1)} \\ F_e^{(2)} \\ \vdots \\ F_e^{(p)} \end{bmatrix} \sum_{s=1}^{i} \overline{e}(s) \\
&+ \begin{bmatrix} F_x^{(1)} \\ F_x^{(2)} \\ \vdots \\ F_x^{(p)} \end{bmatrix} [\overline{x}(i) - \overline{x}(0)] + \sum_{j=1}^{M_R} \begin{bmatrix} F_R^{(1)}(j) \\ F_R^{(2)}(j) \\ \vdots \\ F_R^{(p)}(j) \end{bmatrix} \left[\overline{R}(i+j)\right. \\
&\left. - \overline{R}(j)\right] = \begin{bmatrix} u(0) \\ u(1) \\ \vdots \\ u(p-1) \end{bmatrix} \\
&+ \begin{bmatrix} F_e(1,1) & F_e(1,2) & \cdots & F_e(1,p) \\ F_e(2,1) & F_e(2,2) & \cdots & F_e(2,p) \\ \vdots & \vdots & & \vdots \\ F_e(p,1) & F_e(p,2) & \cdots & F_e(p,p) \end{bmatrix} \sum_{s=1}^{i} \overline{e}(s) \\
&+ \begin{bmatrix} F_x(1,1) & F_x(1,2) & \cdots & F_x(1,p) \\ F_x(2,1) & F_x(2,2) & \cdots & F_x(2,p) \\ \vdots & \vdots & & \vdots \\ F_x(p,1) & F_x(p,2) & \cdots & F_x(p,p) \end{bmatrix} [\overline{x}(i) - \overline{x}(0)] \\
&+ \sum_{j=1}^{M_R} \begin{bmatrix} F_R(1,1,j) & F_R(1,2,j) & \cdots & F_R(1,p,j) \\ F_R(2,1,j) & F_R(2,2,j) & \cdots & F_R(2,p,j) \\ \vdots & \vdots & & \vdots \\ F_R(p,1,j) & F_R(p,2,j) & \cdots & F_R(p,p,j) \end{bmatrix} \left[\overline{R}(i\right. \\
&\left. + j) - \overline{R}(j)\right]
\end{aligned} \tag{34}$$

Moreover, noticing the definitions of $\overline{e}$, $\overline{x}$, and $\overline{R}$, then the above equation can be written as (28). The proof is complete. □

*Remark 3.* In (28), $\sum_{s=1}^{i}\sum_{j=1}^{p} F_e(q,j)e(p(s-1)+j)$ is the sum of the tracking errors, called integrator, which is helpful to eliminate the static error. It can be recognized that the error integral is increasing in terms of the quantity of a period; $\sum_{j=1}^{p} F_x(q,j)[x(p(i-1)+j) - x(-p+j)]$ stands for state feedback, which is formed by the sum of the difference of the state vectors in a period; $\sum_{j=1}^{M_R}\sum_{s=1}^{p} F_R(q,s,j)[R(p(i+j-1)+s) - R(p(j-1)+s)]$ is the previewable reference signal; it is represented by the sum of the difference of the reference vector in a period. All these reflect the periodicity of the periodic system.

*Remark 4.* It should be noted that the numerical simulation can be derived by using (32), where $\overline{u}(0)$ and $\overline{x}(0)$ are the initial values of the input and state, which can be assigned according to the practical condition. Generally, a simple method is to take $\overline{u}(0)$ and $\overline{x}(0)$ as zero vectors and solve $\overline{u}(i)$. In $\overline{u}(i)$, its components $u(pi), u(pi+1), \cdots, u(pi+p-1)$ are the inputs of the original system (1) in one period.

## 6. Some Discussions

First, system (1) is converted into system (9), and then it is transformed into system (16). After that, a time-invariant system which we can deal with can be obtained. In order to ensure the stabilizability and detectability of system (16), a natural idea is that the system represented by each equation in (9) (together with its corresponding observation equation) should be stabilizable and detectable. However, the stabilizability and detectability of (16) do not have necessary relationship to the guaranteed conditions of (9). In fact, the condition that $\left[\begin{smallmatrix} C & 0 \\ I-A_i & B_i \end{smallmatrix}\right]$ has full row rank and $(A_i, B_i)$ is stabilizable is not a necessary condition for $\left[\begin{smallmatrix} \overline{C} & 0 \\ I-\overline{A} & \overline{B} \end{smallmatrix}\right]$ to be of full row rank and $(\overline{A}, \overline{B})$ to be stabilizable. For instance, assuming $p = 2$, $A_0 = \left[\begin{smallmatrix} 1 & 0 \\ 0 & 1 \end{smallmatrix}\right]$, $B_0 = \left[\begin{smallmatrix} 1 \\ 0 \end{smallmatrix}\right]$, $A_1 = \left[\begin{smallmatrix} 1 & 2 \\ 2 & 1 \end{smallmatrix}\right]$, $B_1 = \left[\begin{smallmatrix} 0 \\ 1 \end{smallmatrix}\right]$, $C = \left[\begin{smallmatrix} 1 & 0 \end{smallmatrix}\right]$, it is easy to prove that $(A_0, B_0)$ is not stabilizable. Substituting the above parameters into (3) and (4) yields

$$\begin{aligned}
\overline{A} &= \begin{bmatrix} 0 & A_0 \\ 0 & A_1 A_0 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 2 \\ 0 & 0 & 2 & 1 \end{bmatrix}, \\
\overline{B} &= \begin{bmatrix} B_0 & 0 \\ A_1 B_0 & B_1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 1 & 0 \\ 2 & 1 \end{bmatrix}
\end{aligned} \tag{35}$$

A simple computation gives that $(\overline{A}, \overline{B})$ is stabilizable.

In addition, although the matrix $\left[\begin{smallmatrix} C & 0 \\ I-A_0 & B_0 \end{smallmatrix}\right] = \left[\begin{smallmatrix} 1 & 0 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{smallmatrix}\right]$ is not of full row rank,

$$\begin{bmatrix} \overline{C} & 0 \\ I-\overline{A} & \overline{B} \end{bmatrix} = \left[\begin{array}{cccc|cc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ \hline 1 & 0 & -1 & 0 & 1 & 0 \\ 0 & 1 & 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & -2 & 1 & 0 \\ 0 & 0 & -2 & 0 & 2 & 1 \end{array}\right] \tag{36}$$

has full row rank.

Through the same discussion, it can be seen that $(C, A_i)$ $(i = 0, 1, \cdots, p-1)$ being detectable is not a necessary condition for the detectability of $(\overline{C}, \overline{A})$.

Furthermore, the matrix $\begin{bmatrix} C & 0 \\ I-A_i & B_i \end{bmatrix}$ has full row rank and $(A_i, B_i)$ is stabilizable. $(i = 0, 1, \cdots, p-1)$ cannot guarantee that $\begin{bmatrix} \overline{C} & 0 \\ I-\overline{A} & \overline{B} \end{bmatrix}$ has full row rank and $(\overline{A}, \overline{B})$ is stabilizable. An example is also given as follows.

Suppose $p = 2$, $A_0 = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$, $B_0 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$, $A_1 = \begin{bmatrix} 0 & 1 \\ 1 & 1 \end{bmatrix}$, $B_1 = \begin{bmatrix} 0 \\ 1 \end{bmatrix}$. Using the Popov-Belevitch-Hautus (PBH) criterion [26], it can be seen that both $(A_0, B_0)$ and $(A_1, B_1)$ are proved to be controllable. On the other hand, according to these parameters, we obtain

$$\overline{A} = \begin{bmatrix} 0 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}, \qquad \overline{B} = \begin{bmatrix} 1 & 0 \\ 0 & 0 \\ 0 & 0 \\ 1 & 1 \end{bmatrix} \tag{37}$$

However, $\begin{bmatrix} \lambda I - \overline{A} & \overline{B} \end{bmatrix}$ does not have full row rank for $\lambda = 1$, which implies that $(\overline{A}, \overline{B})$ is not stabilizable.

By means of the duality principle, it is immediately known that $(C, A_i)$ $(i = 0, 1, \cdots, p-1)$ being detectable is not a sufficient condition for ensuring the detectability of $(\overline{C}, \overline{A})$.

## 7. Numerical Simulation

Consider a spacecraft pointing and attitude system described by references [27, 28]. By accurate discretization, the state space model of the spacecraft system is described by the matrices

$$\begin{aligned} A_k &= \begin{bmatrix} 0.9506860 & 0.0429866 & 0.4827320 & -2.5564383 \\ -0.0409684 & 0.9721628 & 1.3617328 & 0.5081454 \\ -0.0122736 & 0.0363280 & -0.8671394 & -0.6014295 \\ -0.0346225 & -0.0072209 & 0.3203622 & -0.8456626 \end{bmatrix}, \\ B_k &= 10^{-5} \begin{bmatrix} 0.2220925 \\ -0.1300536 \\ 0.1877217 \\ 0.0271167 \end{bmatrix} \cos(2\omega\pi k) \\ &\quad + 10^{-5} \begin{bmatrix} 0.5035620 \\ 0.4241087 \\ 0.1218290 \\ 0.3583826 \end{bmatrix} \sin(2\omega\pi k) \\ C_k &= \begin{bmatrix} 1 & 1 & 0 & 0 \end{bmatrix} \end{aligned} \tag{38}$$

Take

$$\begin{aligned} \overline{Q} &= \operatorname{diag}(1, 1, 1, 1, 1, 1, 1, 1, 1), \\ \overline{H} &= \operatorname{diag}\left(10^{-11}, 10^{-11}, 10^{-11}, 10^{-11}, 10^{-11}, 10^{-11}, 10^{-11}, 10^{-11}, 10^{-11}, 10^{-11}\right) \end{aligned} \tag{39}$$

In the reference [28], $\omega$ is taken by $0.00103448\, rad/s$; namely, the period $p$ is very large. In order to compare the effectiveness of the preview controller from the figure more clearly, we choose $\omega = 0.1\, rad/s$; that is, the period $p = 10$.

The assumptions (A1), (A2), and (A3) can be satisfied by calculations. Hence, the corresponding augmented system verifies all the conditions of Theorem 2. Solving the Riccati equation gives

$$F_0 = \begin{bmatrix} -0.5922 & -1.2872 & 0.7490 & -0.1486 & 0.2773 & -0.6032 & 0.1403 & -0.1577 & 0.4933 & 0.0531 & 0 & 0 & \cdots & 0 & 0 & -1.0417 & -1.5995 & 0.2095 & 4.1261 \\ 0.0704 & -0.4209 & -0.6373 & 0.0116 & -0.2068 & 0.5135 & -0.1381 & 0.1243 & -0.1447 & 0.0645 & 0 & 0 & \cdots & 0 & 0 & -0.8133 & -0.8423 & -0.2714 & -0.7137 \\ 0.1026 & 0.0696 & -0.2949 & 0.0193 & -0.0256 & -0.5254 & 0.1505 & -0.1428 & 0.0242 & -0.0339 & 0 & 0 & \cdots & 0 & 0 & -0.3624 & -0.6999 & -1.3878 & 1.0837 \\ 0.0465 & 0.6543 & 0.0143 & -0.7378 & -0.1172 & 0.2164 & -0.2011 & 0.2170 & -0.1561 & -0.0277 & 0 & 0 & \cdots & 0 & 0 & -0.0309 & -0.1886 & 0.2577 & 0.4562 \\ -0.1841 & 0.1306 & 1.2790 & 0.3035 & -1.1468 & -0.6372 & 0.1179 & -0.2353 & 0.3345 & -0.0582 & 0 & 0 & \cdots & 0 & 0 & 0.3015 & -0.3191 & -0.4605 & -0.7393 \\ 0.5089 & 0.0105 & -0.0701 & -1.6037 & -0.1457 & 1.2093 & 0.9169 & -0.3621 & -0.0365 & -0.2405 & 0 & 0 & \cdots & 0 & 0 & -0.1104 & 0.6050 & 0.9249 & 0.0670 \\ -0.4614 & 0.0568 & -0.1578 & 0.2684 & -0.7959 & -0.1957 & 0.4255 & 0.5726 & 0.0127 & 0.2044 & 0 & 0 & \cdots & 0 & 0 & -0.3562 & -0.4025 & -0.8919 & 0.7578 \\ 0.5700 & 0.0141 & 0.0051 & -0.2263 & 0.0357 & -0.6042 & 0.0329 & 0.2045 & 0.0146 & -0.0128 & 0 & 0 & \cdots & 0 & 0 & 0.2891 & 0.5975 & 1.1123 & -0.8488 \\ -0.3235 & 0.1352 & -0.1087 & -0.1951 & -0.0309 & 0.1728 & -0.7840 & 0.1119 & 0.7309 & 0.1372 & 0 & 0 & \cdots & 0 & 0 & -0.3652 & -0.2231 & -0.7568 & 0.5953 \\ 0.7691 & 0.0764 & 0.2009 & -0.0417 & -0.2312 & -0.0282 & -0.0400 & -1.3914 & -0.3096 & 1.1180 & 0 & 0 & \cdots & 0 & 0 & 0.2781 & 0.7928 & 1.5003 & -0.8274 \end{bmatrix} \tag{40}$$

$F_0$ is a matrix with the dimension of $10 \times 50$, where "$\cdots$" represents 0.

$$F_e = \begin{bmatrix} -0.5922 & -1.2872 & 0.7490 & -0.1486 & 0.2773 & -0.6032 & 0.1403 & -0.1577 & 0.4933 & 0.0531 \\ 0.0704 & -0.4209 & -0.6373 & 0.0116 & -0.2068 & 0.5135 & -0.1381 & 0.1243 & -0.1447 & 0.0645 \\ 0.1026 & 0.0696 & -0.2949 & 0.0193 & -0.0256 & -0.5254 & 0.1505 & -0.1428 & 0.0242 & -0.0339 \\ 0.0465 & 0.6543 & 0.0143 & -0.7378 & -0.1172 & 0.2164 & -0.2011 & 0.2170 & -0.1561 & -0.0277 \\ -0.1841 & 0.1306 & 1.2790 & 0.3035 & -1.1468 & -0.6372 & 0.1179 & -0.2353 & 0.3345 & -0.0582 \\ 0.5089 & 0.0105 & -0.0701 & -1.6037 & -0.1457 & 1.2093 & 0.9169 & -0.3621 & -0.0365 & -0.2405 \\ -0.4614 & 0.0568 & -0.1578 & 0.2684 & -0.7959 & -0.1957 & 0.4255 & 0.5726 & 0.0127 & 0.2044 \\ 0.5700 & 0.0141 & 0.0051 & -0.2263 & 0.0357 & -0.6042 & 0.0329 & 0.2045 & 0.0146 & -0.0128 \\ -0.3235 & 0.1352 & -0.1087 & -0.1951 & -0.0309 & 0.1728 & -0.7840 & 0.1119 & 0.7309 & 0.1372 \\ 0.7691 & 0.0764 & 0.2009 & -0.0417 & -0.2312 & -0.0282 & -0.0400 & -1.3914 & -0.3096 & 1.1180 \end{bmatrix}$$

$$F_x = \begin{bmatrix} 0 & 0 & \cdots & 0 & 0 & -1.0417 & -1.5995 & 0.2095 & 4.1261 \\ 0 & 0 & \cdots & 0 & 0 & -0.8133 & -0.8423 & -0.2714 & -0.7137 \\ 0 & 0 & \cdots & 0 & 0 & -0.3624 & -0.6999 & -1.3878 & 1.0837 \\ 0 & 0 & \cdots & 0 & 0 & -0.0309 & -0.1886 & 0.2577 & 0.4562 \\ 0 & 0 & \cdots & 0 & 0 & 0.3015 & -0.3191 & -0.4605 & -0.7393 \\ 0 & 0 & \cdots & 0 & 0 & -0.1104 & 0.6050 & 0.9249 & 0.0670 \\ 0 & 0 & \cdots & 0 & 0 & -0.3562 & -0.4025 & -0.8919 & 0.7578 \\ 0 & 0 & \cdots & 0 & 0 & 0.2891 & 0.5975 & 1.1123 & -0.8488 \\ 0 & 0 & \cdots & 0 & 0 & -0.3652 & -0.2231 & -0.7568 & 0.5953 \\ 0 & 0 & \cdots & 0 & 0 & 0.2781 & 0.7928 & 1.5003 & -0.8274 \end{bmatrix} \tag{41}$$

$F_x$ is a matrix with the dimension of $10 \times 40$, where "$\cdots$" represents 0.

The simulation can be performed in three cases; that is, $M_R = 0, M_R = 2, M_R = 9$. The reference signal is set as

$$R(k) = \begin{cases} 0, & k < 100 \\ \dfrac{1}{50}(k - 100), & 100 \le k \le 200 \\ 2, & k > 200 \end{cases} \tag{42}$$

And the initial conditions are

$$x(-9) = x(-8) = x(-7) = x(-6) = x(-5) = x(-4)$$
$$= x(-3) = x(-2) = x(-1) = x(0) = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \end{bmatrix},$$

$$u(0) = u(1) = u(2) = u(3) = u(4) = u(5) = u(6)$$
$$= u(7) = u(8) = u(9) = 0. \tag{43}$$

Figure 1 is the close-loop output response of system (1), where the black line represents the reference signal.

It follows from Figure 1 that the output can track the reference signal more quickly by using the controller with preview compensation. And it can be found that the longer the preview lengths are taken, the better the tracking effects are. It needs to be pointed out that the oscillations in the figure are caused by the periodicity of the system.

In order to see the tracking performance more clearly, Figure 2 gives the tracking error under the conditions of $M_R = 0, M_R = 2, M_R = 9$, respectively (namely, the preview lengths of $R(k)$ are 0, 20 and 90, respectively).

As it can be seen from Figure 2, the introduction of the preview controller ($M_R = 9$) not only makes the tracking error become smaller but also makes the tracking error tend to zero when $k = 98$. The reason why there appears a significant fluctuation after $k = 191$ is that the reference signal changes at $k = 200$.

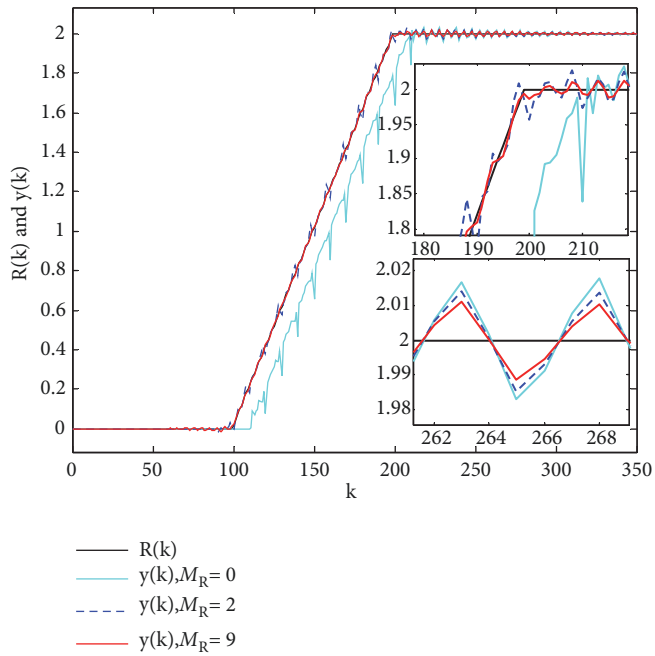


Figure 1: The output responses of the close-loop system ($M_R = 0, M_R = 2, M_R = 9$).

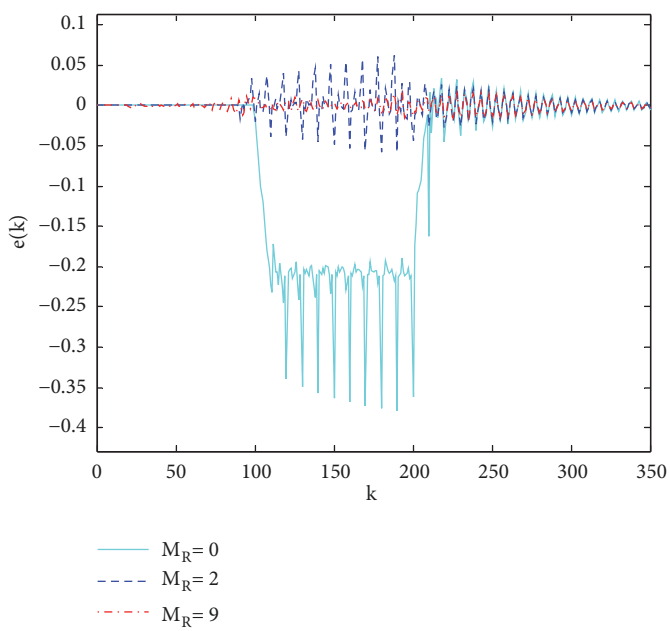


Figure 2: The tracking error.

## 8. Conclusion

Just as the continuous-time linear period system can be transformed into the constant system by a nonsingular transformation, this paper converts the linear discrete-time periodic system into the linear time-invariant system by lifting method. And the original tracking control problem for periodic system (essentially the time-varying system) is equivalently converted to the same problem for the linear time-invariant system. As a result, the controller with the preview compensation for this time-invariant system is designed, and the preview controller for the periodic system is obtained. This current study gives a detailed lifting method to design the controller and carries out the rigorous theoretical deductions. Meanwhile, the study shows that the stabilizability and detectability of the augmented system do not have necessary relationship to the stabilizability and detectability of the subsystem involved in the periodic system. Numerical simulation illustrates that the results of this paper are very effective.

## Data Availability

Data sharing is not applicable to this article as no datasets were generated or analysed during the current study.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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