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Research Article

Time-Inconsistent Preferences, Retirement, and Increasing Life Expectancy

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We study consumption behavior, retirement decisions, and endogenous growth within a dynamic equilibrium when individuals have present-biased preferences. Compared to individual with exponential preferences, individual with hyperbolic preferences will choose to retire early for present-biased preferences but to delay retirement for the initial time preference rate. We extend the benchmark equilibrium model to age-dependent survival law and solve numerically the equilibrium effects. It shows that, at the same age, the consumption-capital ratio may have slightly positive effect on increasing life expectancy before retirement but has a significantly positive effect on it after retirement.

1. Introduction

Most studies in psychology and experimental economics confirm that (quasi-)hyperbolic discounting provides a better answer for future utility than exponential discounting (see e.g., Diamond and Köszegi [1], Zou et al. [2], Holmes [3], and Findley and Caliendo [4]). Some prominent economic topics are delayed saving for retirement and endogenous economic growth. For instance, Bloom et al. [5] find that increased longevity raises aggregate savings rates in countries with universal pension coverage and retirement incentives. Strulik [6] concludes that present-biased preferences, providing the same present value of a constant infinite income stream, are harmless for economic growth. Naturally, this raises questions such as (1) how hyperbolic preferences and time-inconsistent behavior do matter in retirement decisions and (2) how the effects of increasing life expectancy is on aggregation consumption to capital ratio before and after retirement. Most of the classic quasihyperbolic discounting model is beginning with a discrete form model developed by Laibson [7]. For example, earlier selves think that the deciding self tends to choose early retirement and may save less to induce delayed retirement in Diamond and Köszegi [1]; that is, quasihyperbolic discounting can cause dynamic inconsistency. Retirement plans are never time-inconsistent with such preferences in Holmes [3].

In this paper, we extend time-inconsistent consumption problems in the context of endogenous growth and retirement choice following Blanchard-Yarris model (as depicted in Blanchard [8], the survival rate is age-independent). We show that, given the equivalent present value, the retirement is later (earlier) under hyperbolic discounting than under exponential discounting for the initial time preference rate (present-biased preferences). It is so that individuals will choose early retirement under certain circumstances while they do not care about future goals. The lower the quasihyperbolic discount rate is, the later individuals will retire since individuals' savings will be enough to sustain consumption. We also show that the faster the speed of declining impatience is, the older the retirement age is as for hyperbolic discounting increasing the consumption rate.

To show these, we combine Prettner and Canning's [9] endogenous retirement and economic growth model with Strulik's [6] hyperbolic preferences. Firstly, present-biased preferences allow us to use continuous time model to capture dynamics in savings and retirement decisions. Secondly, we extend the standard model of endogenous growth for aggregate capital accumulation and aggregate consumption expenditure. Thirdly, we explore the effects of changes in the mortality rate on the consumption-capital ratio and in mortality rate, the interest rate, and the speed of declining impatience on economic growth rate. Finally, we recalibrate our model using US data and compare the dynamic behaviors of retirement decisions of exponential and hyperbolic preferences.

In addition, we extend our model to age-dependent survival law which is adapted from Boucekkine et al. [10, 11] and Azomahou et al. [12]. In particular, the assumption of age-dependent mortality rate taken in the more realistic modeling is absolutely crucial. Numerically, we find that age-dependent mortality rate leads individual to work longer. What is more, in case of aggregation over cohorts, the consumption to capital ratio has slight effect on age-dependent mortality rate before retirement. But this ratio has a positive effect on the mortality rate after retirement. People are willing to pay for higher consumption expenditures as age increases. Besides, at the same age, the longer individuals' life expectancy is, the lower this ratio is.

Our findings provide a possible theoretical explanation for government intervention in individuals' retirement decisions and increasing life expectancy. These help to align theory with the intuition of demographic structure and social schemes in retirement decisions.

The present paper is organized as follows. Section 2 considers the benchmark model with time-inconsistent preferences merging Blanchard and Yarris structures. Section 3 compares the dynamic behaviors of optimal retirement under exponential and hyperbolic discounting. In Section 4, the model is considered with more realistic demographics, that is, with age-dependent survival probabilities. Section 5 concludes the paper.

2. The Benchmark Model with Time-Inconsistent Preferences

In this section, we first introduce briefly the basic structure of Blanchard-Yarris model with time-inconsistent preferences. That is, the mortality rate is constant (and age-independent) before retirement (How unrealistic is the assumption of a constant λ? Evidence on mortality rates suggests low and approximately constant probabilities at working ages, see Blanchard [8] for example.). We introduce continuous-time consumption models by merging Prettner and Canning [9] with Strulik [6] and then discuss hyperbolic discounting of Laibson [7].

2.1. Individuals with Constant Mortality Rate. In this part, we derive models with hyperbolic preferences to capture optimal consumption-savings and retirement decisions with constant mortality rate. We follow the assumptions in Yaari [13]; i.e., individuals start their working life without capital holdings and there are no bequests.

Individuals enter the labor market as adults at time t_0 and maximize lifetime utility

$$U = \int_{t_0}^{\infty} \left[\log(c) - \chi_t d e^{\lambda(t-t_0)}\right] D(t_0, t)\, e^{-\lambda(t-t_0)} \mathrm{d}t \qquad (1)$$

subject to the budget constraint

$$\dot{k} = \chi_t w + (\lambda + r) k - c, \qquad (2)$$

where λ represents the mortality rate; c, k, w, and r denote consumption, capital, wage rate, and the interest rate, respectively; d is a scaling parameter measuring individuals' unwillingness to work; and χ is an indicator function with a value of 1 when working and zero when retired. According to Frederick and O'Donoghue [14] and Strulik [6], we consider hyperbolic discounting. The discount function in (1) is given by hyperbolic discounting containing exponential case; that is, $\lim_{\beta \longrightarrow 0^+}[1 + \rho_0\beta(t - t_0)]^{-1/\beta} = e^{-\rho_0(t-t_0)}$. Throughout the paper, the present-biased preferences β refer to the short-term discount rate, and the initial time preference rate ρ_0 is the long-term discount rate, according to Laibson [7].)

$$D(t_0, t) = \frac{1}{\left[1 + \beta\rho_0(t - t_0)\right]^{1/\beta}}, \qquad (3)$$

where $\beta \in (0, 1)$ controls the present bias (The smaller the parameter β is, the stronger the present-biased preferences is, and the lower the speed of declining impatience is.) and ρ_0 controls the instantaneous discount rate (or the initial time preference rate) at next instant in time. Generally, the quasihyperbolic discount rate at discount time t to time t_0 is defined as

$$\rho(t_0, t) = -\frac{\dot{D}(t_0, t)}{D(t_0, t)} = \frac{\rho_0}{1 + \beta\rho_0(t - t_0)}. \qquad (4)$$

The Hamiltonian for this problem is

$$\begin{aligned} H(t_0, t) = {} & \left[\log(c) - \chi_t d e^{\lambda(t-t_0)}\right] D(t_0, t)\, e^{-\lambda(t-t_0)} \\ & + \phi(t)\left[\chi_t w + (\lambda + r) k - c\right]. \end{aligned} \qquad (5)$$

The following are the first-order conditions:

$$\begin{aligned} \dot{\phi}(t) &= -\frac{\partial H}{\partial k} = -\phi(t)(\lambda + r), \\ 0 &= \frac{\partial H}{\partial c} = \frac{D(t_0, t)\, e^{-\lambda(t-t_0)}}{c} - \phi(t), \\ \frac{\partial H}{\partial \chi} &= -dD(t_0, t) + \phi(t) w \geq 0 \quad \text{when } \chi = 1, \\ \frac{\partial H}{\partial \chi} &= -dD(t_0, t) + \phi(t) w \leq 0 \quad \text{when } \chi = 0. \end{aligned} \qquad (6)$$

These conditions yield the following:

$$\begin{aligned} \dot{c} &= \left[r - \rho(t_0, t)\right] c, \\ \chi = 1 &\Longleftrightarrow c^{-1} w \geq d e^{\lambda(t-t_0)}. \end{aligned} \qquad (7)$$

It states that consumption expenditure growth is positive if and only if the interest rate exceeds the initial discount rate (it is also similar to the standard neoclassical growth model with an infinite lifetime horizon if $\beta \longrightarrow 0$; see Ramsey [15], for example). Intuitively, individuals prefer to work as long as the additional utility of working longer is able to compensate them for their disutility of delaying retirement, given that the consumption while working is higher than retirement.

Similar to Strulik [6], we obtain individuals' optimal consumption as

$$c(t_0, t) = \frac{k(t_0, t) + \int_t^T w(\tau) e^{-(r+\lambda)(\tau - t)} d\tau}{m}, \quad (8)$$

where $m = \int_{t_0}^{\infty} [1 + \beta\rho_0(\tau - t_0)]^{-1/\beta} e^{-\lambda(\tau - t_0)} d\tau = \int_0^{\infty} (1 + \beta\rho_0 t)^{-1/\beta} e^{-\lambda t} dt \in (1/(\rho_0 + \lambda), 1/\rho_0(1-\beta))$ and T denotes individuals' retirement date.

Since lifetime consumption expenditures are equal to lifetime income, we can calculate consumption from a reparametrization of the budget constraint as follows:

$$\int_{t_0}^{\infty} e^{-(\lambda+r)(t-t_0)} c(t_0, t) \, dt = \int_{t_0}^{T} e^{-(\lambda+r)(t-t_0)} w(t_0, t) \, dt. \quad (9)$$

Integrating and using $c(t_0, t) = c(t_0, t_0) e^{\int_{t_0}^{t} [r - \rho(t_0, s)] ds}$, which follows from the individuals' Euler equation, and $w(t_0, t) = w(t_0, t_0) e^{g(t-t_0)}$, which follows from denoting wage growth by g, we arrive at an expression for the fraction of consumption expenditures to wages at the beginning of working life

$$\frac{c(t_0, t_0)}{w(t_0, t_0)} = \frac{1 - e^{-(\lambda+r-g)(T-t_0)}}{(\lambda + r - g) m}. \quad (10)$$

Intuitively, this expression tells us that if individuals want to consume more or save less in relation to initial income, they will choose to delay retirement. Following Bloom et al. [5], we also introduce the parameter restriction $g < r + \lambda$, which ensures a finite present value of lifetime wage income.

For endogenous retirement the situation is more complex. We denote the retirement ages (or working ages) by $R = T - t_0$ and rewrite the retirement equation as follows:

$$d e^{\lambda R} = \frac{w(t_0, T)}{c(t_0, T)} = \frac{w(t_0, t_0) e^{gR}}{c(t_0, t_0) e^{\int_{t_0}^{T} [r - \rho(t_0, s)] ds}}, \quad (11)$$

which we can be simplify to

$$\frac{c(t_0, t_0)}{w(t_0, t_0)} = \frac{(1 + \beta\rho_0 R)^{1/\beta}}{d} e^{(g - r - \lambda)R}. \quad (12)$$

Substituting for the initial consumption-wage ratio (10) into (12), we have

$$\begin{aligned} & d\left[1 - e^{(g-\lambda-r)R}\right] \\ & \quad = m(\lambda + r - g)(1 + \beta\rho_0 R)^{1/\beta} e^{(g-r-\lambda)R}. \end{aligned} \quad (13)$$

In Figure 1, we sketch $m(\lambda + r - g)(1 + \beta\rho_0 R)^{1/\beta} e^{(g-r-\lambda)R} - d[1 - e^{(g-\lambda-r)R}]$ for varying R and for different speed of declining impatience and discount rate scenarios. To compare this with exponential discounting, Figure 2 depicts $(\lambda + r - g) e^{(g+\delta-r-\lambda)R} - (\lambda + \delta) d[1 - e^{(g-r-\lambda)R}]$ for varying R with the exponential discount rate $\delta = 0.744\%$ according to Prettner and Canning [9]. We obtain the parameter values in the numerical analysis which fit the actual values for the United

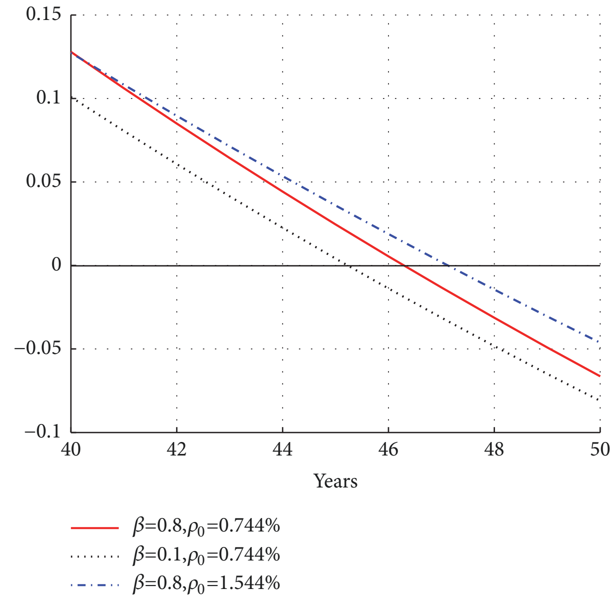

FIGURE 1: Difference in the retirement for varying speed of declining impatience and initial discount rates under hyperbolic preferences.

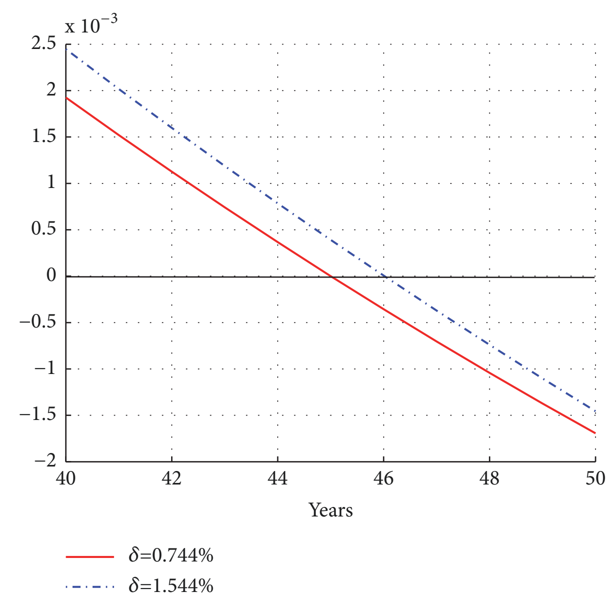

FIGURE 2: Difference in the retirement for varying discount rates under exponential discounting.

States directly from Economagic [16] and World-Bank [17]. That is, $r = 3.25\%$, $g = 1.472\%$, $\lambda = 0.0127$, and $d = 0.719$. (Following Prettner and Canning [9], we reconstruct averages for 2010-2014 to obtain business–cycle adjusted values for banks' prime loan rate ($r = 3.25\%$), discount window primary credit ($\delta = 0.744\%$), per capita gross domestic product (GDP) growth ($g = 1.472\%$), and life expectancy at a birth of 78.7 years. Furthermore, we recalibrate $\lambda = 0.0127$ and $d = 0.719$ in (21) to obtain 78.7 years for life expectancy and a working life duration of 45 years, which corresponds to these figures for the US in 2012 according to OECD [18].)

Figures 1 and 2 emphasize that if the initial or exponential time preference rate (ρ_0 or δ) is high, the individual tends to delay retirement. This is because the individual tends to give up more of the future consumption, resulting in the current consumption being greater than the future consumption. However, when the retirement age is reached, the individual needs to continue to work in order to maintain a higher level of consumption, resulting in delayed retirement. We also find that hyperbolic discounters are choosing to retire later than the exponential discounters. According to Zou et al. [2], one possible explanation is that the hyperbolic discounters have a higher rate of consumption than the exponential discounters, in which case the hyperbolic discounters tend to choose to work longer to maintain higher expenditures. It is worth noting that if the discount rate increased by 0.8 percentage points, the retirement age will be delayed for nearly a year. In addition, Figure 1 illustrates the comparison of different speed of declining impatience. Obviously, the stronger the present-bias preference β is, the earlier the individual retires. This is questionable because (4) shows that the quasihyperbolic discount rate ρ is negatively correlated with β. This may be caused by the assumption of the constant d. Therefore, we will discuss the effects of changes in the speed of declining impatience, the discount rate, and retirement decisions on the unwillingness to work, i.e., $d = d(\beta, \rho_0, R)$, in the next section.

2.2. Aggregation with Constant Mortality Rate. In this part, we derive expressions for aggregate capital accumulation and aggregate consumption expenditures similar to the overlapping generations' framework of Blanchard [8] and Bloom et al. [5].

Consider all cohorts that are alive at a certain instant t and denote the aggregate capital stock by K and aggregate consumption expenditures by C. Following aggregate rules (see Blanchard [8], for instance), we have

$$\begin{aligned} K(t) &:= N\int_{-\infty}^{t} k(t_0, t)\, e^{\lambda(t_0 - t)}\mathrm{d}t_0, \\ C(t) &:= \lambda N\int_{-\infty}^{t} c(t_0, t)\, e^{\lambda(t_0 - t)}\mathrm{d}t_0, \end{aligned} \tag{14}$$

where λ and N represent the mortality rate and the adult population size, respectively, while $k(t_0, t)$ and $c(t_0, t)$ are the capital holdings and consumption levels of its members, respectively.

Similar to Prettner and Canning [9], we account for the demographic structure and derive the following dynamic equations for aggregate capital stock and aggregate consumption, respectively,

$$\begin{aligned} \dot{K}(t) &= W(t) + rK(t) - C(t), \\ \dot{C}(t) &= -\frac{\lambda K(t)}{m} + rC(t) \\ &\quad - \lambda N\int_{-\infty}^{t} \rho(t_0, t)\, c(t_0, t)\, e^{-\lambda(t - t_0)}\mathrm{d}t_0, \end{aligned} \tag{15}$$

where $W = \lambda N\int_{-\infty}^{t} \chi(t_0, t) w(t) e^{-\lambda(t - t_0)}\mathrm{d}t_0$ refers to aggregate wage income.

In particular, let $t = t_0$, and the aggregate Euler equation is given by

$$\frac{\dot{C}(t_0)}{C(t_0)} = r - \rho_0 - \frac{\lambda}{m}\frac{K(t_0)}{C(t_0)}. \tag{16}$$

2.3. Neoclassical Growth Model. This part of the neoclassical economic growth model strictly follows Solow [19] and Prettner and Canning [9]. The economy is populated by all cohorts of measure one. We can write the aggregate production function as $Y = K^{\alpha}(AL)^{1-\alpha}$, where A is the technological frontier of the economy growing at rate $0 < \mathrm{g} = \dot{A}/A$ and $0 < \alpha < 1$ is the elasticity of output with respect to capital. Assuming perfect competition in factor markets, we can write the interest rate as $r = \partial Y/\partial K = \alpha(AL/K)^{1-\alpha}$ and, consequently, $Y/K = r/\alpha$.

2.4. Consequences of Changing Life Expectancy with Constant Mortality Rate. In order to set up the long-run equilibrium, we assume that aggregate income, or economic GDP, consists of wage income and capital income. We also assume that the economy grows at the constant rate g, satisfying $\mathrm{g} = \dot{A}/A = \dot{C}/C = \dot{K}/K = \dot{W}/W$. From (13), (15), and (16), we obtain our model economy as follows:

$$\mathrm{g} = \frac{r}{\alpha} - \xi, \tag{17}$$

$$\mathrm{g} = r - \rho_0 - \frac{\lambda}{m}\frac{1}{\xi}, \tag{18}$$

where ξ denotes the relationship between aggregate consumption expenditures and aggregate capital stock. Solving the system (17) and (18) for ξ and g yields (There are two solution pairs, one of which we can rule out because it involves negative values of ξ.)

$$\begin{aligned} \xi &= \frac{[(1-\alpha)\mathrm{g} + \rho_0] + \sqrt{[(1-\alpha)\mathrm{g} + \rho_0]^2 + 4\alpha\lambda/m}}{2\alpha}, \\ \mathrm{g} &= \frac{r + \alpha(r - \rho_0) - \sqrt{[r - \alpha(r - \rho_0)]^2 + 4\lambda\alpha^2/m}}{2\alpha}. \end{aligned} \tag{19}$$

Next, we can obtain the following propositions.

Proposition 1. *An increase in longevity raises aggregate consumption expenditures.*

Proof. Since the function $e^{-\lambda t}/\lambda$ decreases with respect to mortality rate λ, we can obtain that the above expressions for ξ increases with respect to λ. Note that an increase in longevity is represented by a decrease in mortality, the above proposition holds. □

The intuition for this finding is that as longevity increases, individuals pay more fees in physically and spiritually (such as health care) to meet their own needs.

Proposition 2. *(i) The rate of economic growth is declining in the mortality rate; (ii) the rate of economic growth is increasing in the speed of declining impatience but is decreasing in the initial time preference rate.*

Proof. By the partial differential derivation method, we can get the following inequalities: $\partial g/\partial\lambda < 0, \partial g/\partial\beta > 0$ and $\partial g/\partial\rho_0 < 0$. □

The intuition behind this finding is straightforward. First, in a society where mortality is declining, the population is growing, resulting in additional domestic demand, which has a positive impact on the economy. Second, according to Laibson [7] and Findley and Caliendo [4], β captures the present-bias preference, which can be interpreted as a short-term discount factor, and ρ_0 is the initial time preference rate, which means a long discount factor. If most people in society prefer present value to the future value, the short discount rate or the decline in the long discount rate will bring more short-term investment. This imbalance between short-term investments and long-term investments has a negative impact on economic structure and long-term economic growth, as it will tend to generate excess investment and unnecessary volatility.

3. Comparative Behavior

In this section, we compare the dynamic behaviors of optimal retirement under exponential and hyperbolic discounting. To illustrate this, recall the stream of lifetime utility from Bloom et al. [5] and Prettner and Canning [9]:

$$\widetilde{U} = \int_{t_0}^{\infty} \left[\log(c) - \chi_t \widetilde{d} e^{\lambda(t-t_0)}\right] e^{-(\delta+\lambda)(t-t_0)} \mathrm{d}t, \tag{20}$$

where δ is the exponential discount rate and $\widetilde{d}$ is a scaling parameter measuring individuals' unwillingness to work. Then, the following equation should satisfy the retirement R^*:

$$(\lambda+\delta)\,\widetilde{d}\left[1 - e^{(g-r-\lambda)R^*}\right] = (\lambda + r - g)\, e^{(g+\delta-r-\lambda)R^*}. \tag{21}$$

We first illustrate the effects of the discount rate and retirement age on individuals' unwillingness to work with numerical examples, i.e., $\widetilde{d} = \widetilde{d}(\delta, R)$. Based on (21), we can easily obtain Table 1 following $\lambda = 0.0127$, $g = 1.472\%$, and $r = 3.25\%$. Table 1 displays that exponential discounters postpone retirement as the rate of time preference decreases.

Introducing a format of the statistical curve fitted by multidimensions least square, we assume the following measurement of unwillingness to work under exponential discounting:

$$\widetilde{d} = -0.0556R^* + 392.3476\delta^2 - 13.6851\delta + 3.2999. \tag{22}$$

Next, we study that the retirement age for hyperbolic and exponential preferences is equivalent under a plausible condition. In order to assess this, we extend the assumption following Strulik [6], as follows:

Assumption (Equivalent Present Value). Discounting parameters are such that a discounted infinite stream provides the same present value, i.e.,

$$\int_0^{\infty} e^{-(\delta+\lambda)t}\mathrm{d}t = \int_0^{\infty} \left(1+\rho_0\beta t\right)^{-1/\beta} e^{-\lambda t}\mathrm{d}t. \tag{23}$$

Motivated by (23), which implies $\delta = \rho_0(1-\beta)$, and (22), we introduce a measure of individuals' unwillingness to work under hyperbolic preferences

$$d = f(\beta, \rho_0, R), \tag{24}$$

where $f(\beta,\rho_0,R) = -0.0556R + 392.3476[\rho_0(1-\beta)]^2 - 13.6851\rho_0(1-\beta) + 3.2999$. We can then rewrite (13):

$$\begin{aligned} &f(\beta,\rho_0,R)\left[1 - e^{(g-\lambda-r)R}\right] \\ &\quad = m(\lambda + r - g)\left(1+\beta\rho_0 R\right)^{1/\beta} e^{(g-r-\lambda)R}. \end{aligned} \tag{25}$$

In Figure 3, we describe the effect of retirement age on the speed of declining impatience (or the present-biased preferences) and the initial time preference rate. We see that individuals with strong future-bias preferences will choose to retire late, but the high initial discount rate will lead to early retirement. Numerically, if the speed of declining impatience increases from 0.1 to 0.9, the retirement age will be delayed about a year; the initial time preference rate increased by 0.25 percentage points, retirement age will be about seven months in advance. Given the present value of the equivalence, Figure 4 shows the relationship between the hyperbolic and the exponential discounts corresponding to the retirement age and the discount rate. From the short-term discount rate β, individuals with hyperbolic preference retire later than individuals with exponential preference, but the opposite case is true from the long discount rate ρ_0. More specifically, within short discount rate, individuals often refuse to change due to risk aversion, and there appears to be a strong present-bias preference, in which case the retirement age is six months ahead of schedule. However, from the long-term discount rate, the individual's view of delayed retirement has no significant difference between the current value and the future value, as long as the opportunity to continue to work, in this case the retirement age, will be delayed a year and a half.

We summarize the above findings in the following.

Remark 3. Given equivalent present value (23), we have the following conclusions:

(1) The equilibrium retirement age occurs later under hyperbolic preferences than exponential preferences in the long discount rate, but the situation reverses in the short discount rate.

(2) A strong present-biased preference causes early retirement.

(3) A low discount rate causes delayed retirement.

Next, we show that the main results are robust to the effects of generalizing the instantaneous utility function, disutility of work, the mortality process, the interest rate, the

Table 1: Effects of discount rate and retirement on unwillingness to work.

δ	0.744%	1.244%	1.744%	2.244%	2.744%
R^*	45	44	43	42	41
$\tilde{d}$	0.7191	0.7423	0.7904	0.8571	0.9397

Table 2: Effects of the speed of declining impatience on the optimal duration of working life.

β	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	6
R	43.97	44.00	44.06	44.16	44.28	44.44	44.62	44.83	45.06

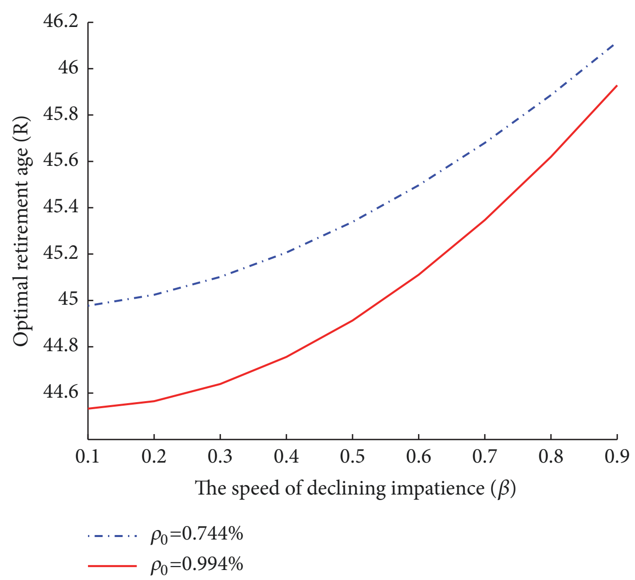

Figure 3: Effects of retirement age on the speed of declining impatience with $\rho_0 = 0.744\%, 0.944\%$.

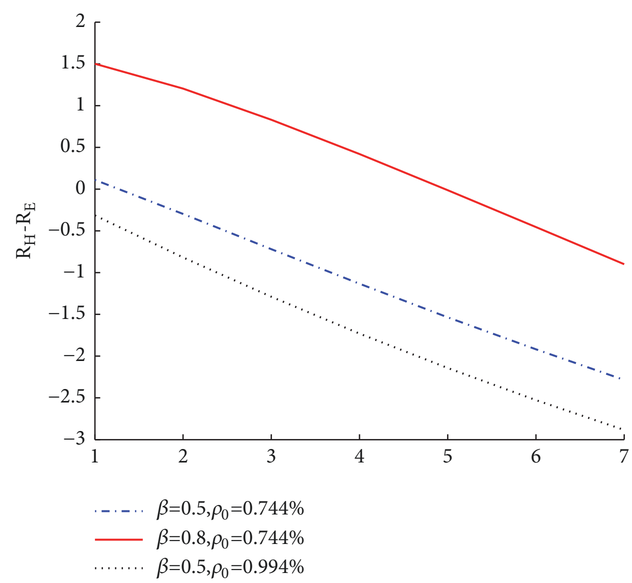

Figure 4: Difference between hyperbolic and exponential preferences in retirement age given the equivalent present value.

discount rate and economic growth. In addition, we assess the sensitivity of our central result with respect to changes in the speed of declining impatience. Results in Table 2 reveal that the speed of declining impatience have a negative effect on retirement age (In this case, we choose g $= 1.63\%$, $\rho_0 = 0.76\%$, and $r = 3.26\%$ for 2015 in the United States according to Economagic [16].).

Remark 4. It is worth noting that when individuals have the same degree of unwillingness to work, the higher discount rate leads to longer working hours. However, when the degree of the unwillingness to work depends on the time preference and retirement, the situation will change; that is, the longer the working time to produce negative effects is driven by the individual choosing to retire early. We measure the degree of the unwillingness to work based on time preference and retirement age. The higher the discount rate is, the earlier the individual chooses to retire. For example, for two jobs with the same remuneration but with different working hours, the individual is clearly willing to take less time with the same incomes of work.

4. Model with a More Realistic Survival Law

In the previous model, we implicitly assume that the mortality rate is age-independent, which is clearly unrealistic after retirement. In this section, we relax this assumption and consider age-dependent survival law. We use a survival law adapted from Boucekkine et al. [10].

The set of individuals are alive in t is an interval of measure ζe^{nt}, with $\zeta \in \mathbb{R}_+$ and $n \in \mathbb{R}$. n is a constant growth rate of population. Each individual has an uncertain lifetime. The unconditional probability for an individual to cohort t_0 of reaching age $t \in [t_0, M - t_0]$ is given by the function $S(t)$,

$$S(t_0, t) = S(t - t_0) = \frac{e^{-\nu(t-t_0)} - \mu}{1 - \mu}, \tag{26}$$

where the maximum age M that an individual can reach is

$$M = \frac{-\log(\mu)}{\nu}. \tag{27}$$

μ is an indicator of survival for young persons, and ν is an indicator of survival for older persons. We suppose $\mu > 1$

and $\nu < 0$, as in the work of Boucekkine et al. [10] to generate a concave survival law as observed in real life. That is, the mortality rate is slightly changed in working ages, but it significantly increases after retirement. Moreover, the unconditional life expectancy is

$$\Gamma = \frac{1}{\nu} + \frac{\mu \log(\mu)}{(1-\mu)\nu}. \tag{28}$$

4.1. Individuals with Age-Dependent Mortality Rate. Under age-dependent survival law, an individual's maximize lifetime utility is

$$\int_{t_0}^{\infty} [\log(c) S(t_0, t) - \chi_t d] D(t_0, t)\, dt \tag{29}$$

subject to the budget constraint

$$\dot{k} = \chi_t w + \left(\nu + \frac{\mu\nu}{e^{-\nu(t-t_0)} - \mu} + r\right) k - c. \tag{30}$$

The Hamiltonian for the above problem is $\widehat{H}(t_0, t) = [\log(c)S(t_0,t) - \chi_t d]D(t_0,t) + \Phi(t)[\chi_t w + (\nu + \mu\nu/(e^{-\nu(t-t_0)} - \mu) + r)k - c]$. The first-order conditions are $\dot{\Phi}(t) = -\partial\widehat{H}/\partial k, \partial\widehat{H}/\partial c = 0$. Then, we have

$$\begin{gathered} \dot{c} = [r - \rho(t_0, t)]\, c, \\ \chi = 1 \Longleftrightarrow c^{-1} w \geq \frac{d(1-\mu)}{e^{-\nu(t-t_0)} - \mu}. \end{gathered} \tag{31}$$

It states that consumption expenditure growth has no effect on age-dependent mortality rate. However, when an individual chooses to work, the lower limit of the ratio of wages to consumption is reduced (e.g., $de^{\nu(t-t_0)} \geq d(1-\mu)/(e^{-\nu(t-t_0)} - \mu)$). Intuitively, due to the impact of the probability of death, an individual chooses low wage income or high consumption levels.

Next, we consider an individual' retirement age based on age-dependent mortality rate. Similarly, we can calculate consumption as follows:

$$\begin{aligned} &\int_{t_0}^{\infty} S(t_0, t)\, e^{-r(t-t_0)} \widehat{c}(t_0, t)\, dt \\ &\quad = \int_{t_0}^{\widehat{T}} S(t_0, t)\, e^{-r(t-t_0)} w(t_0, t)\, dt, \end{aligned} \tag{32}$$

where $\widehat{T}$ is an individual' retirement date, $c(t_0, t) = c(t_0, t_0)e^{(r-\rho)(t-t_0)}$. (For the sake of simplicity, we assume that the discount rate is constant (ρ). And all results derived below hold when we allow for a (quasi-)hyperbolic discount rate.) Then, we have

$$\begin{aligned} &\left(e^{-\nu\widehat{R}} - \mu\right)\left(\frac{1}{\rho+\nu} - \frac{\mu}{\rho}\right) \\ &\quad = d(1-\mu)\left[\frac{1 - e^{-(r+\nu-g)\widehat{R}}}{r+\nu-g} - \frac{\mu\left(1 - e^{-(r-g)\widehat{R}}\right)}{r-g}\right], \end{aligned} \tag{33}$$

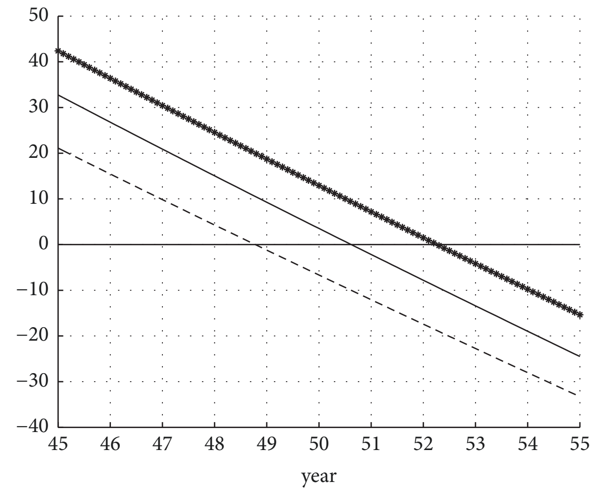

Figure 5: Partial equilibrium effects for different μ and ν on the optimal retirement age $\widehat{R}^*$. The solid line refers to the baseline case, and dashed and star lines indicate in which direction the implicit retirement function shifts for the corresponding parameter change. The optimal duration of the working life is identified by the value at which the retirement function intersects the zero axis.

where $\widehat{R} = \widehat{T} - t_0$ is the optimal retirement age under age-dependent mortality rate.

In Figure 5, we plot $(e^{-\nu\widehat{R}} - \mu)(1/(\rho+\nu) - \mu/\rho) - d(1-\mu)[(1 - e^{-(r+\nu-g)\widehat{R}})/(r+\nu-g) - \mu(1 - e^{-(r-g)\widehat{R}})/(r-g)]$ for varying $\widehat{R}$ and for two different scenarios. The baseline case is indicated by the solid line, and the partial equilibrium adjustments of the retirement function to parameter changes are shown by the dashed or star lines. The parameter values we use are obtained directly from Prettner and Canning [9] and Azomahou et al. [12]; that is, $g = 0.86\%$, $\rho = 2.09\%$, $d = 0.643$, $r = 3.41\%$, $\mu = 5.44$, $\nu = -0.0147$.

For a low value of $\widehat{R}$, the function is positive (people are still working), and for a large value of $\widehat{R}$, the function turns negative (people are retired). The value of $\widehat{R}$ at which the function intersects the zero axis identifies the retirement age $\widehat{R}^*$. The dashed line displays the effect of a decrease in parameter μ by 0.1 points. The star line shows the effect of an increase in parameter ν by 0.01% points. All else equal, we see that individual prefers to work longer—in this case by almost one year—such that the retirement function moves to the right. When parameter μ or ν increases—which means the mortality rate decreases—individual can be alive longer. Individual chooses to spend a part of their additional years of life in retirement and the other part for increasing lifetime labor supply in order to sustain more consumption when retired. It is interesting that individual is willing to work longer when she considers age-dependent survival law, compared to the results in Prettner and Canning [9].

4.2. Aggregation with Age-Dependent Mortality Rate. Based on age-dependent mortality rate, consider all cohorts that are alive at a certain instant t and denote the aggregate capital stock by $\widehat{K}$ and aggregate consumption expenditures by $\widehat{C}$. Following aggregate rules and the size of population at time t

(see Boucekkine et al. [10], Boucekkine et al. [11], Azomahou et al. [12]), we have

$$\widehat{K}(t) = \zeta\pi \int_{-\infty}^{t} k(t_0, t) e^{n(t_0 - t)} dt_0,$$
$$\widehat{C}(t) = \zeta\pi \int_{-\infty}^{t} c(t_0, t) e^{n(t_0 - t)} dt_0, \quad (34)$$

where $\pi = (n(1-\mu) - \mu\nu(1-\mu^{n/\nu}))/n(1-\mu)(n+\nu)$. Then, one has

$$\dot{\widehat{C}}(t) = \zeta\pi c(t,t) + (r - \rho - n)\widehat{C}(t), \quad (35)$$

and

$$\dot{\widehat{K}}(t) = [r - n + \lambda(t)]\widehat{K}(t) + \widehat{W}(t) - \widehat{C}(t), \quad (36)$$

where $\widehat{W}(t) = \zeta\pi \int_{-\infty}^{t} \chi(t_0,t) w(t) e^{n(t_0-t)} dt_0$ refers to aggregate wage income, and $\lambda(t) = -S'(t_0,t)/S(t_0,t)$ refers to mortality rate.

Furthermore, we have

$$c(t_0, t) = (\rho + \lambda(t))[k(t_0,t) + h(t)], \quad (37)$$

where $h(t) = \int_t^T w(\tau) e^{-(r+\lambda(t))(\tau - t)} d\tau$. Therefore,

$$\widehat{C}(t) = (\rho + \lambda(t))\left[\widehat{K}(t) + \widehat{H}(t)\right], \quad (38)$$

where $\widehat{H}(t) = \zeta\pi \int_{-\infty}^{t} h(t) e^{n(t_0-t)} dt_0 = (\zeta\pi/n) h(t)$. Putting everything together, we obtain

$$\frac{\dot{\widehat{C}}(t)}{\widehat{C}(t)} = r - \rho - n[\rho + \lambda(t)]\frac{\widehat{K}(t)}{\widehat{C}(t)}. \quad (39)$$

Therefore, along a balanced growth path, we can write the system as

$$g = \frac{r}{\alpha} + \lambda(t) - n - \xi(t) = r - \rho - n[\rho + \lambda(t)]\frac{1}{\xi(t)}, \quad (40)$$

where $\xi(t) = \widehat{C}(t)/\widehat{K}(t)$ refers to the relationship between aggregate consumption expenditures and aggregate capital stock.

4.3. Consequences of Changing Life Expectancy with Age-Dependent Mortality Rate. Next, we describe the response of an economy's consumption to capital ratio in life expectancy. Solving the system (40) for $\lambda(t)$ yields

$$\xi(t) = \frac{[\rho + r/\alpha + \lambda(t) - r - n] + \sqrt{[\rho + r/\alpha + \lambda(t) - r - n]^2 + 4n[\rho + \lambda(t)]}}{2}. \quad (41)$$

In Figure 6, we plot $\xi(t)$ for varying t and two different scenarios. The baseline case is indicated by the solid line. The adjustments of the consumption to capital ratio function to parameter changes are shown by the dashed and star lines. The parameter values we use are as before, and $n = 0.10, \alpha = 1/3$ are obtained directly from Azomahou et al. [12] and Acemoglu [20], respectively.

The dashed line displays the effect of a decrease in parameter μ by 0.1 points. The star line shows the effect of an increase in parameter ν by 0.01% points. It shows that the consumption to capital ratio may have slight effect on decreasing mortality rate (or increasing life expectancy) before retirement. However, this ratio has a positive effect on increasing life expectancy after retirement. Intuitively, on one hand, people do not care about their health, since they believe that they can be alive to retirement age. On the other hand, people pay more attention on longevity problem and choose to have more consumption expenditures when getting old in order to able to sustain health care. It also displays that as age increases, people are willing to pay for higher consumption levels. Besides, we observe that, at the same age, the longer people's life expectancy is, the lower the consumption to capital ratio is. In fact, people smooth consumption in order to sustain a certain consumption level during their prolonged period of retirement.

5. Conclusion

In this paper, we extend time-inconsistent consumption problems in the context of endogenous growth and retirement choice. In continuous-time models, individuals with hyperbolic preferences will choose to retire early in the short discount rate but to delay retirement in the long discount rate.

A major extension in the theoretical model is the introduction of endogenous growth within a dynamic equilibrium based upon Boucekkine et al. [10] and Prettner and Canning [9]. We have solved a model of this sort without savings. To make it an interesting problem, one has to assume that individuals start their working life without capital holdings and there is no bequest. In benchmark equilibrium framework, we conclude that an increase in longevity raises aggregate consumption expenditures.

Another major extension is the numerical assessment of time inconsistency and age-dependent equilibrium model. They are clearly important in practice, and they change retirement decisions with hyperbolic preferences considerably. We find that, under the assumption of constant unwillingness to work, the higher the rate of time preference is, the later individuals choose to retire regardless of quasihyperbolic or exponential preferences. Considering the unwillingness to work is a function of the speed of declining impatience, the discount rate, and the retirement age, we find that

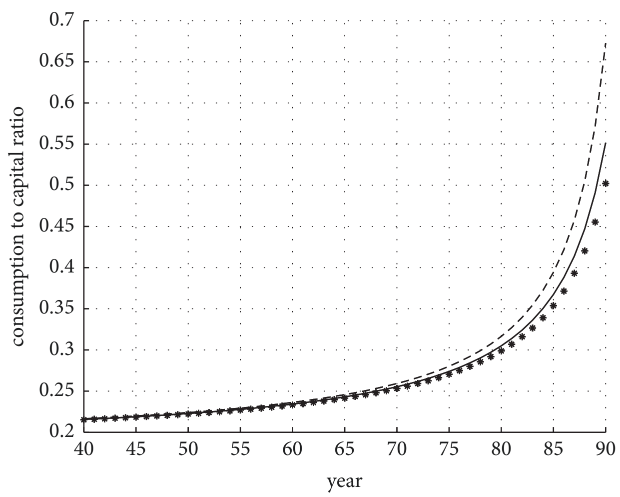

Figure 6: Consumption to capital ratio effects of different μ and ν. The solid line refers to the baseline case, and the dashed and star lines indicate different mortality rate.

individuals with strong future-biased preferences prefer to delayed retirement, but a high initial discount rate will cause early retirement. Consider age-dependent mortality rate, numerically, we give some interesting findings, such as how the consumption to capital ratio has effect on increasing life expectancy before or after retirement. These insights may have implications for individuals' increasing life expectancy and capital-consumption allocations based policy schemes and their effects on retirement decisions.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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