Research Article

Mine Performance Assessment by Means of Stochastic Frontier Analysis

Ioannis E. Tsolas

School of Applied Mathematics and Physics, National Technical University of Athens, Athens, GR 15780, Greece

Correspondence should be addressed to Ioannis E. Tsolas; itsolas@central.ntua.gr

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This paper employs stochastic frontier analysis (SFA) in assessing efficiency at the mine level. An SFA model is derived using annual operational data from the Kardia Field mine of the Greek Public Power Corporation (PPC) S.A. for the 1984-2006 period and the causes of inefficiency are investigated by means of regression techniques. The proposed two-stage model can be used as a diagnostic tool to identify causes of mine inefficiency and as a tool for designing and specifying interventions to improve mine performance.

1. Introduction

Performance measurement in mining deals with the establishment of criteria for determining how well mining production units meet their objectives. Among performance metrics the most frequently employed are labor productivity [1] and efficiency [2].

Productivity is the quotient obtained by dividing the output by each input or the sum of inputs. Productivity measures include [3] partial factor and multifactor or total factor productivity (TFP). In partial productivity measurement, output is being considered to one class of input and the most used partial metric is labor productivity. Multifactor productivity is measured as the ratio of output to a set of combined inputs such as labor capital and land.

The efficiency of a production unit is defined as the ratio of observed to optimal values of either outputs or inputs. From the output maximization perspective this metric involves the ratio of observed output to maximum potential output that can be obtained from the use of inputs, whereas from the input minimization perspective efficiency turns to the ratio of observed input to minimum potential input required to produce the observed output. In the above perspectives the optimum such as either maximum output or minimum input is defined in terms of production possibilities, and the term technical efficiency is used.

Mining labor productivity is most commonly used also as an efficiency indicator. However, the use of a large number of partial productivity measures causes difficulties in comprehending and interpreting if some metrics move in opposite directions. The so-called frontier analysis and the corresponding estimation methodologies or efficiency frontier approaches such as data envelopment analysis (DEA) and stochastic frontier analysis (SFA) overcome the limitations of the partial productivity approaches [2]. DEA and SFA seem to be more appropriate in multi-input multi-output and single-output (or single-input) multi-input (or single-output) setting, respectively. The two main differences of these methodologies are that the DEA is nonparametric and does not take account of noise, whereas the SFA is parametric and accounts for noise [2]. DEA was initiated by Charnes et al. [4] whereas SFA appeared in the literature by Aigner et al. [5] and Meeusen and van den Broeck [6]. For more on the advantages and the disadvantages of the two methodologies and for their use as a tandem of methods in mining the interested reader is referred to Tsolas [2].

The purpose of this paper is to develop an SFA model at the mine level for measuring technical efficiency and identifying the causes of inefficiency by means of regression techniques. The novelty of the paper is the application of SFA for the Kardia lignite mine, located in Greece, to produce estimates of mine efficiency and the identification of causal factors of efficiency by means of regression techniques.

The research unfolds as follows. In the next sections the related works on SFA applications in mining are reviewed,
the SFA model of efficiency measurement is introduced, the data set is presented, and the empirical results are provided and discussed. The last section presents some concluding remarks.

2. Materials and Methods


The above studies focused on the mining industry level; thus a gap identified is the lack of the application of SFA at the mine level. The current study aims to fill this gap in the relevant literature by employing a two-stage approach to measure the technical efficiency at the Kardia Field lignite mine in Stage-1 and to investigate the causes of mine inefficiency by employing regression techniques in Stage-2. The Kardia Field lignite mine is operated by the Public Power Corporation (PPC) S.A., Greece’s leading lignite producer. The production function is assumed to be Cobb-Douglas [14]. It is well known that the translog function [15] has advantages over the Cobb-Douglas function (i.e., variable elasticity of substitution and output elasticity with respect to inputs), but multicollinearity may exist in the translog function [16]. Moreover, our justification for selecting the Cobb-Douglas function form is twofold. Firstly, we want to present an illustration of a two-stage approach to mine efficiency measurement and assessment which has not been examined so far. Secondly, our specification conforms to the approach taken in a seminal study by Wu [7] and the present case is designed to demonstrate how additional information can be extracted from this type of function when it is employed at the mine level.

2.2. Methodological Framework

2.2.1. Stochastic Frontier Analysis. Aigner et al. [5] and Meeusen and van den Broeck [6] both proposed the stochastic frontier approach to estimate a production function taking into account measurement errors and other noise in the data. For a recent survey see Kumbhakar and Lovell [17].

Using the same notation as in Tsolas [2] and assuming that we have data of the yearly activities of the same production unit, the stochastic frontier production function is given by

\[ Y_i = f(x_i, \beta) \exp \{ e_i \} , \]  

where \( Y_i \) is the logarithm of output of the \( i \)th yearly activity \( f(x_i, \beta) \) is the production function \( x_i \) is a vector of logarithms of input quantities of the \( i \)th yearly activity \( \beta \) is a vector of logarithms of input quantities of the \( i \)th yearly activity

\[ e_i = v_i - u_i \] is the error term

\( v_i \) represents the independently and identically distributed (iid) \( N(0, \sigma_v^2) \) random errors, independent of \( u_i \)

\( u_i \) represents nonnegative random variables associated with technical inefficiency in production, iid as half-normal, \( u_i \sim N^+(0, \sigma_u^2) \), \( \sigma_v, \sigma_u \) are the standard deviation parameters of \( v \) and \( u \) respectively.

The estimation of the stochastic frontier along with the inefficiency term involves assumptions on the specification of the distribution of \( u_i \) and the production function form [2].

Since the relevant literature seems to indicate that the estimations are typically robust to the assumed distribution of the one-sided error term [17], we employ the half-normal distribution, although other distributions can be used for the inefficiency disturbance term \( u_i \). The production function, as previously justified, is the Cobb-Douglas. Since estimation procedures yield merely the residuals \( \epsilon \) rather than the inefficiency term \( u_i \), Jondrow et al. [18] showed that if we assume a nonnegative half-normal distribution for \( u_i \) (i.e., \( u_i \sim N^+(0, \sigma_u^2) \)), the distribution of \( u \) given the error term \( e_i = v_i - u_i \), is

\[ f(u | e) = \frac{f(u, e)}{f(e)} = \frac{1}{\sqrt{2\pi} \sigma_v} \exp \left\{ -\frac{(u - \mu_u)^2}{2\sigma_u^2} \right\} \left[ 1 - \Phi \left( -\frac{\mu_u}{\sigma_u} \right) \right] , \]  

where \( \mu_u = -\sigma_u^2/\sigma_v^2, \sigma_u^2 = \sigma_v^2/\sigma_u^2, \Phi(.) \) is the standard normal cumulative distribution function.

Given that \( f(u | e) \) is distributed as \( N^+(\mu_u, \sigma_u^2) \), either the mean \( E(u_i | e_i) \) or the mode \( M(u_i | e_i) \) of this distribution can serve as a point estimator for \( u_i \) according to the following equations, respectively [2]:

\[ E(u_i | e_i) = \mu_u + \sigma_u \frac{\phi(-\mu_u/\sigma_u)}{1 - \Phi(-\mu_u/\sigma_u)} \]  

(3)

\[ M(u_i | e_i) = \begin{cases} \frac{\phi(e\lambda/\sigma)}{1 - \Phi(e\lambda/\sigma)} - \frac{\epsilon \lambda}{\sigma} & \text{if } e \leq 0, \\ 0 & \text{otherwise} \end{cases} \]  

(4)

where \( \lambda = \sigma_u/\sigma\Phi(.) \) and \( \phi(.) \) are the standard normal cumulative distribution and density functions, respectively.
Once point estimates of \( u_i \) are obtained, estimates of the technical efficiency (TE) of each yearly activity can be obtained from the following equation:

\[
TE_i = \exp(-\bar{u}_i)
\]

where \( \bar{u}_i \) is either \( E(u_i \mid \varepsilon_i) \) or \( M(u_i \mid \varepsilon_i) \).

2.2.2 Inefficiency Model. The evaluation of the influence of external or environmental factors on the efficiency is addressed mainly by employing two approaches [7, 19]: (i) a one-step approach where the environmental factors are modeled into SFA and (ii) a two-step approach where the inefficiency is estimated first without including any environmental factors and then inefficiency estimates are regressed against the environmental factors. It is worth noting that the estimates from the two-step procedure are biased if variables in the production function and variables in the inefficiency model are correlated [17, 19].

In the two-step (i.e., two-stage) approach, which is applied here, the first-stage inefficiency score is the dependent variable that is regressed on the explanatory variables according to the model

\[
y_i = f(Z_i, \beta') + u_i', \quad i = 1, 2, \ldots, n,
\]

where \( y_i \) is the dependent variable, \( Z_i \) and \( \beta' \) are vectors of explanatory variables and unknown parameters, respectively, \( u_i' \) is the error term, and \( n \) is the number of observations.

Model (6) can be estimated by means of a censored regression, such as the Tobit model that takes into account the limited nature of the inefficiency scores (i.e., the dependent variable is censored at zero). The advantage of using the Tobit model instead of the usual ordinary least squares linear regression model is that it yields unbiased coefficient estimates for each of the explanatory variables.

When the dependent variable \( Y_i \) is censored at zero, the Tobit model may be described as follows:

\[
y_i^* = \beta' Z_i + u_i', \quad i = 1, 2, \ldots, n,
\]

where \( y_i \) is the dependent variable: \( y_i = y_i^* \) if \( y_i^* > 0 \) and \( y_i = 0 \) if \( y_i^* < 0 \); \( Z_i \) and \( \beta' \) are vectors of explanatory variables and unknown parameters, respectively.

\( u_i' \) is the error term, assumed to be normally and independently distributed with mean zero and a constant variance \( \sigma^2 \).

The number of observations is the number of observations.

Maximum likelihood estimation is used to estimate the parameters of the Tobit model.

### 3. Data Set, Results, and Discussion

#### 3.1 Data Set. The output-input data used in this paper are operational actual data of the Kardia Field lignite mine, over the 1984-2006 period; the sources of data are the annual reports of the mine. The Kardia Field mine being the second biggest lignite mine of PPC S.A. is located in the central part of the Ptolemais area. In the organizational chart of the firm it confronts the Kardia Field mine unit of the West Macedonia Lignite Center Exploitation Department of PPC S.A. [20].

The remaining reserves in 2015 amounted to 300 Mt of lignite. Mining technology is based primarily on the continuous surface mining method with the use of continuous large-scale equipment such as bucket wheel excavators, conveyor belts, and spreaders. In cases where the continuous equipment is not adequate to handle the required earthmoving works, noncontinuous equipment may be used [21].

Data used in this paper were collected as part of a companion paper [22]. The output is the excavated lignite \( Q \) in tons. The main conventional inputs in the lignite mining process are labor, capital, and electrical energy consumed. The inputs used in SFA are the total man-shift hours paid \( L \) measured in 8 hours, electrical energy consumed by bucket wheel excavators \( E \) expressed in kWhs, and overburden removed \( O \) in bank cubic meters. Table 1 provides the descriptive statistics of the above variables.

<table>
<thead>
<tr>
<th>L, 10^3 man-shift hours</th>
<th>O, 10^6 bank m³</th>
<th>E, 10^6 kWhs</th>
<th>Q, 10^6 tons</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min 172.58</td>
<td>29.25</td>
<td>83.68</td>
<td>6.95</td>
</tr>
<tr>
<td>Max 254.63</td>
<td>52.83</td>
<td>167.74</td>
<td>23.34</td>
</tr>
<tr>
<td>Standard deviation 24.53</td>
<td>7.03</td>
<td>33.11</td>
<td>4.47</td>
</tr>
<tr>
<td>Mean 219.03</td>
<td>38.79</td>
<td>119.79</td>
<td>14.67</td>
</tr>
</tbody>
</table>

In surface lignite mining operations the term overburden is used in the context of any material lying over the mineral reserve or between lignite seams. The treatment of overburden as input reflects the cost for its excavation which is necessary for the lignite production. The quantity of excavated overburden is closely related to the stripping ratio (SR). In lignite mining, the SR refers to the bank cubic meters of overburden removed per ton of lignite mined. Another metric that can be used is the incremental stripping ratio (ISR) that is defined as the ratio of the incremental amount of overburden between two consecutive years to the incremental amount of lignite in the same time period. For more on the stripping ratio the interested reader is referred to Kenedy [23].

The capital in terms of bucket wheel excavators’ operating hours has been excluded from the SFA because it has not the expected sign and it is used for the estimation of the capital-to-labor ratio (K/R) that captures the effect of factor intensity [7] in the second stage of analysis. Except for the K/R two more variables are used as environmental factors in the second stage of analysis. These variables include the SR and the vintage (i.e., age) of mine.
### Table 2: Maximum likelihood function estimates, Stage-1.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard error</th>
<th>z-value</th>
<th>p value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant: $\beta_0$:</td>
<td>-42.870</td>
<td>17.340</td>
<td>-2.470</td>
<td>0.013</td>
</tr>
<tr>
<td>$\ln L$: $\beta_1$:</td>
<td>2.174</td>
<td>0.820</td>
<td>2.650</td>
<td>0.008</td>
</tr>
<tr>
<td>$\ln O$: $\beta_2$:</td>
<td>0.694</td>
<td>0.556</td>
<td>1.250</td>
<td>0.212</td>
</tr>
<tr>
<td>$\ln E$: $\beta_3$:</td>
<td>1.112</td>
<td>0.424</td>
<td>2.620</td>
<td>0.009</td>
</tr>
</tbody>
</table>

**Variance parameters for composed error**

\[ \lambda = \frac{\sigma_u}{\sigma_v}; \quad \sigma^2 = \sigma_u^2 + \sigma_v^2. \]

Log likelihood function = 2.047.

Notes: $L$: total man-shift hours paid; $E$: electrical energy consumed by bucket wheel excavators; $O$: overburden removed in bank cubic meters.

### Table 3: Technical efficiency, Stage-1.

<table>
<thead>
<tr>
<th>Panel A: descriptive statistics</th>
<th>Efficiency scores</th>
</tr>
</thead>
<tbody>
<tr>
<td>Min</td>
<td>0.70</td>
</tr>
<tr>
<td>Max</td>
<td>0.93</td>
</tr>
<tr>
<td>Mean</td>
<td>0.86</td>
</tr>
<tr>
<td>Standard deviation</td>
<td>0.05</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Panel B: distribution of efficiency measures</th>
<th>Percentage (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Efficiency scores</td>
<td></td>
</tr>
<tr>
<td>0.80</td>
<td>13.04</td>
</tr>
<tr>
<td>0.85</td>
<td>26.09</td>
</tr>
<tr>
<td>0.90</td>
<td>34.78</td>
</tr>
<tr>
<td>0.95</td>
<td>26.09</td>
</tr>
<tr>
<td>1.00</td>
<td>0.00</td>
</tr>
</tbody>
</table>

3.2. Results. The Cobb-Douglas production function to be estimated could be written as

\[ \ln Q_i = \beta_0 + \beta_1 \ln L_i + \beta_2 \ln O_i + \beta_3 \ln E_i + \varepsilon_i, \quad i = 1, 2, \ldots, n, \tag{8} \]

where $\beta_0, \beta_1, \beta_2, \beta_3$ are unknown parameters; $Q$ is the output; $L$, $O$, and $E$ are the inputs: labor, overburden, and energy, respectively; $\varepsilon_i = y_i - u_i$ is the error term; and $n$ is the number of observations.

Table 2 depicts the results of the maximum likelihood function estimates using annual data on lignite output and total man-shift hours, overburden, and electrical energy consumed under the assumptions of a Cobb-Douglas production function and a nonnegative half-normal distribution for $u_i$.

The estimation results revealed all the inputs, except overburden, were statistically significant in the lignite production. Those with the greatest elasticity were labor and energy. These results are meaningful, because lignite production, though energy-intensive, is also highly labor-intensive. Thus, we can conclude that these two inputs, labor and energy, have a major influence on output in lignite production.

Results concerning the efficiency estimates are presented in Table 3.

In Figure 1 the whole time series of efficiency scores is presented.

The SFA efficiency measures presented above yield the first-stage performance estimates. The variations of performance should be examined further and, therefore, a second-stage analysis is called for because performance may be affected by other explanatory variables. In this study, the Tobit and bootstrapped Tobit regression are used to explore some mine specific factors which are likely to interfere with the performance derived in the first stage of the analysis. The Tobit regression is suggested to model the frontier inefficiency estimates bounded from below. The mine inefficiency measures derived by means of SFA are regressed to identify the impact of a series of explanatory variables. The candidate variables include the SR, K/R, and age of mine (AGE).

The SR is expected to bear a negative relationship to the efficiency because the higher the SR is, and hence the overburden to be cleared prior to excavating the lignite is, the lower the performance should be. The K/R can be used as a proxy to the equipment-manpower mix and it would be interesting to investigate its influence on performance; according to the relevant literature it is expected to bear a positive relationship to efficiency because the higher the capital-labor ratio is, and hence the level of mechanization is, the higher the performance should be [1]. The vintage of the mine is expected, according to the relevant literature [1] to bear a negative relationship to efficiency when the mine is in the final stage of its life cycle, because production levels may depend on the age of the mine. As has already been noted.
Table 4: Results of the Tobit regression for mine inefficiency, Stage-2.

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Standard error</th>
<th>t-value (p value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.0198</td>
<td>0.0240</td>
<td>-0.83 (0.418)</td>
</tr>
<tr>
<td>SR</td>
<td>0.0564</td>
<td>0.0081</td>
<td>6.95 (0.000)</td>
</tr>
<tr>
<td>Sigma</td>
<td>0.0311</td>
<td>0.0047</td>
<td></td>
</tr>
</tbody>
</table>

Log likelihood = 44.158

Panel B: bootstrapped censored Tobit model (dependent variable (inefficiency) = 1-TE)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Coefficient</th>
<th>Bootstrapped standard error</th>
<th>t-value (p value)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Constant</td>
<td>-0.0198</td>
<td>0.0236</td>
<td>-0.84 (0.401)</td>
</tr>
<tr>
<td>SR</td>
<td>0.0564</td>
<td>0.0090</td>
<td>6.24 (0.000)</td>
</tr>
<tr>
<td>Sigma</td>
<td>0.0311</td>
<td>0.0041</td>
<td></td>
</tr>
</tbody>
</table>

Log likelihood = 44.158

Notes: sigma: an estimate of the standard deviation of the error. The bootstrapped Tobit regression was performed with B (number of replications) = 1000 samples.

the estimates of the inefficiency model are biased if variables in the production function and variables in the inefficiency model are correlated. Therefore, since the K/R and AGE are correlated with the production function variables they have been excluded from the analysis.

The results of the analysis explaining the performance scores by the Tobit and bootstrapped Tobit regression are given in Table 4.

3.3. Discussion. As can be seen from Table 2 where the results of the maximum likelihood function estimates are presented, the estimated coefficients for labor, overburden, and electrical energy consumed are positive and significant as well, excluding overburden, indicating that the use of labor and energy is positively associated with the lignite production.

The ratio $\lambda$ indicates that the one-sided error term $u_i$ slightly dominates the symmetric error $v$, so variation in actual production comes from differences in management practice over the time period studied rather than random variability.

According to SFA efficiency estimates vary between 0 and 1, where efficiency equal to 1 indicates perfect technical efficiency. Table 2 depicts that mine efficiency ranges from as low as 0.70 to as high as 0.93 with an average of 0.86. Efficiency distribution shows that 73.9% of the yearly activities have an efficiency score less than 0.90. Since TE scores are calculated from the output maximization perspective, the results imply that the mine would be able to increase lignite output by about 14% using its resources more effectively. The estimated mean efficiency (0.86) is particularly high when compared with the results of Wu [7] but comparable with the findings reported by Shi [11].

It is also observed that the sum of the coefficients of labor, energy, and overburden is significantly greater than one and this may imply that there are gains from economies of scale. This evidence is not in line with the results of previous studies [2, 7] where decreasing returns to scale in the mining industry have been identified.

Table 4 depicts that the SR is significant in explaining inefficiency; the sign of the SR is positive, as expected. This finding is in line with the results of previous studies [1] where the negative relationship of coal mine performance (i.e., labor productivity) with the stripping ratio is reported.

It is worth noting that although the two-stage approach was adopted in this paper, the one-step approach was also performed as an experiment for the benefits of robustness; the results show that the only significant factor in explaining inefficiency is the ISR. The use of the SR did not give statistically significant results. In regard to the estimated coefficients of the production factors which are reported in Table 2, the one-stage model causes important differences for the coefficients of labor and overburden; notably, the derived coefficient for overburden has not the expected sign. The results of the one-step approach are available upon request from the author.

4. Conclusions

The novelty of the paper is proposing a framework for mine performance assessment combining SFA and the Tobit regression to produce estimates of mine efficiency and to identify the causal factors of efficiency. In this study, a two-stage approach to evaluate performance at the mine level is used; the case study concerns the Kardia Field lignite mine of the Greek PPC S.A. A Cobb-Douglas production function SFA model is proposed at Stage-1 to determine the performance of mine yearly activities. The inefficiency term of the model is assumed to follow a half-normal distribution. In Stage-2 an inefficiency model is proposed to identify the mine specific (i.e., environmental) factors that affect performance. This study advances the literature by incorporating SFA and the Tobit regression analysis to identify the drivers that can explain the performance at the mine level. In the light of the results of this study, in Stage-1 we have evidence that mine inefficiency is present and there is room for improvement by increasing output about 14% on average, using resources more effectively. Stage-2 deals with the regression of inefficiency scores on a set of...
environmental factors that capture the characteristics of mine yearly activities. To overcome the problems of the two-stage approach adopted the environmental factors correlated with the components of the production function were excluded from the analysis. Unlike other similar studies, in this study, the bootstrapped Tobit regression was performed for the benefits of robustness; the results show that the only significant factor that explains inefficiency is the stripping ratio. In particular, for the presence of inefficiency and the lower performance of noneffective yearly activities management should take precautions to improve the efficiency paying attention to the factor mentioned above.

The present study has some limitations. First of all, it is expected that a larger data set will help us to generalize results. Therefore, as a future extension of this paper, the Kardia Field mine performance can be evaluated by using the same two-stage procedure with an extensive data set. Moreover, in order to experiment with other production functions a greater number of observations will be essential to the analysis. Second, other findings such as the existence of economies of scale may be investigated further with the use of other competing methods such as DEA.

In regard to further research, future studies could include the use of other SFA models in the framework of Bayesian SFA or the joint use of other competing deterministic methods as a tandem such as DEA to evaluate performance. It should be noted that DEA can be incorporated in the proposed two-stage framework and both SFA and DEA can be used in tandem. In addition, the study of independent sections such as lignite and overburden section which reflect the typical organizational and administrative structure at the lignite mine level is also challenging and opens new avenues for further research. It is also worth noting that the analysis can be easily extended to the firm and industry level to evaluate all mines of the firm under study and all mines at the sector, respectively, if necessary data will be made available.

Data Availability

Data used in this paper were collected as part of a companion paper with PPC’s colleagues [22]. Part of data are included in [22]. Enquiries for data should be made to the author at itsolas@central.ntua.gr.

Conflicts of Interest

The author declares that there are no conflicts of interest regarding the publication of this paper.

References


