Research Article

A Novel TOPSIS Method Based on Improved Grey Relational Analysis for Multiattribute Decision-Making Problem

Wenguang Yang\(^{1,2}\) and Yunjie Wu\(^1\)

\(^1\)School of Automation Science and Electrical Engineering, Beihang University, Beijing 100191, China
\(^2\)College of Science, North China Institute of Science and Technology, Beijing 101601, China

Correspondence should be addressed to Wenguang Yang; yangwenguang@buaa.edu.cn

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Multiattribute decision-making (MADM) problem is difficult to assess because of the large number of attribute indices and the diversity of data distribution. Based on the understanding of data dispersion degree, a new grey TOPSIS method for MADM is studied. The main idea of this paper is to redefine the grey relational analysis through the dispersion of data distribution and redesign the TOPSIS by using the improved grey relational analysis. As a classical multiattribute decision analysis method, traditional TOPSIS does not consider the data distribution of the degree of dispersion and aggregation when it is compared with the optimal and worst alternative solutions. In view of the limitations of traditional TOPSIS, this paper has made two major improvements to TOPSIS. Firstly, the new grey relational analysis is applied to evaluate the grey positive relational degree between each alternative and the optimal solution and compute the grey negative relational degree between each alternative and the worst solution. Secondly, the weights of every attribute index about the optimal and worst solutions are put forward based on the distance standard deviation and the average distance. Finally, the comprehensive grey TOPSIS is utilized to analyze the ranking of weapon selection problem. The numerical results verify the feasibility of the improved grey relational analysis and also highlight the practicability of the grey comprehensive TOPSIS.

1. Introduction

In recent years, with the development of society, the research on multiattribute decision-making (MADM) is becoming more and more concerned about its simplicity, validity, and accuracy [1–9]. Generally speaking, multiattribute decision-making problem often involves complicated external environment and many different attributes. The TOPSIS (techniques for order preference by similarity to ideal solution) is a useful and powerful method for dealing with multiattribute decision-making (MADM) problems which is presented by Hwang and Yoon [10]. The classical TOPSIS approach is based on the suggestion of the chosen alternative which requires consideration of the distance between the positive ideal solution and the negative ideal solution. The most suitable alternative should be closest to the positive ideal solution and the farthest from the negative ideal solution. At present, with the deepening realization about TOPSIS, some extended TOPSIS techniques have been used widely for MADM problems to overcome the vagueness in the judgments made by the assessors [11–21]. In order to overcome the problem of index weight calculation, AHP is the main auxiliary method of TOPSIS, and fuzzy sets are introduced together to facilitate the quantification of the two pairs of indicators [11–13]. Some researchers have improved TOPSIS by using the intuitionistic fuzzy number, triangular fuzzy number, vague set, intuitionistic fuzzy entropy, and other uncertainty tools, which can solve MADM problems with intuitionistic fuzzy environment or linguistic fuzziness [7,14–18].

In fact, in the process of multiattribute decision-making with TOPSIS, there are two main problems to solve. On the one hand, the weights of different indexes need to be determined when the alternatives are compared with the optimal or worst schemes. As a relatively simple method, analytic hierarchy process (AHP) has been successfully
applied to the weight calculation of TOPSIS [11–13, 19], but subjective interference also exists due to expert scoring and pairwise comparison. The technique of analytic network process (ANP) is used to obtain the relative weights to reduce the number of pairwise comparison on the basis of AHP [20]. So the ANP is the development and supplement of AHP. In addition, the method of comprehensive evaluation, principal component analysis, and other methods are all important methods to calculate weights, which are applied in TOPSIS [21–23]. On the other hand, the optimal or worst value for every attribute should be selected, and a quantitative measure method should be constructed to compare with each other. Fuzzy mathematics provides a quantitative calculation method for describing uncertain things [11–15, 18, 19], but there is a lot of uncertainty and subjectivity in the designing of fuzzy sets. Professor Deng proposed the grey system theory which is one of powerful methods for data analysis with partly known and partly unknown information in uncertain environment [24, 25]. Distance standard deviation is an important numerical characteristic which reflects the distribution of data. Very recently, it is worth noting that distance standard deviation is introduced into the grey confidence interval estimation for small samples [26]. This study shows that distance standard deviation contributes to the standard deviation of sample distance of data coincides with this characteristic. Therefore, the sample standard deviation is not enough. Generally speaking, the sample aggregation characteristics reflect the inherent character of data, which should be considered in the calculation of grey relational degree. The numerical characteristics of the attribute data distribution can reflect the degree of dispersion of the data distribution. If the degree of dispersion of the data is small, the distribution is relatively concentrated, and this attribute is relatively important. And the standard deviation of sample distance of data coincides with this characteristic. Therefore, the sample distance standard deviation as an important concept that embodies the degree of distribution density will be brought into the calculation of grey relational degree.

Based on the deepening realization about the generality of multiattribute decision-making problems and the core of TOPSIS method, part of the work has been completed in the following. In Section 2, the sample distance standard deviation is defined between different system behavior sequences, and then the improved grey relational analysis based on sample distance standard deviation is constructed, which paved the way to further discussion. In Section 3, the innovative new TOPSIS method is proposed. The degree of aggregation between different data is served as an important decision factor for TOPSIS method. The grey relational analysis is improved on the basis of the distance standard deviation and applied to quantify the correlation between different sequence data. The distance standard deviation is also used to calculate the weights between the sequence data and the optimal scheme or the worst case. Section 4 gives three applications of the developed approach, and the numerical steps and results are clearly shown. The last section summarizes the whole paper.

2. The Improved Grey Relational Analysis Based on Dispersion Degree

At present, the grey relational analysis has been applied more commonly to many scientific decision-making problems, such as computer science, engineering and other related fields. It provides a tool to evaluate grey relational degree based on distances between reference sequence and comparative sequence. In fact, only considering the maximum and minimum distance between data is not enough. Generally speaking, the sample aggregation characteristics reflect the inherent character of data, which should be considered in the calculation of grey relational degree. The numerical characteristics of the attribute data distribution can reflect the degree of dispersion of the data distribution. If the degree of dispersion of the data is small, the distribution is relatively concentrated, and this attribute is relatively important. And the standard deviation of sample distance of data coincides with this characteristic. Therefore, the sample distance standard deviation as an important concept that embodies the degree of data distribution density will be brought into the calculation of grey relational degree.

Definition 1. For the given system behavior sequence as follows:

\[ X_0 = (x_0(1), x_0(2), \ldots, x_0(n)), \]
\[ X_1 = (x_1(1), x_1(2), \ldots, x_1(n)), \]
\[ \ldots, \]
\[ X_i = (x_i(1), x_i(2), \ldots, x_i(n)), \]
\[ \ldots, \]
\[ X_m = (x_m(1), x_m(2), \ldots, x_m(n)). \]

The distance between \( x_0(k) \) and \( x_i(k) \) is defined as follows:

\[ d(x_0(k), x_i(k)) = |x_0(k) - x_i(k)|. \]

For \( \xi \in (0, 1) \), and for every \( i, j \), let

\[
\gamma(x_0(k), x_i(k)) = \begin{cases} 
\xi \max_j s(X_0, X_j) \\
\frac{\min_j s(X_0, X_j)}{|x_0(k) - x_i(k)| + \xi \max_j s(X_0, X_j)}, \\
\eta^n [\max_k d(x_0(k), x_i(k)) - \min_k d(x_0(k), x_i(k))] e^{-\beta d(x_0(k), x_i(k))}
\end{cases}
\]

\[
\gamma(X_0, X_j) = \frac{1}{n} \sum_{k=1}^n \gamma(x_0(k), x_i(k)),
\]

if \( \min_j s(X_0, X_j) > 0 \) and \( s(X_0, X_j) \neq s(X_0, X_j) \)

(3)

if \( \min_j s(X_0, X_j) = 0 \) or \( s(X_0, X_j) = s(X_0, X_j) \),

(4)
where \[ s(X_0, X_i) = \frac{\sqrt{1/(n-1) \sum_{k=1}^{n} (d(x_0(k), x_i(k)) - m(X_0, X_i))^2}}{\sum_{k=1}^{n} d(x_0(k), x_i(k))} \]

denotes the sample distance standard deviation between \( X_0 \) and \( X_i \),
and \( m(X_0, X_i) = (1/n) \sum_{k=1}^{n} d(x_0(k), x_i(k)) \) is defined as the
average distance between \( X_0 \) and \( X_i \).

Then \( \gamma(X_0, X_i) \) is called grey relational degree with
sample standard distance deviation in which \( \xi \) and \( \eta \) are
called distinguishing coefficient, \( \eta \in (0, 1), \alpha, \beta \in (0, 1) \],
and \( \xi \)'s value satisfies the following requirements:

\[ \xi = \begin{cases} 
\max_j s(X_0, X_j), & \text{if } \min_i s(X_0, X_i) > 0 \\
\min_i s(X_0, X_i) + \max_j s(X_0, X_j), & \text{if } \min_i s(X_0, X_i) = 0.
\end{cases} \] (5)

**Theorem 2.** The improved grey relational degree \( \gamma(X_0, X_i) \)
with sample distance standard deviation \( s(X_0, X_i) \) satisfies the
following three axioms of grey correlation:

1. **The property of normality:** \( 0 < \gamma(X_0, X_i) \leq 1 \).
2. **The property of closeness:** the greater the \( \gamma(X_0, X_i) \), the
closer of \( X_0 \) to \( X_i \).
3. **The property of pair symmetry:** \( \gamma(Y_i, X_i) = \gamma(X_i, Y_i) \).

**Proof.** It is obvious that \( s(X_0, X_i) \geq 0 \) for system behavior
sequence \( X_0, X_1, \ldots, X_i, \ldots, X_m \). So the theorem can be
proved in two cases.

1. When \( \min_i s(X_0, X_i) = 0 \) or \( s(X_0, X_i) = s(X_0, X_i) \),

\[ \gamma(x_0(k), x_i(k)) = \eta \frac{\max_j (d(x_0(k), x_j(k)) - d(x_0(k), x_i(k)) - \beta \max_j (d(x_0(k), x_j(k)) - d(x_0(k), x_i(k)))}{(1/\sqrt{n}) \sum_{k=1}^{n} d(x_0(k), x_i(k))} \] \( 0 < \gamma(x_0(k), x_i(k)) \leq 1 \) for every \( k \), then in these circumstances,
we can obtain the result that \( 0 < \gamma(X_0, X_i) \leq 1 \) by (4).

2. If \( s(X_0, X_i) > 0 \) and \( s(X_0, X_i) \neq s(X_0, X_i) \),
then \( \max_j s(X_0, X_j) \geq \min_i s(X_0, X_i) > 0 \). So, \( 0 < \gamma(X_0, X_i) < 1 \)
by (3). Because \( \min_i s(X_0, X_i) > 0 \), we can receive the result
that \( X_0 \neq X_i \) for every \( i \), then \( \gamma(X_0, X_i) \neq 0 \) in this case.

**Remark 3.** The new grey relational analysis takes full advantage
of the inherent regularity of data distribution and introduces the dispersion of data distribution into them. The
distance standard deviation is more likely to reflect the overall
dispersion of the data distribution than the distance.

**Remark 4.** In the case of \( \min_i s(X_0, X_i) = 0 \) or \( s(X_0, X_i) = s(X_0, X_i) \),
that is, there are two system behavior sequence \( X_0 \) and \( X_i \)
that are parallel or overlapped, the distance between \( X_0 \) and \( X_i \) is fully considered by (3). Expressly if two system
behavior sequence data are parallel, then the smaller the
distance is, the greater the similarity is, and the greater the
grey relational degree is.

### 3. The Novel TOPSIS with Improved Grey Relational Analysis to Solve Multiattribute Decision-Making Problem

This section tries to improve traditional TOPSIS with new grey relational analysis mentioned above. The degree of
dispersion and aggregation between different data will serve as
an important decision factor for TOPSIS. The above-mentioned
grey relational analysis will be used to calculate the
grey relational degree between the alternative scheme and
the optimal or worst case scheme. At the same time, the
standard deviation of distance will be taken as the decisive
factor of weight calculation in order to determine the weight
of each index for MADM problems.

**Step 1.** Standardization of the value of original decision matrix \( X = (x_{ij})_{n \times m} \); this will help eliminating the influence of dimension and magnitude in data processing. The original decision matrix \( X = (x_{ij})_{n \times m} \) can be expressed as follows:

\[ X = \begin{bmatrix} x_{11} & x_{12} & \cdots & x_{1m} \\
x_{21} & x_{22} & \cdots & x_{2m} \\
\vdots & \vdots & \ddots & \vdots \\
x_{n1} & x_{n2} & \cdots & x_{nm} \end{bmatrix}_{n \times m}, \] (6)

where \( x_{ij} \) is the \( j \)th attribute value of the \( i \)th evaluation object.

For multiattribute decision-making problem, the normalized
decision matrix is \( Y = (y_{ij})_{n \times m} \) after standardization of \( X = (x_{ij})_{n \times m} \).

For the benefit attribute

\[ y_{ij} = \frac{x_{ij} - \min_i x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}, \] (7)

For the cost attribute

\[ y_{ij} = \frac{\max_i x_{ij} - x_{ij}}{\max_i x_{ij} - \min_i x_{ij}}, \] (8)

**Step 2.** To pick out the optimal and worst alternatives, the
optimal alternative is denoted as \( \overline{X} = (\overline{x}_1, \overline{x}_2, \ldots, \overline{x}_m) \in \mathbb{R}^m \),
where

\[ \overline{x}_j = \begin{cases} 
\max_i x_{ij}, & \text{if } x_{ij} \text{ is benefit attribute} \\
\min_i x_{ij}, & \text{if } x_{ij} \text{ is cost attribute},
\end{cases} \] (9)

\( j = 1, 2, \ldots, m. \)
The worst alternative denoted as \( X = (\xi_1, \xi_2, \ldots, \xi_m) \in R^m \), where

\[
\sum_j = \begin{cases} 
\max x_{ij}, & \text{if } x_{ij} \text{ is cost attribute} \\
\min_i x_{ij}, & \text{if } x_{ij} \text{ is benefit attribute},
\end{cases}
\]

\[ j = 1, 2, \ldots, m. \] (10)

For the normalized decision matrix \( Y = (y_{ij})_{n \times m} \), the normalized optimal alternative can be denoted as \( \bar{y} = (\bar{y}_1, \bar{y}_2, \ldots, \bar{y}_m) \in R^m \), and the normalized worst alternative can be denoted as \( \bar{y} = (\bar{y}_1', \bar{y}_2', \ldots, \bar{y}_m') \in R^m \). It is not difficult to calculate that \( \bar{y} = (1, 1, \ldots, 1) \in R^m \), and \( Y = (0, 0, \ldots, 0) \in R^m \) by (7) and (8), respectively.

**Step 3.** To calculate the sample distance standard deviation for different attribute, the symbol \( s_{ij} \) used to denote the sample distance standard deviation between the \( j \)th attribute vector and the \( j \)th attribute optimal value, where

\[
s_{ij} = \frac{1}{n-1} \sum_{i=1}^{n} (y_{ij} - \overline{y}_{j})^2, \quad j = 1, 2, \ldots, m.
\]

(11)

The symbol \( s_{2j} \) is used to denote the sample distance standard deviation between the \( j \)th attribute vector and the \( j \)th attribute worst value, where

\[
s_{2j} = \frac{1}{n-1} \sum_{i=1}^{n} (y_{ij} - \overline{y}_{2j})^2, \quad j = 1, 2, \ldots, m.
\]

(12)

**Step 4.** To compute the grey relational degree \( r_{ij}^+ \) and \( r_{ij}^- \). There are some different cases based on (3).

(1) When \( \min_j s_{ij} > 0 \) and \( s_{ij} \neq s_{ik} \) for every \( k, k = 1, 2, \ldots, m \), the symbol \( r_{ij}^+ \) is the grey positive relational degree between \( y_{ij} \) and \( \overline{y}_j \), where

\[
r_{ij}^+ = \frac{\xi \max_j s_{ij}}{|y_{ij} - \overline{y}_j| + \xi \max_j s_{ij}}.
\]

(15)

At the same case, the symbol \( r_{ij}^- \) is the grey negative relational degree between \( y_{ij} \) and \( \overline{y}_j \), where

\[
r_{ij}^- = \frac{\xi \max_j s_{ij}}{|y_{ij} - \overline{y}_j| + \xi \max_j s_{ij}}.
\]

(16)

(2) When \( \min_j s_{ij} = 0 \) or \( s_{ij} = s_{ik} \), for every \( k, k = 1, 2, \ldots, m \), the grey positive relational degree \( r_{ij}^+ \) can be computed as follows:

\[
r_{ij}^+ = r^0 \left[ \max_j (y_{ij} - \overline{y}_j) - \min_j (y_{ij} - \overline{y}_j) \right] e^{-\beta d(y_{ij}, \overline{y}_j)}.
\]

(17)

And the grey negative relational degree \( r_{ij}^- \) between \( y_{ij} \) and \( \overline{y}_j \) can be received as follows:

\[
r_{ij}^- = \eta \left[ \max_j (y_{ij} - \overline{y}_j) - \min_j (y_{ij} - \overline{y}_j) \right] e^{-\beta d(y_{ij}, \overline{y}_j)}.
\]

(18)

**Step 5.** To determine weights \( \overline{w}_j \) and \( w_j \) for the grey positive relational degree and grey negative relational degree, similar to the above, there are two cases below.

(1) When \( \min_j s_{ij} > 0 \), \( s_{ij} \neq s_{ik} \), \( \min_j s_{2j} > 0 \), and \( s_{ij} \neq s_{2k} \) for every \( k, k = 1, 2, \ldots, m \), \( \overline{w}_j \) and \( w_j \) can be obtained as follows:

\[
\overline{w}_j = \frac{1/s_{ij}}{\sum_{j=1}^{m} (1/s_{ij})}, \quad j = 1, 2, \ldots, m,
\]

(19)

\[
w_j = \frac{1/s_{2j}}{\sum_{j=1}^{m} (1/s_{2j})}, \quad j = 1, 2, \ldots, m.
\]

(20)

where the sample distance standard deviation \( s_{ij} \) and \( s_{2j} \) can be determined by (11) and (13).

(2) Otherwise, the weights \( \overline{w}_j \) and \( w_j \) can be calculated as follows:

\[
\overline{w}_j = \frac{1/m_{ij}}{\sum_{j=1}^{m} (1/m_{ij})}, \quad j = 1, 2, \ldots, m,
\]

(21)

\[
w_j = \frac{1/m_{2j}}{\sum_{j=1}^{m} (1/m_{2j})}, \quad j = 1, 2, \ldots, m.
\]

(22)

where the average distance \( m_{ij} \) and \( m_{2j} \) can be determined by (12) and (14).

The smaller the distance standard deviation of attribute data or the average distance is, the stronger the regularity of attribute data is, and the bigger its weight is. On the contrary, the larger the distance standard deviation of attribute data or the average distance is, the larger the distribution range of attribute data is, the weaker the regularity is, and the smaller the weight is. It is obvious to find that the greater the \( \overline{w}_j \) is, the closer alternative attribute value and the optimal alternative attribute value are. Similarly, the weight \( w_j \) has the same nature. The greater the \( w_j \) is, the closer alternative attribute value and the worst value are.

**Step 6.** To calculate the grey TOPSIS grade, the grey positive relational grade \( R_{ij}^+ \) between the \( i \)th evaluation alternative and the optimal solution can be received as follows:

\[
R_{ij}^+ = \frac{\sum_{j=1}^{m} \overline{w}_j r_{ij}^+}{r_{ij}^+}, \quad i = 1, 2, \ldots, n.
\]

(23)
The grey negative relational grade $R_i$ between the $i$th evaluation alternative and the worst solution can be received as follows:

$$R_i = \sum_{j=1}^{m} w_{rij}, \quad i = 1, 2, \ldots, n. \quad (24)$$

And the comprehensive grey TOPSIS evaluation result $R_i$ is defined as follows:

$$R_i = \frac{R_i^+}{R_i^+ + R_i^-}, \quad i = 1, 2, \ldots, n. \quad (25)$$

It is obviously that the comprehensive grey TOPSIS evaluation result $R_i$ reflects the closeness between alternative and positive ideal solution and negative ideal solution. The larger the $R_i^+$ is, the closer $R_i$ is to positive ideal solution, and the smaller the $R_i^-$ is, the farther $R_i$ is to negative ideal solution. The reason behind this is that after the first dimensionless standardized processing of the original data, the improved TOPSIS method is used for evaluation, and the obtained grey relational degree and weights have the function of adaptive adjustment with the change of data. On the one hand, the grey positive relational degree can reflect the closeness between the data and the same attribute data of the negative ideal solution. On the other hand, the weights of different attributes can feedback the importance of the attributes based on the numerical characteristics of the attribute data.

4. **Numerical Results**

In this section, three examples will be used to demonstrate the effectiveness of the methods proposed above. The first example shows that the improved grey relational analysis is more comprehensive than the traditional grey relational analysis, which can avoid some unreasonable results. The second one is used to illustrate the rationality of the improved grey relational analysis. The novel TOPSIS with improved grey relational analysis will be applied for assessment of weapon system in the second example. In addition, Example 3 will illustrate that the method of this paper can avoid the occurrence of the rank reversal phenomenon, which is superior to the traditional TOPSIS method.

**Example 1.** The following example will try to solve the grey relational degree with improved method in this paper. Take four system behavior sequences for example, $X_1 = [0.1, 0.9, 0.1, 0.9, 0.1]$, $X_2 = [1, 0, 1, 0, 1]$, $X_3 = [0.2, 1, 0.2, 1, 0.2]$, and $X_4 = [0.9, 0, 0.9, 0, 0.9]$. Figure 1 illustrates the changing trend curves for different system behavior sequences.

**Solution**

**Step 1.** Because the data of four system behavior sequences are in a relatively small range [0, 1], the data preprocessing can be ignored.

**Step 2.** Compute the average distance $m(X_1, X_i)$ and the sample distance standard deviation $s(X_1, X_i)$.

$$m(X_1, X_2) = 0.9,$$

$$m(X_1, X_3) = 0.1,$$

$$m(X_1, X_4) = 0.84,$$

$$s(X_1, X_2) = 0,$$

$$s(X_1, X_3) = 0,$$

$$s(X_1, X_4) = 0.0548. \quad (27)$$

**Step 3.** According to (3) and (4), the improved grey relational degree $\gamma(X_1, X_i)$ based on discrete degree of data in statistics can be solved as follows:

$$\gamma(X_1, X_2) = 0.6376,$$

$$\gamma(X_1, X_3) = 0.9512,$$

$$\gamma(X_1, X_4) = 0.6500. \quad (28)$$

The results show that $\gamma(X_1, X_3) > \gamma(X_1, X_4) > \gamma(X_1, X_2)$, which indicates $X_1$ is more relevant to $X_3$ compared with
Table 1: The comparison between the improved and classical grey relational analysis.

<table>
<thead>
<tr>
<th>Grey relational degree</th>
<th>$\gamma(X_1, X_2)$</th>
<th>$\gamma(X_1, X_3)$</th>
<th>$\gamma(X_1, X_4)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>The classical grey relational analysis</td>
<td>0.4074</td>
<td>1</td>
<td>0.4270</td>
</tr>
<tr>
<td>The improved grey relational analysis</td>
<td>0.6376</td>
<td>0.9512</td>
<td>0.6500</td>
</tr>
</tbody>
</table>

Table 2: The original multiattribute decision data [27].

<table>
<thead>
<tr>
<th>$M_{\text{max}}$</th>
<th>$M'_{\text{max}}$</th>
<th>$O_{\text{max}}$</th>
<th>$H_{\text{max}}$</th>
<th>$R_{\text{max}}$</th>
<th>$N$</th>
<th>$P$</th>
<th>$T$</th>
<th>$H_{\text{min}}$</th>
<th>$G_{0}$</th>
<th>$R_{\text{min}}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$</td>
<td>6.0000</td>
<td>750</td>
<td>6.0000</td>
<td>24.0000</td>
<td>100</td>
<td>8.0000</td>
<td>0.75</td>
<td>20</td>
<td>0.3000</td>
<td>1000</td>
</tr>
<tr>
<td>$X_2$</td>
<td>5.5000</td>
<td>2300</td>
<td>5.0000</td>
<td>27.0000</td>
<td>90</td>
<td>8.0000</td>
<td>0.8000</td>
<td>15</td>
<td>0.0250</td>
<td>1000</td>
</tr>
<tr>
<td>$X_3$</td>
<td>4.4000</td>
<td>1200</td>
<td>5.0000</td>
<td>27.0000</td>
<td>75</td>
<td>6.0000</td>
<td>0.7600</td>
<td>20</td>
<td>0.0250</td>
<td>1664</td>
</tr>
<tr>
<td>$X_4$</td>
<td>3.0000</td>
<td>420</td>
<td>1.0000</td>
<td>24.5000</td>
<td>32</td>
<td>1.0000</td>
<td>0.7500</td>
<td>40</td>
<td>1.0000</td>
<td>2375</td>
</tr>
<tr>
<td>$X_5$</td>
<td>2.0000</td>
<td>400</td>
<td>2.0000</td>
<td>5.0000</td>
<td>12</td>
<td>3.0000</td>
<td>0.7500</td>
<td>10</td>
<td>0.5000</td>
<td>220</td>
</tr>
<tr>
<td>$X_6$</td>
<td>2.2000</td>
<td>400</td>
<td>2.0000</td>
<td>3.0000</td>
<td>8.0000</td>
<td>3.0000</td>
<td>0.8000</td>
<td>15</td>
<td>0.0250</td>
<td>85</td>
</tr>
<tr>
<td>$X_7$</td>
<td>2.2000</td>
<td>410</td>
<td>2.0000</td>
<td>3.0000</td>
<td>8.0000</td>
<td>3.0000</td>
<td>0.7000</td>
<td>10</td>
<td>0.0500</td>
<td>85</td>
</tr>
</tbody>
</table>

Example 2. In this example, seven surface-to-air missile weapon system alternatives $X_1, X_2, \ldots, X_7$ will be evaluated by grey TOPSIS method in the following. The multiattribute weapon system is evaluated from the combination of traditional grey relational analysis and TOPSIS in reference paper [27], but no convincing calculation index weight is given. In order to obtain the best selection of weapon systems, [28] developed fuzzy analytic hierarchy process by entropy weight to evaluate the weight. Metin, Serkan, and Nevzat [29] utilized AHP and TOPSIS to assess weapon systems. AHP is more difficult because of the comparison between the two factors, then this paper from the distribution of different indicators to consider the dispersion degree and facilitate the given weight. A good surface-to-air missile weapon system depends on a lot of attributes, such as the maximum speed of missile ($M_{\text{max}}$), the maximum speed of target ($M'_{\text{max}}$), the maximum overload of target ($O_{\text{max}}$), the highest boundary of killing range ($H_{\text{max}}$), the farthest boundary of killing range ($R_{\text{max}}$), the number of targets that can simultaneously be shot by one weapon system ($N$), the single shot kill probability of missiles ($P$), the reaction time of missile weapon system ($T$), the lowest boundary of killing range ($H_{\text{min}}$), the launching weight of missiles ($G_{0}$), and the nearest boundary of killing range ($R_{\text{min}}$) as listed in Table 2 [27]. Based on the above facts, the optimal selection of multiple weapons is very important and complicated for evaluation and assessment of weapon system.

Solution

Step 1. The normalized decision matrix $Y = (y_{ij})_{n \times m}$ can be calculated by (7) and (8) as follows:

$$Y = \begin{bmatrix}
1.0000 & 0.1842 & 1.0000 & 0.8750 & 1.0000 & 1.0000 & 0.5000 & 0.6667 & 0.7179 & 0.6004 & 0.6667 \\
0.8750 & 1.0000 & 0.8000 & 1.0000 & 0.8913 & 1.0000 & 1.0000 & 0.8333 & 1.0000 & 0.6004 & 0.4000 \\
0.6000 & 0.4211 & 0.8000 & 1.0000 & 0.7283 & 0.7143 & 0.6000 & 0.6667 & 1.0000 & 0.3105 & 0.4000 \\
0.2500 & 0.0105 & 0 & 0.8958 & 0.2609 & 0 & 0.5000 & 0 & 0 & 0 & 0 \\
0 & 0 & 0.2000 & 0.0833 & 0.0435 & 0.2857 & 0.5000 & 1.0000 & 0.5128 & 0.9410 & 0.9333 \\
0.0500 & 0.2000 & 0 & 0 & 0.2857 & 1.0000 & 0.9867 & 0.9744 & 1.0000 & 0.9333 & 0.9333 \\
0.0500 & 0.0053 & 0.2000 & 0 & 0 & 0.2857 & 0 & 1.0000 & 0.9744 & 1.0000 & 1.0000 
\end{bmatrix}$$

Step 2. Use (9) and (10) to determine the optimal alternative $\bar{Y}$ and worst alternative $\bar{Y}$.

$$\bar{Y} = (1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1), \quad \bar{Y} = (0, 0, 0, 0, 0, 0, 0, 0, 0, 0, 0).$$
Step 3. The average distance and distance standard deviation are calculated, respectively, by (11)-(14).

\[ M_1 = (0.5964, 0.7684, 0.5429, 0.4494, 0.5823, 0.4898, 0.4143, 0.2638, 0.2601, 0.3639, 0.3810), \]
\[ M_2 = (0.4036, 0.2316, 0.4571, 0.5506, 0.4177, 0.5102, 0.5857, 0.7362, 0.7399, 0.6361, 0.6190), \]
\[ S_1 = (0.4189, 0.3733, 0.3952, 0.4921, 0.4422, 0.3943, 0.3436, 0.3566, 0.3752, 0.3810, 0.3706), \]
\[ S_2 = (0.4679, 0.6896, 0.4059, 0.5041, 0.3566, 0.3943, 0.3436, 0.3566, 0.3752, 0.3810, 0.3706), \]

(31)

Step 4. To determine the indicator weights,

\[ \bar{w} = (0.0849, 0.0952, 0.0900, 0.0722, 0.0804, 0.0902, 0.1035, 0.0997, 0.0947, 0.0933, 0.0959), \]
\[ w = (0.0942, 0.0639, 0.1086, 0.0874, 0.0925, 0.1116, 0.1129, 0.0708, 0.0689, 0.0916, 0.0977), \]

(32)

where \( \bar{w} = (\bar{w}_1, \ldots, \bar{w}_{11}), w = (w_1, \ldots, w_{11}) \).

Step 5. The grey positive relational degree \( r^+_{ij} \) and grey negative relational degree \( r^-_{ij} \) can be obtained by (15) and (16), \( i = 1, 2, \ldots, 7; j = 1, 2, \ldots, 11 \). Then, we can get the grey positive relational grade \( R^+_i \) and grey negative relational grade \( R^-_i \).

\[ R^+_1 = 0.6370, \]
\[ R^+_2 = 0.7606, \]
\[ R^+_3 = 0.5232, \]
\[ R^+_4 = 0.2856, \]
\[ R^+_5 = 0.4550, \]
\[ R^+_6 = 0.5826, \]
\[ R^+_7 = 0.5248, \]
\[ R^-_1 = 0.3835, \]
\[ R^-_2 = 0.3510, \]
\[ R^-_3 = 0.4171, \]
\[ R^-_4 = 0.8114, \]
\[ R^-_5 = 0.6250, \]
\[ R^-_6 = 0.6078, \]
\[ R^-_7 = 0.6838. \]

(33)

Step 6. At last, the comprehensive grey TOPSIS evaluation results can be derived as follows:
The comparison between the presented method and reference method.

<table>
<thead>
<tr>
<th>quantization results</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_3$</th>
<th>$R_4$</th>
<th>$R_5$</th>
<th>$R_6$</th>
<th>$R_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>The presented method</td>
<td>0.624</td>
<td>0.684</td>
<td>0.556</td>
<td>0.260</td>
<td>0.421</td>
<td>0.489</td>
<td>0.434</td>
</tr>
<tr>
<td>The reference method [27]</td>
<td>0.638</td>
<td>0.716</td>
<td>0.586</td>
<td>0.281</td>
<td>0.425</td>
<td>0.474</td>
<td>0.431</td>
</tr>
<tr>
<td>TOPSIS</td>
<td>0.689</td>
<td>0.787</td>
<td>0.632</td>
<td>0.274</td>
<td>0.442</td>
<td>0.497</td>
<td>0.451</td>
</tr>
</tbody>
</table>

The ranking results of the comprehensive grey TOPSIS evaluation in this paper.

<table>
<thead>
<tr>
<th>$M_{\text{max}}$</th>
<th>$M'_{\text{max}}$</th>
<th>$Q_{\text{max}}$</th>
<th>$H_{\text{max}}$</th>
<th>$R_{\text{max}}$</th>
<th>$N$</th>
<th>$P$</th>
<th>$T$</th>
<th>$H_{\text{min}}$</th>
<th>$G_0$</th>
<th>$R_{\text{min}}$</th>
<th>evaluation results</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>Drop $X_7$ out of alternative set</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$X_1$</td>
<td>1</td>
<td>0.1842</td>
<td>1</td>
<td>0.875</td>
<td>1</td>
<td>0</td>
<td>0.6667</td>
<td>0.7179</td>
<td>0.6004</td>
<td>0.7143</td>
<td>0.5557</td>
<td>2</td>
</tr>
<tr>
<td>$X_2$</td>
<td>0.875</td>
<td>1</td>
<td>0.8</td>
<td>1</td>
<td>0.8913</td>
<td>1</td>
<td>1</td>
<td>0.8333</td>
<td>1</td>
<td>0.6004</td>
<td>0.4286</td>
<td>0.6387</td>
</tr>
<tr>
<td>$X_3$</td>
<td>0.6</td>
<td>0.4211</td>
<td>0.8</td>
<td>1</td>
<td>0.7283</td>
<td>0.7143</td>
<td>0.2</td>
<td>0.667</td>
<td>1</td>
<td>0.3105</td>
<td>0.4286</td>
<td>0.4982</td>
</tr>
<tr>
<td>$X_4$</td>
<td>0.25</td>
<td>0.0105</td>
<td>0</td>
<td>0.8958</td>
<td>0.2609</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0</td>
<td>0.2339</td>
<td>0.4982</td>
</tr>
<tr>
<td>$X_5$</td>
<td>0</td>
<td>0</td>
<td>0.2</td>
<td>0.0833</td>
<td>0.0435</td>
<td>0.2857</td>
<td>0</td>
<td>1</td>
<td>0.5128</td>
<td>0.941</td>
<td>1</td>
<td>0.3098</td>
</tr>
<tr>
<td>$X_6$</td>
<td>0.05</td>
<td>0</td>
<td>0.2</td>
<td>0</td>
<td>0.2857</td>
<td>0.9867</td>
<td>0.9744</td>
<td>1</td>
<td>1</td>
<td>0.4698</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Add a new alternative $X_8$ to the original set

| $X_1$              | 1                 | 0.1842            | 1                 | 0.875             | 1   | 0  | 0.6667 | 0.7179           | 0.6004 | 0.6667           | 0.6445            | 2   |
| $X_2$              | 0.875             | 1                 | 0.8               | 1                 | 0.8913 | 1   | 1   | 0.8333 | 1       | 0.6004           | 0.4              | 0.6733 | 1 |
| $X_3$              | 0.6               | 0.4211            | 0.8               | 1                 | 0.7283 | 0.7143 | 0.6  | 0.667  | 1       | 0.3105           | 0.4              | 0.5467 | 3 |
| $X_4$              | 0.25              | 0.0105            | 0                 | 0.8958            | 0.2609 | 0   | 0   | 0     | 0       | 0                | 0.2577            | 0.5467 | 3 |
| $X_5$              | 0                 | 0                 | 0.2               | 0.0833            | 0.0435 | 0.2857 | 0.5  | 1     | 0.5128 | 0.941            | 0.9333           | 0.4314 | 7 |
| $X_6$              | 0.05              | 0                 | 0.2               | 0                 | 0.2857 | 1   | 0.9867 | 0.9744 | 1      | 1                | 0.4301           | 0.4794 | 5 |
| $X_7$              | 0.05              | 0.0053            | 0.2               | 0                 | 0.2857 | 0   | 1   | 0.9744 | 1      | 1                | 0.4940           | 0.4794 | 6 |
| $X_8$              | 0.05              | 0                 | 0.22              | 0                 | 0.3571 | 1   | 1   | 0.9744 | 1      | 1                |                  | 0.4940 | 4 |

In the second case, the weights $\overline{w}_j$ and $\overline{w}_j$ for the grey positive relational degree and grey negative relational degree can be determined as follows:

$$\overline{w} = (0.0884, 0.0955, 0.0908, 0.0803, 0.0855, 0.0900, 0.0758, 0.1019, 0.0953, 0.0994, 0.0971),$$

$$w = (0.0907, 0.0916, 0.0910, 0.0895, 0.0903, 0.0909, 0.0886, 0.0922, 0.0915, 0.0920, 0.0917).$$

In the second case, the weights $\overline{w}_j$ and $\overline{w}_j$ for the grey positive relational degree and grey negative relational degree can be determined as follows:

$$\overline{w} = (0.0849, 0.0974, 0.0922, 0.0698, 0.0795, 0.0937, 0.0987, 0.1008, 0.0969, 0.0921, 0.0939),$$

$$w = (0.0923, 0.0642, 0.1150, 0.0940, 0.0896, 0.1264, 0.1022, 0.0697, 0.0690, 0.0865, 0.0911).$$

The above weights are two sets of weights given after considering both positive and negative ideal solutions, which can be adjusted automatically according to the characteristics of data distribution.

Based on Example 2, the ranking results of the original alternative set are

$$R_2 > R_1 > R_3 > R_6 > R_7 > R_5 > R_4.$$
Similarly, we can obtain the following results with dropping $X_7$ out of alternative set:

$$R_2 > R_1 > R_3 > R_6 > R_5 > R_4.$$  

(38)

Then for the last case, we can receive the following ranking:

$$R_2 > R_1 > R_3 > R_8 > R_6 > R_7 > R_5 > R_4.$$  

(39)

It is not difficult to find that the ranking of the original alternatives is unchanged under the two different circumstances, so there is no rank reversal. We believe that no rank reversal is a normal phenomenon, so the novel TOPSIS method in the paper is more in line with people’s thinking logic.

Since the improved TOPSIS method in this paper has two sets of attribute weights during the evaluation process, the weights can be dynamically adjusted with the change of data distribution density no matter in reducing the evaluation objects or increasing the evaluation objects. The final result of the evaluation presented to us is that the original data will not have the phenomenon of rank reversal. Correspondingly, the traditional TOPSIS method is prone to rank reversal because the attribute weights cannot be adjusted dynamically. The specific information under the two different circumstances is shown in Table 4.

5. Conclusions

The traditional TOPSIS method is limited because it only takes the distance between the data into account, without introducing the degree of data dispersion. In addition, the classical grey relational analysis method does not study the aggregation and dispersion degree of different series of data. Based on this, a novel grey TOPSIS method developed from an improved grey relational analysis and a new indicator weight determination method, which well evaluated different objects. The improved grey relational analysis method makes full use of the discrete degree of the different sequence data and considers the overall distribution of the data, so as to avoid the shortage of the maximum and minimum distance of data. At the same time, the concept of sample distance standard deviation is proposed to solve the problem of different indicator weights determination. And there examples have successfully tested the effectiveness of the proposed method. Especially, the ranking of multiple weapons with MADM typical features verifies the feasibility for practical MADM problem. In the process of solving MADM, the present method in this paper overcomes the rank reversal problem of classical TOPSIS. It is worth noting that the proposed TOPSIS method depends on quantitative test data. As a future work, qualitative data and semantic fuzziness will also be introduced into the improved TOPSIS method to solve more complex multiattribute decision problems. In addition, we will reveal the necessity of introducing grey relational analysis into MADM problems further.

Data Availability

Data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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References


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