Hindawi
Mathematical Problems in Engineering
Volume 2019, Article ID 8725610, 12 pages
https://doi.org/10.1155/2019/8725610

Research Article

Improved Delay-Dependent Stability and Stabilization Conditions of T-S Fuzzy Time-Delay Systems via an Augmented Lyapunov-Krasovskii Functional Approach

Zifan Gao,[1] **Jiaxiu Yang**,[1] **and Shuqian Zhu**[2]

[1] *School of Mathematical Sciences, Shanxi University, Taiyuan 030006, China*
[2] *School of Mathematics, Shandong University, Jinan 250100, China*

Correspondence should be addressed to Shuqian Zhu; sqzhu@sdu.edu.cn

Received 9 March 2019; Revised 4 June 2019; Accepted 13 June 2019; Published 4 July 2019

Academic Editor: Libor Pekař

Copyright © 2019 Zifan Gao et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

This paper develops some improved stability and stabilization conditions of T-S fuzzy system with constant time-delay and interval time-varying delay with its derivative bounds available, respectively. These conditions are presented by linear matrix inequalities (LMIs) and derived by applying an augmented Lyapunov-Krasovskii functional (LKF) approach combined with a canonical Bessel-Legendre (B-L) inequality. Different from the existing LKFs, the proposed LKF involves more state variables in an augmented way resorting to the form of the B-L inequality. The B-L inequality is also applied in ensuring the positiveness of the constructed LKF and the negativeness of derivative of the LKF. By numerical examples, it is verified that the obtained stability conditions can ensure a larger upper bound of time-delay, the larger number of Legendre polynomials in the stability conditions can lead to less conservative results, and the stabilization condition is effective, respectively.

1. Introduction

T-S fuzzy time-delay systems are well-recognized by the integration of time-delay systems and fuzzy systems which are often employed to model several nonlinear systems in practice [1]. Such systems have two appealing advantages: T-S fuzzy system is capable of modeling the nonlinear systems [2] and the unavoidable time-delay phenomenon is explicitly incorporated in the system [1]. When both nonlinearity and time-delay phenomena are considered, T-S fuzzy system with time-delay indeed offers a feasible system representation. For such a system, much work has been done in the past few decades to achieve stability and performance conditions, delay-dependent ones in particular [3–8]. The presence of time-delay in the fuzzy system can be either constant or time-varying delay, and systems with time-varying delay are more difficult to be handled than constant time-delay case. In the light of flourishing research on linear time-delay systems, more and more interesting stability results have been published recently for T-S fuzzy systems with time-varying delays [3–8]. Since these delay-dependent stability and performance conditions are only sufficient conditions, the admissible maximum upper bound of time-delay computed by the conditions is commonly treated as an essential index to evaluate the conservatism of the conditions. Thus, a primary purpose of delay-dependent stability conditions is to search for the admissible maximum upper bound of time-delay as large as possible while ensuring the system stability and performance.

Recalling the existing delay-dependent stability results on T-S fuzzy time-delay systems, one can see that a LKF approach is prevalent and well-studied. The basic idea is to derive the condition by estimating the time-derivative of the constructed LKF which satisfies the conditions of the LKF theorem in [9]. Thus, the construction of the LKF and the estimation method of its derivative play a key role in developing less conservative stability conditions. In the derivative operation process of the LKF, various integral inequalities are established to produce an estimation as tight as possible, such as Jensen-type inequality [1, 3, 4], Wirtinger-type inequality [5, 10], B-L inequality [11–14], and reciprocally convex inequality with free weighting

matrices [7, 15, 16]. Moreover, different types of LKFs such as fuzzy weighting-dependent LKFs [17, 18] and augmented LKFs [16, 19] are proposed for T-S fuzzy system. Recently, there are increasing works on exploring the connection or relationship between the process of integral inequality and the construction of the LKF [20], and they confirm that the augmented LKF approach can be considered as a competitive method [19] and the B-L inequality can cover Jensen-type inequality and Wirtinger-type inequality as special cases [11–13]. Accordingly, it is expected to develop some less conservative results for T-S fuzzy system with time-delay by updating the augmented LKF approach together with the B-L inequality, which mainly inspires the current research. More recently, for linear time-delay systems, the augmented LKF together with B-L inequalities are introduced to develop stability results [11–14]. However, this method in [11–14] is not presented for stabilization problem of T-S fuzzy systems with time-delay. More importantly, the augmented LKF in [13, 14, 16, 19] requires all Lyapunov matrix variables to be positive and the delay-induced convex combination in the augmented LKF is not sufficiently used, which leaves us much room for improvement.

In summary, the contributions of this paper are mainly in two aspects as follows.

(i) Augmented LKFs based on the form of B-L inequality are proposed to achieve less conservative stability conditions of T-S fuzzy systems with constant or time-varying delay. In particular, for the time-varying delay case, free-weighting matrices are technically introduced in the process of the positive definiteness of the LKF and the negative definiteness of its derivative resorting to a reciprocally convex method.

(ii) Both stability and stabilization conditions of T-S fuzzy systems with constant or time-varying delay are expressed by tractable LMIs. The advantages of the proposed stability conditions over than those in the existing literature and the design validity are shown in the examples.

Notation. $\mathbb{R}^n$ is the n-dimensional Euclidean space. $\mathbb{S}^n$ (respectively, $\mathbb{S}^n_+$) denotes a set of real symmetric (respectively, positive definite) matrices with $n \times n$ dimensional. For a square matrix X, $\mathrm{Sym}\{X\}$ means the sum of X and X^T, i.e., $\mathrm{Sym}\{X\} = X + X^T$. For two positive integers i, j $(i \geq j)$, we define $\binom{i}{j} = i!/[(i-j)!j!]$. We use $col\{\ldots\}$ to represent a column vector and $\mathrm{diag}\{\ldots\}$ to denote the block-diagonal matrix. The symmetric term in a symmetric matrix is denoted by "$\star$". $Co\{q_1, q_2\}$ is a polytope generated by two vertices q_1 and q_2.

2. System Description and Preliminaries

Consider the time-delay system with fuzzy rules as follows.

Rule l. IF $\delta_1(t)$ is J_{l1}, $\delta_2(t)$ is $J_{l2}, \cdots$, and $\delta_r(t)$ is J_{lr}, THEN

$$\dot{x}(t) = A_{l0}x(t) + A_{l1}x(t-d(t)) + B_l u(t), \tag{1}$$

where $l \in \mathbb{T} \triangleq \{1, 2, \ldots, s\}$, and $\delta_1(t), \delta_2(t), \ldots, \delta_r(t)$ are the premise variables; $N_{l1}, N_{l2}, \ldots, N_{lr}$ are the fuzzy sets; $x(t) \in \mathbb{R}^n$ and $u(t) \in \mathbb{R}^m$ represent the state vector and control input, respectively; A_{l0}, A_{l1}, and B_l are known system matrices. $d(t)$ is the time-delay that can be either constant (let $d(t) \equiv d_0$) or time-varying. For the time-varying case, it satisfies

$$\left(d(t), \dot{d}(t)\right) \in \mathscr{D}_1 = [0, d_0] \times [d_m, d_M]. \tag{2}$$

where d_0 is the upper bound of time-delay and d_m and d_M are the interval bounds of the derivative of $d(t)$. We define $x(t) = \phi(t)$ where $t \in [-d_0, 0]$ as the initial condition of system (1).

According to the technique of fuzzy blending, system (1) can be inferred as the following overall system:

$$\begin{aligned}\dot{x}(t) &= \sum_{l=1}^{s} o_l(\delta(t))\left[A_{l0}x(t) + A_{l1}x(t-d(t)) + B_l u(t)\right],\end{aligned} \tag{3}$$

where $o_l(\delta(t))$ is the normalized membership function; $v_l(\delta(t)) = \prod_{m=1}^{r} J_{lm}(\delta_m(t))$ is the membership function of the fuzzy set $\{J_{lm}\}$ $(m = 1, 2, \ldots, r)$, where $J_{lm}(\delta_m(t))$ is the grade of membership of $\delta_m(t)$ in J_{lm} and

$$\begin{aligned} o_l(\delta(t)) &= \frac{v_l(\delta(t))}{\sum_{l=1}^{s} v_l(\delta(t))}, \\ \sum_{l=1}^{s} o_l(\delta(t)) &= 1. \end{aligned} \tag{4}$$

Consider the following fuzzy controller for lth rule.

Rule l. IF $\delta_1(t)$ is J_{l1}, $\delta_2(t)$ is $J_{l2}, \cdots$, and $\delta_r(t)$ is J_{lr}, THEN

$$u(t) = K_l x(t), \tag{5}$$

where K_l $(l \in \mathbb{T})$ is the fuzzy controller gain. The fuzzy controller is given by

$$u(t) = \sum_{l=1}^{s} o_l(\delta(t)) K_l x(t). \tag{6}$$

Substituting (6) into (3), we obtain the following closed-loop system:

$$\begin{aligned}\dot{x}(t) &= \sum_{l=1}^{s}\sum_{m=1}^{s} o_l o_m \left[\left(A_{l0} + B_l K_m\right)x(t) + A_{l1}x(t-d(t))\right].\end{aligned} \tag{7}$$

To end this section, we give the following lemmas which will be used in deriving our main result.

Lemma 1 (see [14]). *For a matrix $T \in \mathbb{S}^n_+$, an integer $N \geq 0$, a matrix $W \in \mathbb{R}^{(N+1)n\times(N+1)n}$, two scalars p and q with $q > p$, and a vector function $x : [p, q] \longrightarrow \mathbb{R}^n$ such that the integrations below are well defined; then*

$$\begin{aligned} -\int_p^q \dot{x}^{\mathrm{T}}(s) T \dot{x}(s)\, ds \leq \xi_N^{\mathrm{T}} \Big[&\mathscr{M}_N^{\mathrm{T}} \mathscr{L}_N^{\mathrm{T}} W + W^{\mathrm{T}} \mathscr{L}_N \mathscr{M}_N \\ &+ (q-p) W^{\mathrm{T}} \mathscr{T}_N^{-1} W \Big] \xi_N, \end{aligned} \tag{8}$$

where

$$\mathscr{T}_N = \operatorname{diag}\{T, 3T, \ldots, (2N+1)T\}, \tag{9}$$

$$\mathscr{L}_N = \begin{bmatrix} I & 0 & \cdots & 0 \\ I & (-1)^1\binom{1}{1}\binom{1+1}{1} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ I & (-1)^1\binom{N}{1}\binom{N+1}{1} & \cdots & (-1)^N\binom{N}{N}\binom{N+N}{N} \end{bmatrix}, \tag{10}$$

$$\mathscr{M}_N = \begin{bmatrix} I & -I & 0 & 0 & \cdots & 0 \\ 0 & -I & I & 0 & \cdots & 0 \\ 0 & -I & 0 & 2I & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & -I & 0 & 0 & \cdots & NI \end{bmatrix}, \tag{11}$$

$$\begin{aligned} \xi_N &= col\{x(q), x(p), \iota_1, \ldots, \iota_N\}, \\ \iota_k &= \int_p^q \frac{(q-p)^{k-1}}{(q-p)^k} x(s)\,ds, \quad k = 1, 2, \ldots, N. \end{aligned} \tag{12}$$

Lemma 2 (see [12]). *For given Q_1 and $Q_2 \in \mathbb{S}_+^n$, if one can find matrices $U_1, U_2 \in \mathbb{R}^{n\times n}$ and $H_1, H_2 \in \mathbb{S}^n$ such that*

$$\begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} - \epsilon \begin{bmatrix} H_1 & U_1 \\ U_1^T & 0 \end{bmatrix} - (1-\epsilon)\begin{bmatrix} 0 & U_2 \\ U_2^T & H_2 \end{bmatrix} \geq 0, \tag{13}$$

satisfied for $\epsilon = 0$ and 1, then one has

$$\begin{aligned} \begin{bmatrix} \frac{1}{\epsilon}Q_1 & 0 \\ 0 & \frac{1}{1-\epsilon}Q_2 \end{bmatrix} &\geq \begin{bmatrix} Q_1 & 0 \\ 0 & Q_2 \end{bmatrix} + (1-\epsilon)\begin{bmatrix} H_1 & U_2 \\ U_2^T & 0 \end{bmatrix} \\ &\quad + \epsilon \begin{bmatrix} 0 & U_1 \\ U_1^T & H_2 \end{bmatrix}, \end{aligned} \tag{14}$$

which holds for any $\epsilon \in (0, 1)$.

3. Stability Conditions of T-S Fuzzy Time-Delay Systems

In this section, we establish some asymptotic stability conditions of system (6) with two cases of time delay: constant delay and time-varying delay satisfying (2).

3.1. Constant Delay Case. Suppose that $d(t) \equiv d_0 > 0$. Choose the following augmented LKF candidate:

$$\begin{aligned} \mathscr{V}_N^c(t, x_t) &= x_{0N}^T(t) R_{0N} x_{0N}(t) \\ &\quad + \int_{t-d_0}^t \omega^T(s) S_0 \omega(s)\,ds \\ &\quad + d_0 \int_{-d_0}^0 \int_{t+\theta}^t \dot{x}^T(s) T_0 \dot{x}(s)\,ds\,d\theta, \end{aligned} \tag{15}$$

where

$$\begin{aligned} &x_{0N}(t) \\ &\quad = col\{x(t), x(t-d_0), d_0\varsigma_1(t), \ldots, d_0\varsigma_N(t)\}, \\ &\varsigma_k(t) = \frac{1}{d_0}\int_{t-d_0}^t \left(\frac{t-s}{d_0}\right)^{k-1} x(s)\,ds, \quad k = 1, \ldots, N. \\ &\omega(t) = col\{\dot{x}(t), x(t)\}, \end{aligned} \tag{16}$$

and then we can derive the following stability condition.

Theorem 3. *For any given integer $N > 0$ and a positive constant d_0, if there exist $R_{0N} \in \mathbb{S}_+^{(N+2)n}$, $S_0 \in \mathbb{S}_+^{2n}$, $T_0 \in \mathbb{S}_+^n$ and W_{0N} and M_i $(i = 1, 2)$ of appropriate dimensions such that for $l, m = 1, 2, \ldots, s$*

$$\Xi_{Nlm} + \Xi_{Nml} < 0, \quad l \leq m \tag{17}$$

hold, then system (7) with constant delay is asymptotically stable, where

$$\begin{aligned} \Xi_{Nlm} &= \begin{bmatrix} \Psi_{Nlm} & W_{0N}^T \\ \star & -\mathscr{T}_{0N} \end{bmatrix} < 0, \\ \Psi_{Nlm} &= \operatorname{Sym}\{\psi_{1N}^T R_{0N} \psi_{2N}\} + \psi_3^T S_0 \psi_3 - \psi_4^T S_0 \psi_4 \\ &\quad + \Lambda_{0N}^T W_{0N} + W_{0N}^T \Lambda_{0N} + d_0^2 g_3^T T_0 g_3 \\ &\quad + \operatorname{Sym}\{(g_1^T M_1 + g_3^T M_2)\mathscr{E}_{lm}\}, \\ \mathscr{T}_{0N} &= \operatorname{diag}\{T_0, 3T_0, \ldots, (2N+1)T_0\}, \\ \mathscr{E}_{lm} &= (A_{l0} + B_l K_m) g_1 + A_{l1} g_2 - g_3, \\ \psi_{1N} &= col\{g_1, g_2, d_0 g_5, \ldots, d_0 g_{N+4}\}, \\ \psi_{2N} &= col\{g_3, g_4, g_1 - g_2, g_5 - g_2, 2g_6 \\ &\quad - g_2, \ldots, (N-1) g_{N+3} - g_2)\}, \\ \psi_3 &= col\{g_3, g_1\}, \\ \psi_4 &= col\{g_4, g_2\}, \\ \Lambda_{0N} &= \mathscr{L}_N \mathscr{M}_N col\{g_1, g_2, g_5, \ldots, g_{N+4}\}, \end{aligned} \tag{18}$$

with $\mathscr{L}_N$ and $\mathscr{M}_N$ being defined in Lemma 1, and $g_k = [0_{n\times(k-1)n} \;\; I_{n\times n} \;\; 0_{n\times(N+4-k)n}]$, $k = 1, 2, \ldots, N+4$.

Proof. The derivative of $\mathscr{V}_N^c(t, x_t)$ is given by

$$\begin{aligned} \dot{\mathscr{V}}_N^c(t, x_t) &= \dot{x}_{0N}^T(t) R_{0N} x_{0N}(t) + x_{0N}^T(t) R_{0N} \dot{x}_{0N}(t) \\ &\quad + \omega^T(t) S_0 \omega(t) \\ &\quad - \omega^T(t-d_0) S_0 \omega(t-d_0) \\ &\quad + d_0^2 \dot{x}(t)^T T_0 \dot{x}(t) \\ &\quad - d_0 \int_{t-d_0}^t \dot{x}^T(s) T_0 \dot{x}(s)\,ds. \end{aligned} \tag{19}$$

Denote

$$\eta(t) = col\{x(t), x(t-d_0), \dot{x}(t), \dot{x}(t-d_0), \varsigma_1(t), \ldots, \varsigma_N(t)\}. \tag{20}$$

Then

$$\begin{aligned} x_{0N} &= \psi_{1N}\eta(t), \\ \dot{x}_{0N} &= \psi_{2N}\eta(t), \\ \omega(t) &= \psi_3\eta(t), \\ \omega(t-d_0) &= \psi_4\eta(t). \end{aligned} \tag{21}$$

Apply Lemma 1 to deal with the integral term in (19), and we have

$$\begin{aligned} -d_0\int_{t-d_0}^{t}\dot{x}^T(s)T_0\dot{x}(s)\,ds &\le d_0\eta^T(t) \\ &\cdot\left[\Lambda_{0N}^T\widetilde{W}_{0N} + \widetilde{W}_{0N}^T\Lambda_{0N} + d_0\widetilde{W}_{0N}^T\mathcal{T}_{0N}^{-1}\widetilde{W}_{0N}\right]\eta(t). \end{aligned} \tag{22}$$

Let $W_{0N} = d_0\widetilde{W}_{0N}$. (22) can be rewritten as

$$\begin{aligned} -d_0\int_{t-d_0}^{t}\dot{x}^T(s)T_0\dot{x}(s)\,ds &\le \eta^T(t) \\ &\cdot\left[\Lambda_{0N}^T W_{0N} + W_{0N}^T\Lambda_{0N} + W_{0N}^T\mathcal{T}_{0N}^{-1}W_{0N}\right]\eta(t). \end{aligned} \tag{23}$$

For any matrices M_i $(i = 1, 2)$ with appropriate dimensions, the following equation holds for system (7)

$$\begin{aligned} &\sum_{l=1}^{s}\sum_{m=1}^{s} o_l o_m \eta^T(t)\left\{\left[g_1^T M_1 + g_3^T M_2\right]\right. \\ &\quad\left.\cdot\left[(A_{l0} + B_l K_m)g_1 + A_{l1}g_2 - g_3\right]\right\}\eta(t) = 0. \end{aligned} \tag{24}$$

Then combining (19) with (23) and (24) yields

$$\begin{aligned} \dot{\mathcal{V}}_N^c(t, x_t) &\le \sum_{l=1}^{s}\sum_{m=1}^{s} o_l(\delta(t))\,o_m(\delta(t))\,\eta^T(t) \\ &\cdot\left[\Psi_{Nlm} + W_{0N}^T\mathcal{T}_{0N}^{-1}W_{0N}\right]\eta(t) = \sum_{l=1}^{s}\sum_{m=1}^{s} o_l(\delta(t)) \\ &\cdot o_m(\delta(t))\,\eta^T(t)\,\overline{\Psi}_{Nlm}\eta(t). \end{aligned} \tag{25}$$

If LMI (17) is satisfied, by Schur complement, one has $\overline{\Psi}_{Nlm} < 0$, which implies that $\dot{\mathcal{V}}_N^c(t, x_t) < 0$. The proof of Theorem 3 is completed. □

3.2. Time-Varying Delay Case. For simplicity, we denote equations as follows:

$$\begin{aligned} \nu_k(t) &= \frac{1}{d(t)}\int_{t-d(t)}^{t}\left(\frac{t-s}{d(t)}\right)^{k-1} x(s)\,ds, \\ \upsilon_k(t) &= \frac{1}{d_0 - d(t)}\int_{t-d_0}^{t-d(t)}\left(\frac{t-d(t)-s}{d_0-d(t)}\right)^{k-1} x(s)\,ds, \end{aligned} \tag{26}$$

where $k = 1, 2, \ldots, N$.

In order to make full use of B-L inequality, we construct the following augmented LKF for system (3)

$$\mathcal{V}_N^v(t, x_t) = \sum_{i=1}^{4}\mathcal{V}_{Ni}^v(t, x_t), \tag{27}$$

with

$$\begin{aligned} \mathcal{V}_{N1}^v(t, x_t) &= x_{1N}^T(t)R_{1N}x_{1N}(t), \\ \mathcal{V}_{N2}^v(t, x_t) &= d(t)\,x_{2N}^T(t)R_{2N}x_{2N}(t) \\ &\quad + (d_0 - d(t))\,x_{3N}^T(t)R_{3N}x_{3N}(t), \\ \mathcal{V}_{N3}^v(t, x_t) &= \int_{t-d(t)}^{t}\omega^T(s)S_1\omega(s)\,ds \\ &\quad + \int_{t-d_0}^{t-d(t)}\omega^T(s)S_2\omega(s)\,ds, \\ \mathcal{V}_{N4}^v(t, x_t) &= d_0\int_{-d_0}^{0}\int_{t+\theta}^{t}\dot{x}^T(s)T_1\dot{x}(s)\,ds\,d\theta, \end{aligned} \tag{28}$$

where $R_{1N} \in \mathbb{S}^{(2N+3)n}$, $R_{2N}, R_{3N} \in \mathbb{S}_+^{(N+3)n}$, $S_1, S_2 \in \mathbb{S}_+^{2n}$, $T_1 \in \mathbb{S}_+^n$, and

$$\begin{aligned} x_{1N}(t) &= col\{x(t), x(t) - x(t-d(t)), d(t) \\ &\quad\cdot\nu_1(t), x(t-d(t)) - x(t-d_0), (d_0 - d(t)) \\ &\quad\cdot\upsilon_1(t), d(t)\nu_2(t), \ldots, d(t)\nu_N(t), (d_0 - d(t)) \\ &\quad\cdot\upsilon_2(t), \ldots, (d_0 - d(t))\upsilon_N(t)\}, \\ x_{2N}(t) &= col\{x(t), x(t-d(t)), x(t-d_0), \nu(t)\}, \\ x_{3N}(t) &= col\{x(t), x(t-d(t)), x(t-d_0), \upsilon(t)\}, \\ \nu(t) &= col\{\nu_1(t), \nu_2(t), \ldots, \nu_N(t)\}, \\ \upsilon(t) &= col\{\upsilon_1(t), \upsilon_2(t), \ldots, \upsilon_N(t)\}, \end{aligned} \tag{29}$$

and $\omega(t)$ is defined in (16).

Firstly, we deal with the positive definiteness of $\mathcal{V}_N^v(t, x_t)$. Proposition 5 is proposed through which the positive definiteness of the LKF can be proved.

Remark 4. The augmented term $\mathcal{V}_{N1}^v(t, x_t)$ of the proposed LKF is constructed based on the form of B-L inequality, which contributes to reducing the conservatism of the conditions. As proved in [21], (i) the augmented LKF approach is useful for conservatism-reducing, (ii) for the same nonaugmented LKF, the use of Wirtinger-based inequality and Jensen-based inequality does not make a difference in conservatism-reducing, and (iii) for an augmented LKF, the use of Wirtinger-based inequality is better than Jensen-based inequality to obtain less conservative results. As these inequalities are the special cases of B-L inequality, the proposed augmented LKF is significant and effective in deriving the main results. For time-varying delay case, delay-product-type terms of LKF are introduced in [21]. Clearly, these terms in [21] are special cases of the terms in $\mathcal{V}_{N2}^v(t, x_t)$, which are proposed based on the form of B-L inequality.

Proposition 5. *For a given scalar $d_0 > 0$ and any given integer $N > 0$, if there exist $R_{1N} \in \mathbb{S}^{(2N+3)n}$, $R_{2N}, R_{3N} \in \mathbb{S}_+^{(N+3)n}$, $S_1, S_2 \in \mathbb{S}_+^{2n}$, $T_1 \in \mathbb{S}_+^{n}$ and $G \in \mathbb{R}^{2n\times 2n}$ such that*

$$\Theta_N^0 = R_{1N} + \frac{1}{d_0}\begin{bmatrix} 0_{n\times n} & 0_{n\times 4n} & 0_{n\times(2N-2)n} \\ \star & \Theta_0 & 0_{4n\times(2N-2)n} \\ \star & \star & 0_{(2N-2)n\times(2N-2)n} \end{bmatrix} > 0, \quad (30)$$

$$\Theta_0 = \begin{bmatrix} S_1 & G \\ G^{\mathrm{T}} & S_2 \end{bmatrix} \geq 0. \quad (31)$$

are satisfied, and then there exist $\kappa_1 > 0$ and $\kappa_2 > 0$ such that $\mathscr{V}_N^v(t, x_t)$ satisfies

$$\kappa_1 \|x(t)\|^2 \leq \mathscr{V}_N^v(t, x_t) \leq \kappa_2 \|x_t(\theta)\|_{\mathbb{W}}^2, \quad (32)$$

where $x_t(\theta) = x(t+\theta)$ and $\dot{x}_t(\theta) = \dot{x}(t+\theta)$ with $\theta \in [-d_0, 0]$, and $\mathbb{W}$ represents the space of functions $x_t(\theta)$ and $\dot{x}_t(\theta)$ with $\|x_t(\theta)\|_{\mathbb{W}}^2 = \max_{\theta\in[-d_0,0]}\|x_t(\theta)\|^2 + \int_{-d_0}^{0}\|x_t(\theta)\|^2 d\theta$.

Proof. By using Jensen's inequality to $\mathscr{V}_{N3}^v(t, x_t)$, one has

$$\int_{t-d(t)}^{t} \omega^T(s) S_1 \omega(s)\, ds \geq \frac{1}{d(t)}\eta_1^T(t) S_1 \eta_1(t), \quad (33)$$

$$\int_{t-d_0}^{t-d(t)} \omega^T(s) S_2 \omega(s)\, ds \geq \frac{1}{d_0 - d(t)}\eta_2^T(t) S_2 \eta_2(t), \quad (34)$$

where

$$\eta_1(t) = \begin{bmatrix} x(t) - x(t-d(t)) \\ \int_{t-d(t)}^{t} x(s)\, ds \end{bmatrix}, \qquad \eta_2(t) = \begin{bmatrix} x(t-d(t)) - x(t-d_0) \\ \int_{t-d_0}^{t-d(t)} x(s)\, ds \end{bmatrix}. \quad (35)$$

Then substituting (33) and (34) into (27), we obtain

$$\mathscr{V}_N^v(t, x_t) \geq x_{1N}^T(t)\left(R_{1N} + \overline{S}_N\right) x_{1N}(t) + \mathscr{V}_{N2}^v(t, x_t) + \mathscr{V}_{N4}^v(t, x_t), \quad (36)$$

where

$$\overline{S}_N = \begin{bmatrix} 0_{n\times n} & 0_{n\times 2n} & 0_{n\times 2n} & 0_{n\times(2N-2)n} \\ \star & \frac{1}{d(t)}S_1 & 0_{2n\times 2n} & 0_{2n\times(2N-2)n} \\ \star & \star & \frac{1}{d_0 - d(t)}S_2 & 0_{2n\times(2N-2)n} \\ \star & \star & \star & 0_{(2N-2)n\times(2N-2)n} \end{bmatrix}. \quad (37)$$

If there exists a matrix $G \in \mathbb{R}^{2n\times 2n}$ such that (31) is satisfied, similar to [12], using Lemma 2 with $H_1 = H_2 = 0$ and $U_1 = U_2 = G$ to $\begin{bmatrix} (1/d(t))S_1 & 0_{2n\times 2n} \\ \star & (1/(d_0 - d(t)))S_2 \end{bmatrix}$, together with $R_{2N} > 0$, $R_{3N} > 0$ and $T_1 > 0$, then one has $\mathscr{V}_N^v(t, x_t) \geq x_{1N}^T(t)\Theta_N^0 x_{1N}(t)$. If (30) and $R_{2N} > 0$, $R_{3N} > 0$ and $T_1 > 0$ are satisfied, and there exists $\kappa_1 > 0$ such that $\mathscr{V}_N^v(t, x_t) \geq \kappa_1\|x(t)\|^2$, which verifies the first inequality of (32). Similar to [12], the proof of the second inequality in (32) can be obtained, and the specific steps are omitted for brevity. This ends the proof. □

Remark 6. Some extended reciprocally convex inequalities compared to Lemma 2 are proposed in [22, 23], and some improved results with less complexity [22] or less conservatism [23] are provided. It would be interesting to incorporate these inequalities with the proposed augmented LKF method to achieve some potential results in future work. The computing complexity of conditions and stochastic feature of systems [24] also could be studied.

Secondly, the negative definiteness of $\dot{\mathscr{V}}_N^v(t, x_t)$ is shown by the following proposition.

Proposition 7. *For given scalars $d_0 > 0$, d_m and d_M and any given integer $N > 0$, there exists $\kappa_3 > 0$ such that the functional $\mathscr{V}_N^v(t, x_t)$ satisfies $\dot{\mathscr{V}}_N^v(t, x_t) \leq -\kappa_3\|x(t)\|^2$ if there exist $R_{1N} \in \mathbb{S}^{(2N+3)n}$, $R_{2N}, R_{3N} \in \mathbb{S}_+^{(N+3)n}$, $S_1, S_2 \in \mathbb{S}_+^{2n}$, $T_1 \in \mathbb{S}_+^{n}$, and W_{1N}, W_{2N} and $\widetilde{M}_i$ $(i = 1, 2, 3, 4)$ of appropriate dimensions such that for $d_m \leq \mu \leq d_M$, the following LMIs hold for $l, m = 1, 2, \ldots, r$:*

$$\widetilde{\Xi}_{Nlm}\left(d(t), \dot{d}(t)\big|_{d(t)=0, \dot{d}(t)=\mu}\right) + \widetilde{\Xi}_{Nml}\left(d(t), \dot{d}(t)\big|_{d(t)=0, \dot{d}(t)=\mu}\right) < 0, \quad l \leq m \quad (38)$$

$$\widetilde{\Xi}_{Nlm}\left(d(t), \dot{d}(t)\big|_{d(t)=d_0, \dot{d}(t)=\mu}\right) + \widetilde{\Xi}_{Nml}\left(d(t), \dot{d}(t)\big|_{d(t)=d_0, \dot{d}(t)=\mu}\right) < 0, \quad l \leq m \quad (39)$$

where

$$\widetilde{\Xi}_{Nlm}\left(d(t), \dot{d}(t)\big|_{d(t)=0}\right) = \begin{bmatrix} \widetilde{\Psi}_{Nlm}\left(0, \dot{d}(t)\right) & W_{2N}^{\mathrm{T}} \\ \star & -\mathscr{T}_{1N} \end{bmatrix} < 0,$$

$$
\begin{aligned}
&\widetilde{\Xi}_{Nlm}\left(d(t), \dot{d}(t)\big|_{d(t)=d_0}\right) = \begin{bmatrix} \widetilde{\Psi}_{Nlm}\left(d_0, \dot{d}(t)\right) & W_{1N}^{\mathrm{T}} \\ \star & -\mathscr{T}_{1N} \end{bmatrix} < 0, \\
&\widetilde{\Psi}_{Nlm}\left(d(t), \dot{d}(t)\right) = \Phi_{1N} + \Phi_{2N} + \Phi_3 + \Phi_{4N} + \Phi_{5lm}, \\
&\mathscr{T}_{1N} = \operatorname{diag}\left\{T_1, 3T_1, \ldots, (2N+1)T_1\right\}, \\
&\Phi_{1N} = \operatorname{Sym}\left\{\varphi_{1N}^{\mathrm{T}} R_{1N} \varphi_{2N}\right\}, \\
&\Phi_{2N} = \dot{d}(t)\left(\varphi_{3N}^{\mathrm{T}} R_{2N} \varphi_{3N} - \varphi_{4N}^{\mathrm{T}} R_{3N} \varphi_{4N}\right) + \operatorname{Sym}\left\{\varphi_{3N}^{\mathrm{T}} R_{2N} \varphi_{5N} + \varphi_{4N}^{\mathrm{T}} R_{3N} \varphi_{6N}\right\} + d_0^2 \widetilde{g}_6^{\mathrm{T}} T_1 \widetilde{g}_6, \\
&\Phi_3 = \varphi_7^{\mathrm{T}} S_1 \varphi_7 - \left(1 - \dot{d}(t)\right) \varphi_8^{\mathrm{T}} \left(S_1 - S_2\right) \varphi_8 - \varphi_9^{\mathrm{T}} S_2 \varphi_9, \\
&\Phi_{4N} = \operatorname{Sym}\left\{\Lambda_{1N}^{\mathrm{T}} W_{1N} + \Lambda_{2N}^{\mathrm{T}} W_{2N}\right\}, \\
&\Phi_{5lm} = \operatorname{Sym}\left\{\left[\widetilde{g}_1^{\mathrm{T}} \widetilde{M}_1 + \widetilde{g}_2^{\mathrm{T}} \widetilde{M}_2 + \widetilde{g}_3^{\mathrm{T}} \widetilde{M}_3 + \widetilde{g}_4^{\mathrm{T}} \widetilde{M}_4\right] \mathscr{F}_{lm}\right\}, \\
&\mathscr{F}_{lm} = \left(A_{l0} + B_l K_m\right) \widetilde{g}_1 + A_{l1} \widetilde{g}_2 - \widetilde{g}_6, \\
&\varphi_{1N} \\
&\quad = col\left\{\widetilde{g}_1, \widetilde{g}_1 - \widetilde{g}_2, d(t)\widetilde{g}_7, \widetilde{g}_2 - \widetilde{g}_3, \left(d_0 - d(t)\right)\widetilde{g}_{N+7}, d(t)\widetilde{g}_8, \ldots, d(t)\widetilde{g}_{N+6}, \left(d_0 - d(t)\right)\widetilde{g}_{N+8}, \ldots, \left(d_0 - d(t)\right)\widetilde{g}_{2N+6}\right\}, \\
&\varphi_{2N} = col\left\{\widetilde{g}_6, \widetilde{g}_6 - \left(1 - \dot{d}(t)\right)\widetilde{g}_4, \ell_1, \left(1 - \dot{d}(t)\right)\widetilde{g}_4 - \widetilde{g}_5, \vartheta_1, \ell_2, \ldots, \ell_N, \vartheta_2, \ldots, \vartheta_N\right\}, \\
&\varphi_{3N} = col\left\{\widetilde{g}_1, \widetilde{g}_2, \widetilde{g}_3, \widetilde{g}_7, \ldots, \widetilde{g}_{N+6}\right\}, \\
&\varphi_{4N} = col\left\{\widetilde{g}_1, \widetilde{g}_2, \widetilde{g}_3, \widetilde{g}_{N+7}, \ldots, \widetilde{g}_{2N+6}\right\}, \\
&\varphi_{5N} = col\left\{d(t)\widetilde{g}_6, d(t)\left(1 - \dot{d}(t)\right)\widetilde{g}_4, d(t)\widetilde{g}_5, \lambda_1, \ldots, \lambda_N\right\}, \\
&\varphi_{6N} = col\left\{\left(d_0 - d(t)\right)\widetilde{g}_6, \left(d_0 - d(t)\right)\left(1 - \dot{d}(t)\right)\widetilde{g}_4, \left(d_0 - d(t)\right)\widetilde{g}_5, \gamma_1, \ldots, \gamma_N\right\}, \\
&\varphi_7 = col\left\{\widetilde{g}_6, \widetilde{g}_1\right\}, \\
&\varphi_8 = col\left\{\widetilde{g}_4, \widetilde{g}_2\right\}, \\
&\varphi_9 = col\left\{\widetilde{g}_5, \widetilde{g}_3\right\}, \\
&\Lambda_{1N} = \mathscr{L}_N \mathscr{M}_N col\left\{\widetilde{g}_1, \widetilde{g}_2, \widetilde{g}_7, \ldots, \widetilde{g}_{N+6}\right\}, \\
&\Lambda_{2N} = \mathscr{L}_N \mathscr{M}_N col\left\{\widetilde{g}_2, \widetilde{g}_3, \widetilde{g}_{N+7}, \ldots, \widetilde{g}_{2N+6}\right\}, \\
&\ell_k = \begin{cases} -\left(1 - \dot{d}(t)\right)\widetilde{g}_2 + \widetilde{g}_1, & k = 1 \\ -\left(1 - \dot{d}(t)\right)\widetilde{g}_2 + (k-1)\left(\widetilde{g}_{k+5} - \dot{d}(t)\widetilde{g}_{k+6}\right), & k = 2, 3, \ldots, N \end{cases} \\
&\vartheta_k = \begin{cases} -\widetilde{g}_3 + \left(1 - \dot{d}(t)\right)\widetilde{g}_2, & k = 1 \\ -\widetilde{g}_3 + (k-1)\left(1 - \dot{d}(t)\right)\widetilde{g}_{N+k+5} + (k-1)\dot{d}(t)\widetilde{g}_{N+k+6}, & k = 2, 3, \ldots, N \end{cases} \\
&\lambda_k = \begin{cases} -\left(1 - \dot{d}(t)\right)\widetilde{g}_2 + \widetilde{g}_1 - \dot{d}(t)\widetilde{g}_7, & k = 1 \\ -\left(1 - \dot{d}(t)\right)\widetilde{g}_2 + (k-1)\widetilde{g}_{k+5} - k\dot{d}(t)\widetilde{g}_{k+6}, & k = 2, 3, \ldots, N \end{cases} \\
&\gamma_k = \begin{cases} -\widetilde{g}_3 + \left(1 - \dot{d}(t)\right)\widetilde{g}_2 + \dot{d}(t)\widetilde{g}_{N+7}, & k = 1 \\ -\widetilde{g}_3 + \left(1 - \dot{d}(t)\right)(k-1)\widetilde{g}_{N+k+5} + k\dot{d}(t)\widetilde{g}_{N+k+6}, & k = 2, 3, \ldots, N \end{cases} \\
&\widetilde{g}_k = \left[0_{n\times(k-1)n} \quad I_{n\times n} \quad 0_{n\times(2N+6-k)n}\right], \quad k = 1, 2, \ldots, 2N+6
\end{aligned}
\tag{40}
$$

and $\mathscr{L}_N$, $\mathscr{M}_N$ are defined in (10) and (11), respectively.

Proof. For $\mathscr{V}_N^v(t,x_t)$ in (27), we compute its derivative and obtain

$$\dot{\mathscr{V}}_N^v(t,x_t)=\sum_{i=1}^{4}\dot{\mathscr{V}}_{Ni}^v(t,x_t), \tag{41}$$

where

$$\begin{aligned}
\dot{\mathscr{V}}_{N1}^v(t,x_t)&=\operatorname{Sym}\left\{x_{1N}^T(t)R_{1N}\dot{x}_{1N}(t)\right\},\\
\dot{\mathscr{V}}_{N2}^v(t,x_t)&=\dot{d}(t)x_{2N}^T(t)R_{2N}x_{2N}(t)-\dot{d}(t)x_{3N}^T(t)\\
&\quad\cdot R_{3N}x_{3N}(t)\\
&\quad+\operatorname{Sym}\left\{d(t)x_{2N}^T(t)R_{2N}\dot{x}_{2N}(t)\right.\\
&\quad\left.+(d_0-d(t))x_{3N}^T(t)R_{3N}\dot{x}_{3N}(t)\right\},\\
\dot{\mathscr{V}}_{N3}^v(t,x_t)&=\omega^T(t)S_1\omega(t)-\left(1-\dot{d}(t)\right)\omega^T(t\\
&\quad-d(t))(S_1-S_2)\omega(t-d(t))-\omega^T(t-d_0)S_2\omega(t\\
&\quad-d_0),\\
\dot{\mathscr{V}}_{N4}^v(t,x_t)&=d_0^2\dot{x}^T(t)T_1\dot{x}(t)\\
&\quad-d_0\int_{t-d_0}^{t}\dot{x}^T(s)T_1\dot{x}(s)\,ds.
\end{aligned} \tag{42}$$

Denote

$$\begin{aligned}
\zeta(t)&=col\left\{x(t),x(t-d(t)),x(t-d_0),\zeta_0(t)\right\},\\
\zeta_0(t)&\\
&=col\left\{\dot{x}(t-d(t)),\dot{x}(t-d_0),\dot{x}(t),\nu(t),\upsilon(t)\right\}.
\end{aligned} \tag{43}$$

We can rewrite $\dot{\mathscr{V}}_N^v(t,x_t)$ in (41) as follows:

$$\begin{aligned}
&\dot{\mathscr{V}}_N^v(t,x_t)\\
&\quad=\sum_{l=1}^{s}o_l(\delta(t))\zeta^T(t)(\Phi_{1N}+\Phi_{2N}+\Phi_3)\zeta(t)\\
&\qquad-d_0\int_{t-d(t)}^{t}\dot{x}^T(s)T_1\dot{x}(s)\,ds\\
&\qquad-d_0\int_{t-d_0}^{t-d(t)}\dot{x}^T(s)T_1\dot{x}(s)\,ds.
\end{aligned} \tag{44}$$

Using Lemma 1 to estimate the upper bounds of the above integral terms, respectively, we get

$$\begin{aligned}
&-d_0\int_{t-d(t)}^{t}\dot{x}^T(s)T_1\dot{x}(s)\,ds\le d_0\zeta^T(t)\left[\Lambda_{1N}^T\widetilde{W}_{1N}\right.\\
&\quad\left.+\widetilde{W}_{1N}^T\Lambda_{1N}+d(t)\widetilde{W}_{1N}^T\mathscr{T}_{1N}^{-1}\widetilde{W}_{1N}\right]\zeta(t),
\end{aligned} \tag{45}$$

$$\begin{aligned}
&-d_0\int_{t-d_0}^{t-d(t)}\dot{x}^T(s)T_1\dot{x}(s)\,ds\le d_0\zeta^T(t)\left[\Lambda_{2N}^T\widetilde{W}_{2N}\right.\\
&\quad\left.+\widetilde{W}_{2N}^T\Lambda_{2N}+(d_0-d(t))\widetilde{W}_{2N}^T\mathscr{T}_{1N}^{-1}\widetilde{W}_{2N}\right]\zeta(t).
\end{aligned} \tag{46}$$

Setting $W_{1N}=d_0\widetilde{W}_{1N}$ and $W_{2N}=d_0\widetilde{W}_{2N}$. Then, according to (45) and (46), we yield

$$\begin{aligned}
\mathscr{Z}(t)&\le\zeta^T(t)\left[\operatorname{Sym}\left\{\Lambda_{1N}^TW_{1N}+\Lambda_{2N}^TW_{2N}\right\}\right.\\
&\quad\left.+\sigma W_{1N}^T\mathscr{T}_{1N}^{-1}W_{1N}+(1-\sigma)W_{2N}^T\mathscr{T}_{1N}^{-1}W_{2N}\right]\zeta(t),
\end{aligned} \tag{47}$$

where $\sigma=d(t)/d_0$ and

$$\begin{aligned}
\mathscr{Z}(t)&=-d_0\int_{t-d(t)}^{t}\dot{x}^T(s)T_1\dot{x}(s)\,ds\\
&\quad-d_0\int_{t-d_0}^{t-d(t)}\dot{x}^T(s)T_1\dot{x}(s)\,ds.
\end{aligned} \tag{48}$$

Similar to the proof of Theorem 3, we introduce the following equation:

$$\begin{aligned}
&\sum_{l=1}^{s}\sum_{m=1}^{s}o_lo_m\zeta^T(t)\left\{\left[\widetilde{g}_1^T\widetilde{M}_1+\widetilde{g}_2^T\widetilde{M}_2+\widetilde{g}_3^T\widetilde{M}_3+\widetilde{g}_6^T\widetilde{M}_4\right]\right.\\
&\quad\left.\cdot\left[(A_{l0}+B_lK_m)\widetilde{g}_1+A_{l1}\widetilde{g}_2-\widetilde{g}_6\right]\right\}\zeta(t)=0.
\end{aligned} \tag{49}$$

Thus, according to (44), (45), (46), (47), (48), and (49) we have

$$\begin{aligned}
\dot{\mathscr{V}}_N^v(t,x_t)&<\sum_{l=1}^{s}\sum_{m=1}^{s}o_lo_m\zeta^T(t)\left\{\widetilde{\Psi}_{Nlm}\left(d(t),\dot{d}(t)\right)\right.\\
&\quad\left.+\sigma W_{1N}^T\mathscr{T}_{1N}^{-1}W_{1N}+(1-\sigma)W_{2N}^T\mathscr{T}_{1N}^{-1}W_{2N}\right\}\zeta(t)\\
&=\sum_{l=1}^{s}\sum_{m=1}^{s}o_lo_m\zeta^T(t)\widehat{\Psi}_{Nlm}\left(d(t),\dot{d}(t)\right)\zeta(t).
\end{aligned} \tag{50}$$

It can be clearly seen that the matrix $\widehat{\Psi}_{Nlm}(d(t),\dot{d}(t))$ is a convex combination in $d(t)$ and $\dot{d}(t)$. Similar to the two-dimensional convex combination method in [16], if LMIs (38) and (39) are satisfied, one can easily derive $\widehat{\Psi}_{Nlm}(d(t),\dot{d}(t))<0$ for $(d(t),\dot{d}(t))\in\mathscr{D}_1$. The proof of Proposition 7 is completed. □

Now, we establish the stability conditions of the T-S fuzzy system (7) based on Propositions 5 and 7 as follows.

Theorem 8. *For given scalars $d_0>0$, d_m and d_M and any given integer $N>0$, if there exist $R_{1N}\in\mathbb{S}^{(2N+3)n}$, $R_{2N},R_{3N}\in\mathbb{S}_+^{(N+3)n}$, $S_1,S_2\in\mathbb{S}_+^{2n}$, $T_1\in\mathbb{S}_+^n$, $G\in\mathbb{R}^{2n\times2n}$ and $\widetilde{M}_i$ ($i=1,2,3,4$), W_{1N} and W_{2N} of appropriate dimensions such that the LMIs (30), (31) and the LMIs as follows*

$$\begin{aligned}
&\widetilde{\Xi}_{Nlm}\left(d(t),\dot{d}(t)\big|_{d(t)=0,\dot{d}(t)=d_m}\right)\\
&\quad+\widetilde{\Xi}_{Nml}\left(d(t),\dot{d}(t)\big|_{d(t)=0,\dot{d}(t)=d_m}\right)<0,\quad l\le m
\end{aligned}$$

$$\begin{aligned}
&\widetilde{\Xi}_{Nlm}\left(d(t),\dot{d}(t)\big|_{d(t)=0,\dot{d}(t)=d_M}\right)\\
&\quad+\widetilde{\Xi}_{Nml}\left(d(t),\dot{d}(t)\big|_{d(t)=0,\dot{d}(t)=d_M}\right)<0,\quad l\le m\\
&\widetilde{\Xi}_{Nlm}\left(d(t),\dot{d}(t)\big|_{d(t)=d_0,\dot{d}(t)=d_m}\right)\\
&\quad+\widetilde{\Xi}_{Nml}\left(d(t),\dot{d}(t)\big|_{d(t)=d_0,\dot{d}(t)=d_m}\right)<0,\quad l\le m\\
&\widetilde{\Xi}_{Nlm}\left(d(t),\dot{d}(t)\big|_{d(t)=d_0,\dot{d}(t)=d_M}\right)\\
&\quad+\widetilde{\Xi}_{Nml}\left(d(t),\dot{d}(t)\big|_{d(t)=d_0,\dot{d}(t)=d_M}\right)<0.\quad l\le m
\end{aligned}\tag{51}$$

are satisfied for $m,l=1,2,\ldots,s$, then system (7) with time-varying delay $d(t)$ subject to (2) is asymptotically stable.

For the case of time-varying delay, [12, 14] propose a refined delay set $\mathscr{D}_2$ which has been proven to derive less conservative results than delay set $\mathscr{D}_1$. Taking the refined delay set $\mathscr{D}_2 = Co\{(0,0),(0,d_M),(d_0,d_m),(d_0,0)\}$ into account and using Theorem 8, we obtain a stability condition as follows.

Theorem 9. *For given scalars $d_0>0$, d_m and d_M and any given integer $N>0$, if there exist $R_{1N}\in\mathbb{S}^{(2N+3)n}$, $R_{2N},R_{3N}\in\mathbb{S}_+^{(N+3)n}$, $S_1,S_2\in\mathbb{S}_+^{2n}$, $T_1\in\mathbb{S}_+^{n}$, $G\in\mathbb{R}^{2n\times 2n}$ and $\widetilde{M}_i$ $(i=1,2,3,4)$, W_{1N} and W_{2N} of appropriate dimensions such that (30), (31) are satisfied and the following LMIs hold for $l,m=1,2,\ldots,s$, then system (7) is asymptotically stable for $(d(t),\dot{d}(t))\in\mathscr{D}_2$*

$$\begin{aligned}
&\widetilde{\Xi}_{Nlm}\left(d(t),\dot{d}(t)\big|_{d(t)=0,\dot{d}(t)=0}\right)\\
&\quad+\widetilde{\Xi}_{Nml}\left(d(t),\dot{d}(t)\big|_{d(t)=0,\dot{d}(t)=0}\right)<0,\quad l\le m\\
&\widetilde{\Xi}_{Nlm}\left(d(t),\dot{d}(t)\big|_{d(t)=0,\dot{d}(t)=d_M}\right)\\
&\quad+\widetilde{\Xi}_{Nml}\left(d(t),\dot{d}(t)\big|_{d(t)=0,\dot{d}(t)=d_M}\right)<0,\quad l\le m\\
&\widetilde{\Xi}_{Nlm}\left(d(t),\dot{d}(t)\big|_{d(t)=d_0,\dot{d}(t)=d_m}\right)\\
&\quad+\widetilde{\Xi}_{Nml}\left(d(t),\dot{d}(t)\big|_{d(t)=d_0,\dot{d}(t)=d_m}\right)<0,\quad l\le m\\
&\widetilde{\Xi}_{Nlm}\left(d(t),\dot{d}(t)\big|_{d(t)=d_0,\dot{d}(t)=0}\right)\\
&\quad+\widetilde{\Xi}_{Nml}\left(d(t),\dot{d}(t)\big|_{d(t)=d_0,\dot{d}(t)=0}\right)<0.\quad l\le m
\end{aligned}\tag{52}$$

4. Fuzzy Controller Design

In this section, controller design conditions will be given by two cases of the time delay.

4.1. Constant Delay Case. Based on Theorem 3, the fuzzy control gains of system (7) with constant delay can be derived from the following theorem.

Theorem 10. *For given scalars $d_0>0$, ϱ, and a given integer $N>0$, if there exist $\overline{R}_{0N}\in\mathbb{S}_+^{(N+2)n}$, $\overline{S}_0\in\mathbb{S}_+^{2n}$, $\overline{T}_0\in\mathbb{S}_+^{n}$ and $\overline{H}_m$ $(m=1,2,\ldots,s)$, $\overline{W}_{0N}$ and $\overline{X}$ of appropriate dimensions such that the following LMIs hold for $l,m=1,2,\ldots,s$, then the closed-loop system (7) with constant delay $d(t)\equiv d_0$ is asymptotically stable*

$$\overline{\Xi}_{Nlm}+\overline{\Xi}_{Nml}<0,\quad l\le m\tag{53}$$

where

$$\begin{aligned}
\overline{\Xi}_{Nlm}&=\begin{bmatrix}\overline{\Psi}_{Nlm} & \overline{W}_{0N}^{\mathrm{T}}\\ \star & -\overline{\mathscr{T}}_{0N}\end{bmatrix}<0,\\
\overline{\mathscr{T}}_{0N}&=\operatorname{diag}\left\{\overline{T}_0,3\overline{T}_0,\ldots,(2N+1)\overline{T}_0\right\},\\
\overline{\Psi}_{Nlm}&=\operatorname{Sym}\left\{\psi_{1N}^{\mathrm{T}}\overline{R}_{0N}\psi_{2N}\right\}+\psi_3^{\mathrm{T}}\overline{S}_0\psi_3-\psi_4^{\mathrm{T}}\overline{S}_0\psi_4\\
&\quad+\Lambda_{0N}^{\mathrm{T}}\overline{W}_{0N}+\overline{W}_{0N}^{\mathrm{T}}\Lambda_{0N}+d_0^2g_3^{\mathrm{T}}\overline{T}_0g_3\\
&\quad+\operatorname{Sym}\left\{\left(\varrho g_1^{\mathrm{T}}+g_3^{\mathrm{T}}\right)\overline{\mathscr{E}}_{lm}\right\},\\
\overline{\mathscr{E}}_{lm}&=\left(A_{l0}\overline{X}^{\mathrm{T}}+B_l\overline{H}_m\right)g_1+A_{l1}\overline{X}^{\mathrm{T}}g_2-\overline{X}^{\mathrm{T}}g_3,
\end{aligned}\tag{54}$$

with ψ_{1N}, ψ_{2N}, ψ_3, ψ_4, and Λ_{0N} being defined in Theorem 3. Then, the fuzzy control gains can be calculated by $K_m=\overline{H}_m\overline{X}^{-\mathrm{T}}$ $(m=1,2,\ldots,s)$.

Proof. Pre- and postmultiplying both sides of (17) with $\operatorname{diag}\{\overline{X},\overline{X},\ldots,\overline{X}\}_{(N+4)n}$ and its transpose, introducing $\overline{X}=M_2^{-1}$, $M_1=\varrho M_2$, and defining $\overline{X}_1R_{0N}\overline{X}_1^T=\overline{R}_{0N}$, $\overline{X}_2S_0\overline{X}_2^T=\overline{S}_0$, $\overline{X}_3\mathscr{T}_{0N}\overline{X}_3^T=\overline{\mathscr{T}}_{0N}$, $\overline{X}T_0\overline{X}^T=\overline{T}_0$, $\overline{X}_1=\operatorname{diag}\{\overline{X},\ldots,\overline{X}\}_{(N+2)n}$, $\overline{X}_2=\operatorname{diag}\{\overline{X},\overline{X}\}$ and $\overline{X}_3=\operatorname{diag}\{\overline{X},\ldots,\overline{X}\}_{(N+1)n}$, we obtain the LMI (53). This completes the proof. □

4.2. Time-Varying Delay Case. For the case of $(d(t),\dot{d}(t))\in\mathscr{D}_2$, the next theorem is given to obtain the fuzzy control gains.

Theorem 11. *For a given integer $N>0$, given parameters $d_0>0$, d_m and d_M, and given scalars ρ_i $(i=1,2,3)$, if there exist $\widehat{R}_{1N}\in\mathbb{S}^{(2N+3)n}$, $\widehat{R}_{2N},\widehat{R}_{3N}\in\mathbb{S}_+^{(N+3)n}$, $\widehat{S}_1,\widehat{S}_2\in\mathbb{S}_+^{2n}$, $\widehat{T}_1\in\mathbb{S}_+^{n}$, $\widehat{G}\in\mathbb{R}^{2n\times 2n}$ and $\widehat{H}_m$ $(m=1,2,\ldots,s)$, $\widehat{X}$, $\widehat{W}_{1N}$ and $\widehat{W}_{2N}$ of appropriate dimensions such that the following LMIs hold for $l,m=1,2,\ldots,s$, then the closed-loop system (7) with time-varying delay $(d(t),\dot{d}(t))\in\mathscr{D}_2$ is asymptotically stable*

$$\begin{aligned}
&\widehat{\Xi}_{Nlm}\left(d(t),\dot{d}(t)\big|_{d(t)=0,\dot{d}(t)=0}\right)\\
&\quad+\widehat{\Xi}_{Nml}\left(d(t),\dot{d}(t)\big|_{d(t)=0,\dot{d}(t)=0}\right)<0,\quad l\le m
\end{aligned}$$

$$\begin{aligned}
&\widehat{\Xi}_{Nlm}\left(d(t),\dot{d}(t)\big|_{d(t)=0,\dot{d}(t)=d_M}\right)\\
&\quad+\widehat{\Xi}_{Nml}\left(d(t),\dot{d}(t)\big|_{d(t)=0,\dot{d}(t)=d_M}\right)<0,\quad l\le m\\
&\widehat{\Xi}_{Nlm}\left(d(t),\dot{d}(t)\big|_{d(t)=d_0,\dot{d}(t)=d_m}\right)\\
&\quad+\widehat{\Xi}_{Nml}\left(d(t),\dot{d}(t)\big|_{d(t)=d_0,\dot{d}(t)=d_m}\right)<0,\quad l\le m\\
&\widehat{\Xi}_{Nlm}\left(d(t),\dot{d}(t)\big|_{d(t)=d_0,\dot{d}(t)=0}\right)\\
&\quad+\widehat{\Xi}_{Nml}\left(d(t),\dot{d}(t)\big|_{d(t)=d_0,\dot{d}(t)=0}\right)<0,\quad l\le m
\end{aligned}\tag{55}$$

where

$$\begin{aligned}
&\widehat{\Xi}_{Nlm}\left(d(t),\dot{d}(t)\big|_{d(t)=0}\right)=\begin{bmatrix}\widehat{\Psi}_{Nlm}\left(0,\dot{d}(t)\right) & \widehat{W}_{2N}^{\mathrm{T}}\\ \star & -\widehat{\mathscr{T}}_{1N}\end{bmatrix}\\
&\quad<0,\\
&\widehat{\Xi}_{Nlm}\left(d(t),\dot{d}(t)\big|_{d(t)=d_0}\right)\\
&\quad=\begin{bmatrix}\widehat{\Psi}_{Nlm}\left(d_0,\dot{d}(t)\right) & \widehat{W}_{1N}^{\mathrm{T}}\\ \star & -\widehat{\mathscr{T}}_{1N}\end{bmatrix}<0,\\
&\widehat{\Psi}_{Nlm}\left(d(t),\dot{d}(t)\right)\\
&\quad=\widehat{\Phi}_{1N}+\widehat{\Phi}_{2N}+\widehat{\Phi}_{3}+\widehat{\Phi}_{4N}+\widehat{\Phi}_{5lm},\\
&\widehat{\mathscr{T}}_{1N}=\operatorname{diag}\left\{\widehat{T}_1,3\widehat{T}_1,\ldots,(2N+1)\widehat{T}_1\right\},\\
&\widehat{\Phi}_{1N}=\operatorname{Sym}\left\{\varphi_{1N}^{\mathrm{T}}\widehat{R}_{1N}\varphi_{2N}\right\},\\
&\widehat{\Phi}_{2N}\\
&\quad=\dot{d}(t)\left(\varphi_{3N}^{\mathrm{T}}\widehat{R}_{2N}\varphi_{3N}-\varphi_{4N}^{\mathrm{T}}\widehat{R}_{3N}\varphi_{4N}\right)\\
&\qquad+\operatorname{Sym}\left\{\varphi_{3N}^{\mathrm{T}}\widehat{R}_{2N}\varphi_{5N}+\varphi_{4N}^{\mathrm{T}}\widehat{R}_{3N}\varphi_{6N}\right\}\\
&\qquad+d_0^2\widetilde{g}_6^{\mathrm{T}}\widehat{T}_1\widetilde{g}_6,\\
&\widehat{\Phi}_3=\varphi_7^{\mathrm{T}}\widehat{S}_1\varphi_7-\left(1-\dot{d}(t)\right)\varphi_8^{\mathrm{T}}\left(\widehat{S}_1-\widehat{S}_2\right)\varphi_8-\varphi_9^{\mathrm{T}}\widehat{S}_2\varphi_9,\\
&\widehat{\Phi}_{4N}=\operatorname{Sym}\left\{\Lambda_{1N}^{\mathrm{T}}\widehat{W}_{1N}+\Lambda_{2N}^{\mathrm{T}}\widehat{W}_{2N}\right\},\\
&\widehat{\Phi}_{5lm}=\operatorname{Sym}\left\{\left[\widetilde{g}_1^{\mathrm{T}}\rho_1+\widetilde{g}_2^{\mathrm{T}}\rho_2+\widetilde{g}_3^{\mathrm{T}}\rho_3+\widetilde{g}_6^{\mathrm{T}}\right]\widehat{\mathscr{F}}_{lm}\right\},\\
&\widehat{\mathscr{F}}_{lm}=\left(A_{l0}\widehat{X}^{\mathrm{T}}+B_l\widehat{H}_m\right)\widetilde{g}_1+A_{l1}\widehat{X}^{\mathrm{T}}\widetilde{g}_2-\widehat{X}^{\mathrm{T}}\widetilde{g}_6.
\end{aligned}\tag{56}$$

The proof of Theorem 11 is similar to the one of Theorem 10 which is omitted here for brevity. Moreover, the fuzzy control gains are given by $\widehat{K}_m=\widehat{H}_m\widehat{X}^{-T}$ $(m=1,2,\ldots,s)$.

Table 1: Maximum Delay Bound For Constant Delay.

Method	$d_0(s)$	Number of variables
Corollary 1 in [25]	1.6341	$4.5n^2+2.5n$
Theorem 1 in [26]	1.9538	$6n^2+3n$
Theorem 1 in [18]	2.0477	$14.5n^2+4.5n$
Theorem 3 (N=1)	2.0291	$19n^2+3n$
Theorem 3 (N=2)	2.0477	$30.5n^2+3.5n$
Theorem 3 (N=3)	2.0481	$45n^2+4n$

5. Numerical Examples

In this section, we give two numerical examples to verify the effectiveness of the proposed methods, where Example 1 is widely used for the comparison of the admissible delay upper bound computed by the delay-dependent stability conditions of this paper and some existing results, and Example 2 is employed to confirm the validity of the controller design conditions.

5.1. Example 1. Consider system (3) with $s=2$, $u(t)=0$ and system matrices as follows:

$$\begin{aligned}
A_{10}&=\begin{bmatrix}-0.2 & 0\\ 0 & -0.9\end{bmatrix},\\
A_{11}&=\begin{bmatrix}-1 & 0\\ -1 & -1\end{bmatrix},\\
A_{20}&=\begin{bmatrix}-1 & 0.5\\ 0 & -1\end{bmatrix},\\
A_{21}&=\begin{bmatrix}-1 & 0\\ 0.1 & -1\end{bmatrix}.
\end{aligned}\tag{57}$$

5.1.1. Constant Delay Case. When $d(t)\equiv d_0$, using Theorem 3 for different $N\in\{1,2,3,4\}$ and the results of some recent literature, the obtained admissible maximum time-delay upper bounds d_0 are listed in Table 1 for comparison. Meanwhile, the computing complexity of the methods is compared by listing the number of decision variables. We can clearly see that for an integer $N\ge 1$, Theorem 3 proposed in this paper can provide a larger upper bound d_0 than some existing literature, e.g., $1.6341(s)$ in [25] and $1.9538(s)$ in [26], which verifies that Theorem 3 is less conservative. But the computing complexity of Theorem 3 is larger than others.

5.1.2. Time-Varying Delay Case. Set the lower and upper bounds of the derivative of time delay to be $d_m=-0.1$ and $d_M=0.1$. Considering $(d(t),\dot{d}(t))\in\mathscr{D}_1$, one can compute the admissible upper bound $d_0=2.0291(s)$ by Theorem 8 with $N=1$. Using the existing results in [27–30], the obtained maximum delay bounds are listed in Table 2 for comparison. The results for $N=2$, 3 and 4 are also shown in Table 2. From Table 2, it is clear that Theorem 8 produces a larger

Table 2: Maximum Delay Bound with $d_M = -d_m = 0.1$.

Method	$d_0(s)$	Number of variables
Theorem 2 in [27]	1.4847	$59n^2 + 12n$
Theorem 2 in [28]	1.5332	$23n^2 + 5n$
Theorem 1 in [29] (m=3)	1.8090	$16.5n^2 + 7.5n$
Theorem 1 in [30]	1.8447	$49n^2 + 5n$
Theorem 8 (N=1)	1.9491	$73n^2 + 9n$
Theorem 8 (N=2)	1.9690	$122n^2 + 11n$
Theorem 8 (N=3)	1.9700	$185n^2 + 13n$
Theorem 8 (N=4)	1.9711	$262n^2 + 15n$

Table 3: Maximum Delay Bound with $d_M = -d_m = 0.1$.

Method	$d_0(s)$	Number of variables
Theorem 9 ($N = 1$)	2.0291	$73n^2 + 9n$
Theorem 9 ($N = 2$)	2.0477	$122n^2 + 11n$
Theorem 9 ($N = 3$)	2.0481	$185n^2 + 13n$

delay upper bound than Theorem 2 in [27] and others listed in Table 2, which implies that Theorem 8 it is less conservative.

Assume that $(d(t), \dot{d}(t)) \in \mathscr{D}_2$. Then we use Theorem 9 for different $N \in \{1, 2, 3, 4\}$. The obtained maximum delay bounds are given in Table 3, from which one can see that Theorem 9 delivers a larger delay upper bound than Theorem 8. Tables 2 and 3 also show that both Theorems 8 and 9 ensure a larger d_0 with increasing N.

Furthermore, to check that the system can tolerate the time-delay limited by the proposed results, we employ the simulation by Matlab. In simulation, let $o_1(x_1(t)) = 1/(1 + e^{-2x_1(t)})$ and $o_2(x_1(t)) = e^{-2x_1(t)}/(1+e^{-2x_1(t)})$. Then we choose $d_0 = 2.0481(s)$ and set the initial condition $\phi(t) = [1.5 \quad 1]^T$ for $t \in [-d_0, 0]$. We depict the states $x_1(t)$ and $x_2(t)$ by Figure 1, which confirms asymptotically stable responses.

5.2. Example 2. Consider system (7) with s=2 and system matrices as follows:

$$A_{10} = \begin{bmatrix} 0 & 0.6 \\ 0 & 1 \end{bmatrix},$$
$$A_{11} = \begin{bmatrix} 0.5 & 0.9 \\ 0 & 2 \end{bmatrix},$$
$$B_1 = \begin{bmatrix} 1 \\ 1 \end{bmatrix},$$
$$A_{20} = \begin{bmatrix} 1 & 0 \\ 1 & 0 \end{bmatrix},$$
$$A_{21} = \begin{bmatrix} 0.9 & 0 \\ 1 & 1.6 \end{bmatrix},$$
$$B_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{58}$$

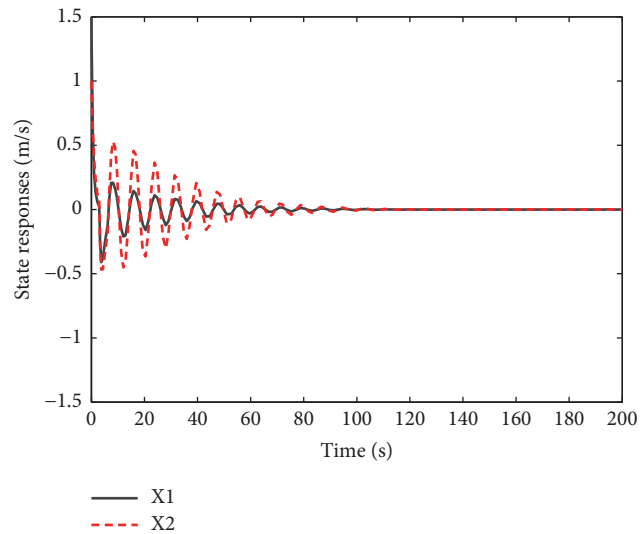

Figure 1: The state responses of the system with $d(t) \equiv d_0 = 2.0481(s)$.

Obviously, when $d_0 = 0$ and without any control input, both the subsystems of the above system are unstable, because $A_{10} + A_{11}$ and $A_{20} + A_{21}$ are not Hurwitz matrices; that is, it is an unstable system. Now we demonstrate that the designed controllers by Theorems 10 and 11 are effective, respectively.

5.2.1. Constant Delay Case. Using Theorem 10 with $N = 2$, $\varrho = 0.5$, the admissible maximum upper bound of constant delay can be computed to be $d_0 = 1.1(s)$, and the corresponding control gains are as follows:

$$K_1 = [67.6465 \quad -194.6495],$$
$$K_2 = [65.3394 \quad -189.3471]. \tag{59}$$

According to the feasibility of Theorem 10, one can say that the controller (6) with the proposed gains (59) ensures the stability of the closed-loop system.

Similar to Example 1, we let $o_1(x_1(t)) = 1/(1+e^{-2x_1(t)})$ and $o_2(x_1(t)) = e^{-2x_1(t)}/(1 + e^{-2x_1(t)})$ and set the initial condition $\phi(t) = [1.5 \quad 1]^T$ for $t \in [-d_0, 0]$. The state responses of the closed-loop system under the controller (6) with gains (59) are demonstrated by Figure 2 which shows a positive result. From Figure 2, one can say that closed-loop system under the controller (6) with gains (59) is asymptotically stable. Then it can be concluded that Theorem 10 is effective.

5.2.2. Time-Varying Delay Case. Let $h_M = -h_m = 0.1$. Then, according to Theorem 11 with $N = 1$, $\rho_1 = 0.5$, $\rho_2 = 0.2$ and $\rho_3 = 0.2$, one can deliver the admissible maximum bound $d_0 = 1.3176(s)$ while the control gains are

$$K_1 = [148.7390 \quad -420.4591],$$
$$K_2 = [148.6048 \quad -422.3894]. \tag{60}$$

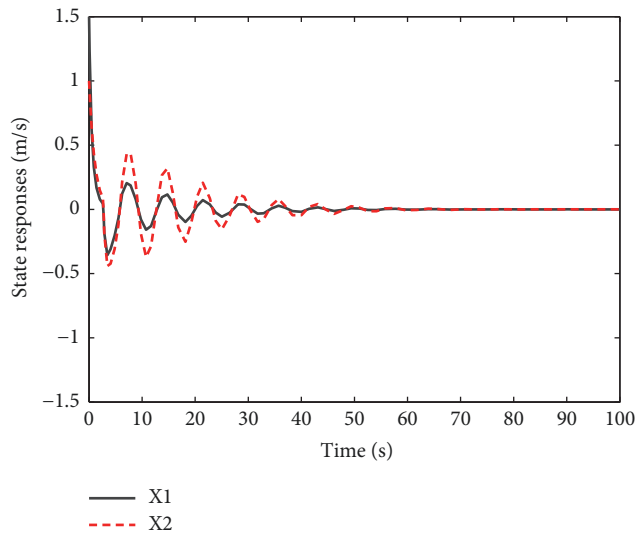

Figure 2: The state responses of the closed-loop system with $d(t) \equiv d_0 = 1.1(s)$.

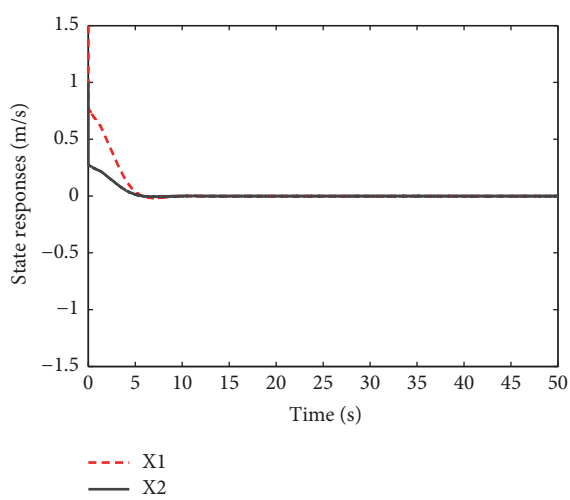

Figure 3: The state responses of the closed-loop system with $d(t) = 1.2176 + 0.1\sin(t)$.

From the feasibility of Theorem 11, it can be considered that the closed-loop system can be stabilized by the controller (6) with the proposed gains (60).

For further confirmation, the normalized membership functions are set as $o_1(x_1(t)) = 1/(1+e^{-2x_1(t)})$ and $o_2(x_1(t)) = e^{-2x_1(t)}/(1+e^{-2x_1(t)})$. Besides, we introduce the initial condition $\phi(t) = [1.5 \;\; 1]^T$ for $t \in [-d_0, 0]$ and choose the time-varying delay $d(t) = 1.2176 + 0.1\sin(t)$. The state responses in Figure 3 shows the positive effect of the controller (6) with gains (60) on the stabilization of the closed-loop system, which demonstrates the effectiveness of Theorem 11.

6. Conclusion

In this paper, we have proposed an augmented LKF approach to derive some improved stability and stabilization conditions of fuzzy system with constant delay and interval time-varying delay with its derivative bounds available, respectively. In particular, the proposed LKF have been constructed on the basis of the form of the B-L inequality. Two numerical examples have illustrated the improvements of the obtained conditions comparing with some existing recently results and the design validity of fuzzy controllers.

Data Availability

The data used to support the findings of this study are included within the article. These data include three tables in the manuscript, which are used to make comparisons between the proposed conditions with the existing results.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported in part by the Natural Science Foundation of China under Grant 61773238, the Fundamental Research Funds of Shandong University, and the Program for the Outstanding Innovative Teams of Higher Learning Institutions of Shanxi.

References

[1] C. Lin, Q. Wang, T. H. Lee, and Y. He, *LMI Approach to Analysis and Control of Takagi-Sugeno Fuzzy Systems with Time Delay*, Springer, Berlin, Germany, 2007.

[2] K. Tanaka and H. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*, John Wiley & Sons, Inc, 2001.

[3] C. Peng and M. Fei, "An improved result on the stability of uncertain T-S fuzzy systems with interval time-varying delay," *Fuzzy Sets and Systems*, vol. 212, pp. 97–109, 2013.

[4] F. O. Souza, V. C. S. Campos, and R. M. Palhares, "On delay-dependent stability conditions for Takagi-Sugeno fuzzy systems," *Journal of The Franklin Institute*, vol. 351, no. 7, pp. 3707–3718, 2014.

[5] O. M. Kwon, M. J. Park, J. H. Park, and S. M. Lee, "Stability and stabilization of T-S fuzzy systems with time-varying delays via augmented Lyapunov-Krasovskii functionals," *Information Sciences*, vol. 372, pp. 1–15, 2016.

[6] D. Zhang, Q.-L. Han, and X. Jia, "Network-based output tracking control for T-S fuzzy systems using an event-triggered communication scheme," *Fuzzy Sets and Systems*, vol. 273, pp. 26–48, 2015.

[7] D. Zhang, Q.-L. Han, and X. Jia, "Network-based output tracking control for a class of T-S fuzzy systems that can not be stabilized by nondelayed output feedback controllers," *IEEE Transactions on Cybernetics*, vol. 45, no. 8, pp. 1511–1524, 2015.

[8] D. Zhang, Z. Zhou, and X. Jia, "Networked fuzzy output feedback control for discrete-time Takagi-Sugeno fuzzy systems with sensor saturation and measurement noise," *Information Sciences*, vol. 457/458, pp. 182–194, 2018.

[9] K. Gu, V. L. Kharitonov, and J. Chen, *Stability of Time-Delay Systems*, Birkhauser, Boston, Mass, USA, 2003.

[10] C.-K. Zhang, Y. He, L. Jiang, and M. Wu, "Stability analysis for delayed neural networks considering both conservativeness and complexity," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 27, no. 7, pp. 1486–1501, 2016.

[11] A. Seuret and F. Gouaisbaut, "Hierarchy of LMI conditions for the stability analysis of time-delay systems," *Systems & Control Letters*, vol. 81, pp. 1–7, 2015.

[12] A. Seuret and F. Gouaisbaut, "Stability of linear systems with time-varying delays using bessel–legendre inequalities," *IEEE Transactions on Automatic Control*, vol. 63, no. 1, pp. 225–232, 2018.

[13] K. Liu, A. Seuret, and Y. Xia, "Stability analysis of systems with time-varying delays via the second-order Bessel–Legendre inequality," *Automatica*, vol. 76, pp. 138–142, 2017.

[14] X.-M. Zhang, Q.-L. Han, and Z. Zeng, "Hierarchical type stability criteria for delayed neural networks via canonical bessel-legendre inequalities," *IEEE Transactions on Cybernetics*, vol. 48, no. 5, pp. 1660–1671, 2018.

[15] X.-M. Zhang, Q.-L. Han, and J. Wang, "Admissible delay upper bounds for global asymptotic stability of neural networks with time-varying delays," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 11, pp. 5319–5329, 2018.

[16] D. Zhang, Q.-L. Han, and X.-M. Zhang, "Network-based modeling and proportional-integral control for direct-drive-wheel systems in wireless network environments," *IEEE Transactions on Systems Science and Cybernetics*, 2019.

[17] F. Souza, L. Mozelli, and R. Palhares, "On stability and stabilization of T–S fuzzy time-delayed systems," *IEEE Transactions on Fuzzy Systems*, vol. 17, no. 6, pp. 1450–1455, 2009.

[18] X. Zhao, C. Lin, B. Chen, and Q. Wang, "A novel Lyapunov–Krasovskii functional approach to stability and stabilization for T–S fuzzy systems with time delay," *Neurocomputing*, vol. 313, pp. 288–294, 2018.

[19] X.-M. Zhang, Q.-L. Han, A. Seuret, and F. Gouaisbaut, "An improved reciprocally convex inequality and an augmented Lyapunov-Krasovskii functional for stability of linear systems with time-varying delay," *Automatica*, vol. 84, pp. 221–226, 2017.

[20] X. Zhang, Q. Han, A. Seuret, F. Gouaisbaut, and Y. He, "Overview of recent advances in stability of linear systems with time-varying delays," *IET Control Theory & Applications*, vol. 13, no. 1, pp. 1–16, 2019.

[21] C. Zhang, Y. He, L. Jiang, and M. Wu, "Notes on stability of time-delay systems: bounding inequalities and augmented lyapunov-krasovskii functionals," *IEEE Transactions on Automatic Control*, vol. 62, no. 10, pp. 5331–5336, 2017.

[22] C. Zhang, Y. He, L. Jiang, M. Wu, and Q. Wang, "An extended reciprocally convex matrix inequality for stability analysis of systems with time-varying delay," *Automatica*, vol. 85, pp. 481–485, 2017.

[23] A. Seuret, K. Liu, and F. Gouaisbaut, "Generalized reciprocally convex combination lemmas and its application to time-delay systems," *Automatica*, vol. 95, pp. 488–493, 2018.

[24] S. Q. Zhu, Q.-L. Han, and C. H. Zhang, "l1-gain performance analysis and positive filter design for positive discrete-time Markov jump linear systems: a linear programming approach," *Automatica*, vol. 50, no. 8, pp. 2098–2107, 2014.

[25] C. Peng, Y.-C. Tian, and E. Tian, "Improved delay-dependent robust stabilization conditions of uncertain T-S fuzzy systems with time-varying delay," *Fuzzy Sets and Systems*, vol. 159, no. 20, pp. 2713–2729, 2008.

[26] Z. Zhang, C. Lin, and B. Chen, "New stability and stabilization conditions for T–S fuzzy systems with time delay," *Fuzzy Sets and Systems*, vol. 263, pp. 82–91, 2015.

[27] L. Li, X. Liu, and T. Chai, "New approaches on H∞ control of T-S fuzzy systems with interval time-varying delay," *Fuzzy Sets and Systems*, vol. 160, no. 12, pp. 1669–1688, 2009.

[28] O. M. Kwon, M. J. Park, S. M. Lee, and J. H. Park, "Augmented Lyapunov-Krasovskii functional approaches to robust stability criteria for uncertain Takagi-Sugeno fuzzy systems with time-varying delays," *Fuzzy Sets and Systems*, vol. 201, pp. 1–19, 2012.

[29] H.-B. Zeng, J. H. Park, J.-W. Xia, and S.-P. Xiao, "Improved delay-dependent stability criteria for T-S fuzzy systems with time-varying delay," *Applied Mathematics and Computation*, vol. 235, pp. 492–501, 2014.

[30] F. Yang, S. Guan, and D. Wang, "Quadratically convex combination approach to stability of T–S fuzzy systems with time-varying delay," *Journal of The Franklin Institute*, vol. 351, no. 7, pp. 3752–3765, 2014.


Advances in Operations Research
Advances in Decision Sciences
Journal of Applied Mathematics
The Scientific World Journal
Journal of Probability and Statistics
International Journal of Mathematics and Mathematical Sciences
Hindawi
Submit your manuscripts at
www.hindawi.com
Journal of Optimization
International Journal of Engineering Mathematics
International Journal of Analysis
Journal of Complex Analysis
Advances in Numerical Analysis
Mathematical Problems in Engineering
International Journal of Differential Equations
Discrete Dynamics in Nature and Society
International Journal of Stochastic Analysis
Journal of Mathematics
Journal of Function Spaces
Abstract and Applied Analysis
Advances in Mathematical Physics