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Research Article

A New Model for Deriving the Priority Weights from Hesitant Triangular Fuzzy Preference Relations

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Fuzzy preference relation is a common tool to express the uncertain preference information of decision maker in the process of decision making. However, the traditional fuzzy preference relation will fail under hesitant fuzzy environment as the membership has a single value. In addition, it is very difficult to obtain the precise membership values. Therefore, a new model of fuzzy preference relation is proposed in this paper. Firstly, the concept of hesitant triangular fuzzy preference relation is defined and its properties are investigated based on the concepts of hesitant fuzzy set, hesitant triangular fuzzy set, fuzzy preference relation, and hesitant fuzzy preference relation. Then, the steps of applying this novel model are offered for the case of determining the weights of failure modes. Finally, an example is used to illustrate the proposed model.

1. Introduction

Having received extensive attention in the last few decades, preference relation is a widely used and effective tool to express the preference of decision makers over the alternatives in decision making [1, 2]. There are two kinds of preference relations, i.e., fuzzy preference relation [2] and multiplicative preference relation [3]. Saaty [4] originally proposed the multiplicative preference relation and used the 1/9-9 scale to measure the intensity of the pairwise comparison between two different alternatives. The fuzzy preference relation was proposed by Orlovsky [2], and the 0-1 scale was employed to describe the assessment information of decision maker by comparing the alternatives. Since the emergence of preference relation, extensive research has been conducted and some meaningful conclusions have been drawn, including the consensus models and methodologies [5, 6], the consistency methods [7, 8], the priority methodologies [9–11], and the incomplete values-determined methods [12, 13]. Previous researches have shown that fuzzy preference relation and multiplicative preference relation have been extended to more general forms of fuzzy set, such as interval-valued environment [11, 14] and linguistic environment [15]. To depict the intensities of both preferences and non-preferences, Xu [16] introduced the intuitionistic fuzzy values based on intuitionistic fuzzy set (IFS) [17], where intuitionistic fuzzy values are composed of a membership degree, a nonmembership degree, and a hesitancy degree [18]. The exact values of the three membership degrees in IFS are however difficult to be determined in some practical applications. Atanassov and Gargov [19] introduced the interval-valued intuitionistic fuzzy sets (IVIFSs) which permit decision makers to use interval values rather than point values to express their information. Despite the superiority of IVIFSs in describing fuzziness and uncertainty compared with other preference relations, IVIFSs are not applicable to the situation of several possible preference values commonly seen in many practical problems. For this limitation, Torra and Narukawa [20, 21] introduced the hesitant fuzzy sets (HFSs), a novel and recent extension of fuzzy sets. Because the membership function of HFSs involves a set of possible values, HFSs are considered as a more powerful tool to manage the imprecise and vague information of decision maker in the process of decision making.

Since the proposing of HFS, it has attracted much attention, and a lot of relevant studies have been conducted in the last few years. Torra [21] discussed the relationship between HFS and other kinds of fuzzy sets and showed that

the envelope of HFS was actually an IFS. Xia and Xu [22] gave the mathematical representation of HFS and defined some useful operations and aggregation operators for hesitant fuzzy information. Xu and Xia [23] proposed a variety of measures for hesitant fuzzy sets, e.g., distance, similarity, and correlation. Xu and Xia [24] also introduced the concepts of entropy and cross-entropy for hesitant fuzzy information and discussed the relationships among them. In order to cluster the massive evaluation information provided by different experts, Chen and Xu [25] proposed a series of correlation coefficient formulae for HFSs and applied them to calculating the degrees of correlation among HFSs. Rodríguez, R.M., et al. [26] made an overview on hesitant fuzzy sets including concept, extension, aggregation operators, measures, and applications like decision making, evaluation, and clustering. To deal with the case of quantitative settings, Zhao and Lin [27] introduced the concept of hesitant fuzzy linguistic term set (HFLTS). The theory of HFLTSs is very useful in objectively dealing with situations in which experts are hesitant in providing linguistic assessments. Recently, HFLTS has garnered considerable attention from researchers. Wei et al. [28] proposed a hesitant fuzzy LWA operator, a hesitant fuzzy LOWA operator, and comparison methods for the HFLTS. Liu and Rodríguez [29] presented a new representation of the HFLTS by means of a fuzzy envelope to carry out the computing with words processes. Liao et al. [28] investigated the correlation measures and correlation coefficients of HFLTSs and proposed several different types of correlation coefficients for HFLTSs such as intuitionistic fuzzy sets and hesitant fuzzy sets. To incorporate distribution information of hesitant fuzzy sets, Wu and Xu [30] defined the concept of possibility distribution for an HFLTS and proposed the corresponding operators and consensus measure. For further applications of HFLTSs to decision making, Zhu and Xu [31] developed a concept of hesitant fuzzy linguistic preference relations (HFLPRs) as a tool to collect and present the decision makers' preferences.

Inspired by the superiority of FHS in expressing human's hesitancy and based on the concept of fuzzy preference relation, fuzzy preference relations [32] are the most common tools to express decision makers' preferences over alternatives in decision making. A lot of studies have been done about them, such as the consistency methods [8], the group consensus methods [33], the priority methods [34, 35], and the incomplete values-determined methods [12]. Although different kinds of preference relations have already been successfully applied for decision making under hesitant fuzzy information, there have been few papers which have considered hesitation in a linguistic environment. Zhu and Xu [31] defined the concept of hesitant fuzzy linguistic preference relation (HFLPR). Zhang and Wu [36] defined the multiplicative consistency of an HFLPR. Wang and Xu [37] presented some consistency measures for an extended HFLPR (EHFLPR). In group decision making (GDM), preference relations are popular and powerful techniques for decision maker preference modeling [38]. The use of hesitant information in pairwise comparisons enriches the flexibility of qualitative decision making, and hesitant fuzzy linguistic preference relations (HFLPR) are one of the most commonly used. Aiming to deal with the linguistic preferences given by the decision makers, various linguistic models have been presented, such as the 2-tuple linguistic model, the symbolic model, the fuzzy number based model, the type-2 fuzzy sets based model, and the granular method [39–43]. Taking into account that the supplied preferences should satisfy some transitive properties and a consensus reaching process, Wu and Xu [44] developed separate consistency and consensus processes to deal with HFLPR individual rationality and group rationality. Furthermore, in the context of probability hesitant fuzzy preference relations (PHFPR), Wu and Xu [45] proposed an optimization based consistency improvement process to deal with the inconsistencies in a given PHFPR. In the proposed approaches, consensus measures based on the distances between the individuals were computed on three levels: an alternative pair level, an alternatives level, and a preference relations level. Hesitant fuzzy linguistic term sets (HFLTSs) can represent a much broader range of linguistic data. Wu and Xu [46] developed compromise solutions for multiple-attribute group decision making (MAGDM) using HFLTS and proposed two models to derive a compromise solution for the MAGDM problems. One is based on the VIKOR method and the other is based on the TOPSIS method.

It is obviously that hesitant fuzzy preference relation can aggregate more useful information to describe the uncertain and valueless situations in a more deep and objective way than fuzzy preference relation. But the set of the possible values of hesitant fuzzy elements are exact and crisp. However, under many conditions, in the process of trying to determine the relative importance via pairwise comparison, it is inadequate or insufficient to use the membership with a set of possible exact and crisp values due to the complexities and uncertainties of real problems. In addition, the estimation is inaccurate due to the inherent vague thought of human. Besides, the hesitant fuzzy elements of the hesitant fuzzy set are still crisp numbers, which are not easy to obtain because of human's hesitancy in actual life. Fortunately, triangular fuzzy is a method of transforming vague and uncertain linguistic variables into definite values. Furthermore triangular fuzzy number can solve the contradiction that the performance of the evaluated object cannot be measured accurately but can only be evaluated by natural language. So, the membership of hesitant fuzzy preference relation with a set of possible triangular fuzzy values is more objective than that with a set of exact and crisp values. In this paper, hesitant triangular fuzzy preference relation is proposed to overcome the limitation of HFPR. The rest of this paper is organized as follows. In Section 2, an important algorithm of fuzzy AHP is introduced, along with some basic knowledge on hesitant fuzzy set, hesitant triangular fuzzy set, fuzzy preference relation, and hesitant fuzzy preference relation. In Section 3, the model of hesitant triangular preference relation is proposed and the steps for applying the model are offered. In Section 4, an example is used to illustrate the proposed model. And finally, conclusions are given.

2. Preliminaries

In this section, an important algorithm of fuzzy AHP is introduced, along with some basic knowledge on hesitant

fuzzy set (HFS), hesitant triangular fuzzy set (HTFS), fuzzy preference relation (FPR), and hesitant fuzzy preference relation (HFPR).

2.1. Fuzzy AHP. In the following, Chang's extent analysis method [47] is explained. Let $X = (x_1, x_2, \cdots, x_n)$ be an object set and $U = (g_1, g_2, \cdots, g_m)$ be a goal set. According to Chang's method, the object is considered one by one, and extent analysis is carried out for each goal. Therefore, there are m extent analysis values for each object, shown as below:

$$\widetilde{M}_{xi}^1, \widetilde{M}_{xi}^2, \cdots, \widetilde{M}_{xi}^m, \quad i = 1, 2, \cdots, n, \tag{1}$$

where $\widetilde{M}_{xi}^j$ $(i = 1, 2, \cdots, n,\ j = 1, 2, \cdots, m)$ are triangular fuzzy numbers.

Next, the steps of Chang's extent analysis are demonstrated.

Step 1. The value of fuzzy synthetic extent with respect to the ith object is defined as

$$S_i \approx \sum_{j=1}^{m} \widetilde{M}_{xi}^j \otimes \left[\sum_{i=1}^{n} \sum_{j=1}^{m} \widetilde{M}_{xi}^j \right]^{-1}. \tag{2}$$

For obtaining $\sum_{j=1}^{m} \widetilde{M}_{xi}^j$, the fuzzy addition operation of m extent analysis values is performed on a particular matrix, i.e.,

$$\sum_{j=1}^{m} \widetilde{M}_{xi}^j = \left(\sum_{j=1}^{m} l_j, \sum_{j=1}^{m} m_j, \sum_{j=1}^{m} u_j \right) \tag{3}$$

and to obtain $\sum_{i=1}^{n} \sum_{j=1}^{m} \widetilde{M}_{xi}^j$, the fuzzy addition operation of $\widetilde{M}_{xi}^j$ $(j = 1, 2, \cdots, m)$ values is performed, i.e.,

$$\sum_{i=1}^{n} \sum_{j=1}^{m} \widetilde{M}_{xi}^j = \left(\sum_{i=1}^{n} l_i, \sum_{i=1}^{n} m_i, \sum_{i=1}^{n} u_i \right) \tag{4}$$

then, the inverse of above vector is calculated, i.e.,

$$\left[\sum_{i=1}^{n} \sum_{j=1}^{m} \widetilde{M}_{xi}^j \right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n} u_i}, \frac{1}{\sum_{i=1}^{n} m_i}, \frac{1}{\sum_{i=1}^{n} l_i} \right). \tag{5}$$

Step 2. Assuming that $\widetilde{M}_1$ and $\widetilde{M}_2$ are two triangular fuzzy numbers, the degree of possibility of $\widetilde{M}_2 = (l_2, m_2, u_2) \geq \widetilde{M}_1 = (l_1, m_1, u_1)$ is defined as

$$V\left(\widetilde{M}_2 \geq \widetilde{M}_1\right) = \sup\left[\min\left(\widetilde{M}_1(x), \widetilde{M}_2(x)\right)\right]. \tag{6}$$

This expression can be equivalently expressed as follows:

$$V\left(\widetilde{M}_2 \geq \widetilde{M}_1\right) = hgt\left(\widetilde{M}_1 \cap \widetilde{M}_2\right) = \widetilde{M}_2(d) = \begin{cases} 1, & \text{if } m_2 \geq m_1, \\ 0, & \text{if } l_1 \geq u_2, \\ \dfrac{l_1 - u_2}{(m_2 - u_2) - (m_1 - l_1)}, & \text{otherwise.} \end{cases} \tag{7}$$

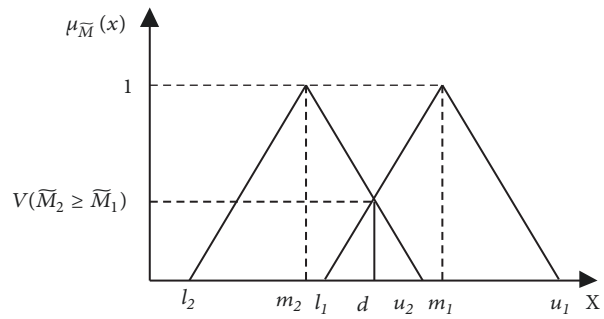

Figure 1: The intersection between $\widetilde{M}_1$ and $\widetilde{M}_2$.

Here d is the abscissa value of the highest intersection point D between $\widetilde{M}_1$ and $\widetilde{M}_2$, as shown in Figure 1. In order to compare $\widetilde{M}_1$ and $\widetilde{M}_2$, both values of $V(\widetilde{M}_1 \geq \widetilde{M}_2)$ and $V(\widetilde{M}_2 \geq \widetilde{M}_1)$ are required.

Step 3. The degree of possibility for a convex fuzzy number to be greater than k convex fuzzy numbers $\widetilde{M}_i$ $(i = 1, 2, \cdots, k)$ can be defined by

$$V\left(\widetilde{M} \geq \widetilde{M}_1, \widetilde{M}_2, \cdots, \widetilde{M}_k\right) = \min V\left(\widetilde{M} \geq \widetilde{M}_i\right), \quad i = 1, 2, \cdots, k. \tag{8}$$

Step 4. $W = (\min V(S_1 \geq S_k), \min V(S_2 \geq S_k), \cdots, \min V(S_n \geq S_k))^T$ is the weight vector for $k = 1, 2, \cdots, n$.

2.2. Concepts of HFS, HTFS, and FPR

Definition 1 (see [20, 21]). Let X be a fixed set; a hesitant fuzzy set on X is in terms of a function h that when applied to X returns a subset of $[0, 1]$, which can be represented as the following mathematical symbol by Xia and Xu [22]:

$$E = \left(\langle x, h_E(x) \rangle \mid x \in X\right), \tag{9}$$

where $h_E(x)$ is a set of values in $[0, 1]$, denoting the possible membership degree of the element $x \in X$ to the set E. For convenience, Xia and Xu [22] took $h = h_E(x)$ as a hesitant fuzzy element (HFE) and H as the set of all HFE.

Definition 2 (see [22]). For a HFE h, $s(h) = (1/\#h) \sum_{\gamma \in h} \gamma$ is called the score function of h, where $\#h$ represents the number of elements in h. For two HFEs h_1 and h_2, if $s(h_1) > s(h_2)$, then $h_1 > h_2$; if $s(h_1) = s(h_2)$, then $h_1 = h_2$.

Definition 3 (see [27]). Let X be a fixed set; a hesitant triangular fuzzy set (HTFS) on X is in terms of a function that when applied to each x in X returns a subset of values in $[0, 1]$, which can be expressed by

$$E = \left(\left\langle x, \widetilde{h}_{E(x)} \right\rangle \mid x \in X\right), \tag{10}$$

where $\widetilde{h}_{E(x)}$ is a set of possible triangular fuzzy values in $[0, 1]$, denoting the possible membership degrees of the element $x \in X$ to the set E. For convenience, Zhao and Lin [27] called

$\widetilde{h}_{E(x)} = \widetilde{h} = (\gamma^L, \gamma^M, \gamma^R)$ as a hesitant triangular fuzzy element (HTFE).

Based on Definition 3 and the operational principle of HFS, Zhao and Lin [27] defined some new operations on HTFE $\widetilde{h} = (\gamma^L, \gamma^M, \gamma^R)$, $\widetilde{h}_1 = ({\gamma_1}^L, {\gamma_1}^M, {\gamma_1}^R)$, and $\widetilde{h} = ({\gamma_2}^L, {\gamma_2}^M, {\gamma_2}^R)$:

(1) $\widetilde{h}^\lambda = \bigcup_{\gamma\in\widetilde{h}}\{((\gamma^L)^\lambda, (\gamma^M)^\lambda, (\gamma^R)^\lambda)\}$;

(2) $\lambda\widetilde{h} = \bigcup_{\gamma\in\widetilde{h}}\{(1-(1-\gamma^L)^\lambda, 1-(1-\gamma^M)^\lambda, 1-(1-\gamma^R)^\lambda)\}$;

(3) $\widetilde{h}_1 \oplus \widetilde{h}_2 = \bigcup_{\gamma_1\in\widetilde{h}_1,\gamma_2\in\widetilde{h}_2}\{({\gamma_1}^L + {\gamma_2}^L - {\gamma_1}^L{\gamma_2}^L, {\gamma_1}^M + {\gamma_2}^M - {\gamma_1}^M{\gamma_2}^M, {\gamma_1}^R + {\gamma_2}^R - {\gamma_1}^R{\gamma_2}^R)\}$;

(4) $\widetilde{h}_1 \otimes \widetilde{h}_2 = \bigcup_{\gamma_1\in\widetilde{h}_1,\gamma_2\in\widetilde{h}_2}\{({\gamma_1}^L{\gamma_2}^L, {\gamma_1}^M{\gamma_2}^M, {\gamma_1}^R{\gamma_2}^R)\}$.

Based on the above definition, Zhao and Lin [27] also defined the score function of $\widetilde{h}$:

$$s(\widetilde{h}) = \frac{1}{\#\widetilde{h}}\sum_{\gamma\in\widetilde{h}}\gamma = \left(\frac{1}{\#\widetilde{h}}\sum_{\gamma\in\widetilde{h}}\gamma^L, \frac{1}{\#\widetilde{h}}\sum_{\gamma\in\widetilde{h}}\gamma^M, \frac{1}{\#\widetilde{h}}\sum_{\gamma\in\widetilde{h}}\gamma^R\right), \quad (11)$$

where $\#\widetilde{h}$ is the number of triangular fuzzy values in $\widetilde{h}$, and $s(\widetilde{h})$ is a triangular fuzzy value in $[0,1]$. For two HTFEs $\widetilde{h}_1$ and $\widetilde{h}_2$, if $s(\widetilde{h}_1) \geq s(\widetilde{h}_2)$, then $\widetilde{h}_1 \geq \widetilde{h}_2$.

Definition 4 (see [27]). Let $\widetilde{h}_j$ $(j = 1, 2, \cdots, n)$ be a collection of HTFEs. The hesitant triangular fuzzy weighted averaging (HTFWA) operator is a mapping $\widetilde{H}^n \longrightarrow \widetilde{H}$.

$$\begin{aligned} HTFWA(\widetilde{h}_1, \widetilde{h}_2, \cdots, \widetilde{h}_n) &= \bigoplus_{j=1}^{n}(\omega_j\widetilde{h}_j) \\ &= \bigcup_{\widetilde{\gamma}_1\in\widetilde{h}_1,\widetilde{\gamma}_2\in\widetilde{h}_2,\cdots,\widetilde{\gamma}_n\in\widetilde{h}_n}\left\{\left(1-\prod_{j=1}^{n}(1-{\gamma_j}^L)^{\omega_j}, 1\right.\right. \\ &\quad \left.\left. -\prod_{j=1}^{n}(1-{\gamma_j}^M)^{\omega_j}, 1-\prod_{j=1}^{n}(1-{\gamma_j}^R)^{\omega_j}\right)\right\} \end{aligned} \quad (12)$$

where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ represents the weight vector of $\widetilde{h}_j$ $(j = 1, 2, \cdots, n)$, and $\omega_j > 0$, $\sum_{j=1}^{n}\omega_j = 1$. In particular, if $\omega = (1/n, 1/n, \cdots, 1/n)^T$, then the HTFWA operator degrades to the hesitant triangular fuzzy averaging (HTFA) operator.

$$\begin{aligned} HTFA(\widetilde{h}_1, \widetilde{h}_2, \cdots, \widetilde{h}_n) &= \bigoplus_{j=1}^{n}\left(\frac{1}{n}\widetilde{h}_j\right) \\ &= \bigcup_{\widetilde{\gamma}_1\in\widetilde{h}_1,\widetilde{\gamma}_2\in\widetilde{h}_2,\cdots,\widetilde{\gamma}_n\in\widetilde{h}_n}\left\{\left(1-\prod_{j=1}^{n}(1-{\gamma_j}^L)^{1/n}, 1\right.\right. \\ &\quad \left.\left. -\prod_{j=1}^{n}(1-{\gamma_j}^M)^{1/n}, 1-\prod_{j=1}^{n}(1-{\gamma_j}^R)^{1/n}\right)\right\} \end{aligned} \quad (13)$$

Definition 5 (see [27]). Let $\widetilde{h}_j$ $(j = 1, 2, \cdots, n)$ be a collection of HTFEs. The hesitant triangular fuzzy weighted geometric (HTFWG) operator is a mapping $\widetilde{H}^n \longrightarrow \widetilde{H}$.

$$\begin{aligned} HTFWG(\widetilde{h}_1, \widetilde{h}_2, \cdots, \widetilde{h}_n) &= \bigoplus_{j=1}^{n}(\widetilde{h}_j)^{w_j} \\ &= \bigcup_{\widetilde{\gamma}_1\in\widetilde{h}_1,\widetilde{\gamma}_2\in\widetilde{h}_2,\cdots,\widetilde{\gamma}_n\in\widetilde{h}_n}\left\{\left(\prod_{j=1}^{n}({\gamma_j}^L)^{w_j}, \prod_{j=1}^{n}({\gamma_j}^M)^{w_j},\right.\right. \\ &\quad \left.\left.\prod_{j=1}^{n}({\gamma_j}^R)^{w_j}\right)\right\} \end{aligned} \quad (14)$$

where $\omega = (\omega_1, \omega_2, \cdots, \omega_n)^T$ represents the weight vector of $\widetilde{h}_j$ $(j = 1, 2, \cdots, n)$, and $\omega_j > 0$, $\sum_{j=1}^{n}\omega_j = 1$. In particular, if $\omega = (1/n, 1/n, \cdots, 1/n)^T$, then the HTFWG operator degrades to the hesitant triangular fuzzy geometric (HTFG) operator.

$$\begin{aligned} HTFG(\widetilde{h}_1, \widetilde{h}_2, \cdots, \widetilde{h}_n) &= \bigoplus_{j=1}^{n}(\widetilde{h}_j)^{w_j} \\ &= \bigcup_{\widetilde{\gamma}_1\in\widetilde{h}_1,\widetilde{\gamma}_2\in\widetilde{h}_2,\cdots,\widetilde{\gamma}_n\in\widetilde{h}_n}\left\{\left(\prod_{j=1}^{n}({\gamma_j}^L)^{1/n}, \prod_{j=1}^{n}({\gamma_j}^M)^{1/n},\right.\right. \\ &\quad \left.\left.\prod_{j=1}^{n}({\gamma_j}^R)^{1/n}\right)\right\}. \end{aligned} \quad (15)$$

Definition 6 (see [48]). Let $X = \{x_1, x_2, \cdots, x_n\}$ be a finite set of alternatives; then $R = (r_{ij})_{n\times n}$ is called an additive FPRR on the product set $X \times X$ with the membership function u_R: $X \times X \longrightarrow [0,1]$, $u_R(x_i, x_j) = r_{ij}$, satisfying $r_{ij} + r_{ji} = 1$, $i, j = 1, 2, \cdots, n$.

Usually, a matrix $R = (r_{ij})_{n\times n}$ denotes the preference relation between two alternatives, and r_{ij} denotes the preference degree of x_i over x_j. In particular, $r_{ij} = 0$ implies that x_j is totally preferred to x_i, and $r_{ij} = 0.5$ indicates that there is no difference between x_i and x_j.

2.3. Concepts of Hesitant Fuzzy Preference Relations. Let $X = \{x_1, x_2, \cdots, x_n\}$ be a fixed set of alternatives; then $A = (a_{ij})_{n\times n}$ is called the fuzzy preference relation matrix [2] on $X \times X$ with $a_{ij} \geq 0$, $a_{ij} + a_{ji} = 1$, $i, j = 1, 2, \cdots, n$, where a_{ij} denotes the degree that the alternative x_i is prior to x_j. In particular, $a_{ij} = 0.5$ means the same importance of x_i and x_j, which can be expressed by $x_i = x_j$; $0 \leq a_{ij} < 0.5$ means that x_j is more important than x_i, expressed by $x_i \prec x_j$, and the smaller the value of a_{ij}, the more important x_j to x_i; on the contrary, $0.5 < a_{ij} \leq 1$ means that x_i is more important than x_j, denoted by $x_i \succ x_j$, and the larger the value of a_{ij}, the more important x_i to x_j. Obviously, the value of a_{ij} indicates that fuzzy preference relation is a certain value between 0 and 1. If a decision group containing several

Table 1: 0.1-0.9 scale values and the means.

Fuzzy number	Meaning
0.5	Equally preferred
0.6	Moderately preferred
0.7	Strongly preferred
0.8	Very strongly preferred
0.9	Extremely preferred
Other values between 0 and 1	Intermediate values used to present compromise

experts is authorized to estimate the degrees of x_i preferred to x_j, there are several possible values, because of the fuzziness and hesitancy of experts and the unpredictable information. For instance, some experts provide a_{ij}^1, some provide a_{ij}^2, and the others provide a_{ij}^3. Therefore, it is clear that $a_{ij}^1, a_{ij}^2, a_{ij}^3 \in [0,1]$, and a_{ij} is called HFE $a_{ij} = \{a_{ij}^1, a_{ij}^2, a_{ij}^3\}$, which implies the preference relation of x_i over x_j. From the above analysis, it can be concluded that HFPR is a novel generalization of fuzzy preference relation, whose fundamental units can be evaluated using the 0.1-0.9 scale described in Table 1.

Definition 7 (see [49, 50]). Let $X = \{x_1, x_2, \cdots, x_n\}$ be a fixed set; then a hesitant fuzzy preference relation A on X is expressed by a matrix $A = (a_{ij})_{n\times n} \subset X \times X$, where $a_{ij} = \{a_{ij}^s, s = 1, 2, \cdots, l_{a_{ij}}\}$ is a HFE denoting all possible degrees to which x_i is preferred to x_j. Additionally, a_{ij} should satisfy the following requirements.

$$a_{ij}^{\sigma(s)} + a_{ji}^{\sigma(l_{a_{ij}}-s+1)} = 1, \quad i, j = 1, 2, \cdots, n, \tag{16}$$

$$a_{ii} = \{0.5\}, \quad i = 1, 2, \cdots, n, \tag{17}$$

$$l_{a_{ij}} = l_{a_{ji}}, \quad i, j = 1, 2, \cdots, n \tag{18}$$

where the values in a_{ij} are assumed to be arranged in an increasing order, $a_{ij}^{\sigma(s)}$ denotes the sth largest value in a_{ij}, and $l_{a_{ij}}$ denotes the number of values in a_{ij}. Obviously, (16) is the reciprocal condition, which means that if $a_{ij} = \{a_{ij}^s, s = 1, 2, \cdots, l_{a_{ij}}\}$ is known, then a_{ji} can be easily obtained by $a_{ji} = \{1 - a_{ij}^s, s = 1, 2, \cdots, l_{a_{ij}}\}$; (17) implies that there is no difference between x_i and x_j. If there is a single value in a_{ij}, $(i, j = 1, 2, \cdots, n)$, then the hesitant fuzzy preference relation degrades to the usual fuzzy ones.

3. Proposed Methodology

3.1. Hesitant Triangular Fuzzy Preference Relations (HTFPR) Model. In many practical situations, due to the incompleteness and uncertainties of information as well as the hesitancy of humans, it is not easy for decision makers to express their preferences between two alternatives with exact and crisp values in traditional HFS. Hence, triangular fuzzy set can model real-life decision problems in a more suitable and sufficient way than real numbers.

Definition 8. Let $X = \{x_1, x_2, \cdots, x_n\}$ be a fixed set; then an additive hesitant triangular fuzzy preference relation $\widetilde{H}$ on X is expressed by a matrix $\widetilde{H} = (\widetilde{h}_{ij})_{n\times n} \subset X \times X$, where $\widetilde{h}_{ij} = \{(\gamma_{ij}^L, \gamma_{ij}^M, \gamma_{ij}^R)^s, \ s = 1, 2, \cdots, l_{\widetilde{h}_{ij}}\}$ is a hesitant triangular fuzzy element (HTFE) denoting all possible degrees to which x_i is preferred to x_j. Moreover, $\widetilde{h}_{ij}$ satisfies the following requirements.

(1)

$$\begin{aligned} \left(\gamma^L\right)_{ij}^{\sigma(s)} + \left(\gamma^R\right)_{ji}^{\sigma(l_{h_{ij}}-s+1)} &= 1, \\ \left(\gamma^M\right)_{ij}^{\sigma(s)} + \left(\gamma^M\right)_{ji}^{\sigma(l_{h_{ij}}-s+1)} &= 1, \\ \left(\gamma^R\right)_{ij}^{\sigma(s)} + \left(\gamma^L\right)_{ji}^{\sigma(l_{h_{ij}}-s+1)} &= 1, \\ & i, j = 1, 2, \cdots, n; \end{aligned} \tag{19}$$

(2)

$$\widetilde{h}_{ii} = \left\{\left(\gamma_{ii}^L, \gamma_{ii}^M, \gamma_{ii}^R\right)\right\} = \{(0.5, 0.5, 0.5)\} = \{0.5\}, \quad i = 1, 2, \cdots, n; \tag{20}$$

(3)

$$l_{\widetilde{h}_{ij}} = l_{\widetilde{h}_{ji}}, \quad i, j = 1, 2, \cdots, n. \tag{21}$$

Here the values in $\widetilde{h}_{ij}$ are assumed to be arranged in an increasing order, $\widetilde{h}_{ij}^{\sigma(s)}$ denotes the sth largest value in $\widetilde{h}_{ij}$, and $l_{\widetilde{h}_{ij}}$ denotes the number of values in $\widetilde{h}_{ij}$. Obviously, (19) is the reciprocal condition, which means that if $\widetilde{h}_{ij} = \{(\gamma_{ij}^L, \gamma_{ij}^M, \gamma_{ij}^R)^{\sigma(s)}, \ s = 1, 2, \cdots, l_{\widetilde{h}_{ij}}\}$ is known, then $\widetilde{h}_{ji}$ can be obtained easily by $\widetilde{h}_{ji} = \{(1 - \gamma_{ij}^R, 1 - \gamma_{ij}^M, 1 - \gamma_{ij}^L)^{\sigma(l_{\widetilde{h}_{ij}}-s+1)}, \ s = 1, 2, \cdots, l_{\widetilde{h}_{ij}}\}$. Equation (20) implies that there is no difference between x_i and x_j. If there is a single value in $\widetilde{h}_{ij}$, $(i, j = 1, 2, \cdots, n)$, then hesitant triangular fuzzy preference relation degrades to the usual triangular fuzzy ones.

Example 9. Let $X = \{x_1, x_2, x_3\}$; a decision group containing several experts is authorized to provide the preference values of x_1 over x_2. There are several possible results among the experts. Some experts provide (0.2, 0.3, 0.4), while others provide (0.3, 0.4, 0.5). Then according to Definition 6, the degree of x_2 is preferred to x_1, which should be (1-0.5 = 0.5, 1-0.4 = 0.6, 1-0.3 = 0.7) and (1-0.4 = 0.6, 1-0.3 = 0.7, 1-0.2 = 0.8), respectively. In this case, $\widetilde{h}_{12} = \{(0.2, 0.3, 0.4), (0.3, 0.4, 05)\}$ is called a HTFE, denoting the preference information about x_1 preference to x_2. The preference information about x_2 over x_1 is denoted by a HTFE $\widetilde{h}_{21} = \{(0.5, 0.6, 0.7), (0.6, 0.7, 0.8)\}$. Similarly, let $\widetilde{h}_{13} = \{(0.5, 0.6, 0.7)\}$ and $\widetilde{h}_{23} = \{(0.2, 0.4, 0.5), (0.5, 0.6, 0.7), (0.5, 0.7, 0.8)\}$; then, we can correspondingly obtain $\widetilde{h}_{31} = \{(0.3, 0.4, 0.5)\}$ and $\widetilde{h}_{32} = \{(0.2, 0.3, 0.5), (0.3, 0.4, 0.5)(0.5, 0.6, 0.8)\}$.

Based on above analysis, the hesitant triangular fuzzy preference relation $\widetilde{H} = (\widetilde{h}_{ij})_{n\times n} \subset X \times X$ is presented in Table 2.

3.2. The Steps of HTRPR. The proposed hesitant triangular fuzzy preference relation model can be used to determine

Table 2: The hesitant triangular fuzzy preference relation $\widetilde{H}$.

	x_1	x_2	x_3
x_1	$\{0.5\}$	$\{(0.2, 0.3, 0.4), (0.3, 0.4, 0.5)\}$	$\{(0.5, 0.6, 0.7)\}$
x_2	$\{(0.5, 0.6, 0.7), (0.6, 0.7, 0.8)\}$	$\{0.5\}$	$\{(0.2, 0.4, 0.5), (0.5, 0.6, 0.7), (0.5, 0.7, 0.8)\}$
x_3	$\{(0.3, 0.4, 0.5)\}$	$\{(0.2, 0.3, 0.5), (0.3, 0.4, 0.5), (0.5, 0.6, 0.8)\}$	$\{0.5\}$

the weights of failure modes of a work system. The steps are shown below.

Step 1. A committee comprised of several experts is set up to identify the potential failure modes of a work system and to provide their preference information via pairwise comparison. The assessment preference values are denoted by hesitant triangular fuzzy elements h_{ij} $(i, j = 1, 2, \cdots, n)$, and the HTFPR matrix $\widetilde{H} = (\widetilde{h}_{ij})_{n\times n}$ is constructed.

Step 2. HTFWA (or HTFWG) operator is used to aggregate all h_{ij} $(j = 1, 2, \cdots, n)$ which correspond to the alternative x_i.

Step 3. The score values $s(\widetilde{h}_i)$ of x_i $(i = 1, 2, \cdots, n)$ are calculated using (11).

Step 4. Chang's fuzzy AHP method is used to obtain the weights of the failure modes. Based on the fuzzy synthetic extent proposed by Chang [31], $\sum_{j=1}^{m} \widetilde{M}_{xi}^{j}$ can be treated as $s(\widetilde{h}_i)$, and $\sum_{i=1}^{n}\sum_{j=1}^{m} \widetilde{M}_{xi}^{j}$ can be treated as $\sum_{i=1}^{n} s(\widetilde{h}_i)$. In this case, the value of HTF synthetic extent with respect to the ith alternative can be expressed by

$$S_i' \approx s\left(\widetilde{h}_i\right) \otimes \left[\sum_{i=1}^{n} s\left(\widetilde{h}_i\right)\right]^{-1}, \tag{22}$$

where $\otimes$ denotes the extended multiplication of two hesitant triangular fuzzy numbers, i.e.,

$$\sum_{i=1}^{n} s\left(\widetilde{h}_i\right) = \left(\sum_{i=1}^{n} \frac{1}{\#\widetilde{h}} \sum_{\gamma\in\widetilde{h}} \gamma_i^{L}, \sum_{i=1}^{n} \frac{1}{\#\widetilde{h}} \sum_{\gamma\in\widetilde{h}} \gamma_i^{M}, \sum_{i=1}^{n} \frac{1}{\#\widetilde{h}} \sum_{\gamma\in\widetilde{h}} \gamma_i^{R}\right), \tag{23}$$

and

$$\left[\sum_{i=1}^{n} s\left(\widetilde{h}_i\right)\right]^{-1} = \left(\frac{1}{\sum_{i=1}^{n}\left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}\gamma^{R}}, \frac{1}{\sum_{i=1}^{n}\left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}\gamma^{M}}, \frac{1}{\sum_{i=1}^{n}\left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}\gamma^{L}}\right). \tag{24}$$

Similarly, (6), (7), and (8) can be equivalently expressed as

$$V\left(s\left(\widetilde{h}_2\right) \geq s\left(\widetilde{h}_1\right)\right) = \sup\left[\min\left(s\left(\widetilde{h}_1(x)\right), s\left(\widetilde{h}_2(x)\right)\right)\right], \tag{25}$$

$$V\left(s\left(\widetilde{h}_2\right) \geq s\left(\widetilde{h}_1\right)\right) = hgt\left(s\left(\widetilde{h}_1\right) \bigcap s\left(\widetilde{h}_2\right)\right) = s\left(\widetilde{h}_2(d)\right)$$

$$= \begin{cases} 1, & \text{if } \frac{1}{\#\widetilde{h}}\sum_{\gamma\in\widetilde{h}}\gamma_2^{M} \geq \frac{1}{\#\widetilde{h}}\sum_{\gamma\in\widetilde{h}}\gamma_1^{M}, \\ 0, & \text{if } \frac{1}{\#\widetilde{h}}\sum_{\gamma\in\widetilde{h}}\gamma_1^{L} \geq \frac{1}{\#\widetilde{h}}\sum_{\gamma\in\widetilde{h}}\gamma_2^{R}, \\ \dfrac{\left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}\gamma_1^{L} - \left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}\gamma_2^{R}}{\left(\left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}\gamma_2^{M} - \left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}\gamma_2^{R}\right) - \left(\left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}\gamma_1^{M} - \left(1/\#\widetilde{h}\right)\sum_{\gamma\in\widetilde{h}}\gamma_1^{L}\right)}, & \text{otherwise.} \end{cases} \tag{26}$$

and

$$V\left(s\left(\widetilde{h}\right) \geq s\left(\widetilde{h}_1\right), s\left(\widetilde{h}_2\right), \cdots, s\left(\widetilde{h}_k\right)\right) = \min V\left(s\left(\widetilde{h}\right) \geq s\left(\widetilde{h}_i\right)\right), \quad i = 1, 2, \cdots, k \tag{27}$$

thus, $W = (\min V(S_1' \geq S_k'), \min V(S_2' \geq S_k'), \cdots, \min V(S_n' \geq S_k'))^T$ is the weight vector for $k = 1, 2, \cdots, n$ of failure modes.

4. Case Illustration

This section provides an example of a practical case involving the assessment of the potential failure modes of a kind of typical amusement rides-roller coaster, to illustrate the proposed model. Then a comparative analysis is conducted between the proposed model and the hesitant fuzzy preference relation [49] to show its superiority.

Amusement rides are a very popular recreational activity and take many forms in structure and movement. For example, a device or vehicle may be driven or guided by a power equipment or operator on a track or slide, and sometimes it walks through or bounces on an air bounce, funhouse, or maze [51]. Usually, amusement rides are classified into two categories, i.e., "fixed site ride" and "mobile ride" [51, 52] which are operated in fixed site and mobile forms, respectively [53]. Generally, the injuries and failure of amusement ride are considered to be infrequent by public, but are noteworthy when they occur. Carrying out quantitative and qualitative safety assessment of amusement ride draws much attention of the public and is important for continuous improvement [54–56]. As a kind of typical amusement ride, roller coaster is very popular among young people and is located at outdoor midways generally, as well as big amusement parks or theme parks. It is classified as 'fixed site ride', and its safe and reliable operation is also worthy of researching. To find out the major potential failure modes and derive the priority of them, it is important to conduct the safety management of roller coasters.

4.1. The Proposed Method. In the following, the proposed method is used to obtain the metal-frame major potential failure modes and its priority of roller coaster.

Step 1. An expert team consisting of three cross-functional members was organized to identify the failure modes of a roller coaster and to prioritize them via pairwise comparison. According to the expert team, the major potential failure modes were identified as weld metal cracking, corrosion, joint angle loosening, deformation, and wear denoted by x_i $(i = 1, 2, 3, 4, 5)$. Because of the team members' limited expertise in the problem domain, lack of knowledge or data, and other reasons, it is difficult for experts to reach a consensus to evaluate these five potential failure modes and their relative importance weights precisely. So, the values of preference relations taking the form of HTFE by experts are more reasonable for real practice. Then, the expert team compare these five potential failure modes and provide their preference values denoted by $\widetilde{H} = (\widetilde{h}_{ij})_{5\times5}$, as expressed in Table 3, where $\widetilde{h}_{ij}$ $(i, j = 1, 2, 3, 4, 5)$ are in the form of HTFPRs.

Step 2. The preference relation information given in matrix $\widetilde{H}$ and the HTFWA (or HTFWG) operator are used to aggregate all h_{ij} $(j = 1, 2, \cdots, n)$ which correspond to the failure modes x_i. Taking the failure modes weld metal cracking x_1 as an example, since $\omega = (1/n, 1/n, \cdots, 1/n)^T$, we have

$$\begin{aligned}
\widetilde{h}_1 &= HTFWA = HTFA\left(\widetilde{h}_{11}, \widetilde{h}_{12}, \widetilde{h}_{13}, \widetilde{h}_{14}, \widetilde{h}_{15}\right) \\
&= \bigoplus_{j=1}^{5}\left(\frac{1}{5}\widetilde{h}_{1j}\right) = \bigcup_{\widetilde{\gamma}_1\in\widetilde{h}_1, \widetilde{\gamma}_2\in\widetilde{h}_2, \widetilde{\gamma}_3\in\widetilde{h}_3, \widetilde{\gamma}_4\in\widetilde{h}_4, \widetilde{\gamma}_5\in\widetilde{h}_5} \left\{\left(1\right.\right. \\
&\quad - \prod_{j=1}^{5}\left(1-\gamma_{1j}{}^{L}\right)^{1/5}, 1 - \prod_{j=1}^{n}\left(1-\gamma_{1j}{}^{M}\right)^{1/5}, 1 \\
&\quad \left.\left. - \prod_{j=1}^{n}\left(1-\gamma_{1j}{}^{R}\right)^{1/5}\right)\right\} \\
&= \{\{0.5\}, \{(0.5, 0.7, 0.8), (0.6, 0.8, 0.9)\}, \\
&\quad \{(0.7, 0.8, 0.9), (0.6, 0.9, 1.0)\}, \{(0.6, 0.7, 0.8)\}, \\
&\quad \{(0.6, 0.8, 0.9)\}\} \\
&= \{(0.6272, 0.7175, 0.8179), (0.6562, 0.7540, 1), \\
&\quad (0.6481, 0.7395, 0.8415), (0.6755, 0.7732, 1)\}.
\end{aligned} \tag{28}$$

Step 3. The score values $s(\widetilde{h}_i)$ $(i = 1, 2, 3, 4, 5)$ of the overall hesitant triangular fuzzy preference relation values $\widetilde{h}_i$ $(i = 1, 2, 3, 4, 5)$ are calculated:

$$\begin{aligned}
s\left(\widetilde{h}_1\right) &= \frac{1}{\#\widetilde{h}_1}\sum_{\gamma\in\widetilde{h}_1}\gamma_1 \\
&= \left(\frac{1}{\#\widetilde{h}_1}\sum_{\gamma\in\widetilde{h}_1}\gamma_1{}^{L}, \frac{1}{\#\widetilde{h}_1}\sum_{\gamma\in\widetilde{h}_1}\gamma_1{}^{M}, \frac{1}{\#\widetilde{h}_1}\sum_{\gamma\in\widetilde{h}_1}\gamma_1{}^{R}\right) \\
&= (0.6518, 0.7461, 0.9149), \\
s\left(\widetilde{h}_2\right) &= (0.3471, 0.4344, 0.5174), \\
s\left(\widetilde{h}_3\right) &= (0.4902, 0.5589, 0.6591), \\
s\left(\widetilde{h}_4\right) &= (0.3480, 0.4262, 0.5176), \\
s\left(\widetilde{h}_5\right) &= (0.3708, 0.4654, 0.5695).
\end{aligned} \tag{29}$$

Step 4. The weights of the failure modes are obtained.

Firstly, by applying (22), we have

$$\begin{aligned}
S_1' &= s\left(\widetilde{h}_1\right) \otimes \left[\sum_{i=1}^{5} s\left(\widetilde{h}_i\right)\right]^{-1} \\
&= (0.6518, 0.7461, 0.9149) \\
&\quad \otimes \left(\frac{1}{3.2612}, \frac{1}{2.707}, \frac{1}{2.2963}\right) \\
&\approx (0.1999, 0.2756, 0.3984), \\
S_2' &= (0.3471, 0.4344, 0.5174) \\
&\quad \otimes \left(\frac{1}{3.2612}, \frac{1}{2.707}, \frac{1}{2.2963}\right) \\
&\approx (0.1064, 0.1605, 0.2253), \\
S_3' &= (0.4902, 0.5589, 0.6591) \\
&\quad \otimes \left(\frac{1}{3.2612}, \frac{1}{2.707}, \frac{1}{2.2963}\right) \\
&\approx (0.1503, 0.2065, 0.287),
\end{aligned}$$

Table 3: The hesitant triangular fuzzy preference relation matrix $\widetilde{H}$ of failure modes.

	weld metal cracking	corrosion	joint angle loosening	deformation	wear
weld metal cracking	$\{0.5\}$	$\{(0.5, 0.7, 0.8), (0.6, 0.8, 0.9)\}$	$\{(0.7, 0.8, 0.9), (0.6, 0.9, 1.0)\}$	$\{(0.6, 0.7, 0.8), (0.7, 0.8, 0.9)\}$	$\{(0.6, 0.8, 0.9)\}$
corrosion	$\{(0.1, 0.2, 0.4), (0.2, 0.3, 0.5)\}$	$\{0.5\}$	$\{(0.3, 0.4, 0.5), (0.6, 0.7, 0.8)\}$	$\{(0.3, 0.4, 0.5)\}$	$\{(0.2, 0.4, 0.5), (0.3, 0.4, 0.5)\}$
fatigue	$\{(0, 0.1, 0.4), (0.1, 0.2, 0.3)\}$	$\{(0.2, 0.3, 0.4), (0.5, 0.6, 0.7)\}$	$\{0.5\}$	$\{(0.6, 0.7, 0.8), (0.7, 0.8, 0.9)\}$	$\{(0.6, 0.7, 0.8)\}$
deformation	$\{(0.1, 0.2, 0.3), (0.2, 0.3, 0.4)\}$	$\{(0.5, 0.6, 0.7)\}$	$\{(0.1, 0.2, 0.3), (0.2, 0.3, 0.4)\}$	$\{0.5\}$	$\{(0.2, 0.3, 0.5), (0.4, 0.5, 0.6)\}$
wear	$\{(0.1, 0.2, 0.4)\}$	$\{(0.5, 0.6, 0.7), (0.5, 0.6, 0.8)\}$	$\{(0.2, 0.3, 0.4)\}$	$\{(0.4, 0.5, 0.6), (0.5, 0.7, 0.8)\}$	$\{0.5\}$

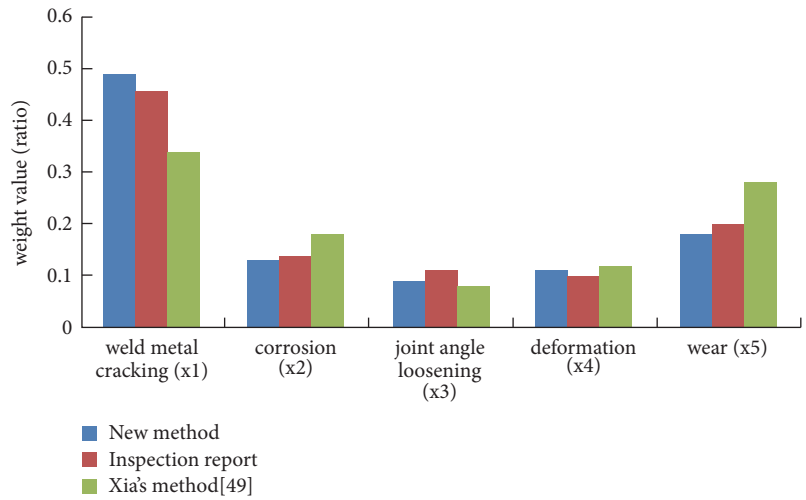

FIGURE 2: Comparison of the new method, Xia's method, and the inspection reports.

$$\begin{aligned} S_4' &= (0.348, 0.4262, 0.5176) \\ &\quad \otimes \left(\frac{1}{3.2612}, \frac{1}{2.707}, \frac{1}{2.2963}\right) \\ &\approx (0.1067, 0.1575, 0.2251), \\ S_5' &= (0.4593, 0.5414, 0.6522) \\ &\quad \otimes \left(\frac{1}{3.2612}, \frac{1}{2.707}, \frac{1}{2.2963}\right) \\ &\approx (0.1408, 0.2, 0.284). \end{aligned} \tag{30}$$

Then, using (25) and (26), we have

$$\begin{aligned} &V\left(S_1' \geq S_2'\right) = 1, \\ &V\left(S_1' \geq S_3'\right) = 1, \\ &V\left(S_1' \geq S_4'\right) = 1, \\ &V\left(S_1' \geq S_2'\right) = 1, \\ &V\left(S_2' \geq S_1'\right) = 0.26, \\ &V\left(S_2' \geq S_3'\right) = 0.62, \\ &V\left(S_2' \geq S_4'\right) = 1, \\ &V\left(S_2' \geq S_5'\right) = 0.68, \\ &V\left(S_3' \geq S_1'\right) = 0.19, \\ &V\left(S_3' \geq S_2'\right) = 1, \\ &V\left(S_3' \geq S_4'\right) = 1, \\ &V\left(S_3' \geq S_5'\right) = 1, \\ &V\left(S_4' \geq S_1'\right) = 0.21, \\ &V\left(S_4' \geq S_2'\right) = 0.98, \\ &V\left(S_4' \geq S_3'\right) = 0.61, \\ &V\left(S_4' \geq S_5'\right) = 0.67, \\ &V\left(S_5' \geq S_1'\right) = 0.37, \\ &V\left(S_5' \geq S_2'\right) = 1, \\ &V\left(S_5' \geq S_3'\right) = 0.95, \\ &V\left(S_5' \geq S_3'\right) = 1. \end{aligned} \tag{31}$$

Finally, using (27), the weight vector of these five potential failure modes can be obtained, i.e., $W_{FM} = (0.49, 0.13, 0.09, 0.11, 0.18)^T$. Thus, the priority ranking of the five potential failure modes follows the order of $FM_{x_1} \succ FM_{x_5} \succ FM_{x_2} \succ FM_{x_4} \succ FM_{x_3}$, as shown in Figure 2. The result indicates that the most important failure mode is the *weld metal cracking*, followed by *wear, corrosion, deformation,* and *joint angle loosening,* and the *weld metal cracking* should be given the top priority for correction.

4.2. The Hesitant Fuzzy Preference Relation Method. In the following, the hesitant fuzzy preference relation method [49] is used to obtain the metal-frame major potential failure modes and its priority for the same case as above. Based on Table 3, the same expert team gave the hesitant fuzzy preference relation matrix H of failure modes as shown in Table 4. And using the hesitant fuzzy preference relation method, we can obtain the weight vector of these five potential failure modes as $W_{FM} = (0.42, 0.36, 0.16, 0.21, 0.38)^T$ (the detailed solution process can be referred to in [49]). Thus, the priority ranking of the five potential failure modes follows the order of $FM_{x_1} \succ FM_{x_5} \succ FM_{x_2} \succ FM_{x_4} \succ FM_{x_3}$, as shown in Figure 2.

Table 4: The hesitant fuzzy preference relation matrix H of failure modes.

	weld metal cracking	corrosion	joint angle loosening	deformation	wear
weld metal cracking	$\{0.5\}$	$\{0.5, 0.6, 0.7, 0.8, 0.9\}$	$\{0.6, 0.7, 0.8, 0.9\}$	$\{0.6, 0.7, 0.8, 0.9\}$	$\{0.6, 0.8, 0.9\}$
corrosion	$\{0.1, 0.2, 0.3, 0.4, 0.5\}$	$\{0.5\}$	$\{0.4, 0.5, 0.6, 0.7, 0.8\}$	$\{0.3, 0.4, 0.5\}$	$\{0.2, 0.3, 0.4, 0.5\}$
joint angle loosening	$\{0.1, 0.2, 0.3, 0.4, 0.5\}$	$\{0.2, 0.3, 0.4, 0.5, 0.6\}$	$\{0.5\}$	$\{0.6, 0.7, 0.8, 0.9\}$	$\{0.6, 0.7, 0.8\}$
deformation	$\{0.1, 0.2, 0.3, 0.4\}$	$\{0.5, 0.6, 0.7\}$	$\{0.1, 0.2, 0.3, 0.4)\}$	$\{0.5\}$	$\{0.2, 0.3, 0.4, 0.5, 0.6\}$
wear	$\{0.1, 0.2, 0.4\}$	$\{0.5, 0.6, 0.7, 0.8\}$	$\{0.2, 0.3, 0.4\}$	$\{0.4, 0.5, 0.6, 0.7, 0.8\}$	$\{0.5\}$

4.3. Comparative Analysis of the Two Methods. According to the inspection reports of failure modes of roller coasters coverage for the latest five-year period by the China Special Equipment Inspection Institute, the normalized proportions of these five failure modes are 0.46, 0.14, 0.11, 0.10, and 0.20, which means the priority ranking of them follows the order of $FM_{x_1} \succ FM_{x_5} \succ FM_{x_2} \succ FM_{x_3} \succ FM_{x_4}$, as shown in Figure 2. And Figure 2 shows that the results of the new method and Xia's method are pretty similar to those of the inspection reports, except for the priority ranking of FM_{x_3} and FM_{x_4}. The difference of weight values (ratio) of FM_{x_3} and FM_{x_4} is very slight, which has no practical significance and can be neglected from the point of view of engineering. Furthermore, it is obvious from Figure 2 that the gaps between the new method and the inspection reports are smaller than that between the Xia's method and the inspection reports. Through the above comparative analysis, we can see that the new method is even more reasonable for reality.

5. Conclusion

In this paper, hesitant triangular fuzzy preference relation is proposed and is then applied to determine the priority of potential failure models of a work system. The known hesitant triangular fuzzy weighted averaging aggregation operators are adopted to aggregate the individual preferences and the score function is applied to calculate the score values of each alternative, based on which, the weights of each alternative are obtained using Chang's fuzzy AHP method. Therefore, the priority of potential failure models in both qualitative and quantitative settings is obtained. Finally, an illustrative example is provided to demonstrate the proposed method. In the future, we shall focus on the application of the proposed model in decision making to other areas.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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References

[1] T. L. Saaty, "How to make a decision: the analytic hierarchy process," *European Journal of Operational Research*, vol. 48, no. 1, pp. 9–26, 1990.

[2] S. A. Orlovsky, "Decision-making with a fuzzy preference relation," *Fuzzy Sets and Systems*, vol. 1, no. 3, pp. 155–167, 1978.

[3] T. Saaty, "Modeling unstructured decision problems—the theory of analytical hierarchies," *Mathematics and Computers in Simulation*, vol. 20, no. 3, pp. 147–158, 1978.

[4] T. L. Saaty, "Axiomatic foundation of the analytic hierarchy process," *Management Science*, vol. 32, no. 7, pp. 841–855, 1986.

[5] S. Alonso, E. Herrera-Viedma, F. Chiclana, and F. Herrera, "A web based consensus support system for group decision making problems and incomplete preferences," *Information Sciences*, vol. 180, no. 23, pp. 4477–4495, 2010.

[6] Y. Dong, G. Zhang, W.-C. Hong, and Y. Xu, "Consensus models for AHP group decision making under row geometric mean prioritization method," *Decision Support Systems*, vol. 49, no. 3, pp. 281–289, 2010.

[7] J. Aguarón and J. M. Moreno-Jiménez, "The geometric consistency index: Approximated thresholds," *European Journal of Operational Research*, vol. 147, no. 1, pp. 137–145, 2003.

[8] J. Ma, Z.-P. Fan, Y.-P. Jiang, J.-Y. Mao, and L. Ma, "A method for repairing the inconsistency of fuzzy preference relations," *Fuzzy Sets and Systems*, vol. 157, no. 1, pp. 20–33, 2006.

[9] Z. S. Xu, "Goal programming models for obtaining the priority vector of incomplete fuzzy preference relation," *International Journal of Approximate Reasoning*, vol. 36, no. 3, pp. 261–270, 2004.

[10] Z. Xu and Q. Da, "A least deviation method to obtain a priority vector of a fuzzy preference relation," *European Journal of Operational Research*, vol. 164, no. 1, pp. 206–216, 2005.

[11] Z. Xu and J. Chen, "Some models for deriving the priority weights from interval fuzzy preference relations," *European Journal of Operational Research*, vol. 184, no. 1, pp. 266–280, 2008.

[12] F. Chiclana, E. Herrera-Viedma, F. Alonso, and S. Herrera, "Cardinal consistency of reciprocal preference relations: a characterization of multiplicative transitivity," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 1, pp. 14–23, 2009.

[13] R. Vetschera, "Deriving rankings from incomplete preference information: a comparison of different approaches," *European Journal of Operational Research*, vol. 258, no. 1, pp. 244–253, 2017.

[14] S. Genç, F. E. Boran, D. Akay, and Z. S. Xu, "Interval multiplicative transitivity for consistency, missing values and priority weights of interval fuzzy preference relations," *Information Sciences*, vol. 180, no. 24, pp. 4877–4891, 2010.

[15] F. E. Boran, D. Akay, and R. R. Yager, "An overview of methods for linguistic summarization with fuzzy sets," *Expert Systems with Applications*, vol. 61, pp. 356–377, 2016.

[16] Z. Xu, "Intuitionistic fuzzy aggregation operators," *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 6, pp. 1179–1187, 2007.

[17] K. Atanassov, *lntuitionistic fuzzy sets , presented at the VII ITKRs Session*, Bulgaria, Sofia, 1983.

[18] H. Liao and Z. Xu, "Priorities of intuitionistic fuzzy preference relation based on multiplicative consistency," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 6, pp. 1669–1681, 2014.

[19] K. Atanassov and G. Gargov, "Interval valued intuitionistic fuzzy sets," *Fuzzy Sets and Systems*, vol. 31, no. 3, pp. 343–349, 1989.

[20] Y. N. Vicenc Torra, "On hesitant fuzzy sets and decision," in *Proceedings of the 18th IEEE International Conference on Fuzzy Systems*, pp. 1378–1382, Jeju Island, South Korea, 2009.

[21] V. Torra, "Hesitant fuzzy sets," *International Journal of Intelligent Systems*, vol. 25, no. 6, pp. 529–539, 2010.

[22] M. Xia and Z. Xu, "Hesitant fuzzy information aggregation in decision making," *International Journal of Approximate Reasoning*, vol. 52, no. 3, pp. 395–407, 2011.

[23] Z. S. Xu and M. Xia, "Distance and similarity measures for hesitant fuzzy sets," *Information Sciences*, vol. 181, no. 11, pp. 2128–2138, 2011.

[24] Z. S. Xu and M. M. Xia, "Hesitant fuzzy entropy and cross-entropy and their use in multiattribute decision-making," *International Journal of Intelligent Systems*, vol. 27, no. 9, pp. 799–822, 2012.

[25] N. Chen, Z. Xu, and M. Xia, "Correlation coefficients of hesitant fuzzy sets and their applications to clustering analysis," *Applied Mathematical Modelling*, vol. 37, no. 4, pp. 2197–2211, 2013.

[26] R. M. Rodríguez, L. Martínez, V. Torra, Z. S. Xu, and F. Herrera, "Hesitant fuzzy sets: state of the art and future directions," *International Journal of Intelligent Systems*, vol. 29, no. 6, pp. 495–524, 2014.

[27] X. Zhao, R. Lin, and G. Wei, "Hesitant triangular fuzzy information aggregation based on Einstein operations and their application to multiple attribute decision making," *Expert Systems with Applications*, vol. 41, no. 4, pp. 1086–1094, 2014.

[28] C. Wei, N. Zhao, and X. Tang, "Operators and comparisons of hesitant fuzzy linguistic term sets," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 3, pp. 575–585, 2014.

[29] H. Liu and R. M. Rodríguez, "A fuzzy envelope for hesitant fuzzy linguistic term set and its application to multicriteria decision making," *Information Sciences*, vol. 258, pp. 220–238, 2014.

[30] Z. Wu and J. Xu, "Possibility distribution-based approach for MAGDM with hesitant fuzzy linguistic information," *IEEE Transactions on Cybernetics*, vol. 46, no. 3, pp. 694–705, 2016.

[31] B. Zhu and Z. Xu, "Consistency measures for hesitant fuzzy linguistic preference relations," *IEEE Transactions on Fuzzy Systems*, vol. 22, no. 1, pp. 35–45, 2014.

[32] H. Liao, Z. Xu, and X.-J. Zeng, "Qualitative decision making with correlation coefficients of hesitant fuzzy linguistic term sets," *Knowledge-Based Systems*, vol. 76, no. 1, pp. 127–138, 2015.

[33] E. Herrera-Viedma, S. Alonso, F. Chiclana, and F. Herrera, "A consensus model for group decision making with incomplete fuzzy preference relations," *IEEE Transactions on Fuzzy Systems*, vol. 15, no. 5, pp. 863–877, 2007.

[34] Y.-M. Wang, Z.-P. Fan, and Z. Hua, "A chi-square method for obtaining a priority vector from multiplicative and fuzzy preference relations," *European Journal of Operational Research*, vol. 182, no. 1, pp. 356–366, 2007.

[35] Y.-M. Wang and Z.-P. Fan, "Group decision analysis based on fuzzy preference relations: logarithmic and geometric least squares methods," *Applied Mathematics and Computation*, vol. 194, no. 1, pp. 108–119, 2007.

[36] Z. Zhang and C. Wu, "On the use of multiplicative consistency in hesitant fuzzy linguistic preference relations," *Knowledge-Based Systems*, vol. 72, pp. 13–27, 2014.

[37] H. Wang and Z. Xu, "Some consistency measures of extended hesitant fuzzy linguistic preference relations," *Information Sciences*, vol. 297, pp. 316–331, 2015.

[38] R. Ureña, F. Chiclana, J. A. Morente-Molinera, and E. Herrera-Viedma, "Managing incomplete preference relations in decision making: a review and future trends," *Information Sciences*, vol. 302, no. 1, pp. 14–32, 2015.

[39] A. Hatami-Marbini and M. Tavana, "An extension of the Electre I method for group decision-making under a fuzzy environment," *Omega* , vol. 39, no. 4, pp. 373–386, 2011.

[40] L. Martínez and F. Herrera, "An overview on the 2-tuple linguistic model for computing with words in decision making: extensions, applications and challenges," *Information Sciences*, vol. 207, pp. 1–18, 2012.

[41] J. M. Mendel and D. R. Wu, *Perceptual Computing Aiding People in Making Subjective Judgments*, John Wiley and Sons, New Jersey, NJ, USA, 2010.

[42] W. Pedrycz, R. Al-Hmouz, A. Morfeq, and A. S. Balamash, "Building granular fuzzy decision support systems," *Knowledge-Based Systems*, vol. 58, pp. 3–10, 2014.

[43] J. Pang and J. Liang, "Evaluation of the results of multi-attribute group decision-making with linguistic information," *Omega* , vol. 40, no. 3, pp. 294–301, 2012.

[44] Z. Wu and J. Xu, "Managing consistency and consensus in group decision making with hesitant fuzzy linguistic preference relations," *Omega*, vol. 65, pp. 28–40, 2016.

[45] Z. Wu, B. Jin, and J. Xu, "Local feedback strategy for consensus building with probability-hesitant fuzzy preference relations," *Applied Soft Computing*, vol. 67, pp. 691–705, 2018.

[46] Z. Wu, J. Xu, X. Jiang, and etal., "Two MAGDM models based on hesitant fuzzy linguistic term sets with possibility distributions: VIKOR and TOPSIS," *Information Sciences*, vol. 473, pp. 101–120, 2019.

[47] D. Chang, "Applications of the extent analysis method on fuzzy AHP," *European Journal of Operational Research*, vol. 95, no. 3, pp. 649–655, 1996.

[48] T. Tanino, "Fuzzy preference orderings in group decision making," *Fuzzy Sets and Systems*, vol. 12, no. 2, pp. 117–131, 1984.

[49] M. Xia and Z. Xu, "Managing hesitant information in GDM problems under fuzzy and multiplicative preference relations ," *International Journal of Uncertainty, Fuzziness and Knowledge-Based Systems*, vol. 21, no. 6, pp. 865–897, 2013.

[50] Y. Xu, F. J. Cabrerizo, and E. Herrera-Viedma, "A consensus model for hesitant fuzzy preference relations and its application in water allocation management," *Applied Soft Computing*, vol. 58, pp. 265–284, 2017.

[51] K. Woodcock, "Rider errors and amusement ride safety: observation at three carnival midways," *Accident Analysis and Prevention*, vol. 39, no. 2, pp. 390–397, 2007.

[52] K. Woodcock, "Content analysis of 100 consecutive media reports of amusement ride accidents," *Accident Analysis and Prevention*, vol. 40, no. 1, pp. 89–96, 2008.

[53] K. Woodcock, "Amusement ride injury data in the United States," *Safety Science*, vol. 62, pp. 466–474, 2014.

[54] Y. L. Wang, W. K. Song, and W. M. Lin, "The method of risk assessmet of large amusement rides," *China Special Equipment Safety*, vol. 30, no. 4, pp. 58–62, 2014.

[55] K. Woodcock, "Human factors and use of amusement ride control interfaces," *International Journal of Industrial Ergonomics*, vol. 44, no. 1, pp. 99–106, 2014.

[56] X. D. Zhang, C. J. Zhuang, and C. J. Yang, "On the safety evaluation of the large-type circular amusement rides based on the BP neural network," *Journal of Safety and Environment*, vol. 4, pp. 28–31, 2016.