Research Article

L_1 Input-Output Finite-Time Control of Positive Switched Nonlinear Systems

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In this paper, the problem of L_1 input-output finite-time control of positive switched nonlinear systems with time-varying and distributed delays is investigated. Nonlinear functions considered in this paper are located in a sector field. Firstly, the proof of the positivity of switched positive nonlinear systems with time-varying and distributed delays is given, and the concept of L_1 input-output finite-time stability (L_1 IO-FTS) is firstly introduced. Then, by constructing multiple co-positive-type nonlinear Lyapunov functions and using the average dwell time (ADT) approach, a state feedback controller is designed and sufficient conditions are derived to guarantee the corresponding closed-loop system is L_1 IO-FTS. Such conditions can be easily solved by linear programming. Finally, a numerical example is provided to demonstrate the effectiveness of the proposed method.

1. Introduction

In the past decades, positive switched systems have been paid much attention due to its broad applications in many areas such as communication systems [1], biological systems [2], and systems theory [3–6]. Many meaningful results have been presented in the literatures [7–11], and the references therein. Most of related results are mainly concerned with positive switched linear systems. In practice, many systems are nonlinear, such as physical systems, chemical systems, and transport systems. Although the study of dynamic behavior of positive switched nonlinear systems is not an easy work, because it is difficult to define the positivity of a nonlinear system, and few effective control techniques with respect to positive switched nonlinear systems are proposed. There are already some available results about positive switched nonlinear systems [12–16].

However, the literatures mentioned above mainly focus on asymptotic stability, but in many practical applications, one is more interested in the behavior of the systems within a finite-time interval. Reference [17] firstly defined the concept of finite-time stability (FTS) for linear deterministic systems. For positive switched nonlinear systems, only [18] considered the guaranteed cost finite-time

control problem. Recently, the definition of input-output finite-time stability (IO-FTS) is proposed in [19], which is fully consistent with the definition of FTS. IO-FTS means that given a class of norm bounded input signals over a specified time interval $[0, T]$, the outputs of the system do not exceed an assigned threshold during such time intervals, and it does not necessarily require the inputs and outputs to belong to the same class. It is noteworthy that the concept of IO-FTS is different from that of input-output stability, the later deals with the behavior of a system within an infinite time interval. As we know, only a few results considered the IO-FTS problem of positive switched linear systems [20–22]. Reference [20] extended IO-FTS to positive switched linear systems with state delays, but the time-varying delay and distributed delay were not considered. References [21, 22] discussed the input-output finite-time control problem of positive switched systems and uncertain positive impulsive switched systems, respectively. However, for positive switched nonlinear systems, the IO-FTS problem has not been studied.

Moreover, multiple time delays have great effect on the systems, such as oscillation and instability. On the other hand, compared with H_∞ performance, L_1 gain performance is more suitable to describe the performances of the

systems with positive properties [23–25]. When taking multiple time delays and L_1 gain performance into account, the problem of choosing an appropriate nonlinear copositive Lyapunov function and analyzing the L_1 IO-FTS of the positive switched nonlinear systems will be more complex and challenging, which motivates our study.

In this paper, we are interested in the problem of L_1 IO-FTS for a class of positive switched nonlinear systems with time-varying and distributed delays via ADT. The proposed system model is more general such that the systems dealt with in [6, 13, 17, 21–23] can be regarded as special forms. The main contributions of this paper are (1) the proof of the positivity of positive switched nonlinear systems with time-varying and distributed delays is given, (2) integrating L_1 performance index into IO-FTS, the concept of L_1 IO-FTS is for the first time extended to positive switched nonlinear systems with time-varying and distributed delays, and (3) a state feedback controller is designed and sufficient conditions for L_1 IO-FTS of the corresponding closed-loop system are given. Such conditions can be easily solved by linear programming. The rest of this paper is organized as follows. In Section 2, problem statements and necessary lemmas are given. Main results are given in Section 3. A numerical example is provided in Section 4. Section 5 is the conclusion.

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)}f(x(t)) + A_{d\sigma(t)}f(x(t-d(t))) + A_{b\sigma(t)}f\left(\int_{t-\tau_2}^t x(s)ds\right) + G_{\sigma(t)}u(t) + B_{\sigma(t)}w(t), \\ y(t) = C_{\sigma(t)}g(x(t)), \\ x(\theta) = \varphi(\theta), \quad \theta \in [-\tau_1, 0], \end{cases} \quad (1)$$

where $x(t) \in R^n$ is the system state, $u(t) \in R^m$ and $y(t) \in R^s$ represent the control input and output, respectively. $f(x) = (f_1(x_1), f_2(x_2), \dots, f_n(x_n))^T \in R^n$, $g(x) = (g_1(x_1), g_2(x_2), \dots, g_n(x_n))^T \in R^n$. The function $\sigma(t) : [0, \infty)$ is the system switching signal, and it is continuous from the right everywhere for a switching sequence, $0 \leq t_0 \leq t_1 \leq \dots, N$ denotes the number of subsystems. $t_0 = 0$ is the initial time, t_k denotes the k th switching instant. $\forall p \in \mathbb{N}$, A_p , A_{dp} , A_{bp} , B_p , C_p , and G_p are constant matrices with appropriate dimensions. The differentiable function $d(t) \geq 0$ represents time-varying delay, which satisfies $\dot{d}(t) \leq h < 1$, τ_2 is the distributed delays, where τ_2 and h are known positive constants. $\varphi(\theta)$ is the initial condition on $[-\tau_1, 0]$, $\tau_1 = \max\{d(t), \tau_2\}$. $w(t) \in R^l$ is the exogenous disturbance and is defined as

$$W_1 = \left\{ w(\cdot) \in L_{1,[0,T]} : \int_0^T \|w(t)\| dt \leq \varepsilon \right\}, \quad (2)$$

with a known scalar $\varepsilon > 0$.

Next, we will present some definitions and lemmas for the positive switched nonlinear systems (1).

Definition 1. System (1) is said to be positive if for any switching signals $\sigma(t)$, any initial conditions $\varphi(\theta) \geq 0$,

Notations. The representation $A \succ 0$ ($\geq 0, < 0, \leq 0$) means that $a_{ij} > 0$ ($\geq 0, < 0, \leq 0$), which is applicable to a vector. $A \succ B$ ($A \geq B$) means that $A - B \succ 0$ ($A - B \geq 0$). The symbols R , R^n , and $R^{n \times n}$ denote the set of real numbers, the space of the vectors of n -tuples of real numbers, the space of $n \times n$ matrices with real numbers, respectively. R_+^n is the n -dimensional nonnegative (positive) vector space. 1_l denotes the l -dimensional vector $[1, \dots, 1]^T$. A^T denotes the transpose of matrix A . Let $x \in R^n$, 1-norm $\|x\|$ is defined by $x = \sum_{k=1}^n |x_k|$. $L_{1,[0,T]}$ denotes the space of absolute integrable vector-valued functions on the interval $[0, T]$; that is, $s(t) \in L_{1,[0,T]}$ if $\int_0^T \|s(t)\| dt < \infty$ holds. $L_{\infty,[0,T]}$ denotes the space of the uniformly bounded vector-valued functions on the interval $[0, T]$; that is, $s(t) \in L_{\infty,[0,T]}$ if $\max_{t \in [0,T]} \|w(t)\| < \infty$ holds. Matrices are assumed to have compatible dimensions for calculating if their dimensions are not explicitly stated.

2. Preliminaries and Problem Statements

Consider a class of positive switched nonlinear systems with time-varying and distributed delays:

$\theta \in [-\tau_1, 0]$, $w(t) \geq 0$, the corresponding trajectory satisfies $x(t) \geq 0$ and $y(t) \geq 0$ for all $t \geq 0$.

Definition 2 (see [20]). A is called a Metzler matrix if the off-diagonal entries of matrix A are nonnegative.

Definition 3 (see [20]). For any $T_2 > T_1 \geq 0$, let $N_\sigma(T_1, T_2)$ denote the switching number of $\sigma(t)$, over the interval $[T_1, T_2)$, for given $T_\alpha > 0$ and $N_0 \geq 0$, if the inequality

$$N_\sigma(T_1, T_2) \leq N_0 + \frac{T_2 - T_1}{T_\alpha} \quad (3)$$

holds, then the positive constant T_α is called an ADT, and N_0 is called a chattering bound. Generally speaking, $N_0 = 0$ is chosen in this paper.

Assumption 1. The nonlinear functions $f(x)$ and $g(x)$ lie in sector fields satisfying

$$m_1 x_i^2 \leq f_i(x_i) x_i \leq m_2 x_i^2, \quad (4)$$

$$m_3 x_i^2 \leq g_i(x_i) x_i \leq m_4 x_i^2, \quad (5)$$

for $x_i \in R$ and $i = 1, 2, \dots, n$, where $0 < m_1 \leq m_2, 0 < m_3 \leq m_4$, and $f_i(0) = 0, g_i(0) = 0$.

Remark 1. The system model (1) is a more general form. If $A_{bp} = 0$, then system (1) is turned into the system in [13]. If $m_1 = m_2 = m_3 = m_4 = 1$ (it means $f_i(x_i) = x_i$, $g_i(x_i) = x_i$), then system (1) is transformed into the systems in [21, 22]. Moreover, if $m_1 = m_2 = m_3 = m_4 = 1$ and $A_{bp} = A_{dp} = 0$, then system (1) will be turned into the systems in [6, 10, 17, 23].

Remark 2. Assumption 1 is a necessary assumption, because it takes full advantage of the characteristics of nonnegative states of positive switched nonlinear systems.

Lemma 1 (see [25]). Let $A \in R^{n \times n}$ be a Metzler matrix, then there exists a vector $v > 0$ such that $Av < 0$.

Lemma 2. Under Assumption 1. System (1) is positive if and only if A_p , $\forall p \in \underline{N}$, are Metzler matrices and $\forall p \in \underline{N}$, $A_{dp} \geq 0$, $A_{bp} \geq 0$, $B_p \geq 0$, $C_p \geq 0$, and $G_p \geq 0$.

Proof (Necessity). Firstly, let $\varphi(\theta) = 0$ on $\theta \in [-\tau, 0]$ and $w(t) = 0$. Then, $\dot{x}(0) = G_p u(0)$ for some $p \in \underline{N}$. By definition 1, $\dot{x}(0) \geq 0$ for every $u(0) \geq 0$, which means that $G_p \geq 0$ for each $p \in \underline{N}$. In the same way, we can get $B_p \geq 0$.

Secondly, we prove that A_p , $\forall p \in \underline{N}$, are Metzler matrices. Let $u(t) = 0$ and $w(t) = 0$. Suppose there exists an element $a_p^{ik} < 0$, $i \neq k$. From the system (1), we have

$$\begin{aligned} \dot{x}_i(t) = & \sum_{j=1, j \neq i, j \neq k}^n a_p^{ij} f_j(x_j(t)) + a_p^{ii} f_i(x_i(t)) \\ & + a_p^{ik} f_k(x_k(t)) + \sum_{j=1}^n a_{dp}^{ij} f_j(x_j(t-d(t))) \\ & + \sum_{j=1}^n \left(a_{bp}^{ij} f_j \left(\int_{t-\tau_2}^t x_j(s) ds \right) \right). \end{aligned} \quad (6)$$

We can see that $x_i(t) < 0$ is possible if $f_i(x_k(t)) = 0$, $f_k(x_k(t)) \neq 0$, and a_p^{ik} takes a small value enough. Thus, $x_i(t^+) < 0$, which brings a contradiction with the positivity of the system (1). So, A_p , $\forall p \in \underline{N}$, are Metzler matrices.

Thirdly, we prove $A_{dp} \geq 0$ by the same way. Suppose there exists an element $a_{dp}^{ik} < 0$, then we have

$$\begin{aligned} \dot{x}_i(t) = & \sum_{j=1}^n a_p^{ij} f_j(x_j(t)) + \sum_{j=1, j \neq k}^n \left(a_{dp}^{ij} f_j(x_j(t-d(t))) \right) \\ & + a_{dp}^{ik} f_k(x_k(t-d(t))) + \sum_{j=1}^n \left(a_{bp}^{ij} f_j \left(\int_{t-\tau_2}^t x_j(s) ds \right) \right). \end{aligned} \quad (7)$$

It is easy to get that $\dot{x}_i(t) < 0$ is possible if a_{dp}^{ik} takes a small value enough. Furthermore, $x_i(t^+) < 0$, which brings a contradiction with the positivity of the system (1). So, $A_{dp} \geq 0$.

Finally, suppose there exists an element $a_{bp}^{ik} < 0$, similar to the above process, we have

$$\begin{aligned} \dot{x}_i(t) = & \sum_{j=1}^n a_p^{ij} f_j(x_j(t)) + \sum_{j=1}^n a_{dp}^{ij} f_j(x_j(t-d(t))) \\ & + \sum_{j=1, j \neq k}^n \left(a_{bp}^{ij} f_j \left(\int_{t-\tau_2}^t x_j(s) ds \right) \right) \\ & + a_{bp}^{ik} f_k \left(\int_{t-\tau_2}^t x_k(s) ds \right). \end{aligned} \quad (8)$$

In the same way, it is possible that $\dot{x}_i(t) < 0$ if a_{bp}^{ik} takes a small value enough. It shows that system (1) is not positive. Based on these four points addressed above, the necessity is obtained.

2.1. Sufficiency. Let $\varphi(t) \geq 0$. To prove that $x(t) \geq 0$ for all $t \geq 0$, it is sufficient to ensure that the vector $\dot{x}(t)$ does not point towards the outside of R_+^n whenever $x(t)$ is on the boundary of R_+^n . This is equivalent to verifying that the components of the vector $\dot{x}(t)$ corresponding to the zero components of $x(t) \geq 0$ are nonnegative. Denoting by Ω the set of indices of such components, that is, $x_i(t) = 0$ for $i \in \Omega$. Then, for some $p \in \underline{N}$, we have

$$\begin{aligned} \dot{x}_i(t) = & \sum_{j \notin \Omega} a_p^{ij} f_j(x_j(t)) + \sum_{j=1}^n \left(a_{dp}^{ij} f_j(x_j(t-d(t))) \right) \\ & + \sum_{j=1}^n \left(a_{bp}^{ij} f_j \left(\int_{t-\tau_2}^t x_j(s) ds \right) \right) + g_p^{ij} u_j(t) \\ & + b_p^{ij} w_j(t), \quad i \in \Omega, \end{aligned} \quad (9)$$

where a_p^{ij} , a_{dp}^{ij} , a_{bp}^{ij} , g_p^{ij} , b_p^{ij} are the i th row j th column element of A_p , A_{dp} , A_{bp} , G_p , B_p , respectively. From the condition (4), it follows from $\varphi(\theta) \geq 0$ that $f_j(x_j(t)) \geq 0$ for $t \in [-\tau_1, T_f]$. According to system (1), $a_p^{ij} \geq 0$ for $i \neq j$, $a_{dp}^{ij} \geq 0$, $a_{bp}^{ij} \geq 0$, $g_p^{ij} \geq 0$, $b_p^{ij} \geq 0$. So, we have $\dot{x}_i(t) \geq 0$ for $u(t) \in R_+^m$ and $w(t) \geq 0$. This means $x(t) \geq 0$ for $\varphi(t) \geq 0$. From (5), we have $g(x(t)) \geq 0$ for $x(t) \geq 0$. Combining this with $C_p \geq 0$, it yields $y(t) \geq 0$.

That reveals the system (1) is positive under any switching signals if and only if A_i are Metzler matrices, $A_{dp} \geq 0$, $A_{bp} \geq 0$, $B_p \geq 0$, $C_p \geq 0$, and $G_p \geq 0$, $\forall i \in \underline{N}$.

From the above, we can conclude the system (1) is positive.

Remark 3. Lemma 2 proves the positivity of a new class of switched nonlinear systems, which plays a key role in our latter work. From Assumption 1, if $f(x) \geq 0$ and $g(x) \geq 0$ for $x \geq 0$, then Lemma 2 still holds. That is to say, the system (1) is positive under any switching signals if and only if it consists of a family of positive nonlinear subsystems.

Next, we will give the definitions of IO-FTS and L_1 IO-FTS for the positive switched nonlinear system (1).

Definition 4. (IO-FTS) Consider zero initial condition ($x(0) = 0$), for a given time constant T_f , disturbances signals W_1 defined by (2) and a vector $\delta > 0$, the system (1) is said to be IO-FTS with respect to $(\delta, T_f, \varepsilon, \sigma(t))$, if

$$w(t) \in W_1 \implies y^T \delta \leq 1, \quad \forall t \in [0, T_f]. \quad (10)$$

If the disturbance $w(t)$ satisfies $w(t) \in W_2$, where W_2 is defined as

$$W_2 = \left\{ w(\cdot) \in L_{\infty, [0, T]} : \max_{t \in [0, T]} w(t) \leq \varepsilon \right\}, \quad (11)$$

then, we give Definition 5.

Definition 5. (IO-FTS) Consider zero initial condition ($x(0) = 0$), for a given time constant T_f , disturbances signals W_2 , and a vector $\delta > 0$. System (1) is said to be IO-FTS with respect to $(\delta, T_f, \varepsilon, \sigma(t))$, if

$$w(t) \in W_2 \implies y^T \delta \leq 1 \quad \forall t \in [0, T_f]. \quad (12)$$

Remark 4. In Definition 4 and 5, two classes of exogenous disturbances are norm bounded integrable signals W_1 and the uniformly bounded signals W_2 , respectively. Because of the similarity of the proof process, we only focus on the former in this paper.

Definition 6. (L_1 IO-FTS) Under the zero initial condition ($\varphi(\theta) = 0$), for a given time constant T_f , the system (1) is said to be L_1 IO-FTS with respect to $(\delta, T_f, \varepsilon, \sigma(t))$, if the following conditions are satisfied: (1) system (3) is IO-FTS with respect to $(\delta, T_f, \varepsilon, \sigma(t))$; (2) the output $y(t)$ satisfies

$$\int_0^{T_f} e^{-\beta t} \|y(t)\| dt \leq \eta \int_0^{T_f} \|w(t)\| dt, \quad (13)$$

$$\begin{cases} \dot{x}(t) = A_{\sigma(t)} f(x(t)) + A_{d\sigma(t)} f(x(t-d(t))) + A_{b\sigma(t)} f\left(\int_{t-\tau_2}^t x(s) ds\right) + B_{\sigma(t)} w(t), \\ y(t) = C_{\sigma(t)} g(x(t)), \\ x(\theta) = \varphi(\theta) \quad \theta \in [-\tau_1, 0]. \end{cases} \quad (14)$$

In this subsection, we concern with the IO-FTS analysis of positive switched nonlinear system (14). The following theorem gives some sufficient conditions of IO-FTS for system (14) via the ADT technique.

Theorem 1. Consider the system (14), for given constants T_f , γ , λ_p , $p \in \mathbb{N}$, and a vectors $\delta > 0$, if there exist positive vectors $v_p \in \mathbb{R}^n$, $\rho_p \in \mathbb{R}^n$, $q_p \in \mathbb{R}^n$, $p \in \mathbb{N}$, such that the following inequalities hold:

$$A_p^T v_p + \rho_p + \tau_1 q_p - \frac{\lambda_p}{m_2} v_p < 0, \quad (15)$$

$$A_{dp}^T v_p - (1-h)\rho_p < 0, \quad (16)$$

$$m_2 A_{bp}^T v_p - m_1 q_p < 0, \quad (17)$$

$$\varepsilon \gamma < e^{-\lambda T_f}, \quad (18)$$

$$m_4 C_p^T \delta < v_p, \quad (19)$$

$$v_p \leq \mu v_q, \quad (20)$$

$$\rho_p \leq \mu \rho_q,$$

$$q_p \leq \mu q_q.$$

where $\beta > 0$, $\eta > 0$, and $w(t)$ satisfies (2).

Remark 5. In Definition 6, L_1 -gain performance index provides a more useful description for the disturbance attenuation performance of positive switched systems. Integrating L_1 -gain performance index into IO-FTS, the concept of L_1 IO-FTS is more suitable to describe the system behavior during a specified time interval.

The aim of this paper is to design a state feedback controller $u(t)$ and find a class of switching signals $\sigma(t)$ for positive switched nonlinear system (1) such that the corresponding closed-loop system is L_1 IO-FTS.

3. Main Results

3.1. IO-FTS Analysis. Consider the system (1) with $u(t) = 0$, the system is described by

$$B_p^T v_p \leq \gamma 1_l, \quad (21)$$

where $v_p = [v_{p1}, v_{p2}, \dots, v_{pn}]^T$, $\rho_p = [\rho_{p1}, \rho_{p2}, \dots, \rho_{pn}]^T$, $q_p = [q_{p1}, q_{p2}, \dots, q_{pn}]^T$, $\lambda = \max_{p \in \mathbb{N}} \{\lambda_p\}$, $\mu (\mu \geq 1)$ satisfies (20), then the system (14) is IO-FTS for switching signal $\sigma(t)$ with the ADT scheme:

$$T_\alpha > T_\alpha^* = \frac{T_f \ln \mu}{\ln(e^{-\lambda T_f}) - \ln(\varepsilon \gamma)}. \quad (22)$$

Proof. Construct the multiple co-positive-type nonlinear Lyapunov functional for the system (14):

$$V_{\sigma(t)}(t) = V_{\sigma(t)}(t, x(t)), \quad (23)$$

the form of each $V_p(t, x(t))$, $\forall p \in \mathbb{N}$ can be given by

$$\begin{aligned} V_p(t) &= V_p(t, x(t)) = x^T(t) v_p \\ &+ \int_{t-d(t)}^t e^{\lambda_p(t-s)} f^T(x(s)) \rho_p ds \\ &+ \int_{-\tau_1}^0 \int_{t+\theta}^t e^{\lambda_p(t-s)} f^T(x(s)) q_p ds d\theta, \end{aligned} \quad (24)$$

where v_p, ρ_p , and $q_p \in \mathbb{R}_+^n$, $\forall p \in \mathbb{N}$.

Along the trajectory of system (14), we have

$$\begin{aligned}
\dot{V}_p(t) = & f^T(x(t))A_p^T v_p + f^T(x(t-d(t)))A_{dp}^T v_p \\
& + f^T\left(\int_{t-\tau_2}^t x(s)ds\right)A_{bp}^T v_p \\
& + \lambda_p \int_{t-d(t)}^t e^{\lambda_p(t-s)} f^T(x(s))\rho_p ds \\
& - (1-\dot{d}(t))e^{\lambda_p d(t)} f^T(x(t-d(t)))\rho_p \\
& + \lambda_p \int_{-\tau_1}^0 \int_{t+\theta}^t e^{\lambda_p(t-s)} f^T(x(s))\varrho_p ds d\theta \\
& + \tau_1 f^T(x(t))\varrho_p - \int_{-\tau_1}^0 e^{-\lambda_p \theta} f^T(x(t+\theta))\varrho_p d\theta \\
& + w^T(t)B_p^T v_p + f^T(x(t))\rho_p.
\end{aligned} \tag{25}$$

Consider (4) and $x(t) \geq 0$, $m_1 x(t) \leq f(t) \leq m_2 x(t)$ can be obtained, then (25) is transformed into

$$\begin{aligned}
\dot{V}_p(t) = & \lambda_p V_p(t) + f^T(x(t))\left[A_p^T v_p + \rho_p + \tau_1 \varrho_p\right] - \lambda_p x^T(t)v_p \\
& + f^T(x(t-d(t)))\left[A_{dp}^T v_p - (1-\dot{d}(t))\rho_p\right] \\
& + f^T\left(\int_{t-\tau_2}^t x(s)ds\right)A_{bp}^T v_p \\
& - \int_{-\tau_1}^0 e^{-\lambda_p \theta} f^T(x(t+\theta))\varrho_p d\theta + w^T(t)B_p^T v_p \\
\leq & \lambda_p V_{\sigma(t)}(t) + f^T(x(t))\left[A_p^T v_p + \rho_p + \tau_1 \varrho_p - \frac{\lambda_p}{m_2} v_p\right] \\
& + f^T(x(t-d(t)))\left[A_{dp}^T v_p - (1-h)\rho_p\right] \\
& + m_2 \int_{t-\tau_2}^t x^T(s)ds A_{bp}^T v_p \\
& - \int_{t-\tau_1}^t f^T(x(s))\varrho_p ds + w^T(t)B_p^T v_p \\
\leq & \lambda_p V_{\sigma(t)}(t) + f^T(x(t))\left[A_p^T v_p + \rho_p + \tau_1 \varrho_p - \frac{\lambda_p}{m_2} v_p\right] \\
& + f^T(x(t-d(t)))\left[A_{dp}^T v_p - (1-h)\rho_p\right] \\
& + m_2 \int_{t-\tau_2}^t x^T(s)ds A_{bp}^T v_p \\
& - m_1 \int_{t-\tau_1}^t x^T(s)\varrho_p ds + w^T(t)B_p^T v_p.
\end{aligned} \tag{26}$$

Noting that $\tau_1 = \max\{d(t), \tau_2\}$, we have

$$\begin{aligned}
& m_2 \int_{t-\tau_2}^t x^T(s)ds A_{bp}^T v_p - m_1 \int_{t-\tau_1}^t x^T(s)\varrho_p ds \\
& \leq m_2 \int_{t-\tau_1}^t x^T(s)ds A_{bp}^T v_p - m_1 \int_{t-\tau_1}^t x^T(s)\varrho_p ds \\
& = \int_{t-\tau_1}^t x^T(s)(m_2 A_{bp}^T v_p - m_1 \varrho_p)ds.
\end{aligned} \tag{27}$$

According to (27), (26) can be rewritten as

$$\begin{aligned}
\dot{V}_p(t) - \lambda_p V_p(t) \leq & f^T(x(t))\left[A_p^T v_p + \rho_p + \tau_1 \varrho_p - \frac{\lambda_p}{m_2} v_p\right] \\
& + f^T(x(t-d(t)))\left[A_{dp}^T v_p - (1-h)\rho_p\right] \\
& + \int_{t-\tau_1}^t x^T(s)(m_2 A_{bp}^T v_p - m_1 \varrho_p)ds \\
& + w^T(t)B_p^T v_p.
\end{aligned} \tag{28}$$

From (15)–(17) and (21), we get

$$\dot{V}_p(t) - \lambda_p V_p(t) \leq w^T(t)B_p^T v_p \leq \gamma \|w(t)\|. \tag{29}$$

Integrating both sides of (29) during the period $[t_k, t]$ for $t \in [t_k, t_{k+1})$ leads to

$$V_{\sigma(t)}(t) \leq e^{\lambda_{\sigma(t_k)}(t-t_k)} V_{\sigma(t_k)}(t_k) + \int_{t_k}^t e^{\lambda_{\sigma(t_k)}(t-s)} \gamma \|w(s)\| ds. \tag{30}$$

From (20) and (24), we get

$$V_{\sigma(t_k)}(t_k) \leq \mu V_{\sigma(t_k^-)}(t_k^-). \tag{31}$$

Denote $t_1, \dots, t_N$, as the switching instants over the interval $[0, T_f)$. From (2), (30), and (31), we have

$$\begin{aligned}
V_{\sigma(t)}(t) & \leq e^{\lambda_{\sigma(t_k)}(t-t_k)} V_{\sigma(t_k)}(t_k) + \int_{t_k}^t e^{\lambda_{\sigma(t_k)}(t-s)} \gamma \|w(s)\| ds \\
& \leq \mu e^{\lambda(t-t_k)} V_{\sigma(t_k^-)}(t_k^-) + \int_{t_k}^t e^{\lambda(t-s)} \gamma \|w(s)\| ds \\
& \leq \mu e^{\lambda(t-t_k)} \left[e^{\lambda(t_k-t_{k-1})} V_{\sigma(t_{k-1})}(t_{k-1}) + \int_{t_{k-1}}^{t_k} e^{\lambda(t_k-s)} \gamma \|w(s)\| ds \right] \\
& \quad + \int_{t_k}^t e^{\lambda(t-s)} \gamma \|w(s)\| ds \\
& \leq \mu^2 e^{\lambda(t-t_k)} \left[e^{\lambda(t_k-t_{k-1})} V_{\sigma(t_{k-1}^-)}(t_{k-1}^-) + \int_{t_{k-1}}^{t_k} e^{\lambda(t_k-s)} \gamma \|w(s)\| ds \right] \\
& \quad + \int_{t_k}^t e^{\lambda(t-s)} \gamma \|w(s)\| ds \\
& \leq \dots \\
& \leq \mu^k e^{\lambda t} V_{\sigma(0)}(0) + \mu^k \int_0^{t_1} e^{\lambda(t-s)} \gamma \|w(s)\| ds + \mu^{k-1} \\
& \quad \cdot \int_{t_1}^{t_2} e^{\lambda(t-s)} \gamma \|w(s)\| ds + \dots + \int_{t_k}^t e^{\lambda(t-s)} \gamma \|w(s)\| ds \\
& \leq \mu^N e^{\lambda T_f} V_{\sigma(0)}(0) + \int_0^t \mu^{N_{\sigma(t)}(s,t)} e^{\lambda(t-s)} \gamma \|w(s)\| ds \\
& \leq \mu^N e^{\lambda T_f} V_{\sigma(0)}(0) + \mu^N \int_0^{T_f} e^{\lambda T_f} \gamma \|w(s)\| ds \\
& \leq \mu^N e^{\lambda T_f} (V_{\sigma(0)}(0) + \varepsilon \gamma).
\end{aligned} \tag{32}$$

From (24), (32) and the zero initial condition $x(0) = 0$, we have

$$V_{\sigma(t)}(t) \geq x^T(t)v_p, \tag{33}$$

$$V_{\sigma(t)}(t) \leq \mu^N e^{\lambda T_f} \varepsilon \gamma. \tag{34}$$

From (3), (33) and (34), one has

$$x^T(t)v_p \leq e^{(\lambda + (\ln \mu / T_\alpha))T_f} \varepsilon \gamma. \quad (35)$$

According to (22) and (35), we can easily obtain

$$x^T(t)v_p < 1. \quad (36)$$

By (5) and (19), we get

$$y^T(t)\delta = g^T(x(t))C_p^T \delta \leq m_4 x^T(t)C_{\sigma(t)}^T \delta < x^T(t)v_p. \quad (37)$$

From (36) and (37), we obtain

$$y^T(t)\delta < 1. \quad (38)$$

Thus, the proof is completed.

3.2. L_1 IO-FTS Performance Analysis. In this section, we will consider the problem of L_1 IO-FTS for system (14).

Theorem 2. Consider the system (14), for given constants T_f , λ_p , $p \in \mathbb{N}$, η , and a vector $\delta > 0$, if there exist positive vectors $v_p \in R^n$, $\rho_p \in R^n$, $q_p \in R^n$, $p \in \mathbb{N}$, such that (16), (17), (19), (20), and the following inequalities hold,

$$A_p^T v_p + \rho_p + \tau_1 q_p - \frac{\lambda_p}{m_2} v_p + \frac{m_4}{m_1} C_p^T \delta < 0, \quad (39)$$

$$B_p^T v_p < \eta 1_I, \quad (40)$$

$$\varepsilon \eta < e^{-\lambda T_f}, \quad (41)$$

where $v_p = [v_{p1}, v_{p2}, \dots, v_{pn}]^T \sqrt{2}$, $\rho_p = [\rho_{p1}, \rho_{p2}, \dots, \rho_{pn}]^T$, $q_p = [q_{p1}, q_{p2}, \dots, q_{pn}]^T$, $\lambda = \max_{p \in \mathbb{N}} \{\lambda_p\}$, $\mu (\mu \geq 1)$ satisfies (20), then the system (14) is L_1 IO-FTS for any switching signal $\sigma(t)$ with the ADT scheme

$$T_\alpha > T_\alpha^* = \max \left\{ \frac{T_f \ln \mu}{\ln(e^{-\lambda T_f}) - \ln(\varepsilon \eta)}, \frac{\ln \mu}{\eta} \right\}. \quad (42)$$

Proof. (39) can be easily derived from (15). Let $\eta > \gamma$ in Theorem 1, from (16), (17), (19), (20), (39), (40), and (41), we can obtain that the system (14) is IO-FTS with respect to $(\delta, T_f, \varepsilon, \sigma(t))$.

Choosing the multiple co-positive-type Lyapunov functional (24). Similar to the proof process of Theorem 1, from (16), (17), and (39), we have

$$\dot{V}_p(t) - \lambda_p V_p(t) + \|y(t)\| - \eta \|w(t)\| < 0. \quad (43)$$

Denoting $\nabla(s) = \eta \|w(s)\| - \|y(s)\|$ and integrating both sides of (43) from t_k to t for $t \in [t_k, t_{k+1})$, it gives rise to

$$V_{\sigma(t)}(t) \leq e^{\lambda_{\sigma(t_k)}(t-t_k)} V_{\sigma(t_k)}(t_k) + \int_{t_k}^t e^{\lambda_{\sigma(t_k)}(t-s)} \nabla(s) ds. \quad (44)$$

Similar to the proof process of (32), for any $t \in [0, T_f]$, we can obtain

$$V_{\sigma(t)}(t) \leq \mu^{N_{\sigma(t)}(0,t)} e^{\lambda t} V_{\sigma(0)}(0) + \int_0^t e^{\lambda(t-s)} \mu^{N_{\sigma(t)}(s,t)} \nabla(s) ds. \quad (45)$$

Considering the zero initial condition, we have

$$\int_0^t e^{\lambda(t-s)} \mu^{N_{\sigma(t)}(s,t)} \|y(s)\| ds < \eta \int_0^t e^{\lambda(t-s)} \mu^{N_{\sigma(t)}(s,t)} \|w(s)\| ds. \quad (46)$$

Multiplying both sides of (46) by $\mu^{-N_{\sigma(t)}(0,t)}$ leads to

$$\int_0^t e^{\lambda(t-s)} \mu^{-N_{\sigma(t)}(0,s)} \|y(s)\| ds < \eta \int_0^t e^{\lambda(t-s)} \mu^{-N_{\sigma(t)}(0,s)} \|w(s)\| ds. \quad (47)$$

Noting that $\mu > 1$, $N_{\sigma(t)}(0,s) \leq (s/t)$, and $T_\alpha > (\ln \mu / \lambda)$, we obtain that $0 < N_{\sigma(t)}(0,s) \leq (s/t) \leq (\lambda s / \ln \mu)$; that is, $e^{-\lambda s} \leq \mu^{-N_{\sigma(t)}(0,s)} < 1$. Then (47) can be turned into

$$\int_0^t e^{\lambda(t-s)} e^{-\lambda s} \|y(s)\| ds < \eta \int_0^t e^{\lambda(t-s)} \|w(s)\| ds. \quad (48)$$

Let $t = T_f$, then multiplying both sides of (48) by $e^{-\lambda T_f}$ yields

$$\int_0^{T_f} e^{-2\lambda s} \|y(s)\| ds < \eta \int_0^{T_f} \|w(s)\| ds. \quad (49)$$

Denoting $\beta = 2\lambda$, (49) can be rewritten as

$$\int_0^{T_f} e^{-\beta s} \|y(s)\| ds < \eta \int_0^{T_f} \|w(s)\| ds. \quad (50)$$

Thus, the proof is completed.

3.3. L_1 IO-FTS Controller Design. In this subsection, we consider the design of L_1 IO-FTS controller. Considering system (1), under the controller $u(t) = K_{\sigma(t)} f(x(t))$, the corresponding closed-loop system is given by

$$\begin{cases} \dot{x}(t) = (A_{\sigma(t)} + G_{\sigma(t)} K_{\sigma(t)}) f(x(t)) + A_{d\sigma(t)} f(x(t-d(t))) + A_{b\sigma(t)} f\left(\int_{t-\tau_2}^t x(s) ds\right) + B_{\sigma(t)} w(t), \\ y(t) = C_{\sigma(t)} g(x(t)), \\ x(\theta) = \varphi(\theta), \quad \theta \in [-\tau_1, 0]. \end{cases} \quad (51)$$

By Lemma 2, to guarantee the positivity of system (51), $A_p + G_p K_p$ should be Metzler matrices.

Theorem 3. Consider the system (51), for given constants T_f , λ_p , $p \in \underline{N}$, and a vector $\delta > 0$, if there exist positive vectors $v_p \in R^n$, $\rho_p \in R^n$, $q_p \in R^n$, h_p , $p \in \underline{N}$, such that (16), (17), (19), (20), (40)–(42), and the following inequalities hold:

$$A_p + G_p K_p \text{ are Metzler matrices,} \quad (52)$$

$$A_p^T v_p + \rho_p + \tau_1 q_p - \frac{\lambda_p}{m_2} v_p + \frac{m_4}{m_1} C_p + h_p < 0, \quad (53)$$

where $h_p = K_p^T G_p^T v_p$, $v_p = [v_{p1}, v_{p2}, \dots, v_{pn}]^T$, $\rho_p = [\rho_{p1}, \rho_{p2}, \dots, \rho_{pn}]^T$, and $q_p = [q_{p1}, q_{p2}, \dots, q_{pn}]^T$; then, under the ADT scheme (42), the resulting closed-loop system (51) is L_1 IO-FTS with respect to $(\delta, T_f, \varepsilon, \sigma(t))$, where $\mu (\mu \geq 1)$ satisfies (20).

Proof. Replacing A_p in (39) with $A_p + G_p K_p$, then letting $h_p = K_p^T G_p^T v_p$, similar to Theorem 2, we can get (53); then, the resulting closed-loop system (51) is L_1 IO-FTS with ADT scheme (42).

The proof is completed.

Next, an algorithm is presented to obtain the feedback gain matrices K_p , $p \in \underline{S}$.

Remark 6. There is not a systemic method to adjust these parameters; the selection is generally by experience.

4. Numerical Examples

Example 1. Consider system (1) with the parameters as follows:

$$\begin{aligned} A_1 &= \begin{bmatrix} -1.6 & 0.3 \\ 0.2 & -1.2 \end{bmatrix}, \\ A_{d1} &= \begin{bmatrix} 0.06 & 0.02 \\ 0.08 & 0.06 \end{bmatrix}, \\ A_{b1} &= \begin{bmatrix} 0.02 & 0.01 \\ 0.01 & 0.02 \end{bmatrix}, \\ G_1 &= \begin{bmatrix} 0.4 \\ 0.2 \end{bmatrix}, \\ A_2 &= \begin{bmatrix} -1.1 & 0.1 \\ 0.2 & -0.8 \end{bmatrix}, \\ A_{d2} &= \begin{bmatrix} 0.01 & 0.05 \\ 0.03 & 0.04 \end{bmatrix}, \\ A_{b2} &= \begin{bmatrix} 0.01 & 0.02 \\ 0.02 & 0.01 \end{bmatrix}, \\ G_2 &= \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, \\ B_1 &= \begin{bmatrix} 0.04 \\ 0.05 \end{bmatrix}, \\ B_2 &= \begin{bmatrix} 0.02 \\ 0.03 \end{bmatrix}, \end{aligned}$$

$$\begin{aligned} C_1 &= \begin{bmatrix} 0.03 & 0.01 \\ 0.04 & 0.04 \end{bmatrix}, \\ C_2 &= \begin{bmatrix} 0.03 & 0.04 \\ 0.02 & 0.02 \end{bmatrix}, \end{aligned} \quad (54)$$

where $f_i(x_i(t)) = x_i(t) + (x_i(t)/(x_i^2(t) + 2))$, $g_i(x_i(t)) = x_i(t)$; then, we get $m_1 = 1$, $m_2 = 2$, $m_3 = m_4 = 1$. Choosing $d(t) = 0.1 \sin(t) + 0.2$, $w(t) = 0.05e^{-0.5t}$, $\tau_2 = 0.1$, $\tau_1 = \max\{d(t), \tau_2\} = 0.3$, $h = 0.1$, $\lambda_1 = \lambda_2 = 0.5$, $\eta = 0.5$, $\varepsilon = 0.01$, $\mu = 1.5$, $\delta = [1.25 \ 1.75]^T$. Solving the inequalities in Theorem 3 by linear programming, we get

$$\begin{aligned} v_1 &= \begin{bmatrix} 5.4493 \\ 4.4656 \end{bmatrix}, \\ \rho_1 &= \begin{bmatrix} 2.3319 \\ 0.5649 \end{bmatrix}, \\ q_1 &= \begin{bmatrix} 15.9376 \\ 12.7097 \end{bmatrix}, \\ v_2 &= \begin{bmatrix} 7.3646 \\ 6.2864 \end{bmatrix}, \\ \rho_2 &= \begin{bmatrix} 2.2885 \\ 0.6289 \end{bmatrix}, \\ q_2 &= \begin{bmatrix} 15.3861 \\ 13.9692 \end{bmatrix}, \\ h_1 &= \begin{bmatrix} 0.7444 \\ 0.2401 \end{bmatrix}, \\ h_2 &= \begin{bmatrix} 0.7191 \\ 0.5509 \end{bmatrix}. \end{aligned} \quad (55)$$

By $h_p = K_p^T G_p^T v_p$, we obtain

$$\begin{aligned} K_1 &= [0.3415 \ 0.2688], \\ K_2 &= [0.4899 \ 0.2921]. \end{aligned} \quad (56)$$

It is easy to verify that $A_p + G_p K_p$ are Metzler matrices. Then, from (42), we can obtain $T_\alpha^* = 0.8105$. Choosing $T_\alpha = 0.82 > T_\alpha^*$. The simulation results are shown in Figures 1–3, where the $x(0) = [0, 0]^T$. Figure 1 shows the state trajectories of the closed-loop system (1). The switching signal $\sigma(t)$ is depicted in Figure 2. Figure 3 plots the evolution of $y^T(t)\delta$ of system (1). From the simulation results, we know $y^T(t)\delta < 1$, which implies the effectiveness of our proposed method.

Example 2. A practical price dynamics model is considered as an uncertain positive switched system with time-varying delays in [3]. In an economic system, affected by the national macroeconomic regulation and control policy, the relation

- (1) Step 1. By adjusting the parameters λ_p and solving (16), (17), (19), (20), (40)–(42), (52), and (53) via linear programming, positive vectors ν_p , ρ_p , q_p , and h_p can be obtained.
- (2) Step 2. Substituting ν_p and h_p into $\tilde{h}_p = K_p^T G_p^T \nu_p$, K_p can be obtained. According to (42), T_α can also be obtained.
- (3) Step 3. If the gain K_p satisfies (52), then, K_p is admissible. Otherwise, return to Step 1.

ALGORITHM 1

FIGURE 1: State trajectories of closed-loop system (1).

between supply and demand always presents nonlinear behavior. So, it is more suitable to describe the price dynamic model by positive switched nonlinear systems [18]. Consider the parameters as follows:

$$\begin{aligned}
 A_1 &= \begin{bmatrix} 0.4 & 0.2 \\ 0.3 & -2.5 \end{bmatrix}, \\
 A_{d1} &= \begin{bmatrix} 0.1 & 0.3 \\ 0.2 & 0.2 \end{bmatrix}, \\
 G_1 &= \begin{bmatrix} 0.3 \\ 0.4 \end{bmatrix}, \\
 A_2 &= \begin{bmatrix} -1.5 & 0.3 \\ 0.2 & 0.2 \end{bmatrix}, \\
 A_{d2} &= \begin{bmatrix} 0.2 & 0.1 \\ 0.3 & 0.1 \end{bmatrix}, \\
 G_2 &= \begin{bmatrix} 0.2 \\ 0.3 \end{bmatrix}, \\
 B_1 &= \begin{bmatrix} 0.3 \\ 0.2 \end{bmatrix}, \\
 B_2 &= \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \\
 C_1 &= \begin{bmatrix} 0.2 & 0.3 \\ 0.1 & 0.1 \end{bmatrix}, \\
 C_2 &= \begin{bmatrix} 0.3 & 0.2 \\ 0.2 & 0.1 \end{bmatrix}.
 \end{aligned} \tag{57}$$

FIGURE 2: Switching signal of system (1).

where $f_i(x_i(t)) = x_i(t) + (x_i(t)/(x_i(t) + 2))$, $g_i(x_i(t)) = x_i(t)$; then, we get $m_1 = 1$, $m_2 = 2$, $m_3 = m_4 = 1$. Choosing $d(t) = 0.1 \cos(t) + 0.2$, $w(t) = 0.05e^{-0.5t}$, $\tau_1 = 0.3$, $h = 0.2$, $\lambda_1 = \lambda_2 = 0.5$, $\eta = 0.5$, $\varepsilon = 0.01$, $\mu = 1.3$, $\delta = [1 \ 2]^T$. Solving the inequalities in Theorem 3 by linear programming, we get

$$\begin{aligned}
 \nu_1 &= \begin{bmatrix} 0.8927 \\ 0.9982 \end{bmatrix}, \\
 \rho_1 &= \begin{bmatrix} 1.2218 \\ 0.2905 \end{bmatrix}, \\
 q_1 &= \begin{bmatrix} 0.8398 \\ 0.1101 \end{bmatrix}, \\
 \nu_2 &= \begin{bmatrix} 1.3957 \\ 1.3527 \end{bmatrix}, \\
 \rho_2 &= \begin{bmatrix} 0.9671 \\ 0.2873 \end{bmatrix}, \\
 q_2 &= \begin{bmatrix} 0.7778 \\ 0.1292 \end{bmatrix}, \\
 \tilde{h}_1 &= \begin{bmatrix} 0.1202 \\ 0.5457 \end{bmatrix}, \\
 \tilde{h}_2 &= \begin{bmatrix} 0.0172 \\ 0.0814 \end{bmatrix}.
 \end{aligned} \tag{58}$$

By $\tilde{h}_p = K_p^T G_p^T \nu_p$, we obtain

FIGURE 3: Evolution of $y^T(t)\delta$ of system (1). FIGURE 6: Evolution of $y^T(t)\delta$ of system (1).

FIGURE 4: State trajectories of closed-loop system (1).

FIGURE 5: Switching signal of system (1).

$$\begin{aligned} K_1 &= \begin{bmatrix} 0.1791 & 0.8060 \end{bmatrix}, \\ K_2 &= \begin{bmatrix} 0.0246 & 0.1143 \end{bmatrix}. \end{aligned} \quad (59)$$

It is easy to verify that $A_p + G_p K_p$ are Metzler matrices. Then, from (42), we can obtain $T_\alpha^* = 0.7483$. Choosing $T_\alpha = 0.75 > T_\alpha^*$. The simulation results are shown in Figures 4–6, where the $x(0) = [0, 0]^T$. Figure 4 shows the state trajectories of the closed-loop system (1). The switching signal $\sigma(t)$ is depicted in Figure 5. Figure 6 plots the evolution of $y^T(t)\delta$ of system (1). From the simulation results, we know $y^T(t)\delta < 1$, which implies the effectiveness of our proposed method.

5. Conclusions

This paper has investigated the problem of L_1 IO-FTS for positive switched nonlinear systems with time-varying and distributed delays. Firstly, the concept of L_1 IO-FTS for positive switched nonlinear systems is firstly proposed. Then, by constructing nonlinear Lyapunov–Krasovskii functions, a state feedback controller is designed. Based on the ADT approach, some sufficient conditions are obtained to guarantee that the closed-loop system is L_1 IO-FTS.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that there are no conflicts of interests regarding the publication of this paper.

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