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Research Article

Modeling the Energy Harvested by an RF Energy Harvesting System Using Gamma Processes

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In an Energy Harvesting system (EHS) the gamma process is used to model the electromagnetic energy received from radiofrequency (RF) radiation. The stochastic characterization of the harvested energy as a continuous-time stochastic process, namely, gamma process, is obtained from the Nakagami-m fading model, which describes the signal reception in a large amount of types of radiofrequency channels. Using the gamma process, some performance measures of the EHS system are obtained. Also, a transmission policy subject to different fading conditions is considered.

1. Introduction

The energy present in the environment, from sources as wind, solar energy, vibration, and electromagnetic radiofrequency (RF), can be harvested to power systems as low and ultralow power electronics and sensor networks [1]. Nowadays, Energy Harvesting Systems (EHS) are emerging as a promising technology for charging batteries used to supply ultra low power micro controllers, sensors, and wearable systems [2].

In an EHS, the energy is incoming from an exogenous source and is managed using two points of view

(i) Harvest-store-use (HSU) where a battery stores the harvested energy before its future use. It requires an energy storage device or battery along with an appropriate management of the stored energy.

(ii) Harvest-use (HU) where the harvested energy is not stored and it is directly consumed.

Many works in the literature deal with the HSU architectures due to their interest in commercial applications. Although HU architecture literature is not very common, it is a suitable technology for limited battery systems. For a HSU system, two approaches have been developed taking into account the capacity of storage of the battery: infinite and finite capacity. Assuming infinite capacity means that the harvested energy can always be stored in the battery. This infinity assumption is valid for devices with large storage capacity compared to the incoming energy [3]. On the other hand, for a device with finite capacity, the harvested energy can be stored up to a limit. Beyond this limit, incoming energy cannot be stored and would be lost [4].

From the system theoretical perspective, the harvested energy can be modeled as a deterministic or a stochastic process. In a deterministic model, the incoming energy and its fluctuation are known in advance. These models are suitable for energy sources with predictable or with low fluctuation power sources. For example, assuming no cloud coverage, the sun can be modeled as a deterministic source of energy.

In a RF energy harvesting system, the energy is captured from the electromagnetic spectrum, from sources as radio FM, wireless radio (for example, Wi-Fi communication systems, mobile communications systems), TV broadcasting, or cognitive wireless sensor systems [5, 6]. One of the advantages of the RF harvesting is that the energy is ubiquitous and permanently available; in contrast, energy levels are low, except in the surroundings of the transmitter, because the power of the electromagnetic wave is in inverse proportion to the square of the distance to the transmitter [7]. Other kinds of ambient sources for harvesting as mechanical, thermal, or acoustics are nor ubiquitous but with levels of energy greater than electromagnetic in many cases [8].

In practice, the amount of energy that can be harvested from the environment is generally a random quantity [9]. For example, when RF harvested energy is used, the harvested energy is randomly distributed since it depends on the channel fading [10]. Channel fading is a random adverse phenomenon that affects the signal and energy reception due to multipath and shadowing. Some authors have characterized the RF power received by an energy harvester node assuming a specific distribution for the fading. For example, if the fading follows a Rayleigh distribution and assuming Gamma shadowing, and the RF received power a the antenna can be modelled using a sum of gamma distributions [11].

In this paper, we focus on the distribution probability for the energy harvested. In the literature, the stochastic models used to describe the energy harvested by the system include Poisson processes [12], Markov models [13], or compound Poisson processes [14, 15].

For a compound Poisson process, energy arrivals occur at random times (following a Poisson process) and in random amounts. Using a compound Poisson process model, the total harvested energy at time t can be expressed as

$$Y(t) = \sum_{i=1}^{N(t)} D_i, \tag{1}$$

where $N(t)$ represents the number of energy arrivals at time t and D_i the harvested energy at the i-th arrival. Under the HSU architecture, to characterize the storage model in the battery, some authors use the so-called compound Poisson dam storage process [14, 15]. So, the energy stored in the battery at time t is given by

$$X(t) = X(0) + \sum_{i=0}^{N(t)} D_i - \int_0^t p(X(s))\, ds, \tag{2}$$

where $\int_0^t p(X(s))ds$ represents the total energy expenditure until time t being $p(\cdot)$ the power delivered. In the case of a finite capacity battery E_K ($E_K < \infty$), expression (2) incorporates a process $Z(t)$ ($Z(0) = 0$) called reflection process, that it is null at zero, nondecreasing, continuous almost everywhere as follows:

$$X(t) = X(0) + \sum_{i=0}^{N(t)} D_i - \int_0^t p(X(s))\, ds - Z(t). \tag{3}$$

This reflection process ensures that the storage process does not exceed the threshold E_K for any energy arrival,

$$\int_{R^+} (E_K - X(s))\, dZ(s). \tag{4}$$

These stochastic models assume a discrete arrival scheme for the incoming energy (more discrete models of energy arrival can be found in [3, 12]). However, energy harvesting devices receive their energy from external nondiscrete fluctuating sources (such as radio waves) and a continuous model would be a better model when these sources are used. Despite this fact, there are very limited results for the energy harvesting systems with continuous energy arrival except from a few papers such as [16, 17].

Considering a continuous scheme for the energy arrival, in this paper we propose a stochastic process to model the energy harvested by an RF harvesting system. This stochastic process is the gamma process. The reason to choose this process is based on electromagnetic propagation statistical models and its use is justified in the paper. When the electromagnetic RF channel is dominated by nonline-of-sight (LOS) propagation, the energy stored in the battery can be modeled accurately by a gamma process.

Before defining a gamma process, the definition of gamma distribution is first recalled. We say that a random variable X follows a gamma distribution with parameters (α, β) if its probability density function is given by

$$f_X(x) = \frac{\beta^\alpha}{\Gamma(\alpha)} e^{-\beta x} x^{\alpha-1}, \quad x > 0, \tag{5}$$

where $\Gamma(\alpha)$ is the gamma function given by

$$\Gamma(\alpha) = \int_0^\infty x^{\alpha-1} e^{-\alpha x} dx. \tag{6}$$

In mathematical terms, an homogeneous gamma process with shape parameter α and scale parameter β is a continuous-time stochastic process $\{X(t),\ t \geq 0\}$ with the following properties:

(i) $P(X(0) = 0) = 1$.

(ii) Increments $X(t+s) - X(t)$ are independent for all t and s.

(iii) The increments $X(s+t) - X(s)$ follow a gamma distribution with parameters αt and β.

A gamma process is a pure-jump increasing Lévy process mainly used to model the deterioration of a system over the time [18]. Other examples of the application of gamma processes are the theory of water storage by dams and the risk theory [19]. The gamma process can be thought as a compound Poisson process in which the Poisson rate tends to infinity while the sizes of the increments tend to zero in proportion [20]. Although the compound Poisson process has been used to model the harvested energy process, as far as the authors are concerned, the gamma process has not been used to model the harvested energy. An advantage of using this gamma process in the stochastic modeling is that the required mathematical calculations are relatively straightforward. We again emphasize that the harvested energy process is continuous rather than slotted [21].

The general framework of this paper is the following. The use of the gamma process to model the harvested energy is justified. Afterwards, this process models the harvested energy by an EHS. This EHS contains an antenna that gathers energy from the environment, a rectifier, and a battery with finite capacity to store the harvested energy. Furthermore, a device is powered by the battery. Due to the finite capacity of the battery, energy may overflow without being utilized and hence is wasted. To model the energy harvested by the system, a gamma process is used. The battery delivers to the device a deterministic quantity of energy at certain instants of time that we call feeding times. Denoting by $\tau_1, \tau_2, \ldots$ the successive feeding times, we assume a uniform scheduling policy for the feeding times, that is, $\tau_{i+1} - \tau_i = T > 0$ for each $i = 1, 2, \ldots$.

Some constraints are imposed to this system.

(1) The energy stored in the battery at time t, $E_B(t)$, is limited by the battery capacity E_K (that is, $E_B(t) \leq E_K$, $\forall t$).

(2) The energy drawn from the battery can not exceed the energy stored in the battery (*Energy Casualty*).

(3) When the system detects that its stored energy is going to drop below a certain threshold, the feeding process to the device stops. This threshold corresponds to the minimum energy reserve of the battery that can not be consumed. Denoting by E_M this threshold, the stored energy in the battery at time t fulfills $E_M \leq E_B(t) \leq E_K$ for all t.

Under this general framework, a transmission policy is implemented. Renewal process theory is used to analyze the optimal transmission policy.

The objectives of this paper and its main contributions are the following.

(i) Firstly, the use of the gamma process as a probabilistic tool to model the energy harvested by a RF harvesting system is justified. Since the gamma process is a continuous stochastic process, the continuous scheme for the energy arrival in EHS system is extended. Up to our knowledge, there are not works in the literature that deal with the use of the gamma process to model the harvested energy. It is, therefore, the main novelty and contribution of this paper.

(ii) Secondly, to establish an optimal transmission policy to optimize the functioning of the RF harvesting system. For that, probabilistic efficiency measures are obtained. Another novelty of the paper is the approach used to deal with the transmission policy. In this work, renewal techniques are used to find the optimal transmission policy, in particular, the renewal-reward theorem. Although the renewal techniques are not new in the harvesting system literature [22, 23], the use of the renewal reward theorem to achieve a transmission policy is not a common practice in energy harvesting system literature (except some particular cases as [24]).

To obtain these objectives, the paper is structured as follows. In Section 2, the stochastic process used to model the harvested energy is described and its use is justified. Section 3 describes the main assumptions of the system functioning. Sections 4 and 5 obtain probabilistic efficiency measures related to the system functioning to analyze the optimal transmission policy for this system. Section 6 shows some numerical examples associated with the system and finally Section 7 concludes.

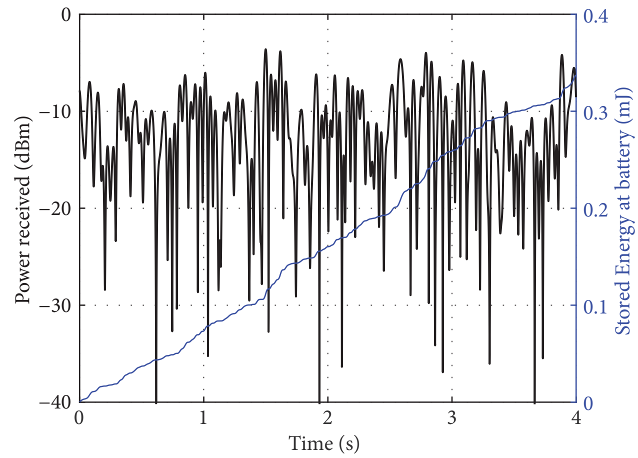

Figure 1: Received power and energy stored by a battery in a simulation of a Rayleigh fading channel environment.

2. Harvested Energy Model as a Gamma Process

In this section, we shall analyze the gamma process as a feasible tool to model the energy harvested by an EHS located in a RF fading channel environment. To obtain a first practical confirmation of this hypothesis, the storage of the energy received by an antenna in a Rayleigh fading environment has been simulated using the Matlab *Communication System Toolbox*™. The channel is modeled with the command `channel = rayleighchan (Ts, FD)` with `Ts` the sampling period equals to 100 μs and `FD` the maximum Doppler shift of 15 Hz that corresponds to a frequency-flat Rayleigh fading channel. We establish a mean received power at antenna output of 80 μW. The instantaneous received power is harvested by a RF rectifier with no efficiency loss obtaining the energy harvested energy at time t as

$$E_H(t) = \int_0^t P_r(\tau)\, d\tau, \tag{7}$$

where $P_r(\tau)$ denotes the instantaneous received power at time τ.

In Figure 1, the simulated received power and the harvested energy by the system in the first $4s$ are represented. This harvested energy can be easily interpreted as the realization of an increasing stochastic process. In particular, we shall show that the harvested energy can be modeled by a gamma process. To validate this assumption, we have performed goodness-of-fit tests to compare the distribution of the sampling increments $E_H(iT) - E_H((i-1)T)$ for different values of T with a gamma distribution of parameters αT and β being $\alpha^{-1} = 35\ ms$ and $\beta^{-1} = 2.8\ \mu J$. These statistical goodness-of-fit tests characterize the discrepancy between the simulated harvested energy and the expected values under a theoretical gamma distribution [25]. The Kolmogorov-Smirnov test is

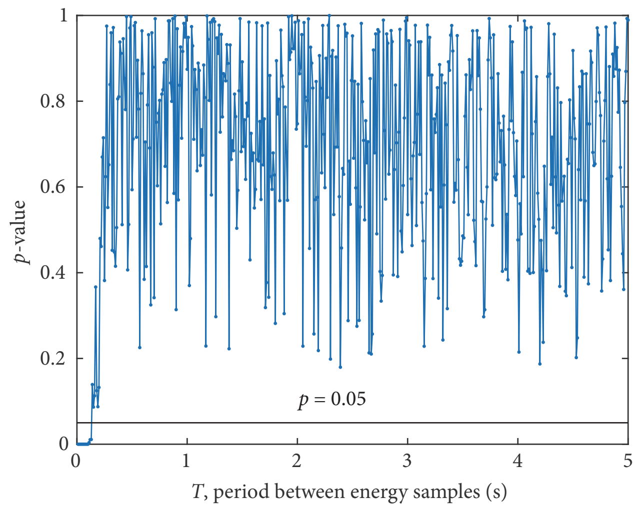

FIGURE 2: P-values of the Kolmogorov-Smirnov test for the increments in the energy stored in a battery by channel simulation.

used to validate the gamma distribution of the increments. The p-values of the test are represented in Figure 2. As we can see, Figure 2 shows how well the gamma distribution fits the observed data for $T \geq 140\ \mu s$ (approximately 4 times the value of α^{-1} or 2 times the value of the inverse of the maximum Doppler shift). In general, if the increments of time are large compared to the length of the fading, we can consider that the harvested and stored energy can be modeled by a gamma process. In practice T_0, the value of the increment of T from which the simulated process fits as a gamma process is proportional to the correlation length of the fading. In any case, the length of the fading is tiny enough (usually of order of milliseconds) compared to the order of magnitude of the times in which the EHS supplies energy to other devices. Consequently, considering fine and coarse time increments, harvested and stored energy is well modeled by a gamma process except for very fine time requirements.

Nakagami-m fading model is a very general model which fits a lot of different empirical measurements for signal reception levels in different multipaths and multisignal reception. This distribution includes and generalizes Rayleigh fading (as the distribution used in the previous example) used for nonline-of-sight propagation models (NLOS). It also approximates well other set of channel variations dominated by line-of-sight (LOS) propagation as Rician fading. The density function of the Nakagami-m model with parameters m and Ω is

$$f(x) = 2\left(\frac{m}{\Omega}\right)^{m} \frac{x^{2m-1}}{\Gamma(m)} \exp\left(-\frac{mx^2}{\Omega}\right), \tag{8}$$

for $x \geq 0$, where $\Gamma(m)$ is the gamma function given by (6). The Nakagami-m distribution has two parameters, a shape parameter $m \geq 1/2$ and a second parameter $\Omega > 0$ controlling the spread. It is well known that if Y follows a Nakagami-m distribution with parameters m and Ω, Y^2 follows a gamma distribution with parameters $(\alpha, \beta) = (m, \Omega/m)$ [26].

Since the received power is proportional to the square of the signal amplitude, assuming a signal amplitude modeled as a Nakagami-m distribution, the received power P_r at time t is gamma distributed with parameters (α_P, β_P) with $\beta_P = \alpha_P/\overline{P_r}$ being $\overline{P_r}$ the mean received power.

Assuming that there is not efficiency loss, the harvested energy by the EHS at time t is given by

$$E_H(t) = \int_0^t P_r(\tau)\, d\tau, \tag{9}$$

where $P_r(\tau)$ corresponds to the received power at time τ. In the sequel of this work, we denote by $\{E_H(t),\ t \geq 0\}$ the harvested energy process.

2.1. $\{E_H(t),\ t \geq 0\}$ as a Gamma Process. Let $t_1, t_2, \ldots, t_n$ be a partition of $P_r(\tau)$ in $[0, t]$ as follows:

$$\begin{aligned} n &= \left\lfloor \frac{t}{\Delta w} \right\rfloor, \\ t_i &= \left(i - \frac{1}{2}\right) \Delta w, \end{aligned} \tag{10}$$

where $\lfloor \cdot \rfloor$ denotes the floor function. Considering this partition we approximate $E_H(t)$ as

$$E_H(t) = \int_0^t P_r(\tau)\, d\tau \cong \sum_{i=1}^{n} P_r(t_i)\, \Delta w. \tag{11}$$

Since $P_r(t_i)$ follows a gamma distribution with parameters α_P and β_P, then the approximation of $E_H(t)$ given by (11) also follows a gamma distribution with parameters αt and β with

$$\begin{aligned} \alpha &= \frac{\alpha_P}{\Delta w}, \\ \beta &= \frac{\beta_P}{\Delta w} \end{aligned} \tag{12}$$

since the sum of independent random and gamma distributed variables with the same scale parameter is also gamma distributed (*gamma additivity property*). Notice that

$$\overline{P_r} = \frac{\alpha}{\beta} = \frac{\alpha_P}{\beta_P}. \tag{13}$$

Obviously, in this approximation, the parameter α of the gamma process depends on the length Δw of discretization of (11), which looks inconsistent. Nevertheless, neither small Δw nor large Δw can be used. Firstly, large values of Δw might invalidate the Simpson rule in (11). Also, small values of Δw might imply that $P_r(t_i)$ and $P_r(t_{i+1})$ are correlated and the assumption of independence in the gamma additivity property is violated. For small intervals of time, the fading of the channel is highly correlated; otherwise, the independence assumption can be assumed as true. The exact computation of α is out of the scope of this paper. In general, α^{-1} is inversely proportional to the shape factor m of the Nakagami-m fading channel and directly proportional to the correlation length of the fading (because the length of the fading is strongly

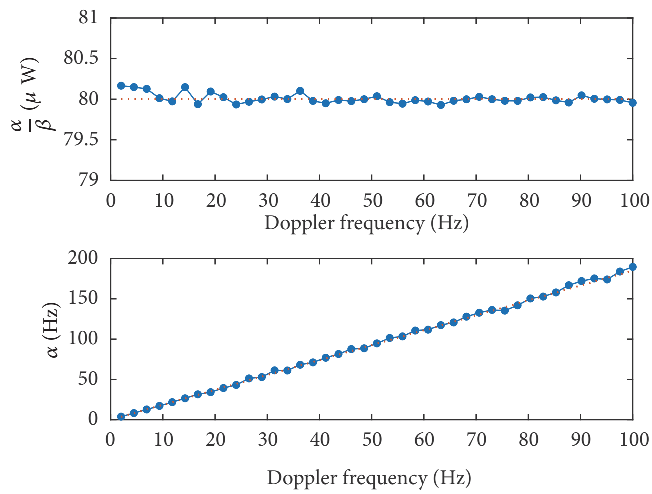

FIGURE 3: Relationship between gamma process parameters and Doppler frequency shift in a Rayleigh channel environment.

related to the best Δw). In the example shown in Figures 1 and 2, we obtained the values of $\overline{P_r} = \alpha/\beta = 80\ \mu W$, that is, equal to the power reception provided in the simulation and $\alpha = (35ms)^{-1} \simeq 28.6$ Hz which is approximately the double of the maximum Doppler frequency shift. Since the fading time variations are very fast compared to the orders of magnitude of time used in the functioning of the harvesting system, we can consider that the discrete approximation in (11) can be used in continuous mode without loss of accuracy. In fact, for the example given in Figure 2, we showed that, for intervals length bigger than 140 ms, the hypothesis of gamma process fits well.

For the same received power at antenna output, in a Rayleigh Channel environment, simulations have been performed to obtain an empiric relation between the parameters α and β of the gamma process (used to model the harvested energy) and the fading length. In the case of the Rayleigh Channel simulation provided by Matlab, the Doppler frequency shift, `FD`, is the parameter which is inversely proportional to the fading length. Figure 3 shows the α and β parameters estimated for the gamma process versus the Doppler shift varying from 2 to 100 Hz. A total of 5000 simulations of 10s of time length have been performed for each Doppler shift frequency. In Figure 3, we can see that α/β is actually equal to the mean power received (80 μW) independently of the Doppler shift and the α parameter varies linearly with the Doppler frequency shift, approximately $\alpha =$ 1.85 `FD`.

Therefore, for a Nakagami-m or a Rayleigh RF channel, the process $\{E_H(t),\ t \geq 0\}$ is modeled using an homogeneous gamma process that fulfills the following properties:

(i) $P(E_H(0) = 0) = 1$.

(ii) Increments $E_H(T+s) - E_H(T)$ are independent for all T and s.

(iii) The harvested energy in an interval of amplitude T, $E_H(s+T) - E_H(s)$, follows a gamma distribution with parameters αT and β with density

$$f_{E_H(T)}(x) = \frac{\beta^{\alpha T}}{\Gamma(\alpha T)} x^{\alpha T - 1} \exp(-\beta x), \tag{14}$$

for $x > 0$ where we recall that $\overline{P_r} = \alpha/\beta$ corresponds to the mean received power.

The mean level of harvested energy and the variance in an time interval of amplitude T is given by

$$\begin{aligned} \mathbb{E}[E_H(T)] &= \overline{P_r} T, \\ \operatorname{var}(E_H(T)) &= \alpha^{-1} T \overline{P_r}^2. \end{aligned} \tag{15}$$

In the remainder of this work, the expectation operator $\mathbb{E}(\cdot)$ is used to avoid confusion between energy and expectation. So, we have a simplified model for the harvesting process using a continuous cumulative stochastic process.

3. System Description

We assume a harvesting system which contains an antenna, a RF rectifier and a battery to store the harvested energy. The battery powers a device. The system functioning is described as follows:

(1) We suppose that no efficiency loss is present in the RF rectifier and the battery.

(2) The fading of the channel is modeled using a Nakagami-m distribution with density given in (8).

(3) Let $E_H(t)$ be the harvested energy at the output of the rectifier at time t, and $\{E_H(t),\ t \geq 0\}$ follows an homogeneous gamma process with parameters α and β, where the ratio $\alpha/\beta = \overline{P_r}$ corresponds to the mean received power.

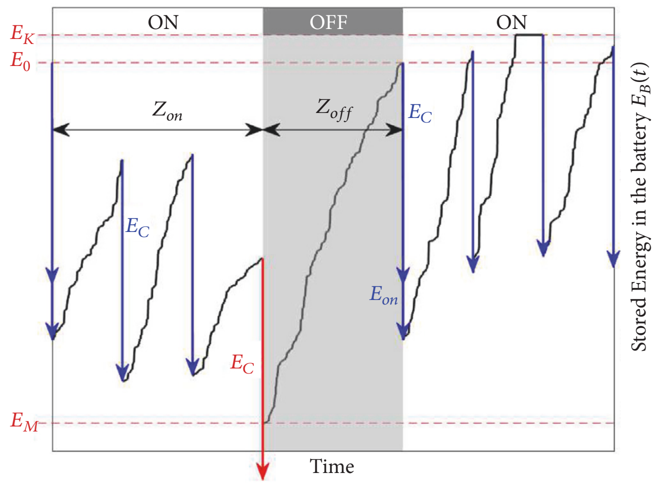

Figure 4: Evolution of the energy stored in the battery due to the operation of the EHS. Blue arrows correspond to feasible feeding tasks. Red arrows correspond to unfeasible feeding tasks.

(4) Let $E_B(t)$ be the stored energy in the battery at time t with $E_B(0) = E_0$, where E_0 denotes the initial stored energy. For operational reasons, the stored energy in the battery must always exceed a threshold E_M.

(5) We assume that the capacity of the battery is finite and equals to E_K, hence

$$E_M \leq E_B(t) \leq E_K, \quad \forall t. \tag{16}$$

The battery wear is assumed to be much slower than the time scale of the problem.

(6) The device can be in two modes: ON and OFF depending on its *feeding process*. In ON mode, the device is operational and powered by the battery. In OFF mode the device is turned off and the feeding process of the device stops.

(7) *Feeding times*. At certain instantaneous times named *feeding times* the device consumes E_C units of energy performing, for example, an instantaneous measurement. Let $T_0, T_1, \ldots, T_n$ be the feeding times. We consider a block-based policy for the feeding times where they are scheduled periodically during an ON state with $T_i - T_{i-1} = T > 0$ for $i = 1, 2, \ldots$. We assume a large enough value of T to consider that the gamma process model for the harvested energy is satisfied.

(8) The standby consumption of the device is negligible compared to the periodic consumption.

(9) *Device turn on*. At the time of the turn on, the device needs to perform transitory tasks that consume E_{on} energy units. The turn on instant is also considered a feeding time. At short, there is an instantaneous requirement of energy of $E_{on} + E_C$ units in the turn on of the device. We assume that

$$E_0 \geq E_C + E_{on} + E_M, \tag{17}$$

i.e., the device turn on is ensured.

(10) Let $E_B(T_n^-)$ be the stored energy in the battery right before the feeding time T_n for $n = 1, 2, \ldots$

(i) If $E_B(T_n^-) - E_C \geq E_M$, then the device is properly powered and it is called a *feasible feeding task*.

(ii) If $E_B(T_n^-) - E_C < E_M$, the feeding of the device starts but, when the system detects that the stored energy is going to drop below E_M, the feeding process is stopped. In this case, the device is not properly powered and it is called an *unfeasible feeding task*. Furthermore, there is a waste of $E_B(T_n^-) - E_M$ energy units. Right after an *unfeasible feeding task*, the energy stored in the battery is equal to the threshold E_M and the device enters in OFF mode until the energy stored in the battery exceeds again the level E_0 (see Figure 4).

Denoting by $E_B(T_n^+)$ the energy stored in the battery right after T_n, we get that

$$E_B(T_n^+) = \max\left(E_B(T_n^-) - E_C, E_M\right). \tag{18}$$

Notice that condition $E_M = 0$ implies that as long as there is energy in the battery, the device remains in ON mode.

Figure 4 shows a realization of the process $\{E_B(t), t \geq 0\}$ that represents the energy stored in the battery. Initially, the energy stored in the battery is equal to E_0 units and the device consumes $E_{on} + E_C$ energy units. Furthermore, the device consumes E_C units of energy each T units of time whenever the device is in ON mode. In red color, an *unfeasible feeding task* is shown since the energy stored in the battery was not enough to feed the device fulfilling the energy constraint given in (16). Right after an *unfeasible feeding task*, the OFF period starts and it remains in this mode until the energy stored in the battery exceeds the threshold E_0.

At the activation of the device, its consumption is equal to $E_{on} + E_C$. Assuming an activation at $t = 0$, we get that

$$E_B(0^+) = E_0 - E_{on} - E_C. \tag{19}$$

To fulfill the constraint given in (16), E_0 has to verify

$$E_M + E_{on} + E_C < E_0. \tag{20}$$

This lower limit is imposed by (17). Whenever (16) is fulfilled in $[0, t]$, the total energy consumed by the device due to the feeding tasks at time t, $E_D(t)$, is equal to

$$E_D(t) = \left(\left\lfloor \frac{t}{T} \right\rfloor + 1\right) E_C, \tag{21}$$

and the total energy stored in the battery at time t can be expressed as

$$E_B(t) = E_0 - E_{on} + E_H(t) - E_D(t), \tag{22}$$

with expectation

$$\mathbb{E}(E_B(t)) = E_0 - E_{on} + \overline{P_r} t - \left(\left\lfloor \frac{t}{T} \right\rfloor + 1\right) E_C. \tag{23}$$

Let $\{E_H^{(i)}(t),\ t \geq 0\}$ be replicas of the process $\{E_H(t),\ t \geq 0\}$ for $i = 1, 2, \ldots$. Starting from $t = 0$, the system evolves as follows. For $0 \leq t < T$, we get that

$$E_B(t) = E_H^{(1)}(t) + E_B(0^+), \quad 0 \leq t < T. \tag{24}$$

The harvested energy in $[0, T)$ is $E_H^{(1)}(T)$ with density function given by (14). Right before T, the energy stored in the battery is equal to

$$E_B(T^-) = \min\left(E_H^{(1)}(T) + E_B(0^+), E_K\right) \tag{25}$$

where $E_B(0^+)$ is given by (19) and E_K denotes the battery capacity with $E_K > E_M + E_C$. The overflow in $(0, T)$ is produced when

$$E_H^{(1)}(T) + E_B(0^+) > E_K, \tag{26}$$

with probability given by

$$\int_{E_K - E_B(0^+)}^{\infty} f_{E_H(T)}(x)\, dx, \tag{27}$$

where $f_{E_H(T)}(x)$ is given by (14). At time T, if $E_B(T^-) - E_C > E_M$, the feeding task is performed properly and the device remains in ON mode. Otherwise, the feeding task is unfeasible and the device enters in OFF mode. Hence, the energy stored in the battery right after the feeding time T is given by

$$E_B(T^+) = \max(E_B(T^-) - E_C, E_M), \tag{28}$$

where $E_B(T^-)$ is given by (25).

Let V_i be the following event:

$$V_i = \{E_B(iT^-) - E_C > E_M)\}, \quad i = 1, 2, \ldots, \tag{29}$$

that corresponds to a feasible feeding task at time iT. Given $E_B(T^-) - E_C \geq M$, next feeding time is scheduled at time $2T$ and the energy harvested in $[T, 2T)$ is given by $E_H^{(2)}(2T) - E_H^{(2)}(T)$ that follows a gamma distribution with density shown in (14). Hence, right before $2T$, the energy stored in the battery is given by

$$\begin{aligned} &E_B(2T^-) \\ &\quad = \min\left(E_B(T^+) + E_H^{(2)}(2T) - E_H^{(2)}(T), E_K\right) \mathbf{1}_{V_1} \end{aligned} \tag{30}$$

where $\mathbf{1}$ denotes the indicator function which equals 1 if the argument is true and 0 otherwise and V_1 is given by (29). At time $2T$, condition

$$E_B(2T^-) - E_C > E_M \tag{31}$$

corresponds to a feasible feeding task. Otherwise, an unfeasible feeding task takes place consuming $E_B(2T^-) - E_M$ energy units. Hence, the energy stored in the battery right after $2T$ is given by

$$E_B(2T^+) = \max(E_B(2T^-) - E_C, E_M) \mathbf{1}_{V_1}, \tag{32}$$

where V_1 is given by (29).

In a general setting, we assume that the feeding tasks performed at times $0, T, 2T, 3T, \ldots iT$ have been feasible. Then, right before feeding time $(i+1)T$, the energy stored in the battery is given by

$$\begin{aligned} E_B((i+1)T^-) = {} & \min\left(E_B(iT^+) + E_H^{(i+1)}((i+1)T)\right. \\ & \left. - E_H^{(i+1)}(iT), E_K\right) \mathbf{1}_{\{\bigcap_{j=1}^{i} V_j\}} \end{aligned} \tag{33}$$

and just after the feeding time, we get that the energy stored in the battery is equal to

$$\begin{aligned} &E_B((i+1)T^+) \\ &\quad = \max(E_B((i+1)T^-) - E_C, E_M) \mathbf{1}_{\{\bigcap_{j=1}^{i} V_j\}} \end{aligned} \tag{34}$$

As before, in (33), the variable

$$E_H^{(i+1)}((i+1)T) - E_H^{(i+1)}(iT) \tag{35}$$

is gamma distributed with density given by (14).

4. Time to the First Unfeasible Feeding Task

After the turn on of the device and before the first unfeasible feeding task, the device is in ON mode. After an unfeasible feeding task, the device enters in OFF mode and it remains in this mode until the stored energy reaches again the level E_0. Let Z_{on} be the time from the turn on to the first unfeasible feeding task. Let n_{uf} be the index of the first unfeasible feeding task, that is,

$$n_{uf} = \inf\{i \geq 0,\ E_B(iT^-) - E_C < E_M\}, \tag{36}$$

then $Z_{on} = n_{uf} T$.

Before computing the reliability function of Z_{on}, we define the mixture random variable W_A given by

$$W_A = \begin{cases} E_H(T) + A & E_H(T) + A \le E_K \\ E_K & E_H(T) + A > E_K \end{cases} \tag{37}$$

with $E_K > A$ and where the density of $E_H(T)$ is shown in (14). The distribution of W_A is given by

$$\begin{aligned} P(W_A < w) &= \int_0^{w-A} f_{E_H(T)}(x)\,dx, \quad w < E_K \\ P(W_A = E_K) &= \int_{w-A}^{\infty} f_{E_H(T)}(x)\,dx \\ P(W_A > w) &= 0, \quad w > E_K. \end{aligned} \tag{38}$$

Notice that W_A corresponds to the energy stored in the battery between two feeding times T_{i-1} and T_i if the energy in the battery just after T_{i-1} is equal to A ($A < E_K$).

Let $\overline{F}_{Z_{on}}$ be the reliability function of Z_{on}, that is,

$$\overline{F}_{Z_{on}}(t) = P(Z_{on} \ge t). \tag{39}$$

Next, the analytical formulation of $\overline{F}_{Z_{on}}$ is obtained.

(i) For $0 \le t < T$,

$$\overline{F}_{Z_{on}}(t) = 1, \tag{40}$$

by the condition given in (17).

(ii) For $T \le t < 2T$, we get that

$$\begin{aligned} \overline{F}_{Z_{on}}(t) &= P\left(E_C + E_M \le E_H^{(1)}(T) + E_B(0^+) \le E_K\right) \\ &\quad + P\left(E_H^{(1)}(T) + E_B(0^+) > E_K\right) \\ &= P\left(E_H^{(1)}(T) + E_B(0^+) - E_C \ge E_M\right) \\ &= \int_{E_M+E_c-E_B(0^+)}^{\infty} f_{E_H(T)}(u)\,du, \end{aligned} \tag{41}$$

where $f_{E_H(T)}$ is given by (14). For $T \le t < 2T$, $\overline{F}_{Z_{on}}(t)$ represents the probability of a feasible feeding time at time T.

(iii) For $2T \le t < 3T$, we get that

$$\begin{aligned} \overline{F}_{Z_{on}}(t) &= P\left(\min\left(E_B(2T^-), E_B(T^-)\right) > E_C + C_M\right) \\ &= P\left(E_B(2T^-) > E_C + E_M,\ E_B(T^-) > E_C + E_M\right) \\ &= \int_{E_C+E_M}^{E_K} f_{W_{E_B(0^+)}}(u_1)\,\overline{F}_{W_{u_1-E_C}}(E_C + E_M)\,du_1, \end{aligned} \tag{42}$$

where $f_{W_{E_B(0^+)}}$ and $f_{W_{u_1-E_C}}$ denote the density of probability of $W_{E_B(0^+)}$ and $W_{u_1-E_C}$ given in (37).

(iv) In a general setting, for $iT \le t \le (i+1)T$ with $i \ge 1$, we get that for $iT < t \le (i+1)T$

$$\begin{aligned} \overline{F}_{Z_{on}}(t) &= P\left(\bigcap_{j=1}^{i}\{E_B(jT^-) > E_M + E_C\}\right) \\ &= \int_{E_M+E_C}^{E_K} f_{W_{E_B(0^+)}}(u_1)\,du_1 \left(\int_{E_C+E_M}^{E_K}\int_{E_C+E_M}^{E_K} \right. \\ &\quad \left. \cdots \int_{E_C+E_M}^{E_K}\prod_{n=1}^{i-1} f_{W_{u_n-E_C}}(u_{n+1})\,du_{n+1}\right), \end{aligned} \tag{43}$$

where for $n = 1, 2, \ldots$, $f_{W_{u_n-E_C}}$ denotes the density of the variable $W_{u_n-E_C}$ given in (37).

5. Renewal Process

As in previous sections, the device is switch on at time $t = 0$ and the energy stored in the battery is equal to E_0. The device remains in ON state until the first unfeasible feeding time (see Figure 4). After this unfeasible feeding time, the device enters in OFF mode and the system keeps harvesting energy until the energy stored in the battery exceeds the level E_0, time in which the device again enters in ON mode. Let Z_{off} be the time in which the device is OFF. Next, we analyse the distribution of Z_{off}. By Assumption 10, at the time of the first unfeasible feeding time, that is, $t = n_{uf}T$, where n_{uf} is given by (36), the energy stored in the battery is equal to

$$E_B\left(n_{uf}T^+\right) = E_M. \tag{44}$$

After this unfeasible feeding task, the device remains in OFF mode until the energy stored in the battery reaches the level E_0. Then, Z_{off} follows the same distribution as $\sigma_{E_0-E_M}$, where $\sigma_{E_0-E_M}$ denotes the first hitting time to the level $E_0 - E_M$ of an homogeneous gamma process. The distribution function of Z_{off} is given by

$$F_{Z_{off}}(t) = F_{\sigma_{E_0-E_M}}(t) = \frac{\Gamma\left(\alpha t, (E_0 - E_M)\overline{P_r}/\alpha\right)}{\Gamma(\alpha t)}, \quad t \ge 0, \tag{45}$$

where $\Gamma(\alpha t)$ is given by (6) and $\Gamma(\alpha t, (E_0 - E_M)\overline{P_r}/\alpha)$ denotes the incomplete gamma function given by

$$\Gamma\left(\alpha t, \frac{(E_0 - E_M)\overline{P_r}}{\alpha}\right) = \int_{(E_0-E_M)\overline{P_r}/\alpha}^{\infty} z^{\alpha t-1} e^{-z}\,dz, \tag{46}$$

(see [27, 28] for more details). Although the exact distribution of $F_{Z_{off}}(t)$ is given in (45), it is very difficult to compute in practice. However, some approximations to this distribution are given. A Birnbaum–Saunders (BS) distribution is used to approximate the distribution of the first hitting time in a gamma process [29]. So, we can approximate $Z_{off}(t)$ by a BirnbaumSaunders (BS) distribution with parameters

$$\alpha^* = \frac{1}{\sqrt{(E_0 - E_M)\left(\overline{P_r}/\alpha\right)}},$$
$$\beta^* = \frac{(E_0 - E_M)\,\alpha}{\overline{P_r}}. \tag{47}$$

Furthermore, if

$$\frac{(E_0 - E_M)\,\overline{P_r}}{\sqrt{\alpha}} \gg \frac{(E_0 - E_M)\,\overline{P_r}}{\alpha}, \tag{48}$$

then the distribution of Z_{off} is approximated closely by the inverse Gaussian distribution with parameters

$$\mu = \frac{(E_0 - E_M)\,\overline{P_r}}{\alpha^2},$$
$$\lambda = \frac{(E_0 - E_M)^2\,\overline{P_r}^2}{\alpha^3}. \tag{49}$$

Due to the jump discontinuities of the sample paths of a gamma process, the energy stored in the battery in the OFF period fulfills $E_B(Z_{off}) \geq E_0 - E_M$; hence the overshoot

$$E_B\left(Z_{on} + Z_{off}\right) - E_0 \tag{50}$$

is positive almost surely. To avoid complicated mathematical technicalities, it seems realistic from a practical point of view to disregard the overshoot and consider that the energy stored in the battery right after the end of the OFF period of the device is equal to E_0, that is,

$$E_B\left(Z_{on} + Z_{off}\right) = E_0. \tag{51}$$

As a matter of fact, in most studies, the overshoot of the gamma process is not mentioned at all [30].

A goal of this paper is the optimization of the functioning of this system. The renewal process and its generalization (the quasirenewal process) are useful tools to analyze the optimal transmission policy for a system.

To optimize the transmission policy, a renewal argument is used. We recall that a renewal process is an arrival process in which the interarrival intervals are independent and identically distributed random variables. In this paper, a *renewal cycle* takes place every time that the OFF mode of the device ends. So, let $R_1, R_2, \ldots$ be the time between renewals, where R_i are independent and identically distributed variables with $R_i \equiv_d R$ and where R is given by

$$R = Z_{on} + Z_{off} \tag{52}$$

with expectation

$$\mathbb{E}[R] = \mathbb{E}\left[n_{uf}\right]T + \mathbb{E}\left[Z_{off}\right]. \tag{53}$$

Let n_f be the number of feasible feeding tasks in a renewal cycle; then

$$n_f = n_{uf} - 1, \tag{54}$$

where n_{uf} is given by (36). Under the assumptions of the model

$$P\left(n_f = 0\right) = 0, \tag{55}$$

since the initial consumption of the device is guaranteed. Following the previous results, we get that

$$P\left(n_f = 1\right) = \int_0^{E_M + E_C - E_B(0)} f_{E_H(T)}(x)\,dx, \tag{56}$$

where $f_{E_H(T)}$ is given in (14). In a general setting, for $i \geq 1$, we get that

$$P\left(n_f = i\right) = \overline{F}_{Z_{on}}((i-1)T) - \overline{F}_{Z_{on}}(iT), \tag{57}$$

where $\overline{F}_{Z_{on}}$ is given by (43). It is easy to check that

$$\mathbb{E}\left[n_f\right] = \sum_{i=1}^{\infty} i\left(\overline{F}_{Z_{on}}((i-1)T) - \overline{F}_{Z_{on}}(iT)\right)$$
$$= \sum_{i=0}^{\infty} \overline{F}_{Z_{on}}(iT). \tag{58}$$

Hence, the expected time to a renewal cycle can be expressed as

$$\mathbb{E}[R] = \mathbb{E}\left[n_{uf}\right]T + \mathbb{E}\left[Z_{off}\right]$$
$$= \left(\mathbb{E}\left[n_f\right] - 1\right)T + \mathbb{E}\left[Z_{off}\right]$$
$$= T\left(\sum_{i=0}^{\infty} \overline{F}_{Z_{on}}(iT) - 1\right) + \mathbb{E}\left[Z_{off}\right] \tag{59}$$
$$\simeq T\sum_{i=1}^{\infty} \overline{F}_{Z_{on}}(iT) + \frac{(E_0 - E_M)\,\overline{P_r}}{\alpha^2},$$

where this approximation has been obtained using the inverse Gaussian distribution if $\sqrt{(\alpha)} \gg 1$ and where $\overline{F}_{Z_{on}}$ is given by (43). The expectation also can also be approximated by

$$\mathbb{E}[R] = \mathbb{E}\left[n_{uf}\right]T + \mathbb{E}\left[Z_{off}\right]$$
$$\simeq T\sum_{i=1}^{\infty} \overline{F}_{Z_{on}}(iT) + \frac{(E_0 - E_M)\,\overline{P_r}}{\alpha} + \frac{1}{2\alpha}, \tag{60}$$

using the Birnbaum–Saunders distribution.

Next, we describe an optimal transmission policy for this EHS. By optimal, we mean the policy that maximizes an objective function. Renewal techniques are used to approach the search of the optimal transmission policy. Let $R_1, R_2, \ldots, R_i$ be the successive renewal cycles (as we explained before, a renewal cycle in this paper takes place every time the OFF mode of the device ends). Let $C_1, C_2, \ldots, C_n$ be the total of feasible tasks performed in a renewal cycle. Denoting by $C(t)$ the total of feasible tasks up to time t and using the renewal-reward theorem, we get that

$$\lim_{t \longrightarrow \infty} \frac{C(t)}{t} = \frac{\mathbb{E}(C_1)}{\mathbb{E}(R_1)}. \tag{61}$$

Hence, the limit of the total of feasible tasks per unit time can be reduced to the behavior of this ratio in the first renewal cycle. To deal with the optimal transmission policy, the ratio

$$T_f^{-1} = \frac{\mathbb{E}\left[n_f\right]}{\mathbb{E}\left[R\right]} \tag{62}$$

where $\mathbb{E}[R]$ and $\mathbb{E}[n_f]$ are given by (59) and (58), respectively, is chosen as objective cost function. The optimal transmission policy corresponds to the value of T (the time between feeding times) that maximizes the expected number of feasible feeding tasks per unit time in a renewal cycle, that is,

$$\sup_{T>0}\left(\frac{\mathbb{E}\left[n_f\right]}{\mathbb{E}\left[R\right]}\right). \tag{63}$$

The analytic formulation of T_f^{-1} given in (63) is not straightforward since it involves multidimensional integrals as we can check in (58) and (59). Traditional quadrature methods to solve these multidimensional integrals are outperformed and the Monte Carlo method is chosen to analyze the optimization problem formulated in (63). Monte Carlo method is an adequate method when integrals are multidimensional [31].

Next section shows some numerical examples of the search of the optimal transmission policy using Monte Carlo method. For that, replicas of the first renewal cycle are simulated and the expected values given in (62) are numerically estimated using these simulations. Optimal values for the transmission policy are obtained by inspection considering these estimations.

6. Numerical Examples

We assume a battery with a finite capacity of $E_K = 6$ milijoules (mJ) and an energy threshold of $E_M = 1\ mJ$. This battery powers a device and we assume that its consumption is equal to $E_C = 1\ mJ$ in each feasible feeding task. Due to the transitory effects, at the device turn on, it consumes $E_{on} = 3\ mJ$. The initial stored energy in the battery is equal to $E_0 = 5.3\ mJ$ (see Figure 4 for a better understanding). We consider that the harvested energy comes from an RF channel with Nakagami m-fading and a mean power harvested equals to $\overline{P_r} = 120\ \mu W$.

The model used to describe the harvested energy is an homogeneous gamma process with parameters α and $\beta = \alpha/\overline{P_r}$. Parameter α depends on fading rate. In this numerical example, three different rates have been considered $\alpha^{-1} = 1\ s$, $\alpha^{-1} = 5\ s$, and $\alpha^{-1} = 10\ s$. Notice that α^{-1} is related to the fading length, so the examples show a range of possibilities: from moderate fast fading to slow fading depending on α. A total of 50000 complete renewal cycles for each point have been performed to obtain the results.

In Figure 5, the expected number of feasible feedings per minute, $T_f^{-1} = \mathbb{E}[n_f]/\mathbb{E}[R]$ is shown. For high values of T, almost all the feeding times are feasible since there is enough energy in the battery to feed the device. Instead, high values of T^{-1} cause a lot of unfeasible feeding tasks and, therefore, the repetitive switch off of the device. The maximum value of feasible feeding tasks per unit time corresponds to the optimal policy for the system operation and it is reached at $T^{-1} \simeq 7\ min^{-1}$ (7 feeding times per minute), $T^{-1} \simeq 6.5\ min^{-1}$, and $T^{-1} \simeq 6\ min^{-1}$ according to the value of α^{-1}. It is relevant that, for high values of α^{-1}, the number of feasible feeding tasks decreases. So, for $\alpha^{-1} = 10\,s$ the maximum expected feasible feeding tasks per minute are equal to $T_f^{-1} = 5.23\ tasks/min$, for $\alpha^{-1} = 5s$ are $T_f^{-1} = 5.88\ tasks/min$, and for $\alpha^{-1} = 1\ s$ are $T_f^{-1} = 7\ tasks/min$. As one would hope, the expected number of feasible feeding tasks per unit time is closely related to the RF channel characteristics. In contrast, for low values of T (that is, large number of feeding times per unit time), slow fading channel may be less disadvantageous, as in the present case.

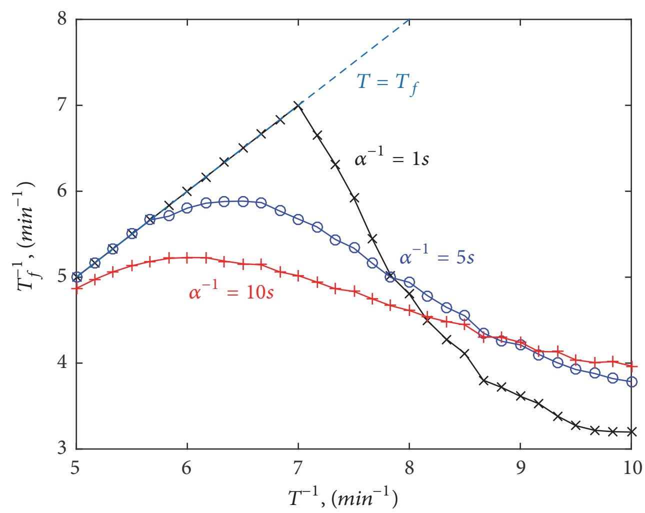

Figure 5: Expected number of feasible tasks per minute for several lengths of fading.

Figure 6 shows the energy lost per unit time (power lost) versus T^{-1}. The lost power in the system is due to three factors: energy not stored in the battery due to its finite capacity (overflow), energy consumed in the device turn on and wasted energy due to the unfeasible feeding tasks. As we could expect, the minimum of the lost power is reached at the values described above ($T^{-1} \simeq [7, 6.5, 6](min^{-1})$) with values of 3.33 μW, 21.91 μW, and 32.85 μW, respectively. Notice that the power received at the antenna is 120 μW consequently, and the efficiency of the system is 97.23%, 81.74%, and 72.63% for $\alpha^{-1} = 1\ s$, $\alpha^{-1} = 5\ s$, and $\alpha^{-1} = 10\ s$, respectively.

Another interesting measure is the ratio of unfeasible feeding tasks, calculated as

$$r_u = \frac{T^{-1} - T_f^{-1}}{T^{-1}}; \tag{64}$$

which is represented in Figure 7. In Figure 7, we can see that for small values of T^{-1} the ratio r_u is reduced. As we expected, the lower α^{-1} (fast fading) the better ratio.

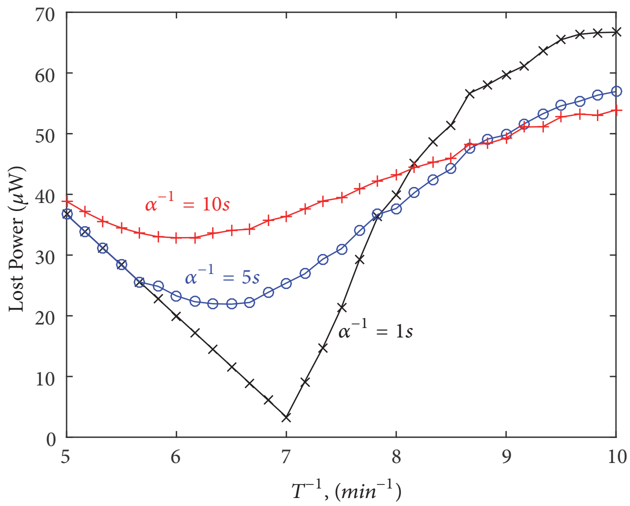

FIGURE 6: Expected lost power (lost energy per unit time) at the EHS for several lengths of fading.

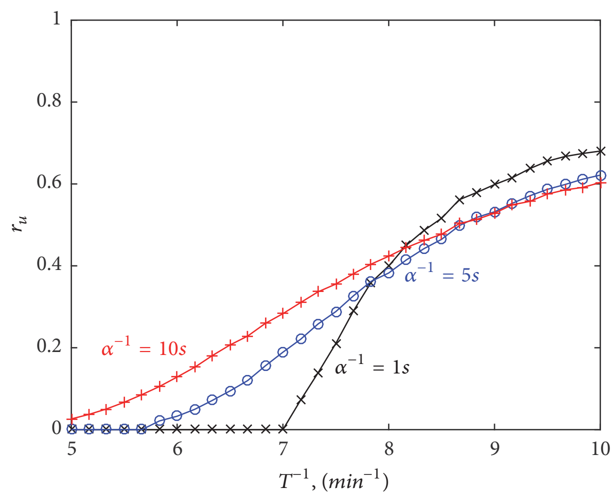

FIGURE 7: Ratio of unfeasible feeding tasks for several lengths of fading.

In general, fast fading channels (that correspond to small values of α^{-1}) are more advantageous for harvesting systems since the storage battery fluctuations are much less susceptible to variations; i.e., the gamma process associated with the harvested energy presents a lower variance ($\mathrm{var}(E_H(t)) = \overline{P_r}^2 t/\alpha$).

7. Conclusions and Further Works

An EHS system that consists of an antenna, a RF rectifier and a battery used to store harvested energy and a device powered by the battery, is considered in this paper.

The harvested energy of ambient RF energy in a NLOS fading environment is modeled using a gamma process. As far as the authors are concerned, this stochastic process is new to the area of energy harvesting. Due to the properties of the gamma process, the mathematical model of the whole system is tractable and some probabilistic quantities are obtained. These quantities are useful for the construction and analysis of a transmission policy of this EHS. Since the functioning of the system can be described using renewal arguments, the optimal scheduling policy is numerically analyzed.

As we showed in the examples, the fading of the signal causes losses in the efficiency of the harvesting system but this loss can be reduced to reasonable limits in fast fading environments. Fast fading channels are more advantageous for harvesting systems since the storage battery are much less susceptible to variations rather than slow fading channels.

In this paper, we have considered a perfect and ideal energy storage. It means that the battery saves in a proper way the energy that it receives. However, energy storage inefficiency can be developed in further works. For example, an inefficiency which occurs at the time of the energy storage can be analyzed. Also, the energy demand of the device has been modeled as a ideal instantaneous process. Further works could consider a continuous demand of energy.

We have also assumed in this paper an uniform transmission policy that schedules the feeding times uniformly. However, a scheduling policy could be considered in future works where the future feeding times are planned taking into account the stored energy in previous feeding times.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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