Research Article

Extended Version of Linguistic Picture Fuzzy TOPSIS Method and Its Applications in Enterprise Resource Planning Systems

Shouzhen Zeng,1,2 Muhammad Qiyas,3 Muhammad Arif,3,4 and Tariq Mahmood4

1School of Management, Fudan University, Shanghai 200433, China
2School of Business, Ningbo University, Ningbo 315211, China
3Department of Mathematics, Abdul Wali Khan University Mardan, Pakistan
4Department of Electronics, University of Engineering and Technology, Taxila Sub-Campus Chakwal, Pakistan

Correspondence should be addressed to Muhammad Arif; marifmaths@awkum.edu.pk

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The main objective of the proposed research in this paper is introducing an extended version of the linguistic picture fuzzy TOPSIS technique and then solving the problems in enterprise resource planning systems. In this article, we use the uncertain information in terms of linguistic picture fuzzy numbers; the decision maker provides membership, neutral, and nonmembership fuzzy linguistic terms to represent uncertain assessments information of alternatives in linguistic multicriteria decision making (LMCDMs). In order to introduce the extended version of TOPSIS method, we defined a new hamming distance measure between two linguistic picture fuzzy numbers. Further, we apply the proposed method to problem of enterprise resource planning systems and discuss numerical implementation of the proposed method of LMCDM.

1. Introduction

In our daily life, we face some tasks and activities to handle with the help of decision making. Basically, decision making is an intellectual process which depends on different spiritual and reasoning processes; in the decision making process we choose a good alternative from the set of all available alternatives. For a single decision maker, it is mostly difficult to examine all related conditions of a problem due to the increasing complication of the socioeconomic environment. In everyday life, many decision making measures take place in group information. However, under more conditions, for the real multiple attribute decision making problems, the decision information about alternatives is usually uncertain or fuzzy due to the increasing complication of the socioeconomic situation and the ambiguity of inherent subjective attribute of human thinking; thus, numerical values are incomplete or partial for exemplary real-life decision problems. However, in practice, the attribute values of decision problems are not ever disposed by crisp information because of the fuzziness, and many of them are more considerable to be stated by fuzzy information; see [1–5], such as fuzzy numbers, intuitionistic fuzzy numbers [6], and linguistic variable [7, 8].

Fuzzy set theory was firstly defined by Zadeh’s in 1965 [9], which is a framework to encounter confusion, ambiguity, and exaggeration and perform a degree of membership for each member of the nature of discussion to a subset of it. Therefore, we have a forum of true values. The degree of nonmembership was added by Atanassov in 1983 to FS; Atanassov defined the notion of intuitionistic fuzzy set (IFS) [10], which is mostly permissible to handle the ambiguity quantification and give the occasion to correctly model the problem based on existing judgment and experience [11]. The two of these structures are assumed to be a soft approach which, in turn, start soft computing and approximate analysis [12].

Later on in 2014, Cuong proposed the concept of picture fuzzy set (PFS) and studied its basic operations and properties. The picture fuzzy set is designated by the membership, neutral, and nonmembership degree. The only restraint is that the sum of the membership, neutral, and nonmembership degrees must not exceed 1. PFS are basically used for those
models, which cannot be expressed in the traditional PS and IFS, like the opinions of the people which have answers of the following types: yes, abstain, no, and refusal. Already some promotion has happened in the research of the PFS theory. In (2014) P. H. Phong discussed the composition of picture fuzzy relation [13], and Singh studied the correlation coefficients for picture fuzzy set and tested the correlation coefficient for clustering analysis with picture fuzzy information. Son et al. developed many novel fuzzy clustering algorithms based on picture fuzzy sets and applications to time series forecasting and weather forecasting (Thong and Son, 2015). Wei [14] proposed picture fuzzy cross-entropy model for multiple attribute decision making problems. S. jun et al. (2017) proposed picture fuzzy set and tested the correlation coefficient for relation [13], and Singh studied the correlation coefficients (2014) P. H. Phong discussed the composition of picture fuzzy set. Promotion has happened in the research of the PFS theory. In (1981) Hwang and Yoon introduced the idea of TOPSIS method; the idea that the selected alternative must be the closest to the positive ideal solution and the farthest from negative ideal solution is an outstanding multicriteria decision making method [17]. In (1996) Triantaphyllou et al. continued the TOPSIS strategy for decision making problem with fuzzy data [18]. Many authors extended TOPSIS methods and applied these on multiple attribute decision making [19, 20]. Chen (2000) stretches out TOPSIS to fuzzy group decision making problems [21]. Ashitani et al. [22] used the TOPSIS method to solve decision making problem with the interval valued fuzzy sets. To deal with multiple attribute decision making problem with intuitionistic fuzzy sets, He and Gong [23] developed a natural generalization of the TOPSIS method. Liu et al. [24] proposed a new TOPSIS method for decision making problems with interval-valued intuitionistic fuzzy data.

The refusal degree of \( Y \) in \( H \) is defined as \( \sigma_H (y) = 1 - a_H (y) - b_H (y) - c_H (y) \), and if \( \sigma_H (y) = 0 \), \( \forall y \in Y \), then \( H \) is reduced to an IFS [10], and if \( \sigma_H (y) = b_H (y) = 0 \) for all \( y \in Y \), \( H \) is degenerated to an FS.

Definition 2 (see [25, 26]). Let \( \tilde{L} = (\xi_0, \xi_1, \ldots, \xi_{\gamma-1}) \) be the absolute order distinct term finite set. Then \( \tilde{L} \) is called a linguistic term set, where \( g \) is the odd cardinality with \( g > 0 \). Generally \( g \) is considered as 3, 5, 7, etc. For example, when \( g = 7 \) the linguistic term set is as follows:

\[
\tilde{L} = (\xi_0, \xi_1, \xi_2, \xi_3, \xi_4, \xi_5, \xi_6) \tag{3}
\]

= (v. poor, poor, slightly poor, fair, slightly good, good, v. good)

If \( \xi_f, \xi_g \in \tilde{L} \), then the following characteristics are satisfied:

(1) The order of set as: \( \xi_f < \xi_g \iff f < g \);

(2) The operator of negation as: \( \text{neg} (\xi_f) = \xi_{g-1-f} \);

(3) Maximum \( (\xi_f, x_g) = \xi_f \), if \( f \geq g \);

(4) Minimum \( (\xi_f, x_g) = \xi_g \), if \( f \leq g \).

Definition 3 (see [16]). Let \( Y \) be a nonempty set; then we defined the picture linguistic number set \( R \) in \( Y \) as

\[
H = \{(y, \langle \xi_0(y), a_H(y), b_H(y), c_H(y) \rangle) \mid y \in Y \} \tag{4}
\]

Which is designated by a linguistic term \( \xi_0(y) \in \tilde{L} \), a positive degree, a neutral degree, and a negative degree \( a_H (y), b_H (y), c_H (y) \in [0, 1] \), respectively, the element \( y \) to \( \xi_0(y) \) with the following condition:

\[
a_H (y) + b_H (y) + c_H (y) \leq 1, \quad \forall y \in Y. \tag{5}
\]

and \( \sigma_0 (y) = 1 - a_H (y) - b_H (y) - c_H (y) \) is said to be the refusal degree of \( y \) to \( \xi_0(y) \) for all \( y \in Y \).

If \( \sigma_H (Y) = 0, \forall y \in Y \), in this case the picture linguistic set is reduced to the intuitionistic linguistic set.

Definition 4. Let \( \xi_a, \xi_b, \xi_c \in \tilde{L}_{[0, g]} \) and \( \alpha = \langle \xi_a, \xi_b, \xi_c \rangle \), if \( a + b + c \leq g \). Then, \( \alpha \) is called the linguistic picture fuzzy numbers on \( \tilde{L}_{[0, g]} \). If \( \xi_a, \xi_b, \xi_c \in \tilde{L} \), then we consider \( \alpha \) the original linguistic picture fuzzy numbers, and virtual linguistic picture fuzzy numbers, otherwise.

\[
\Gamma_{[0, g]} = \langle (\xi_a, \xi_b, \xi_c) \mid \xi_a, \xi_b, \xi_c \in \tilde{L}_{[0, g]} \rangle \text{ represent all linguistic picture fuzzy numbers defined on } \tilde{L}_{[0, g]}.
\]
Definition 5. Let \( \langle \xi_1, \xi_2, \xi_3 \rangle, \langle \xi_3, \xi_2, \xi_1 \rangle, \langle \xi_2, \xi_1, \xi_3 \rangle \in \Gamma_{[0, g]} \). Then, the following operators hold by operations of picture fuzzy sets:

\[
\begin{align*}
\langle \xi_1, \xi_2, \xi_3 \rangle & \cup \langle \xi_3, \xi_2, \xi_1 \rangle = \langle \max (\xi_1, \xi_3), \min (\xi_2, \xi_2), \min (\xi_1, \xi_1) \rangle; \\
\langle \xi_1, \xi_2, \xi_3 \rangle & \cap \langle \xi_3, \xi_2, \xi_1 \rangle = \langle \min (\xi_1, \xi_1), \max (\xi_2, \xi_2), \max (\xi_1, \xi_1) \rangle; \\
\langle \xi_1, \xi_2, \xi_3 \rangle & \ominus \langle \xi_3, \xi_2, \xi_1 \rangle = \langle \xi_1, \xi_2, \xi_3 \rangle; \\
\langle \xi_1, \xi_2, \xi_3 \rangle & = \langle \xi_1, \xi_2, \xi_3 \rangle.
\end{align*}
\]

Definition 6. Let \( \langle \xi_1, \xi_2, \xi_1 \rangle, \langle \xi_1, \xi_2, \xi_1 \rangle, \langle \xi_2, \xi_1, \xi_3 \rangle \in \Gamma_{[0, g]} \), \( \lambda > 0 \). Then we defined the following operations of linguistic picture fuzzy numbers:

\[
\begin{align*}
\langle \xi_1, \xi_2, \xi_3 \rangle & \oplus \langle \xi_1, \xi_2, \xi_3 \rangle = \langle \xi_1, \xi_1, \xi_1 \rangle; \\
\lambda \langle \xi_1, \xi_2, \xi_3 \rangle & = \langle \xi_1, \xi_2, \xi_3 \rangle.
\end{align*}
\]

Definition 7. Let \( \alpha_i = \langle \xi_1, \xi_2, \xi_1 \rangle \in \Gamma_{[0, g]} \) \( (i = 1, 2, \ldots, m) \) and \( \omega_i \) be the weighting vector of \( \alpha_i \), satisfying \( 0 \leq \omega_i \leq 1 \) \( (i = 1, 2, \ldots, m) \) and \( \sum_{i=1}^{m} \omega_i = 1 \). Then, we defined the linguistic picture fuzzy weighted averaging operator as

\[
\text{LPFWA}(\alpha_1, \alpha_2, \ldots, \alpha_m) = \langle g_{\text{PFWA}}(\xi_1, \xi_2, \xi_3) \rangle
\]

Example 8. For \( X = \{x_1, x_2, x_3\} \), \( C = \{c_1, c_2, c_3\} \) be the sets of alternatives and criteria, respectively. The linguistic set \( \hat{I} = \{\xi_0\} \) (nothing), \( \xi_1 \) (very very low), \( \xi_2 \) (very low), \( \xi_3 \) (low), \( \xi_4 \) (medium), \( \xi_5 \) (high), \( \xi_6 \) (very high), \( \xi_7 \) (very very high), \( \xi_8 \) (perfect). The decision makers provide the linguistic picture fuzzy assessments as shown in Table 1.

3. The Distance between Two Linguistic Picture Fuzzy Numbers

In TOPSIS method the distance measure is an important concept. We can define the Hamming distance between two linguistic picture fuzzy sets.
Example II. In Example 8, suppose that \( \omega = \langle 0.3, 0.4, 0.3 \rangle \) are the weight of criteria \( C = \{c_1, c_2, c_3\} \); then the linguistic picture fuzzy decision matrix of the LMCDM is

\[
D = \begin{pmatrix}
\langle c_1 \rangle \langle 0.3 \rangle & \langle c_2 \rangle \langle 0.4 \rangle & \langle c_3 \rangle \langle 0.3 \rangle \\
\langle x_1 \rangle & \langle \xi_{11}, \xi_{12}, \xi_{13} \rangle & \langle \xi_{14}, \xi_{15}, \xi_{16} \rangle \\
\langle x_2 \rangle & \langle \xi_{21}, \xi_{22}, \xi_{23} \rangle & \langle \xi_{24}, \xi_{25}, \xi_{26} \rangle \\
\langle x_3 \rangle & \langle \xi_{31}, \xi_{32}, \xi_{33} \rangle & \langle \xi_{34}, \xi_{35}, \xi_{36} \rangle \\
\end{pmatrix}
\]

(13)

4.2. The Positive and Negative Ideal Solutions of Alternatives.

To determine the positive ideal solution and negative ideal solution of the alternatives, we apply the \( \cup \) and \( \cap \) operation of the linguistic picture fuzzy numbers and the linguistic picture fuzzy weighted averaging operator on the linguistic picture fuzzy matrix.

Operation \( \cup \) and \( \cap \) of linguistic picture fuzzy sets for each column is defined as

\[
\forall c_j = \begin{pmatrix}
\mathbb{W}_{c_{j1}} \mathbb{W}_{c_{j2}} \mathbb{W}_{c_{j3}} \\
\mathbb{W}_{c_{j4}} \mathbb{W}_{c_{j5}} \mathbb{W}_{c_{j6}} \\
\mathbb{W}_{c_{j7}} \mathbb{W}_{c_{j8}} \mathbb{W}_{c_{j9}} \\
\end{pmatrix}
\]

\[
= \begin{pmatrix}
\langle \text{max} (\xi_{i1}, \xi_{i2}, \ldots, \xi_{in}) \rangle, \text{min} (\xi_{i1}, \xi_{i2}, \ldots, \xi_{in}) \\
\text{max} (\xi_{i1}, \xi_{i2}, \ldots, \xi_{in}), \min (\xi_{i1}, \xi_{i2}, \ldots, \xi_{in}) \\
\text{max} (\xi_{i1}, \xi_{i2}, \ldots, \xi_{in}), \min (\xi_{i1}, \xi_{i2}, \ldots, \xi_{in}) \\
\end{pmatrix}
\]

(14)

\[\land c_j = \begin{pmatrix}
\mathbb{W}_{c_{j1}} \mathbb{W}_{c_{j2}} \mathbb{W}_{c_{j3}} \\
\mathbb{W}_{c_{j4}} \mathbb{W}_{c_{j5}} \mathbb{W}_{c_{j6}} \\
\mathbb{W}_{c_{j7}} \mathbb{W}_{c_{j8}} \mathbb{W}_{c_{j9}} \\
\end{pmatrix}
\]

Based on equations (15) and (16), we calculate PIS and NIS of \( D \).

\[
PIS = \begin{pmatrix}
\mathbb{W}_{c_{11}} \mathbb{W}_{c_{12}} \mathbb{W}_{c_{13}} \\
\mathbb{W}_{c_{21}} \mathbb{W}_{c_{22}} \mathbb{W}_{c_{23}} \\
\mathbb{W}_{c_{31}} \mathbb{W}_{c_{32}} \mathbb{W}_{c_{33}} \\
\end{pmatrix} = \text{LPFWA} \left( \mathbb{W}_{c_{11}}, \mathbb{W}_{c_{12}}, \ldots, \mathbb{W}_{c_{m}} \right)
\]

(15)

\[
NIS = \begin{pmatrix}
\mathbb{W}_{c_{11}} \mathbb{W}_{c_{12}} \mathbb{W}_{c_{13}} \\
\mathbb{W}_{c_{21}} \mathbb{W}_{c_{22}} \mathbb{W}_{c_{23}} \\
\mathbb{W}_{c_{31}} \mathbb{W}_{c_{32}} \mathbb{W}_{c_{33}} \\
\end{pmatrix} = \text{LPFWA} \left( \mathbb{W}_{c_{11}}, \mathbb{W}_{c_{12}}, \ldots, \mathbb{W}_{c_{m}} \right)
\]

(16)

Example 12. According to \( D \) in Example 8, we have

\[
\forall c_1 = \text{LPFWA} \left( \langle \xi_{11}, \xi_{12}, \xi_{13} \rangle, \min (\xi_{14}, \xi_{15}, \xi_{16}) \right),
\]

\[
\land c_1 = \text{LPFWA} \left( \langle \xi_{24}, \xi_{25}, \xi_{26} \rangle, \min (\xi_{21}, \xi_{22}, \xi_{23}) \right),
\]

\[
\land c_2 = \text{LPFWA} \left( \langle \xi_{34}, \xi_{35}, \xi_{36} \rangle, \min (\xi_{31}, \xi_{32}, \xi_{33}) \right),
\]

(17)

\[
\forall c_2 = \text{LPFWA} \left( \langle \xi_{41}, \xi_{42}, \xi_{43} \rangle, \min (\xi_{44}, \xi_{45}, \xi_{46}) \right),
\]

\[
\land c_3 = \text{LPFWA} \left( \langle \xi_{51}, \xi_{52}, \xi_{53} \rangle, \min (\xi_{54}, \xi_{55}, \xi_{56}) \right),
\]

(18)
4.3. The Relative Closeness and the Ranking of Alternatives. For the alternatives ranking, we find for every alternative the relative close degree, where the relative close degree is determined by distance between the linguistic picture fuzzy assessment of each alternative and the positive and negative ideal solutions. Based on the decision matrix, we obtain the linguistic picture fuzzy set of each alternative and the positive and negative ideal solutions. Based on Hamming distance between the linguistic picture fuzzy set of individual alternative and the positive and negative alternatives is the following bases "select the shortest distance of each alternative in existing TOPSIS ideal solutions, respectively; formally, it is fulfilled by the distance between the linguistic picture fuzzy set of each alternative which provides the relative close degree, where the relative close degree is determined by distance between the linguistic picture fuzzy set of each alternative and the positive and negative ideal solutions. Based on the decision matrix, we obtain the following results, applying equations (22), (23), and (24):

\[
d(A_j, PIS) = \left| a_j - b_j \right| + \left| b_j - b_p \right| + \left| c_j - c_p \right| + \left| r_j - r_p \right| \tag{20}
\]

\[
d(A_j, NIS) = \left| a_j - a_n \right| + \left| b_j - b_n \right| + \left| c_j - c_n \right| + \left| r_j - r_n \right| \tag{21}
\]

Where \( r_j = g - a_j - b_j - c_j, r_p = g - a_p - b_p - c_p, r_n = g - a_n - b_n - c_n \).

Originating from the TOPSIS method, the ranking of alternatives is the following bases "select the shortest distance of each alternative in existing TOPSIS methods. Based on Hamming distance between the linguistic picture fuzzy set of each alternative and the positive and negative ideal solutions (Eqs. (20) and (21)), we supply for each alternative \( x_i \) the following relative closeness degree \( C(x_i) \):

\[
d_{\text{max}} = \max \left( d(A_1, NIS), d(A_2, NIS), \ldots, d(A_n, NIS) \right) \tag{22}
\]

\[
d_{\text{min}} = \min \left( d(A_1, PIS), d(A_2, PIS), \ldots, d(A_n, PIS) \right) \tag{23}
\]

\[
C(x_i) = \frac{1}{2} \left( \frac{d(A_i, NIS)}{d_{\text{max}}} + \frac{d_{\text{min}}}{d(A_i, PIS)} \right) \tag{24}
\]

Basically, \( C(x_i) \in [0, 1] \) for any alternative \( x_i \); that is, \( C(x_i) \) is increasing for \( d(A_i, NIS) \), and decreasing for \( d(A_i, PIS) \). The alternative rankings are obtained based on the relative closeness degree of alternatives as follows: \( x_j < x_j' \iff C(x_j) \leq C(x_j') \).

\[
x_j < x_j' \iff C(x_j) \leq C(x_j') \tag{25}
\]

Example 13. According to \( D \) in Example 8, and equation (19), we collect the linguistic picture fuzzy assessment of each alternative as follows:

\[
A_1 = LPFWA \{ (\xi_1, \xi_1, \xi_3), (\xi_4, \xi_2, \xi_1), (\xi_1, \xi_2, \xi_3) \}
\]

\[
= \{ \xi_9 \cdot \left( \frac{1}{9} \right) \cdot \left( \frac{1}{9} \right) \cdot \left( \frac{1}{9} \right), \xi_9 \cdot \left( \frac{1}{9} \right) \cdot \left( \frac{1}{9} \right) \cdot \left( \frac{1}{9} \right) \}
\]

\[
\xi_9 \cdot \left( \frac{1}{9} \right) \cdot \left( \frac{1}{9} \right) \cdot \left( \frac{1}{9} \right), \xi_9 \cdot \left( \frac{1}{9} \right) \cdot \left( \frac{1}{9} \right) \cdot \left( \frac{1}{9} \right)
\]

\[
= \{ \xi_2, \xi_5, \xi_2, \xi_7 \}
\]

\[
A_2 = LPFWA \{ (\xi_1, \xi_2, \xi_3), (\xi_3, \xi_5, \xi_1), (\xi_3, \xi_5, \xi_2) \}
\]

\[
= \{ \xi_9 \cdot \left( \frac{1}{9} \right) \cdot \left( \frac{1}{9} \right) \cdot \left( \frac{1}{9} \right), \xi_9 \cdot \left( \frac{1}{9} \right) \cdot \left( \frac{1}{9} \right) \cdot \left( \frac{1}{9} \right) \}
\]

\[
= \{ \xi_2, \xi_5, \xi_2, \xi_7 \}
\]

\[
A_3 = LPFWA \{ (\xi_4, \xi_1, \xi_4), (\xi_4, \xi_1, \xi_5), (\xi_5, \xi_3, \xi_3) \}
\]

\[
= \{ \xi_9 \cdot \left( \frac{1}{9} \right) \cdot \left( \frac{1}{9} \right) \cdot \left( \frac{1}{9} \right), \xi_9 \cdot \left( \frac{1}{9} \right) \cdot \left( \frac{1}{9} \right) \cdot \left( \frac{1}{9} \right) \}
\]

Based on PIS and NIS in Example 12 and Equations (20) and (21), we obtain

\[
d(A_1, PIS) = \frac{2.65 - 4.10 + 2.53 - 1.26 + 1.65 - 2.42 + 2.17 - 3.36 + 0.28}{2} = 0.25\tag{27}
\]

and

\[
d(A_1, NIS) = \frac{2.19 - 4.10 + 2.53 - 1.26 + 1.65 - 2.42 + 2.17 - 3.36 + 0.28}{2} = 0.28\tag{28}
\]

Similarly, \( d(A_2, PIS) = 0.34 \) and \( d(A_2, NIS) = 0.39 \), \( d(A_3, PIS) = 0.21 \) and \( d(A_3, NIS) = 0.25 \).

Now we get the following results, applying equations (22), (23), and (24):

\[
d_{\text{max}} = (0.28, 0.39, 0.25) = 0.39
\]

\[
d_{\text{min}} = (0.25, 0.34, 0.21) = 0.21
\]

\[
C(x_1) = \frac{1}{2} \left( \frac{0.28}{0.39} + \frac{0.21}{0.25} \right) = 0.78
\]

\[
C(x_2) = \frac{1}{2} \left( \frac{0.39}{0.39} + \frac{0.21}{0.34} \right) = 0.79
\]

\[
C(x_3) = \frac{1}{2} \left( \frac{0.25}{0.39} + \frac{0.21}{0.21} \right) = 0.81
\]

According to equation (25), the ranking is as follows:

\[
x_3 > x_2 > x_1
\]

Hence, \( x_3 \) is the best alternative.
Algorithm 14.

**Step 1.** According to the membership, neutral, and non-membership fuzzy linguistic assessments, the decision matrix $D$ of the LMCMDM problem is constructed.

**Step 2.** Calculate $\veps_i$ and $\mu_i$, for each column, using the linguistic picture fuzzy weighted averaging operator to determine the PIS and NIS.

**Step 3.** To find the linguistic picture fuzzy assessment $A_j$ of each $x_j$, use the linguistic picture fuzzy weighted averaging operator.

**Step 4.** Find the Hamming distance $d(A_j, PIS)$ and $d(A_j, NIS)$ between each $A_j$ and PIS (NIS) are calculated; the maximum Hamming distance $d_{\text{max}}^-$ of all $d(A_j, NIS)$ and minimum Hamming distance $d_{\text{min}}^+$ of all $d(A_j, PIS)$ are obtained; then calculate the relative close degree $C(x_j)$ of every alternative $x_j$.

**Step 5.** Rank the alternatives using equation (25).

5. Example

A multinational company chooses an enterprise resource planning system to apply from three candidates $A = (a_1, a_2, a_3)$. To develop other considerable decisions, the Chief Information Officer (CIO) of the company evaluates the applicant enterprise resource planning systems in terms of three criteria, that is, $c_1$ (energy cost), $c_2$ (business), and $c_3$ (procedure complication), where $(0.3, 0.5, 0.2)$ are the weight of the criteria. Since the given criteria are qualitative, the estimated values are in the form of linguistic term, which are shown in Table 2.

**Step 2.** For each column of $D$, we have

$$
\veps_1 = \langle \max (\veps_4, \veps_3, \veps_2), \min (\veps_2, \veps_3, \veps_2), \min (\veps_2, \veps_3, \veps_2) \rangle = \langle \veps_4, \veps_2, \veps_2 \rangle,
$$

$$
\veps_2 = \langle \max (\veps_4, \veps_3, \veps_2), \min (\veps_3, \veps_5, \veps_4), \min (\veps_1, \veps_2, \veps_2) \rangle = \langle \veps_4, \veps_3, \veps_1 \rangle,
$$

$$
\veps_3 = \langle \max (\veps_6, \veps_3, \veps_7), \min (\veps_1, \veps_2, \veps_1), \min (\veps_2, \veps_4, \veps_1) \rangle = \langle \veps_7, \veps_1, \veps_1 \rangle,
$$

$$
\mu_1 = \langle \min (\veps_4, \veps_3, \veps_2), \min (\veps_2, \veps_3, \veps_2), \max (\veps_2, \veps_3, \veps_2) \rangle = \langle \veps_2, \veps_3, \veps_2 \rangle,
$$

$$
\mu_2 = \langle \min (\veps_4, \veps_3, \veps_2), \min (\veps_3, \veps_5, \veps_4), \max (\veps_1, \veps_1, \veps_2) \rangle = \langle \veps_3, \veps_3, \veps_2 \rangle,
$$

$$
\mu_3 = \langle \min (\veps_4, \veps_3, \veps_2), \min (\veps_2, \veps_3, \veps_2), \max (\veps_2, \veps_3, \veps_2) \rangle = \langle \veps_2, \veps_2, \veps_3 \rangle.
$$

Using equations (15) and (16), we find that the positive and negative ideal solutions are

$$
PIS = LPFWA \langle \veps_1, \veps_2, \ldots, \veps_n \rangle
$$

$$
= \{ \veps_{1.99}, \veps_{2.15}, \veps_{1.26} \}
$$

$$
NIS = \{ \veps_{1.99}, \veps_{2.47}, \veps_{2.44} \}
$$

**Step 3.** Using (19), for each row of $D$, then

$$
A_1 = LPFWA \langle (\veps_4, \veps_2, \veps_2), (\veps_4, \veps_3, \veps_1), (\veps_6, \veps_1, \veps_2) \rangle
$$

$$
= \{ \veps_{4.50}, \veps_{2.15}, \veps_{1.45} \}
$$

$$
A_2 = LPFWA \langle (\veps_3, \veps_3, \veps_3), (\veps_2, \veps_5, \veps_1), (\veps_3, \veps_2, \veps_4) \rangle
$$

$$
= \{ \veps_{2.72}, \veps_{3.55}, \veps_{1.87} \}
$$

$$
A_3 = LPGWA \langle (\veps_2, \veps_2, \veps_2), (\veps_3, \veps_4, \veps_2), (\veps_7, \veps_1, \veps_1) \rangle
$$

$$
= \{ \veps_{3.92}, \veps_{2.48}, \veps_{1.74} \}
$$

**Step 4.** Using equations (20) and (21), to find the Hamming distance for each alternative

$$
d(A_1, PIS) = \frac{|4.50 - 4.80| + |2.15 - 2.15| + |1.45 - 1.26| + |0.9 - 0.7|}{2(9)}
$$

$$
d(A_1, PIS) = 0.038
$$

$$
d(A_2, PIS) = 0.236,
$$

$$
d(A_1, NIS) = 0.201,
$$

$$
d(A_3, PIS) = 0.102,
$$

$$
d(A_2, NIS) = 0.215.
using equations (22), (23), and (24), which are as follows:

The maximum and minimum Hamming distance and relative closeness degree of each alternative are obtained by using equations (22), (23), and (24), which are as follows:

\[ d_{\text{max}} = \max (0.278, 0.201, 0.215) = 0.278, \]
\[ d_{\text{min}} = \min (0.038, 0.236, 0.102) = 0.038, \]
\[ C(x_1) = \frac{1}{2} \left( \frac{0.278}{0.278} + \frac{0.038}{0.038} \right) = 1, \quad (35) \]
\[ C(x_2) = \frac{1}{2} \left( \frac{0.201}{0.278} + \frac{0.038}{0.236} \right) = 0.117 \]
\[ C(x_3) = \frac{1}{2} \left( \frac{0.215}{0.278} + \frac{0.038}{0.102} \right) = 0.215 \]

According to equation (25), we rank the alternatives, as \( a_1 > a_3 > a_2 \), so the best alternative is \( A_1 \).

### 6. Conclusion

Inspired by linguistic picture fuzzy numbers, in this paper, uncertain estimate data in linguistic multicriteria decision making are expressed by linguistic picture fuzzy sets on linguistic terms set; then Hamming distance between two linguistic picture fuzzy sets and their properties are given and evaluated. Therefore, the linguistic picture fuzzy set TOPSIS method for LMCDM problems is proposed; different positive ideal solution, the negative ideal solution, and the relative closeness degrees of alternatives are provided; based on the designed algorithm, LMCDM problems with linguistic picture fuzzy sets can be automatically carried out. An example is also utilized to clarify the achievement, usefulness, and effectiveness of the linguistic picture fuzzy set TOPSIS method.

### Data Availability

No data were used to support this study.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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### References


### Table 2

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<tr>
<td>( a_3 )</td>
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<td>( \varepsilon_3, \varepsilon_1, \varepsilon_2 )</td>
<td>( \varepsilon_1, \varepsilon_2, \varepsilon_3 )</td>
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</tbody>
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