Hindawi
Mathematical Problems in Engineering
Volume 2019, Article ID 8493298, 8 pages
https://doi.org/10.1155/2019/8493298

Research Article

Optimal Observer Design of State Delay Systems

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Received 7 November 2018; Revised 6 January 2019; Accepted 16 January 2019; Published 31 January 2019

Academic Editor: Xianming Zhang

This paper proposes a new method to design the observer for state delay systems such that (i) state estimation errors converge to zero quickly and (ii), at the same time, a quadratic performance measurement of the deviation of estimates from the actual states is minimized for reducing large error during the transient period of observation. The proposed new approach fuses the merits of both the orthogonal functions approach and evolutionary optimization. One illustrative example is given to verify the effectiveness and efficiency of the proposed new optimization method on performance improvement of state estimations. From the illustrative example, in addition to the asymptotical convergence of the estimated state errors, the performance index for the proposed optimal design approach is clearly much lower than that of the nonoptimal design method.

1. Introduction

Control systems are commonly used in a variety of machines, products, and processes to regulate system output. Most control system design approaches that exploit state feedback control theory are based on the assumption of having sensors to measure the output of a system under control. In short, all the state variables are assumed to be available outputs. However, for many control systems, state variables are not accessible to direct measurement, or the number of measurement devices is limited, possibly due to cost considerations. In order to effectively use state feedback, developing an approach to estimating the system states is inevitable. As a result, an important problem regarding linear multivariable control systems is designing an observer for a given dynamic system. Therefore, many researchers have proposed various methods for designing observers ([1–21] and references therein). On the other hand, time-delay systems are commonly seen in industrial processes, electrical and mechanical systems, economic and population growth, biological systems, etc. [22–29]. Hence, due to their importance and the many predicaments encountered in analyzing and designing them, many interesting research topics have emerged, such as observer design for time-delay systems [30]. In order for the estimated states to be close and converge fast to the actual states of the observed system, many researchers have proposed various observer design methods for time-delay systems ([16, 30–36] and references therein). These existing observer design methods guarantee that the state estimation errors can asymptotically converge to zero. Although the existing approaches can be applied to improve the convergence of the estimated states, they may lead to unsatisfactory estimation results, since a large estimation error may occur during the transient period of observation [19].

For delay-free systems, some research has been devoted to considering the observer design issue of reducing the large error during the transient period of observation [1–4, 6, 19]. However, to the authors' best knowledge, for the observer design of time-delay systems, there is no extant literature investigating the issue of minimizing estimation errors during the transient period of observation. Therefore, the purpose of this paper is to study the observer design problem for state delay systems by concurrently considering the following two design issues: (i) the state estimation errors quickly converge to zero, and (ii) a quadratic performance measurement of the deviation of the estimates from the actual states is minimized so as to reduce the large error occurring during the transient period of observation.

The remainder of the paper is organized as follows. Section 2 describes the problem statement. A new method of designing an optimal observer for state delay systems is proposed in Section 3. An illustrative example is provided in Section 4. Finally, Section 5 gives some conclusions.

2. Problem Statement

Consider the following time-delay system:

$$\dot{x}(t) = A_1 x(t) + A_2 x(t-h) + Bu(t), \quad (1a)$$

$$y(t) = Cx(t) + Du(t), \quad (1b)$$

$$\text{and } x(t) = \emptyset(t), \quad \text{for } t \in [-h, 0), \quad (1c)$$

with $x(0) = k$, where h is a known constant time delay, k is a constant vector, $x(t)$ is the n-dimensional state vector, $u(t)$ is the r-dimensional control vector, $y(t)$ is the m-dimensional output vector, and $\emptyset(t)$ is a given time function vector. In addition, A_1, A_2, B, and C are constant matrices of appropriate dimensions.

For estimating state vector $x(t)$, the state observer for the system (1a), (1b), and (1c) is designed as [37]:

$$\begin{aligned}\dot{\hat{x}}(t) = {} & A_1 \hat{x}(t) + A_2 \hat{x}(t-h) + Bu(t) \\ & + \overline{L}\left(y(t) - \hat{y}(t)\right),\end{aligned} \quad (2a)$$

$$\hat{y}(t) = C\hat{x}(t) + Du(t), \quad (2b)$$

$$\text{and } \hat{x}(t) = 0, \quad \text{for } t \in [-h, 0], \quad (2c)$$

where $\hat{x}(t)$ is the estimate of $x(t)$, $\hat{y}(t)$ is the estimate of $y(t)$, and $\overline{L}$ is the $n \times m$ observer gain matrix to be designed.

The $n \times 1$ vector of state estimation error is defined as

$$e(t) = x(t) - \hat{x}(t), \quad (3)$$

where $e(t) = e_1(t)$ with $e_1(t)$ denoting the state estimation error vector when t belongs to $[0, h]$, $e(t) = e_2(t)$ with $e_2(t)$ denoting the state estimation error vector when t belongs to $[h, \beta]$, and β is the final time which is long enough for state estimation error $e(t)$ to decrease to almost zero. Consequently, for $0 \le t \le h, e(t-h) = e_1(t-h) = x(t-h) - \hat{x}(t-h) = \emptyset(t-h)$.

It can be shown that

$$\begin{aligned}\dot{e}(t) &= \dot{e}_1(t) + \dot{e}_2(t) = \dot{x}(t) - \dot{\hat{x}}(t) \\ &= \overline{A}e(t) + A_2 e(t-h) \\ &= \overline{A}e_1(t) + A_2 e_1(t-h) + \overline{A}e_2(t) + A_2 e_2(t-h),\end{aligned} \quad (4)$$

in which $\overline{A} = A_1 - \overline{L}C$. That is,

$$\dot{e}_1(t) = \overline{A}e_1(t) + A_2 e_1(t-h), \quad (5)$$

$$\text{and } \dot{e}_2(t) = \overline{A}e_2(t) + A_2 e_2(t-h). \quad (6)$$

A quadratic performance measurement of estimation error is defined as

$$J = \int_0^h e_1^T(t) Q_1 e_1(t)\, dt + \int_h^\beta e_2^T(t) Q_2 e_2(t)\, dt, \quad (7)$$

where the matrices Q_1 and Q_2 are $n \times n$ specified positive definite weighting matrices. In addition, $e^T(t), e_1^T(t)$, and $e_2^T(t)$ denote the transpose vectors of $e(t), e_1(t)$, and $e_2(t)$, respectively. Performance measurement J is exploited to design a proper observer that can avoid large estimation error during the transient period of observation.

The optimal observer design problem for the time-delay system is to directly find the observer gain matrix $\overline{L}$ such that the state estimation error vector $e(t)$ converges to zero, while at the same time the quadratic performance measurement defined by (7) is minimized.

3. Optimal Design of the Observer Gain Matrix

For the state observer in (2a), (2b), and (2c), the problem of guaranteeing the state estimation error vector $e(t)$ converges to zero, under the situation that the $n \times m$ observer gain matrix $\overline{L}$ has been given beforehand, to derive an asymptotic stability criterion for checking whether the state estimation error vector $e(t)$ can asymptotically converge to zero or not. For delay-free systems, some researchers [1–4, 6, 19] have considered the following two issues concerning the design of linear observers: (i) the eigenvalues of the linear observers are specified to satisfy desired asymptotic convergence performance, and (ii) a quadratic performance measurement of state estimation error is minimized so as to reduce the transient estimation error. But, for time-delay systems, the eigenvalues of the linear observers cannot be specified to satisfy desired convergence performance. Hence, in what follows, an LMI-based (linear-matrix-inequality-based) asymptotic stability criterion is presented to analyze whether the state estimation error vector $e(t)$ can asymptotically converge to zero or not, where the observer gain matrix $\overline{L}$ has been specified beforehand.

Theorem 1. *The state estimation error vector $e(t)$ in (4) asymptotically converges to zero when the observer gain matrix $\overline{L}$ has been specified beforehand if, for the specified observer gain matrix $\overline{L}$, an $n \times n$ symmetric positive definite matrix $\overline{P}$ exists such that the following inequality is satisfied:*

$$\overline{A}^T \overline{P} + \overline{P}\,\overline{A} + A_2^T A_2 + \overline{P}^2 < 0, \quad (8)$$

where $\overline{A} = A_1 - \overline{L}C$.

Proof. Let the Lyapunov function candidate for the error dynamic system described by (4) be [38]

$$V(t) = V_1(t) + V_2(t), \quad (9)$$

in which

$$V_1(t) = e^T(t)\, \overline{P} e(t), \quad (10)$$

$$\text{and } V_2(t) = \int_{t-h}^{t} e^T(\theta) A_2^T A_2 e(\theta)\, d\theta. \quad (11)$$

Then, the time derivative of $V(t)$ gives

$$\begin{aligned}\dot{V}(t) &\le e^T(t)\left(\overline{A}^T\overline{P}+\overline{P}\,\overline{A}\right)e(t)\\&\quad+e^T(t-h)A_2^TA_2e(t-h)+e^T(t)\overline{P}^2e(t)\\&\quad+e^T(t)A_2^TA_2e(t)-e^T(t-h)A_2^TA_2e(t-h)\\&=e^T(t)\left(\overline{A}^T\overline{P}+\overline{P}\,\overline{A}+A_2^TA_2+\overline{P}^2\right)e(t).\end{aligned}\tag{12}$$

By using (8), it can be shown that $\dot{V}(t) < 0$. Therefore, the state estimation error vector $e(t)$ converges to zero asymptotically. □

By using the Schur complete formula [39], (8) becomes the following LMI form:

$$\begin{bmatrix}\left(A_1-\overline{L}C\right)^T\overline{P}+\overline{P}\left(A_1-\overline{L}C\right)+A_2^TA_2 & \overline{P}\\ \overline{P} & -I_n\end{bmatrix}<0,\tag{13}$$

where I_n is the $n\times n$ identity matrix. In this form of (13), the powerful LMI approach can be adopted to find the solution $\overline{P}$ of (8) when the $n\times m$ observer gain matrix $\overline{L}$ has been previously specified.

By using the asymptotic stability criterion of (8), when the observer gain matrix $\overline{L}$ has been previously given, only the property of asymptotic convergence of the state estimation error vector $e(t)$ can be guaranteed. However, this observer design result is not satisfactory, as a large estimation error may occur during the transient period of observation [19]. Hence, the problem considered here is how to specify the $n\times m$ observer gain matrix $\overline{L}$ in (2a), (2b), and (2c) so that the constraint of the LMI-based condition of the state estimation error vector asymptotically converging to zero in (13) can be satisfied, while the quadratic performance measurement of estimation error in (7) is simultaneously minimized. The procedure for designing the observer gain matrix $\overline{L}$ can be stated as follows.

Step 1. Check the LMI-based constraint condition of the state estimation error vector asymptotically converging to zero in (13).

Step 2. Minimize the quadratic performance measurement of estimation error in (7) for the error dynamic system described by (4).

In order to help design the observer gain matrix, the orthogonal functions approach (OFA) is employed in this paper. The OFA has been successfully applied to investigate various problems of systems and control [22, 40–42]. The key characteristic of OFA is that it converts integral or differential equations into algebraic equations. As a result, the OFA has become very popular in computation because the dynamic equations of the system can be transformed into a set of algebraic equations whose solutions lead to the solution of the original problem [22, 40–42]. Suppose that the vectors $e_1(t)$, $e_2(t)$ and $\emptyset(t-h)$ can be developed approximately in terms of truncated orthogonal functions (OF) representation as described by (14)-(16), respectively,

$$e_1(t)=\sum_{i=0}^{q-1}e_{1i}T_i(t)=E_1T(t),\tag{14}$$

$$e_2(t)=\sum_{i=0}^{q-1}e_{2i}T_i(t)=E_2T(t),\tag{15}$$

$$\text{and }\emptyset(t-h)=\sum_{i=0}^{q-1}\emptyset_iT_i(t)=\overline{\emptyset}T(t),\tag{16}$$

where q is the number of the terms needed for the OF, $T(t)=[T_0(t),T_1(t),\cdots,T_{q-1}(t)]^T$ represents the $q\times 1$ OF basis vector, while $T_i(t)$ $(i=0,1,\cdots,q-1)$ are the orthogonal functions, and e_{1i}, e_{2i} and $\emptyset_i$ $(i=0,1,\cdots,q-1)$ denote the $n\times 1$ coefficient vectors. In addition, $E_i=[e_{i0},e_{i1},\cdots,e_{i,q-1}]$ $(i=1,2)$, and $\overline{\emptyset}=[\emptyset_0,\emptyset_1,\cdots,\emptyset_{q-1}]$ denote the $n\times q$ coefficient matrices.

For $0\le t\le h$, using the OF representations of $e_1(t)$ and $\emptyset(t-h)$ in (14) and (16), and applying the following integral property of the OF [22, 40]

$$\int_\alpha^t T(\tau)\,d\tau=P_\alpha T(t),\tag{17}$$

(5) can be cast into the form

$$E_1-E_0=\overline{A}E_1P_0+A_2\overline{\emptyset}P_0=\overline{A}E_1P_0+\overline{B},\tag{18}$$

where $E_0=[x(0),0,\cdots,0]$, $\overline{B}=A_2\overline{\emptyset}P_0=[\overline{B}_0,\overline{B}_1,\cdots,\overline{B}_{q-1}]$, and the matrix P_0 is equivalent to the matrix P_α in (17) with $\alpha=0$. P_α is the $q\times q$ integration-operational matrix of the OF, in which the entries of matrix P_α depend on the definite choice of the OF basis vector $T(t)$ [22, 40].

Using the Kronecker product, the solution $\widetilde{E}_1$ of (18) can be obtained as

$$\widetilde{E}_1=\left[I_{nq}-P_0^T\otimes\overline{A}\right]^{-1}\widetilde{E}_0,\tag{19}$$

where $\otimes$ denotes the Kronecker product [43], $\widetilde{E}_1=[e_{10}^T,e_{11}^T,\cdots,e_{1,q-1}^T]^T$, $\widetilde{E}_0=[x^T(0),0^T,\cdots,0^T]^T+\widetilde{B}$, $\widetilde{B}=[\overline{B}_0^T,\overline{B}_1^T,\cdots,\overline{B}_{q-1}^T]^T$, and I_{nq} is the $nq\times nq$ identity matrix.

For $h\le t\le\beta$, applying the OF representation of $e_2(t)$ in (15) and using the integral property of the OF in (17), integrating (6) gives

$$E_2-E_h=\overline{A}E_2P_h+A_2E_2\overline{H}P_h,\tag{20}$$

where $e_2(h)=e_1(h)=E_1T(h)$, $E_h=[e_1(h),0,\cdots,0]$, matrix P_α denotes matrix P_h in (17) with $\alpha=h$, and matrix $\overline{H}$ is the $q\times q$ time-delay-operational matrix of orthogonal functions, in which the elements of the delay matrix $\overline{H}$ depend on the specific choice of the OF basis vector $T(t)$ [22, 40].

Applying the Kronecker product, the solution $\widetilde{E}_2$ of Eq. (20) can be obtained as

$$\widetilde{E}_2=\left[I_{nq}-P_h^T\otimes\overline{A}-\left(\overline{H}P_h\right)^T\otimes A_2\right]^{-1}\widetilde{E}_h,\tag{21}$$

where $\widetilde{E}_h=[e_1^T(h),0^T,\cdots,0^T]^T$.

Now, substituting the truncated OF representations of the estimation error vectors $e_1(t)$ and $e_2(t)$ in (14) and (15) into (7), the quadratic performance measurement of (7) becomes

$$\begin{aligned} J &= \int_0^h (E_1 T(t))^T Q_1 (E_1 T(t))\, dt \\ &\quad + \int_h^\beta (E_2 T(t))^T Q_2 (E_2 T(t))\, dt \\ &= \int_0^h trace\left(T(t) T^T(t) R_1\right) dt \\ &\quad + \int_h^\beta trace\left(T(t) T^T(t) R_2\right) dt \\ &= trace\left(\left(\int_0^h T(t) T^T(t)\, dt\, R_1\right)\right) \\ &\quad + trace\left(\left(\int_h^\beta T(t) T^T(t)\, dt\, R_2\right)\right) \\ &= trace\left(W_h R_1\right) + trace\left(W_\beta R_2\right), \end{aligned} \tag{22}$$

in which $R_1 = E_1^T Q_1 E_1$, $R_2 = E_2^T Q_2 E_2$, and the constant matrices W_h and W_β denote, respectively, the product-integration-matrix of two OF basis vectors having different time intervals [22]. From (19), (21), and (22), it can be seen that the OFA-based computational approach does not limit the sizes of both h and β, where h is the given known constant time delay and β is the final time which is given by the control engineer for desiring state estimation error decreased to almost zero.

From the aforementioned procedures for solving E_1 and E_2 of (18) and (20), respectively, it is clear that if the observer gain matrix $\overline{L}$ is given, then E_1 and E_2 can be ascertained, and thus the quadratic performance measurement value in (22) relevant to the observer gain matrix $\overline{L}$ can be calculated. Given another matrix $\overline{L}$ of observer gain, another performance measurement value in (22) can be obtained. That is, the performance measurement value of the algebraic form in (22) is in fact dependent on the observer gain matrix $\overline{L}$, indicating

$$J = G\left(\bar{l}_{11}, \bar{l}_{12}, \cdots, \bar{l}_{nm}\right), \tag{23}$$

where $\bar{l}_{ij}$ ($i = 1, 2, \cdots, n$; $j = 1, 2, \cdots, m$) represent the elements of the observer gain matrix $\overline{L}$. Thus, the optimal observer design problem of the time-delay system in (1a), (1b), and (1c) is to find the optimal $\bar{l}_{ij}$ for a symmetric positive definite matrix $\overline{P}$ making the LMI in (13) holds (i.e., such that the property of the state estimation error vector $e(t)$ asymptotically converging to zero in the theorem is satisfied). At the same time, the performance measurement of the algebraic form in (22) is minimized. This is equivalent to the constrained static optimization problem:

$$\text{Minimize } J = G\left(\bar{l}_{11}, \bar{l}_{12}, \cdots, \bar{l}_{nm}\right) \tag{24}$$

subject to $|\bar{l}_{ij}| \le \overline{L}_{ij}$ and subject to the constraint of there being a symmetric positive definite matrix $\overline{P}$ to let the LMI in (13) holds for $i = 1, 2, \ldots, n$ and $j = 1, 2, \ldots, m$, in which $\overline{L}_{ij}$ are the given positive real numbers. This means that, by using the OFA, the optimal observer design problem for the time-delay system described by (1a), (1b), and (1c) can be substituted by constrained static optimization problems with constrained algebraic equations. It greatly simplifies the optimal observer design problem, where the state estimation error vector $e(t)$ converges to zero, and at the same time the quadratic performance measurement of (7) is minimized. Both genetic algorithm (GA) and particle swarm optimization (PSO) have acquired considerable attention concerning their ability as effective optimization techniques solving complex problems and have been successfully utilized in many areas [29, 44–46]. The GA is adopted in this paper to deal with the complex problem of searching for the optimal observer gain matrix $\overline{L}$ in the algebraic form of (24) subject to the constraint of there being a matrix $\overline{P}$ such that the LMI in (13) holds, where (24) is a complex nonlinear function having continuous variables. In addition, for the parameter optimization problems of complex nonlinear functions having continuous variables, it has been shown that the Hybrid Taguchi-Genetic Algorithm (HTGA) method can obtain better results than existing approaches [47]. The HTGA method is therefore employed in this paper to design the optimal observer for a time-delay system.

Optimal Observer Design Procedures

Step 1 (parameter setting). Input is as follows: population size $\overline{G}$, number of generations, mutation rate p_m, and crossover rate p_c. Output is as follows: the value of J in (22), and the observer gain matrix $\overline{L}$ of dimension $n \times m$.

Step 2 (initialization). First, randomly generate the initial population with the chromosomes of form $\tilde{\theta} = [\bar{l}_{11}, \bar{l}_{12}, \cdots, \bar{l}_{nm}]$ for executing the HTGA. Next, compute the solutions of $\widetilde{E}_1$ and $\widetilde{E}_2$ by applying (19) and (21), and compute J by using (22) which is defined as the fitness function of the HTGA. Then, the fitness values of the initial population feasible for the LMI-based constraint in (13) are calculated.

Step 3 (HTGA method execution [47]). By integrating (19), (21), and (22), the HTGA method is utilized to search for the optimal observer gain matrix $\overline{L}$, where a penalty on the fitness value is given for the chromosome violating the LMI-based constraint in (13).

Step 4 (whether or not the stopping criteria are met). If yes, go to Step 5. Otherwise, go back to Step 3.

Step 5. The optimal gain matrix $\overline{L}$ of the observer and the optimal value of J are found.

Remark 2. For studying the stability of time-delay systems, both delay-independent and delay-dependent criteria have been proposed in the literature. The purpose of both delay-independent and delay-dependent criteria is to guarantee that the steady-state errors are reduced. But the purpose of the proposed observer design method with the quadratic performance measurement of (7) is to yield the improvement in observer transient error performance. Thus, the key point of this paper is to propose a new observer design method for reducing the large error occurring during the transient period of observation, not to study the stability of time-delay systems. Therefore, this proposed design approach can be expected to give an adequately damped response such that a small overshoot can be obtained; thus, this provides better physical performance characteristics in the time domain. Though the proposed theorem is a delay-independent stability criterion, the proposed observer design method can also combine delay-dependent stability criterion to reduce the large error occurring during the transient period of observation.

Remark 3. To conquer the inherent difficulty of giving a priori proper bound of parameters in which the optimal solution is placed, it is desirable to allow the HTGA to dynamically expand the search space. To this end, the search-space expansion schemes developed in [48] are also integrated into the HTGA approach to design the optimal observer gain matrix $\overline{L}$. In addition, in the HTGA approach, the bound $\overline{L}_{ij}$ of parameters of the observer gain matrix $\overline{L}$ can be specified to avoid the case of a high gain value from practical considerations.

4. Illustrative Example

One example is given in this section to verify the efficiency of the proposed method for designing the optimal observer gain matrix $\overline{L}$ such that (i) the state estimation error vector $e(t)$ converges to zero and (ii), at the same time, the quadratic performance measurement of algebraic (7) is minimized for reducing the large error occurring during the transient period of observation. The following evolutionary environments of the HTGA are adopted in this example; the maximum number of generation is 100, the crossover rate is 0.95, the population size is 30, and the mutation rate is 0.01. The type of orthogonal function used in this example is the shifted Chebyshev series with $q = 12$ [22, 40].

Consider the time-delay system described by (1a), (1b), and (1c), where

$$
\begin{gathered}
A_1 = \begin{bmatrix} 1 & 6 \\ -2.6733 & 1.4 \end{bmatrix}, \\
A_2 = \begin{bmatrix} -8.3 & 2 \\ -8.036 & -0.28 \end{bmatrix}, \\
C = \begin{bmatrix} 1 & -5 \\ 2 & 1 \end{bmatrix}, \\
B = I_2, \\
D = I_2, \\
h = 0.1, \\
x(0) = [1, 1]^T \\
\text{and } x(t) = 0, \quad \text{for } t \in [-h, 0).
\end{gathered} \tag{25}
$$

In this example, $Q_1 = Q_2 = I_2$, and $\beta = 1$. By using the proposed optimal observer design method, and applying the LMI Toolbox [39], the optimal observer gain matrix $\overline{L}$ can be obtained as

$$\overline{L} = \begin{bmatrix} -7.362 & 9.0392 \\ -8.7454 & -1.1492 \end{bmatrix}, \tag{26}$$

and the symmetric positive definite matrix $\overline{P}$ can be derived as

$$\overline{P} = \begin{bmatrix} 7.6262 & -6.2157 \\ -6.2157 & 9.1401 \end{bmatrix}. \tag{27}$$

For the optimal observer, the value of J is 2.4053×10^{-3}.

To the authors' best knowledge, for the observer design of time-delay systems, there is no extant literature investigating the issue of minimizing estimation errors during the transient period of observation. That is, for state delay systems, the extant approaches do not consider to minimizing estimation errors during the transient period of observation. So, no existing methods can be used to compare the proposed optimal design approach. Thus, only the comparisons of the state estimation errors of optimal estimation and nonoptimal estimation are given in the simulation. Letting $\widetilde{W} = \overline{P}\,\overline{L}$ and solving the LMI in (13) to find the matrices $\widetilde{W}$ and $\overline{P}$, then the nonoptimal observer gain matrix $\overline{L} = \overline{P}^{-1}\widetilde{W}$ can be obtained as

$$\overline{L} = \begin{bmatrix} 9.8394 & 29.3123 \\ -0.0232 & 3.9302 \end{bmatrix}, \tag{28}$$

where

$$\overline{P} = \begin{bmatrix} 0.6922 & 3.0692 \times 10^{-16} \\ 3.0692 \times 10^{-16} & 0.6922 \end{bmatrix} \tag{29}$$

and the value of J is 3.7821×10^{-2}. The comparisons of the state estimation errors $e_1(t)$ and $e_2(t)$ for the to-be-estimated states $x_1(t)$ and $x_2(t)$ of optimal estimation and nonoptimal estimation are given in Figures 1 and 2, respectively. It can be seen that, by using the proposed optimal design method, the estimated state errors can quickly converge to zero, and at the same time the transient estimation errors have been significantly reduced during the observation period. Furthermore, the performance measurement value J for the proposed optimal design method is 93.64% lower than that for the nonoptimal design approach only using the LMI method. That is, the proposed optimal observer design method can obtain a smaller estimation error during the transient period of observation.

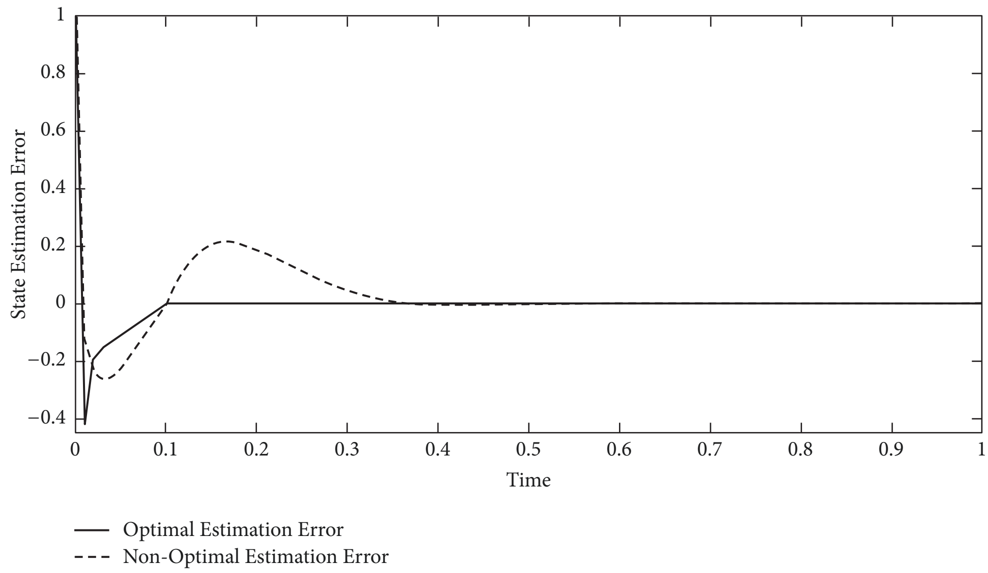

Figure 1: A comparison of state estimation errors $e_1(t)$ for the state $x_1(t)$ of optimal estimation and nonoptimal estimation, respectively.

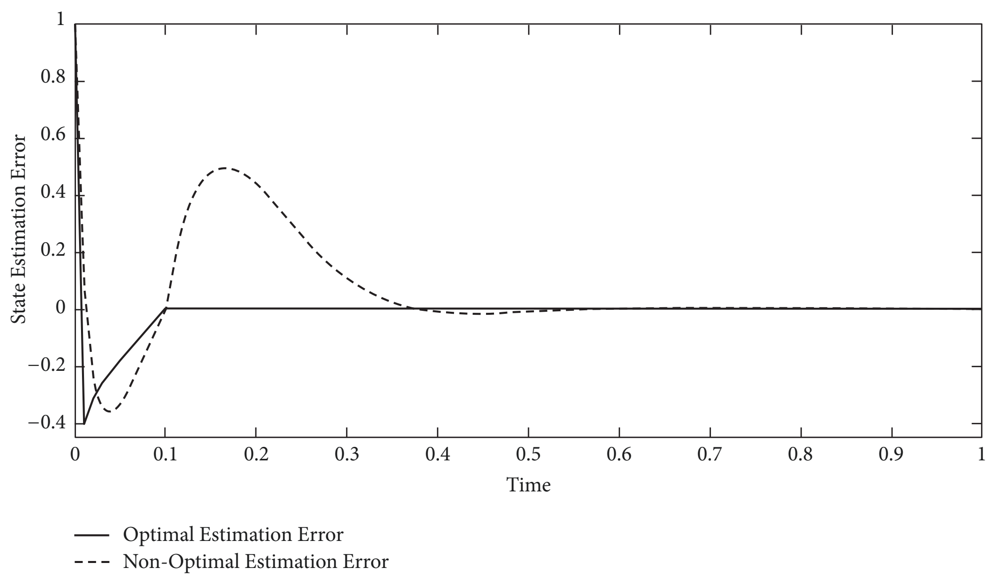

Figure 2: A comparison of state estimation errors $e_2(t)$ for the state $x_2(t)$ of optimal estimation and nonoptimal estimation, respectively.

5. Conclusions

A new approach has been proposed in this paper to design the observer gain matrix $\overline{L}$ for state delay systems such that (i) the constraint of the LMI-based condition of the state estimation error vector asymptotically converging to zero in (13) can be satisfied and (ii) at the same time the quadratic performance measurement of (7) is minimized for reducing the large error occurring during the transient period of observation. One illustrative example has been given to verify the effectiveness and efficiency of the proposed optimization method on the improvement of transient performance of state estimation errors. Based on the simulation results, it is evident that not only do the estimated state errors quickly converge to zero, but also the performance measurement values for the proposed optimal design approach are significantly lower than those of the nonoptimal design approach only using the LMI method. The constraint of the LMI-based condition makes the state estimation error vector asymptotically converge to zero so that the steady-state errors are reduced. Besides, the proposed observer design method with the quadratic performance measurement gives a penalty for the transient error. It has been shown in this paper that there exists a physically realizable, optimally designed observer gain $\overline{L}$ that can yield the improvement in observer transient error performance. Therefore, this proposed design approach can be expected to give an adequately damped response such that a small overshoot can be obtained; thus, this provides better physical performance characteristics in the time domain. Last but not least, to the authors' best knowledge, this paper is a pioneering work in dealing with the observer design issue of transient estimation performance improvement for state delay systems.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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