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Research Article

Extended Version of Linguistic Picture Fuzzy TOPSIS Method and Its Applications in Enterprise Resource Planning Systems

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The main objective of the proposed research in this paper is introducing an extended version of the linguistic picture fuzzy TOPSIS technique and then solving the problems in enterprise resource planning systems. In this article, we use the uncertain information in terms of linguistic picture fuzzy numbers; the decision maker provides membership, neutral, and nonmembership fuzzy linguistic terms to represent uncertain assessments information of alternatives in linguistic multicriteria decision making (LMCDMs). In order to introduce the extended version of TOPSIS method, we defined a new hamming distance measure between two linguistic picture fuzzy numbers. Further, we apply the proposed method to problem of enterprise resource planning systems and discuss numerical implementation of the proposed method of LMCDM.

1. Introduction

In our daily life, we face some tasks and activities to handle with the help of decision making. Basically, decision making is an intellectual process which depends on different spiritual and reasoning processes; in the decision making process we choose a good alternative from the set of all available alternatives. For a single decision maker, it is mostly difficult to examine all related conditions of a problem due to the increasing complication of the socioeconomic environment. In everyday life, many decision making measures take place in group information. However, under more conditions, for the real multiple attribute decision making problems, the decision information about alternatives is usually uncertain or fuzzy due to the increasing complication of the socioeconomic situation and the ambiguity of inherent subjective attribute of human thinking; thus, numerical values are incomplete or partial for exemplary real-life decision problems. However, in practice, the attribute values of decision problems are not ever disposed by crisp information because of the fuzziness, and many of them are more considerable to be stated by fuzzy information; see [1–5], such as fuzzy numbers, intuitionistic fuzzy numbers [6], and linguistic variable [7, 8].

Fuzzy set theory was firstly defined by Zadeh's in 1965 [9], which is a framework to encounter confusion, ambiguity, and exaggeration and perform a degree of membership for each member of the nature of discussion to a subset of it. Therefore, we have a forum of true values. The degree of nonmembership was added by Atanassov in 1983 to FS; Atanassov defined the notion of intuitionistic fuzzy set (IFS) [10], which is mostly permissible to handle the ambiguity quantification and give the occasion to correctly model the problem based on existing judgment and experience [11]. The two of these structures are assumed to be a soft approach which, in turn, start soft computing and approximate analysis [12].

Later on in 2014, Cuong proposed the concept of picture fuzzy set (PFS) and studied its basic operations and properties. The picture fuzzy set is designated by the membership, neutral, and nonmembership degree. The only restraint is that the sum of the membership, neutral, and nonmembership degrees must not exceed 1. PFS are basically used for those

models, which cannot be expressed in the traditional PS and IFS, like the opinions of the people which have answers of the following types: yes, abstain, no, and refusal. Already some promotion has happened in the research of the PFS theory. In (2014) P. H. Phong discussed the composition of picture fuzzy relation [13], and Singh studied the correlation coefficients for picture fuzzy set and tested the correlation coefficient for clustering analysis with picture fuzzy information. Son et al. developed many novel fuzzy clustering algorithms based on picture fuzzy sets and applications to time series forecasting and weather forecasting (Thong and Son, 2015). Wei [14] proposed picture fuzzy cross-entropy model for multiple attribute decision making problems. S. jun et al. (2017) defined picture uncertain linguistic aggregation operators [15]. In 2017 Phong and Cuong introduced the notion of picture linguistic set and handled MCGDM problem under PLNs environment [16].

In 1981 Hwang and Yoon introduced the idea of TOPSIS method; the idea that the selected alternative must be the closest from the positive ideal solution and the farthest from negative ideal solution is an outstanding multicriteria decision making method [17]. In (1996) Triantaphyllou et al. continued the TOPSIS strategy for decision making problem with fuzzy data [18]. Many authors extended TOPSIS methods and applied these on multiple attribute decision making [19, 20]. Chen (2000) stretches out TOPSIS to fuzzy group decision making problems [21]. Ashtiani et al. [22] used the TOPSIS method to solve decision making problem with the interval valued fuzzy sets. To deal with multiple attribute decision making problem with intuitionistic fuzzy sets, He and Gong [23] developed a natural generalization of the TOPSIS method. Liu et al. [24] proposed a new TOPSIS method for decision making problems with interval-valued intuitionistic fuzzy data.

The remainder of the article is structured as follows. In Section 2, we briefly discussed basic definitions and linguistic picture fuzzy numbers. In Section 3, we defined the Hamming distance between the two linguistic picture fuzzy sets and found the distance between the two PFSs. In Section 4, we discussed the framework of the linguistic picture fuzzy set TOPSIS method and explained an algorithm to solve the LMCDM problem with linguistic picture fuzzy environment. In Section 5, we solved an LMCDM problem to expose the achievement and practicality of the linguistic picture fuzzy set TOPSIS method, and we wrote the conclusion in the last section.

2. Preliminaries

Definition 1 (see [12]). Let $Y \neq 0$ be a universal set. Then we defined the picture fuzzy set as

$$H = \{\langle y, a_H(y), b_H(y), c_H(y)\rangle \mid y \in Y\} \tag{1}$$

Where $a_H(y), b_H(y), c_H(y) : Y \longrightarrow [0,1]$. For each $y \in Y$. $a_H(y), b_H(y)$, and $c_H(y)$ are respectively, known as positive degree, neutral degree, and negative degree of Y in R, which satisfy the condition

$$a_H(y) + b_H(y) + c_H(y) \leq 1, \quad \forall y \in Y. \tag{2}$$

The refusal degree of Y in H is defined as $\sigma_H(y) = 1 - a_H(y) - b_H(y) - c_H(y)$, and if $\sigma_H(y) = 0, \forall y \in Y$, then H is reduced to an IFS [10], and if $\sigma_H(y) = b_H(y) = 0$ for all $y \in Y$, H is degenerated to an FS.

Definition 2 (see [25, 26]). Let $\check{L} = (£_0, £_1, \ldots, £_{g-1})$ be the absolute order distinct term finite set. Then $\check{L}$ is called a linguistic term set, where g is the odd cardinality with $g > 0$. Generally g is considered as $3, 5, 7$, etc. For example, when $g = 7$ the linguistic term set is as follows:

$$\check{L} = \langle £_0, £_1, £_2, £_3, £_4, £_5, £_6\rangle \tag{3}$$

= (v. poor, poor, slightly poor, fair, slightly good, good, v. good)

If $£_f, £_g \in \check{L}$, then the following characteristics are satisfied:

(1) The order of set as: $£_f \prec £_g, \Longleftrightarrow f \prec g$;

(2) The operator of negation as: neg $(£_f) = £_{g-1-f}$;

(3) Maximum $(£_f, x_g) = £_f$, if $f \geq g$;

(4) Minimum $(£_f, £_g) = £_y$,if $f \leq g$.

Definition 3 (see [16]). Let Y be a nonempty set; then we defined the picture linguistic number set R in Y as

$$H = \{(y, \langle £_{\theta(Y)}, a_H(y), b_H(y), c_H(y)\rangle) \mid y \in Y\} \tag{4}$$

Which is designated by a linguistic term $£_{\theta(x)} \in \check{L}$, a positive degree,a neutral degree, and a negative degree $a_H(Y), b_H(y), c_H(y) \in [0,1]$, respectively, the element y to $£_\theta(y)$ with the following condition:

$$a_H(y) + b_H(y) + c_H(y) \leq 1, \quad \forall y \in Y. \tag{5}$$

and $\sigma_R(y) = 1 - a_H(y) - b_H(y) - c_H(y)$ is said to be the refusal degree of y to $£_\theta(y)$ for all $y \in Y$.

If $\sigma_H(Y) = 0, \forall y \in Y$, in this case the picture linguistic set is reduced to the intuitionistic linguistic set.

Definition 4. Let $£_a, £_b, £_c \in \check{L}_{[0,g]}$ and $\alpha = \langle £_a, £_b, £_c\rangle$, if $a + b + c \leq g$. Then, α is called the linguistic picture fuzzy numbers on $\check{L}_{[0,g]}$. If $£_a, £_b, £_c \in \check{L}$, then we consider α the original linguistic picture fuzzy numbers, and virtual linguistic picture fuzzy numbers, otherwise.

$\Gamma_{[0,g]} = \langle (£_a, £_b, £_c) \mid £_a, £_b, £_c \in \check{L}_{[0,g]}\rangle$ represent all linguistic picture fuzzy numbers defined on $\check{L}_{[0,g]}$.

Definition 5. Let $\langle £_a, £_b, £_c\rangle, \langle £_{a_1}, £_{b_1}, £_{c_1}\rangle, \langle £_{a_2}, £_{b_2}, £_{c_2}\rangle \in \Gamma_{[0,g]}$. Then, the following operators hold by operations of picture fuzzy sets:

$$\begin{aligned}
&\cdot\ \left\langle £_{a_1}, £_{b_1}, £_{c_1}\right\rangle \cup \left\langle £_{a_2}, £_{b_2}, £_{c_2}\right\rangle \\
&\quad = \left\langle \max\left(£_{a_1}, £_{a_2}\right), \min\left(£_{b_1}, £_{b_2}\right), \min\left(£_{c_1}, £_{c_2}\right)\right\rangle; \\
&\cdot\ \left\langle £_{a_1}, £_{b_1}, £_{c_1}\right\rangle \cap \left\langle £_{a_2}, £_{b_2}, £_{c_2}\right\rangle \\
&\quad = \left\langle \min\left(£_{a_1}, £_{a_2}\right), \min\left(£_{b_1}, £_{b_2}\right), \max\left(£_{c_1}, £_{c_2}\right)\right\rangle; \\
&\cdot\ \langle £_a, £_b, £_c\rangle^c = \langle £_c, £_b, £_a\rangle; \\
&\cdot\ \left\langle £_{a_1}, £_{b_1}, £_{c_1}\right\rangle \subseteq \left\langle £_{a_2}, £_{b_2}, £_{c_2}\right\rangle \text{ iff } £_{a_1} \le £_{a_2},\ £_{b_1} \\
&\quad \ge £_{b_2} \text{ and } £_{c_1} \ge £_{c_2}; \\
&\cdot\ \left\langle £_{a_1}, £_{b_1}, £_{c_1}\right\rangle = \left\langle £_{a_2}, £_{b_2}, £_{c_2}\right\rangle \text{ iff } £_{a_1} = £_{a_2},\ £_{b_1} \\
&\quad = £_{b_2} \text{ and } £_{c_1} = £_{c_2}.
\end{aligned} \tag{6}$$

Definition 6. Let $\langle £_a, £_b, £_c\rangle, \langle £_{a_1}, £_{b_1}, £_{c_1}\rangle, \langle £_{a_2}, £_{b_2}, £_{c_2}\rangle \in \Gamma_{[0,g]}, \lambda > 0$. Then we defined the following operations of linguistic picture fuzzy numbers:

$$\begin{aligned}
&\cdot\ \left\langle £_{a_1}, £_{b_1}, £_{c_1}\right\rangle \oplus \left\langle £_{a_2}, £_{b_2}, £_{c_2}\right\rangle \\
&\quad = \left\langle £_{a_1+a_2-a_1a_2/g}, £_{b_1+b_2-b_1b_2/g}, £_{c_1+c_2-c_1c_2/g}\right\rangle; \\
&\cdot\ \lambda\langle £_a, £_b, £_c\rangle = \left\langle £_{g-g(1-a/g)^\lambda}, £_{g(b/g)^\lambda}, £_{g(c/g)^\lambda}\right\rangle;
\end{aligned} \tag{7}$$

Definition 7. Let $\alpha_i = \langle £_{a_i}, £_{b_i}, £_{c_i}\rangle \in \Gamma_{[0,g]}(i = 1, 2, \ldots, m)$ and ω_i be the weighting vector of α_i, satisfying $0 \le \omega_i \le 1(i = 1, 2, \ldots, m)$ and $\sum_{i=1}^m \omega_i = 1$. Then, we defined the linguistic picture fuzzy weighted averaging operator as

$$\begin{aligned}
&LPFWA(\alpha_1, \alpha_2, \ldots, \alpha_m) \\
&\quad = \left\langle £_{g-g\prod_{i=1}^m (1-a_i/g)^{\omega_i}}, £_{g\prod_{i=1}^m (b_i/g)^{\omega_i}}, £_{g\prod_{i=1}^m (c_i/g)^{\omega_i}}\right\rangle
\end{aligned} \tag{8}$$

As a special case, if $\omega = (1/n, 1/n, \ldots, 1/n)$, then equation (8) is reduced to the following form:

$$\begin{aligned}
&LPFWA(\alpha_1, \alpha_2, \ldots, \alpha_m) \\
&\quad = \left\langle £_{g-g\prod_{i=1}^m (1-a_i/g)^{1/n}}, £_{g\prod_{i=1}^m (b_i/g)^{1/n}}, £_{g\prod_{i=1}^m (c_i/g)^{1/n}}\right\rangle
\end{aligned} \tag{9}$$

Example 8. Let $X = \langle x_1, x_2, x_3\rangle$, $C = \langle c_1, c_2, c_3\rangle$ be the sets of alternatives and criteria, respectively. The linguistic set $\check{L} =$ $£_0$ (nothing), $£_1$ (very very low), $£_2$ (very low), $£_3$ (low), $£_4$ (medium), $£_5$ (high), $£_6$ (very high), $£_7$ (very very high), $£_8$ (perfect). The decision makers provide the linguistic picture fuzzy assessments as shown in Table 1.

3. The Distance between Two Linguistic Picture Fuzzy Numbers

In TOPSIS method the distance measure is an important concept. We can define the following Hamming distance between two linguistic picture fuzzy sets.

Definition 9. Let $A = \langle £_{a_1}, £_{b_1}, £_{c_1}\rangle$ and $B = \langle £_{a_2}, £_{b_2}, £_{c_2}\rangle$ be the two linguistic picture fuzzy numbers. Then we defined the Hamming distance between A and B as

$$d(A, B) = \frac{|a_1 - a_2| + |b_1 - b_2| + |c_1 - c_2| + |\tau_1 - \tau_2|}{2g} \tag{10}$$

Example 10. In Example 8, for $A_1 = \langle £_2, £_3, £_3\rangle$ of x_1, $A_2 = \langle £_1, £_5, £_2\rangle$ of x_2 and $A_3 = \langle £_4, £_1, £_4\rangle$ of x_3, in addition, $\tau_1 = 9-2-3-3 = 1$, $\tau_2 = 9-1-5-2 = 1$, and $\tau_3 = 9-4-1-4 = 0$,

$$\begin{aligned}
d(A_1, A_2) &= \frac{|2-1| + |3-5| + |3-2| + |1-1|}{2(9)} = \frac{4}{18} \\
&= 0.22, \\
d(A_1, A_3) &= \frac{|2-4| + |3-1| + |3-4| + |1-0|}{2(9)} = \frac{6}{18} \\
&= 0.33, \\
d(A_2, A_3) &= \frac{|1-4| + |5-1| + |2-4| + |1-0|}{2(9)} = \frac{10}{18} \\
&= 0.56,
\end{aligned} \tag{11}$$

4. The Linguistic Picture Fuzzy Set TOPSIS Method

To solve the LMCDMs problem, we present the linguistic picture fuzzy set TOPSIS method, which consists of the following steps.

Step 1. Construct the linguistic picture fuzzy decision matrix,

Step 2. With the help of averaging operator, find the positive and negative ideal solutions of alternatives.

Step 3. For every alternative, find the relative closeness degree.

Step 4. Give rank to the alternatives, based on relative closeness degree.

4.1. The Linguistic Picture Fuzzy Decision Matrix. Let $A_{ji} = \langle £_{a_{ji}}, £_{b_{ji}}, £_{c_{ji}}\rangle$ be the linguistic picture fuzzy assessment of the alternatives x_i provided by the decision makers d_Y with respect to the criteria $C = \langle c_1, c_2, \ldots, c_m\rangle$ and let $\omega = \langle \omega_1, \omega_2, \ldots, \omega_m\rangle$ be the weight of criteria. Then the linguistic picture fuzzy decision matrix of a LMCDM is written as

$$\begin{aligned}
D &= \left\langle A_{ji}\right\rangle_{n\times m} \\
&= \begin{pmatrix}
 & C(\omega_1) & \cdots & C(\omega_m) \\
x_1 & \left\langle £_{a_{11}}, £_{b_{11}}, £_{c_{11}}\right\rangle & \cdots & \left\langle £_{a_{1m}}, £_{b_{1m}}, £_{c_{1m}}\right\rangle \\
x_2 & \left\langle £_{a_{21}}, £_{b_{21}}, £_{c_{21}}\right\rangle & \cdots & \left\langle £_{a_{2m}}, £_{b_{2m}}, £_{c_{2m}}\right\rangle \\
\vdots & \vdots & \vdots & \vdots \\
\dot{x}_n & \left\langle £_{a_{n1}}, £_{b_{n1}}, £_{c_{n1}}\right\rangle & \cdots & \left\langle £_{a_{nm}}, £_{b_{nm}}, £_{c_{nm}}\right\rangle
\end{pmatrix}
\end{aligned} \tag{12}$$

Table 1: Linguistic picture assessments of alternatives.

		c_1	c_2	c_3
d	x_1	$\langle £_2, £_3, £_3\rangle$	$\langle £_4, £_2, £_1\rangle$	$\langle £_1, £_3, £_5\rangle$
	x_2	$\langle £_1, £_5, £_2\rangle$	$\langle £_3, £_4, £_1\rangle$	$\langle £_2, £_3, £_4\rangle$
	x_3	$\langle £_4, £_1, £_4\rangle$	$\langle £_4, £_1, £_3\rangle$	$\langle £_3, £_2, £_3\rangle$

Example 11. In Example 8, suppose that $\omega = \langle 0.3, 0.4, 0.3\rangle$ are the weight of criteria $C = \langle c_1, c_2, c_3\rangle$; then the linguistic picture fuzzy decision matrix of the LMCDM is

$$D = \left\langle A_{ji}\right\rangle_{3\times 3} = \begin{pmatrix} & c_1\ \langle 0.3\rangle & c_2\ \langle 0.4\rangle & c_3\ \langle 0.3\rangle \\ x_1 & \langle £_2, £_3, £_3\rangle & \langle £_4, £_2, £_1\rangle & \langle £_1, £_3, £_5\rangle \\ x_2 & \langle £_1, £_5, £_2\rangle & \langle £_3, £_4, £_1\rangle & \langle £_2, £_3, £_4\rangle \\ x_3 & \langle £_4, £_1, £_4\rangle & \langle £_4, £_1, £_3\rangle & \langle £_3, £_2, £_3\rangle \end{pmatrix} \tag{13}$$

4.2. The Positive and Negative Ideal Solutions of Alternatives. To determine the positive ideal solution and negative ideal solution of the alternatives, we apply the $\cup$ and $\cap$ operation of the linguistic picture fuzzy numbers and the linguistic picture fuzzy weighted averaging operator on the linguistic picture fuzzy matrix.

Operation $\cup$ and $\cap$ of linguistic picture fuzzy sets for each column is defined as

$$\begin{aligned} \vee c_j &= \left\langle £_{\vee a_j}, £_{\vee b_j}, £_{\vee c_j}\right\rangle = \bigcup_{j=1}^{n} A_{ji} = \bigcup_{j=1}^{n} \left\langle £_{a_{ji}}, £_{b_{ji}}, £_{c_{ji}}\right\rangle \\ &= \left\langle \max\left(£_{a_{1i}}, £_{a_{2i}}, \ldots, £_{a_{ni}}\right), \min\left(£_{b_{1i}}, £_{b_{2i}}, \ldots, £_{b_{ni}}\right), \right. \\ &\quad \left. \min\left(£_{c_{1i}}, £_{c_{2i}}, \ldots, £_{c_{ni}}\right)\right\rangle \\ \wedge c_j &= \left\langle £_{\wedge a_j}, £_{\wedge b_j}, £_{\wedge c_j}\right\rangle = \bigcap_{j=1}^{n} A_{ji} = \bigcap_{j=1}^{n}{}_{j=1}^{n} \left\langle £_{a_{ji}}, £_{b_{ji}}, \right. \\ &\quad \left. £_{c_{ji}}\right\rangle = \left\langle \min\left(£_{a_{1i}}, £_{a_{2i}}, \ldots, £_{a_{ni}}\right), \right. \\ &\quad \left. \min\left(£_{b_{1i}}, £_{b_{2i}}, \ldots, £_{b_{ni}}\right), \max\left(£_{c_{1i}}, £_{c_{2i}}, \ldots, £_{c_{ni}}\right)\right\rangle \end{aligned} \tag{14}$$

On the basis of $\vee c_j, \wedge c_j$ and weight $\omega = \langle \omega_1, \omega_2, \ldots, \omega_m\rangle$ of criteria $C = \langle c_1, c_2, \ldots, c_m\rangle$, to find the positive and negative ideal solutions of alternatives, using the linguistic picture fuzzy weighted averaging operator,

$$\begin{aligned} PIS &= \left\langle £_{a_p}, £_{b_p}, £_{c_p}\right\rangle = LPFWA\left\langle \vee c_1, \vee c_2, \ldots, \vee c_m\right\rangle \\ &= \left\langle £_{g-g\prod_{i=1}^{m}(1-\vee a_i/g)^{\omega_i}}, £_{g\prod_{i=1}^{m}(\vee b_i/g)^{\omega_i}}, £_{g\prod_{i=1}^{m}(\vee c_i/g)^{\omega_i}}\right\rangle \end{aligned} \tag{15}$$

$$\begin{aligned} NIS &= \left\langle £_{a_n}, £_{b_n}, £_{c_n}\right\rangle = LPFWA\left\langle \wedge c_1, \wedge c_2, \ldots, \wedge c_m\right\rangle \\ &= \left\langle £_{g-g\prod_{i=1}^{m}(1-\wedge a_i/g)^{\omega_i}}, £_{g\prod_{i=1}^{m}(\wedge b_i/g)^{\omega_i}}, £_{g\prod_{i=1}^{m}(\wedge c_i/g)^{\omega_i}}\right\rangle \end{aligned} \tag{16}$$

Example 12. According to D in Example 8, we have

$$\begin{aligned} \vee c_1 &= \bigcup_{j=1}^{3} A_{j1} = \left\langle \max\left(£_2, £_3, £_3\right), \min\left(£_1, £_5, £_2\right), \right. \\ &\quad \left. \min\left(£_4, £_1, £_4\right)\right\rangle = \left\langle £_3, £_1, £_1\right\rangle, \\ \wedge c_1 &= \bigcap_{j=1}^{3} A_{j1} = \left\langle \min\left(£_2, £_3, £_3\right), \min\left(£_1, £_5, £_2\right), \right. \\ &\quad \left. \max\left(£_4, £_1, £_4\right)\right\rangle = \left\langle £_2, £_1, £_4\right\rangle, \\ \vee c_2 &= \bigcup_{j=1}^{3} A_{j2} = \left\langle \max\left(£_4, £_2, £_1\right), \min\left(£_3, £_4, £_1\right), \right. \\ &\quad \left. \min\left(£_4, £_1, £_3\right)\right\rangle = \left\langle £_4, £_1, £_1\right\rangle, \\ \wedge c_2 &= \bigcap_{j=1}^{3} A_{j2} = \left\langle \min\left(£_4, £_2, £_1\right), \min\left(£_3, £_4, £_1\right), \right. \\ &\quad \left. \max\left(£_4, £_1, £_3\right)\right\rangle = \left\langle £_1, £_1, £_4\right\rangle, \\ \vee c_3 &= \bigcup_{j=1}^{3} A_{j3} = \left\langle \max\left(£_1, £_3, £_5\right), \min\left(£_2, £_3, £_4\right), \right. \\ &\quad \left. \min\left(£_3, £_2, £_3\right)\right\rangle = \left\langle £_5, £_2, £_2\right\rangle, \\ \wedge c_3 &= \bigcap_{j=1}^{3} A_{j3} = \left\langle \min\left(£_1, £_3, £_5\right), \min\left(£_2, £_3, £_4\right), \right. \\ &\quad \left. \max\left(£_3, £_2, £_3\right)\right\rangle = \left\langle £_1, £_2, £_3\right\rangle \end{aligned} \tag{17}$$

Based on equations (15) and (16), we calculate PIS and NIS of D,

$$\begin{aligned} PIS &= \left\langle £_{a_p}, £_{b_p}, £_{c_p}\right\rangle = LPFWA\left\langle \vee c_1, \vee c_2, \ldots, \vee c_m\right\rangle \\ &= \left\langle £_{9-9\prod_{i=1}^{3}(1-\vee a_i/9)^{\omega_i}}, £_{9\prod_{i=1}^{3}(\vee b_i/9)^{\omega_i}}, £_{9\prod_{i=1}^{3}(\vee c_i/9)^{\omega_i}}\right\rangle \\ &= £_{9-9[(1-3/9)^{0.3}\times(1-4/9)^{0.4}\times(1-5/9)^{0.3}]}, £_{9[(1/9)^{0.7}\times(2/9)^{0.3}]}, \\ &\quad £_{9[(1/9)^{0.7}\times(2/9)^{0.3}]} = \left\langle £_{4.10}, £_{1.26}, £_{1.22}\right\rangle \\ NIS &= \left\langle £_{a_n}, £_{b_n}, £_{c_n}\right\rangle = LPFWA\left\langle \wedge c_1, \wedge c_2, \ldots, \wedge c_m\right\rangle \\ &= \left\langle £_{g-g\prod_{i=1}^{m}(1-\wedge a_i/g)^{\omega_i}}, £_{g\prod_{i=1}^{m}(\wedge b_i/g)^{\omega_i}}, £_{g\prod_{i=1}^{m}(\wedge c_i/g)^{\omega_i}}\right\rangle \\ &= £_{9-9[(1-2/9)^{0.3}\times(1-1/9)^{0.7}]}, £_{9[(1/9)^{0.7}\times(2/9)^{0.3}]}, \\ &\quad £_{9[(4/9)^{0.7}\times(3/9)^{0.3}]} = \left\langle £_{1.29}, £_{1.22}, £_{3.68}\right\rangle \end{aligned} \tag{18}$$

4.3. The Relative Closeness and the Ranking of Alternatives. For the alternatives ranking, we find for every alternative the relative close degree, where the relative close degree is determined by distance between the linguistic picture fuzzy assessment of each alternative and the positive and negative ideal solutions. Based on the decision matrix, we obtain the linguistic picture fuzzy set A_j of x_j, using the linguistic picture fuzzy weighted averaging operator:

$$\begin{aligned} A_j &= LPFWA\left\langle A_{j1}, A_{j2}, \ldots, A_{jm}\right\rangle \\ &= \left\langle £_{g-g\prod_{i=1}^{m}(1-a_{ji}/g)^{\omega_i}}, £_{g\prod_{i=1}^{m}(b_{ji}/g)^{\omega_i}}, £_{g\prod_{i=1}^{m}(c_{ji}/g)^{\omega_i}}\right\rangle \end{aligned} \tag{19}$$

Based on A_j of x_j, the Hamming distance between the linguistic picture fuzzy set of individul alternative and the positive and negative ideal solutions is obtained utilizing PIS (equation (15)) and NIS (equation (16)) and equation (10):

$$d\left\langle A_j, PIS\right\rangle = \frac{\left|a_j - a_p\right| + \left|b_j - b_p\right| + \left|c_j - c_p\right| + \left|\tau_j - \tau_p\right|}{2g} \tag{20}$$

$$d\left\langle A_j, NIS\right\rangle = \frac{\left|a_j - a_n\right| + \left|b_j - b_n\right| + \left|c_j - c_n\right| + \left|\tau_j - \tau_n\right|}{2g} \tag{21}$$

Where $\tau_j = g - a_j - b_j - c_j$, $\tau_p = g - a_p - b_p - c_p$, $\tau_n = g - a_n - b_n - c_n$

Originating from the TOPSIS method, the ranking of alternatives is the following bases "select the shortest distance and the farthest distance from the positive and negative ideal solutions, respectively"; formally, it is fulfilled by the relative close degree of each alternative in existing TOPSIS methods. Based on Hamming distance between the linguistic picture fuzzy set of each alternative and the positive and negative ideal solutions (Eqs. (20) and (21)), we supply for each alternative x_i the following relative closeness degree $C(x_j)$:

$$d^{-}_{\max} = \max\left\langle d\left(A_1, NIS\right), d\left(A_2, NIS\right), \ldots, d\left(A_n, NIS\right)\right\rangle \tag{22}$$

$$d^{+}_{\min} = \min\left\langle d\left(A_1, PIS\right), d\left(A_2, PIS\right), \ldots, d\left(A_n, PIS\right)\right\rangle \tag{23}$$

$$C\left(x_i\right) = \frac{1}{2}\left\langle \frac{d\left(A_j, NIS\right)}{d^{-}_{\max}} + \frac{d^{+}_{\min}}{d\left(A_j, PIS\right)}\right\rangle \tag{24}$$

Basically, $C(x_i) \in [0, 1]$ for any alternative x_i; that is, $C(x_i)$ is increasing for $d(A_j, NIS)$, and decreasing for $d(A_j, PIS)$. The alternative rankings are obtained based on the relative closeness degree of alternatives as follows: $\forall x_j, x_j^* \in X$,

$$x_j < x_j^* \Longleftrightarrow C\left(x_j\right) \le C\left(x_j\right). \tag{25}$$

Example 13. According to D in Example 8, and equation (19), we collect the linguistic picture fuzzy assessment of each alternative as follows:

$$\begin{aligned} A_1 &= LPFWA\left\{\left\langle £_2, £_3, £_3\right\rangle, \left\langle £_4, £_2, £_1\right\rangle, \left\langle £_1, £_3, £_5\right\rangle\right\} \\ &= \left\langle £_{9-9[(1-2/9)^{0.3}\times(1-4/9)^{0.4}\times(1-1/9)^{0.3}]}, £_{9[(3/9)^{0.7}\times(2/9)^{0.3}]}, \right. \\ &\quad \left. £_{9[(3/9)^{0.3}\times(1/9)^{0.4}\times(5/9)^{0.3}]}\right\rangle = \left\langle £_{2.65}, £_{2.53}, £_{2.17}\right\rangle \\ A_2 &= LPFWA\left\{\left\langle £_1, £_5, £_2\right\rangle, \left\langle £_3, £_4, £_1\right\rangle, \left\langle £_2, £_3, £_4\right\rangle\right\} \\ &= \left\langle £_{9-9[(1-1/9)^{0.3}\times(1-3/9)^{0.4}\times(1-2/9)^{0.3}]}, \right. \\ &\quad \left. £_{9[(5/9)^{0.3}\times(4/9)^{0.4}\times(3/9)^{0.3}]}, £_{9[(2/9)^{0.3}\times(1/9)^{0.4}\times(4/9)^{0.3}]}\right\rangle \\ &= \left\langle £_{2.19}, £_{3.87}, £_{1.71}\right\rangle \\ A_3 &= LPFWA\left\{\left\langle £_4, £_1, £_4\right\rangle, \left\langle £_4, £_1, £_3\right\rangle, \left\langle £_3, £_2, £_3\right\rangle\right\} \\ &= \left\langle £_{9-9[(1-4/9)^{0.7}\times(1-3/9)^{0.3}]}, £_{9[(1/9)^{0.7}\times(2/9)^{0.3}]}, \right. \\ &\quad \left. £_{9[(4/9)^{0.3}\times(3/9)^{0.7}]}\right\rangle = \left\langle £_{3.66}, £_{1.23}, £_{3.25}\right\rangle \end{aligned} \tag{26}$$

Based on PIS and NIS in Example 12 and Equations (20) and (21), we obtain

$$\begin{aligned} &d\left(A_1, PIS\right) \\ &= \frac{|2.65 - 4.10| + |2.53 - 1.26| + |2.17 - 1.22| + |1.65 - 2.42|}{2\,(9)} \\ &= 0.25 \end{aligned} \tag{27}$$

and

$$\begin{aligned} &d\left(A_1, NIS\right) \\ &= \frac{|2.65 - 1.29| + |2.53 - 1.22| + |2.17 - 3.36| + |1.65 - 3.13|}{2\,(9)} \\ &= 0.28 \end{aligned} \tag{28}$$

Similarly, $d(A_2, PIS) = 0.34$ and $d(A_2, NIS) = 0.39$, $d(A_3, PIS) = 0.21$ and $d(A_3, NIS) = 0.25$.

Now we get the following results, applying equations (22), (23), and (24):

$$\begin{aligned} d^{-}_{\max} &= \langle 0.28, 0.39, 0.25\rangle = 0.39 \\ d^{+}_{\min} &= \langle 0.25, 0.34, 0.21\rangle = 0.21 \\ C\left(x_1\right) &= \frac{1}{2}\left\langle \frac{0.28}{0.39} + \frac{0.21}{0.25}\right\rangle = 0.78 \\ C\left(x_2\right) &= \frac{1}{2}\left\langle \frac{0.39}{0.39} + \frac{0.21}{0.34}\right\rangle = 0.79 \\ C\left(x_3\right) &= \frac{1}{2}\left\langle \frac{0.25}{0.39} + \frac{0.21}{0.21}\right\rangle = 0.81 \end{aligned} \tag{29}$$

According to equation (25), the ranking is as follows:

$$x_3 > x_2 > x_1 \tag{30}$$

Hence, x_3 is the best alternative.

Algorithm 14.

Step 1. According to the membership, neutral, and nonmembership fuzzy linguistic assessments, the decision matrix D of the LMCDM problem is constructed.

Step 2. Calculate $\vee c_i$ and $\wedge c_i$, for each column, using the linguistic picture fuzzy weighted averaging operator to determine the PIS and NIS.

Step 3. To find the linguistic picture fuzzy assessment A_j of each x_j, use the linguistic picture fuzzy weighted averaging operator.

Step 4. Find the Hamming distance $d(A_j, PIS)$ and $d(A_j, NIS)$ between each A_j and PIS (NIS) are calculated; the maximum Hamming distance $d^-_{\max}$ of all $d(A_j, NIS)$ and minimum Hamming distance $d^+_{\min}$ of all $d(A_j, PIS)$ are obtained; then calculate the relative close degree $C(x_j)$ of every alternative x_i.

Step 5. Rank the alternatives using equation (25).

5. Example

A multinational company chooses an enterprise resource planning system to apply from three candidates $A = (a_1, a_2, a_3)$. To develop other consierable decisions, the Chief Information Officer (CIO) of the company evaluates the applicant enterprise resource planning systems in terms of three criteria, that is, c_1 (energy cost), c_2 (business), and c_3 (procedure complication), where $(0.3, 0.5, 0.2)$ are the weight of the criteria. Since the given criteria are qualitative, the estimated values are in the form of linguistic term, which are shown in Table 2.

Step 2. For each column of D, we have

$$\begin{aligned}
\vee c_1 &= \langle \max(£_4, £_3, £_2), \min(£_2, £_3, £_2), \\
&\quad \min(£_2, £_3, £_2)\rangle = \langle £_4, £_2, £_2\rangle, \\
\vee c_2 &= \langle \max(£_4, £_2, £_3), \min(£_3, £_5, £_4), \\
&\quad \min(£_1, £_1, £_2)\rangle = \langle £_4, £_3, £_1\rangle, \\
\vee c_3 &= \langle \max(£_6, £_3, £_7), \min(£_1, £_2, £_1), \\
&\quad \min(£_2, £_4, £_1)\rangle = \langle £_7, £_1, £_1\rangle, \\
\wedge c_1 &= \langle \min(£_4, £_3, £_2), \min(£_2, £_3, £_2), \\
&\quad \max(£_2, £_3, £_2)\rangle = \langle £_2, £_2, £_3\rangle, \\
\wedge c_2 &= \langle \min(£_4, £_2, £_3), \min(£_3, £_5, £_4), \\
&\quad \max(£_1, £_1, £_2)\rangle = \langle £_2, £_3, £_2\rangle, \\
\wedge c_3 &= \langle \min(£_4, £_3, £_2), \min(£_2, £_3, £_2), \\
&\quad \max(£_2, £_3, £_2)\rangle = \langle £_2, £_2, £_3\rangle.
\end{aligned} \tag{31}$$

Using equations (15) and (16), we find that the positive and negative ideal solutions are

$$\begin{aligned}
PIS &= LPFWA\langle \vee c_1, \vee c_2, \ldots, \vee c_m\rangle \\
&= £_{9-9[(1-4/9)^{0.3}\times(1-4/9)^{0.5}\times(1-7/9)^{0.2}]}, \\
&\quad £_{9[(2/9)^{0.3}\times(3/9)^{0.5}\times(1/9)^{0.2}]}, £_{9[(2/9)^{0.3}\times(1/9)^{0.5}\times(1/9)^{0.2}]} \\
&= \langle £_{4.80}, £_{2.15}, £_{1.26}\rangle \\
NIS &= \langle £_{a_n}, £_{b_n}, £_{c_n}\rangle = LPFWA\langle \wedge c_1, \wedge c_2, \ldots, \wedge c_m\rangle \\
&= £_{9-9[(1-2/9)^{0.3}\times(1-2/9)^{0.5}\times(1-2/9)^{0.2}]}, \\
&\quad £_{9[(2/9)^{0.3}\times(3/9)^{0.5}\times(2/9)^{0.2}]}, £_{9[(3/9)^{0.3}\times(2/9)^{0.5}\times(3/9)^{0.2}]} \\
&= \langle £_{1.99}, £_{2.47}, £_{2.41}\rangle
\end{aligned} \tag{32}$$

Step 3. Using (19), for each row of D, then

$$\begin{aligned}
A_1 &= LPFWA\langle (£_4, £_2, £_2), (£_4, £_3, £_1), (£_6, £_1, £_2)\rangle \\
&= £_{9-9[(1-4/9)^{0.3}\times(1-4/9)^{0.5}\times(1-6/9)^{0.2}]}, \\
&\quad £_{9[(2/9)^{0.3}\times(3/9)^{0.5}\times(1/9)^{0.2}]}, £_{9[(2/9)^{0.3}\times(1/9)^{0.5}\times(2/9)^{0.2}]} \\
&= \langle £_{4.50}, £_{2.15}, £_{1.45}\rangle \\
A_2 &= LPFWA\langle (£_3, £_3, £_3), (£_2, £_5, £_1), (£_3, £_2, £_4)\rangle \\
&= \langle £_{2.72}, £_{3.55}, £_{1.87}\rangle \\
A_3 &= LPGWA\langle (£_2, £_2, £_2), (£_3, £_4, £_2), (£_7, £_1, £_1)\rangle \\
&= \langle £_{3.92}, £_{2.48}, £_{1.74}\rangle
\end{aligned} \tag{33}$$

Step 4. Using equations (20) and (21), to find the Hamming distance for each alternative

$$\begin{aligned}
&d(A_1, PIS) \\
&= \frac{|4.50 - 4.80| + |2.15 - 2.15| + |1.45 - 1.26| + |0.9 - 0.7|}{2(9)} \\
&= 0.038 \\
&d(A_1, NIS) \\
&= \frac{|4.50 - 1.99| + |2.15 - 2.47| + |1.45 - 2.41| + |0.9 - 2.13|}{2(9)} \\
&= 0.278. \\
&d(A_2, PIS) = 0.236, \\
&d(A_2, NIS) = 0.201, \\
&d(A_3, PIS) = 0.102, \\
&d(A_2, NIS) = 0.215.
\end{aligned} \tag{34}$$

Table 2

		c_1	c_2	c_3
$D =$	a_1	$(£_4, £_2, £_2)$	$(£_4, £_3, £_1)$	$(£_6, £_1, £_2)$
	a_2	$(£_3, £_3, £_3)$	$(£_2, £_5, £_1)$	$(£_3, £_2, £_4)$
	a_3	$(£_2, £_2, £_2)$	$(£_3, £_4, £_2)$	$(£_7, £_1, £_1)$

The maximum and minimum Hamming distance and relative closeness degree of each alternative are obtained by using equations (22), (23), and (24), which are as follows:

$$\begin{aligned} d^-_{\max} &= \max(0.278, 0.201, 0.215) = 0.278, \\ d^+_{\min} &= \min(0.038, 0.236, 0.102) = 0.038, \\ C(x_1) &= \frac{1}{2}\left(\frac{0.278}{0.278} + \frac{0.038}{0.038}\right) = 1, \\ C(x_2) &= \frac{1}{2}\left(\frac{0.201}{0.278} + \frac{0.038}{0.236}\right) = 0.117 \\ C(x_3) &= \frac{1}{2}\left(\frac{0.215}{0.278} + \frac{0.038}{0.102}\right) = 0.215 \end{aligned} \tag{35}$$

According to equation (25), we rank the alternatives, as $a_1 > a_3 > a_2$, so the best alternative is A_1.

6. Conclusion

Inspired by linguistic picture fuzzy numbers, in this paper, uncertain estimate data in linguistic multicriteria decision making are expressed by linguistic picture fuzzy sets on linguistic terms set; then Hamming distance between two linguistic picture fuzzy sets and their properties are given and evaluated. Therefore, the linguistic picture fuzzy set TOPSIS method for LMCDM problems is proposed; different positive ideal solution, the negative ideal solution, and the relative closeness degrees of alternatives are provided; based on the designed algorithm, LMCDM problems with linguistic picture fuzzy sets can be automatically carried out. An example is also utilized to clarify the achievement, usefulness, and effectiveness of the linguistic picture fuzzy set TOPSIS method.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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