

Research Article

Study on Equivalent Viscous Damping Coefficient of Sucker Rod Based on the Principle of Equal Friction Loss

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Equivalent viscous damping coefficient is an important parameter of wave equation for sucker rod string. In this paper, based on the principle of equal friction loss, when the viscous energy consumption and the local damping energy consumption are taken into account, effects of equivalent viscous damping coefficients are obtained. Through deducing energy consumption equation of oil and energy consumption equation of the coupling, theoretical formula for equivalent damping coefficient of sucker rods is received. Results show that the smaller the K is (K is the ratio of sectional area of tubing to sucker rod), the larger the proportion of damping coefficient caused by viscous energy consumption in the equivalent damping coefficient of sucker rod system is. When $K < 0.095$, the proportion of damping coefficient caused by viscous energy consumption is more than 90%. Reducing the sudden change of cross-section area at sucker rod coupling has remarkable effect on reducing damping force of the sucker rod system. The research provides a theoretical basis for the application and design of sucker rod and tubing.

1. Instruction

Sucker rod equipment is widely used in artificial lift wells [1–3]. Its failure modes are various, such as pump leakage, pump blockage, sucker rod fracture, sucker rod eccentric wear, and so on. In order to accurately predict and judge the working condition of oil wells, the sucker rod string wave equation is regularly used to characterize the sucker rod pumping system. The wave equation contains viscous damping coefficient. The magnitude of viscous damping coefficient has a great influence on the results of prediction and diagnosis [4–7].

Since the 1960s, many researchers have carried out the study of equivalent viscous damping coefficient [8]. Now, there are many methods to determine the equivalent viscous damping coefficient, such as experiential algorithm, deduction formula of hypothetical conditions, calculated by surface dynamometer cards, etc. [9–12]. S.G.Gibbs considered that the nonviscous damping force is negligible. Assuming that the suspension point motion is a simple harmonic motion, the average velocity of the rod is expressed by the root mean square value of the instantaneous velocity of the sucker rod, and the dissipated work in one cycle of the rod string

is equal to the dissipated work by the equivalent viscous damping. Based on these assumptions, the formula of the equivalent viscous damping coefficient is derived [13–15]. The formula of sucker rod damping force on the condition of laminar flow ($Re < 2300$) is derived by A.M.Pearvillej, but the formula can only be applied to laminar flow in Poiseuille flow [16]. Based on the principle of equivalent method, the formula of dimensionless equivalent viscous damping coefficient is derived by Rubio D [17]. Zhang Qi used the energy consumption of the upstroke to replace the downstroke energy consumption. By calculating the friction loss caused by the viscous resistance of the sucker rod string in a cycle, the damping coefficient formula suited for various flow patterns is derived based on the principle of equal friction loss [18]. Using the curve method, M.J.Basition proposed that the intersection of the water power/damping coefficient and pump power/damping coefficient curves is the viscous damping coefficient of the sucker rod. Thus, the equivalent viscous damping coefficient is obtained [19]. Based on the Gibbs formula, Everitt, T.A. deduced the formula of viscous damping coefficient by cyclic iteration method. This method can be applied to any material sucker rod, but Gibbs damping coefficient formula can only be applied to

steel sucker rod [20]. On the basis of the sucker rod string wave equation, Sun Renyuan et al. proposed the equivalent viscous damping coefficient formula which was related to the actual pump dynamometer cards by using the numerical integration method and established the corresponding iterative algorithm to calculate the equivalent viscous damping coefficient [21]. I.Steliga and Dong-Yu Wang proposed that the sucker rod dynamic model including the influences of rod-fluid-tubing viscous friction and rod-tubing Coulomb friction is developed. And then, a method for calculating vibration damping of sucker rod string based on indicator diagram is proposed. The periodic variation of vibration damping coefficient is analyzed with surface dynamometer cards and pump dynamometer cards. However, this method can only be used to analyze the dynamometer cards of a specific oil well [22, 23].

All the above studies are aimed at the specific well conditions, and the viscous damping coefficients are calculated when the size of sucker rod and tubing are unchanged. Unfortunately, the energy consumption of sucker rod coupling and rod guides in the process of pumping is not considered. The calculation model needs to be optimized. On the basis of Gibbs formula and Zhang Qi formula, this paper assumes that the fluid is Couette flow in a circular pipe and considers the energy consumption of coupling and rod guides. Based on the principle of equal friction loss, the equivalent viscous damping coefficient of sucker rod is derived. And the influence of sucker rod and tubing size on the equivalent viscous damping coefficient is analyzed, which provides a theoretical basis for sucker rod and tubing matching application.

2. Dynamic Equations of Sucker Rod String

Dynamic differential equation of sucker rod string (Figure 1) with viscous damping coefficient [14] is as follows:

$$\frac{\partial^2 u}{\partial t^2} = a^2 \frac{\partial^2 u}{\partial x^2} - c \frac{\partial u}{\partial t} \quad (1)$$

where u is the displacement of the sucker rod node; x is the distance between the node and the wellhead; t is the run duration of sucker rod.

The suspension point displacement function and load function expressed by Fourier series are used as boundary conditions:

$$D_{x=0} = \frac{\sigma_0}{2} + \sum_0^{\bar{n}} \sigma_n \cos n\omega t + \tau_n \sin n\omega t \quad (2)$$

$$u_{x=0} = \frac{v_0}{2} + \sum_0^{\bar{n}} v_n \cos n\omega t + \delta_n \sin n\omega t \quad (3)$$

where σ_n , τ_n , v_n , and δ_n are Fourier coefficient.

Combining (1)-(3), the displacement function of sucker rod at any depth of x section can be given by

$$u = \frac{\sigma_0}{2EA_r} x + \frac{v_0}{2} + \sum_0^{\bar{n}} O_n \cos n\omega t + P_n \sin n\omega t \quad (4)$$

3. The Principle of Equal Frictional Loss

When the wave equation is used to solve the dynamics of sucker rod string, the damping term is included in the equation. Damping of sucker rod string system is a combination of many factors, such as viscosity, temperature, and flow pattern of oil, so it is difficult to solve the real damping of the system. In general, equivalent damping coefficient is used to replace the real damping. The substituted condition is that the frictional loss done by the viscous damping term in the wave equation is equal to the energy loss in the actual pumping well during a period of motion, and the viscous damping coefficient is called the equivalent viscous damping coefficient. This described the principle of equal frictional loss [13].

$$W_f = W_d \quad (5)$$

where W_f is the frictional loss of sucker rod string during a period of motion and W_d is the energy consumption in actual pumping wells.

$$W_f = \int_0^L \int_0^T c \rho_r A_r \left(\frac{\partial u}{\partial t} \right)^2 dt dx \quad (6)$$

Without considering the Coulomb friction of single-stage rod vertical well, the actual energy consumption includes the viscous energy consumption in the up stroke and down stroke and the local viscous energy consumption of the oil passing through the sucker rod coupling in the actual operation of sucker rod.

Combining (5) and (6), the equivalent viscous damping coefficient is deduced:

$$c = \frac{W_d}{\rho_r A_r \int_0^L \int_0^T (\partial u / \partial t)^2 dt dx} \quad (7)$$

4. Viscous Energy Consumption along the Tubing of Oil

Within a running cycle (Figure 3) of the sucker rod string, the oil moves up with the sucker rod and the direction of net flow is the same as that of the sucker rod during the up stroke. The sucker rod moves downward, but the direction of net flow of oil is upward. Then, the average velocity of oil is opposite to the direction of sucker rod movement on the occasion of the down stroke. Therefore, when calculating the viscous energy consumption along the tubing, the viscous energy consumption of the upstroke and the downstroke should be calculated, respectively, according to the net flow rate of oil.

4.1. Couette Flow of Circular Tube. In the process of pumping, the tubing is always in a static state and the sucker rod is in a moving state. Therefore, the oil flow can be regarded as Couette flow when analyzing the flow of oil in the tubing-rod string annulus [24].

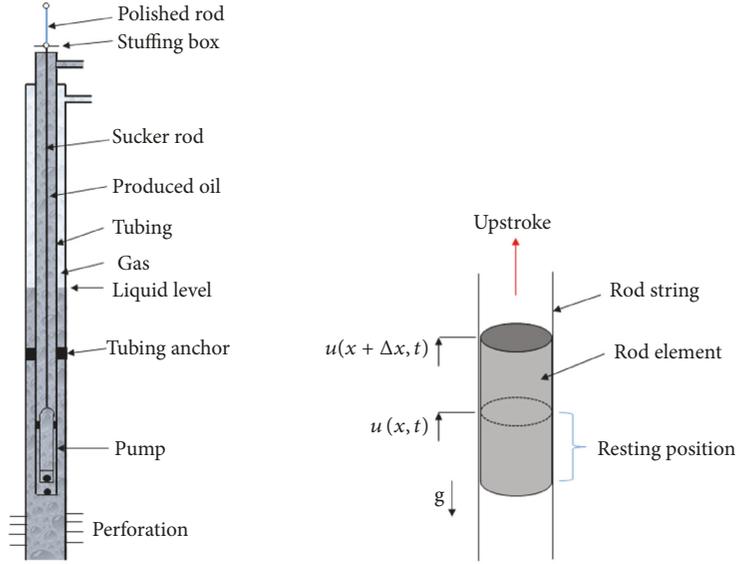


FIGURE 1: The diagram of system and microelement of sucker rod string.

The momentum equation of incompressible fluid with constant physical properties in cylindrical-coordinates system (Figure 2) can be expressed as

$$\begin{aligned} & \frac{\partial V_x}{\partial t} + V_r \frac{\partial V_x}{\partial r} + \frac{V_\varphi}{r} \frac{\partial V_x}{\partial \varphi} + V_x \frac{\partial V_x}{\partial x} \\ &= -\frac{1}{\rho} \frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 V_x}{\partial r^2} + \frac{1}{r^2} \frac{\partial^2 V_x}{\partial \varphi^2} + \frac{\partial^2 V_x}{\partial x^2} + \frac{1}{r} \frac{\partial V_x}{\partial r} \right) \quad (8) \\ &+ g \end{aligned}$$

The function of the energy consumption along the tubing can be expressed as

$$\begin{aligned} \Phi = \rho \nu & \left[2 \left(\frac{\partial V_r}{\partial r} \right)^2 + 2 \left(\frac{1}{r} \frac{\partial V_\varphi}{\partial \varphi} + \frac{1}{r} V_r \right)^2 + 2 \left(\frac{\partial V_x}{\partial x} \right)^2 \right. \\ &+ \left. \left(\frac{1}{r} \frac{\partial V_r}{\partial \varphi} + \frac{\partial V_\varphi}{\partial r} - \frac{1}{r} V_\varphi \right)^2 + \left(\frac{1}{r} \frac{\partial V_x}{\partial \varphi} + \frac{\partial V_\varphi}{\partial x} \right)^2 \right. \\ &+ \left. \left(\frac{\partial V_r}{\partial x} + \frac{\partial V_x}{\partial r} \right)^2 \right] \quad (9) \end{aligned}$$

where if $r = r_0$, then $V_r < \infty$, $V_\varphi < \infty$, and $V_x < \infty$. If $r = r_1$, then $V_r = V_\varphi = V_x = 0$.

To solve the above equations, the oil is regarded as steady one-way parallel flow along the x-axis, which is uniform in the direction of flow and axisymmetric in the axial direction.

$$\begin{aligned} V_x &= w \\ V_r &= V_\varphi = 0 \end{aligned}$$

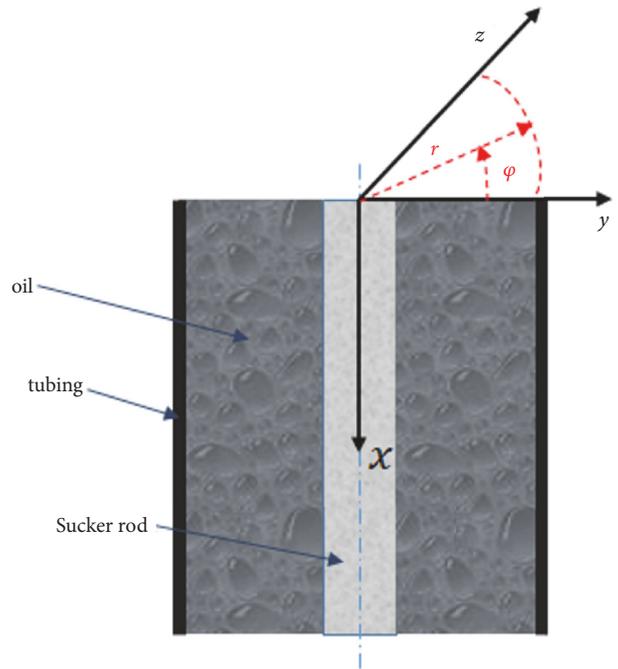


FIGURE 2: Sucker rod string in cylindrical-coordinate system.

$$\frac{\partial}{\partial t} = 0$$

$$\frac{\partial}{\partial \varphi} = 0$$

$$\frac{\partial V_x}{\partial x} = 0$$

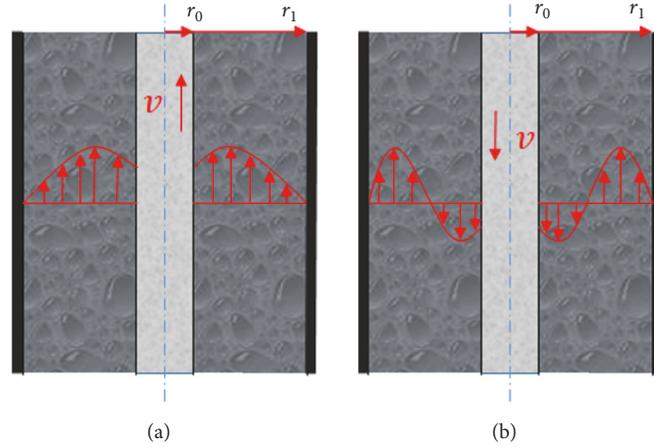


FIGURE 3: Velocity distribution of crude along section. (a) Upstroke and (b) downstroke.

Substituting the above boundary conditions equation (10), the momentum equation (8) can be simplified to

$$-\frac{1}{\rho} \frac{\partial p}{\partial x} + v \left(\frac{\partial^2 w}{\partial r^2} + \frac{1}{r} \frac{\partial w}{\partial r} \right) + g = 0 \quad (11)$$

where p is a function of x and $\partial p / \partial x = dp / dx$.

Define $P = dp / dx - \rho g$, P is the baric gradient of fluid flow except gravity. And $\mu = \rho \nu$.

The momentum equation (11) can be further simplified to

$$P = \mu \left(\frac{d^2 w}{dr^2} + \frac{1}{r} \frac{dw}{dr} \right) \quad (12)$$

Substituting the above boundary conditions equation (10), the energy consumption function along the tubing can be obtained:

$$\Phi = \mu \left(\frac{dw}{dr} \right)^2 \quad (13)$$

By integrating the momentum equation (13), the oil velocity equation can be obtained:

$$w = \frac{1}{4\mu} P r^2 + C_1 \ln r + C_2 \quad (14)$$

Substitute the boundary conditions: $r = r_0$, $w = v$; $r = r_1$, $w = 0$. The coefficient C_1 and C_2 can be obtained:

$$C_1 = \frac{v + (P/4\mu)(r_1^2 - r_0^2)}{\ln(r_0/r_1)} \quad (15)$$

$$C_2 = -\frac{P}{4\mu} r_1^2 - \frac{v + (P/4\mu)(r_1^2 - r_0^2)}{\ln(r_0/r_1)} \ln r_1 \quad (16)$$

Cross-section flow in X direction is as follows:

$$Q = \int_0^{2\pi} \int_{r_0}^{r_1} w r dr d\theta$$

$$= \frac{\pi r_0^4 P}{8\mu} \left[1 - m^4 + \frac{(m^2 - 1)^2}{\ln m} \right] \quad (17)$$

$$+ \pi r_0^2 v \left(\frac{m^2 - 1}{2 \ln m} - 1 \right)$$

$$m = \frac{r_1}{r_0} > 1 \quad (18)$$

According to (17), the cross-section flow in the X direction is the vector sum of differential pressure flow and shear flow. The first half of the formula is differential pressure flow, and the second half is shear flow.

According to (13)-(16) and (18), the equation of energy dissipation per time per length of oil can be derived.

$$N = \int_0^{2\pi} \int_{r_0}^{r_1} \Phi r dr d\theta$$

$$= \frac{\pi P^2 r_0^4}{8\mu} \left[m^4 - 1 - \frac{(m^2 - 1)^2}{\ln m} \right] + \frac{2\pi \mu v^2}{\ln m} \quad (19)$$

4.2. Viscous Energy Consumption of the Upstroke. The direction of oil movement is the same as that of the sucker rod in the upstroke; at the same time the sucker rod and the

oil should be regarded as an integral whole in consideration of the viscous energy consumption. The viscous energy consumption is caused by the work loss by the oil shear force and the hydraulic loss along the oil flow. The actual flow rate of the oil pumping system is equal to the volume of the liquid discharged by the pump per time. When calculating the viscous energy consumption, the flow rate should be considered as the actual flow rate of the oil pumping system.

Without considering the elastic deformation of the sucker rod, the volume of the liquid discharged from the upstroke per time can be expressed as

$$q = \pi (r_p^2 - r_0^2) v_{x=L} \quad (20)$$

where r_p is the radius of the downhole pump plunger.

According to the continuity equation of oil flow, the cross-section flow rate in the X direction is equal to the volume of liquid discharged by the pump per time. Combine (17) and (20). P can be given by

$$P = \frac{4\mu}{r_0^2} \frac{2(m_2^2 - 1)v_{x=L} \ln m - v(m^2 - 1 - 2 \ln m)}{(1 - m^4) \ln m + (m^2 - 1)^2} \quad (21)$$

$$m_2 = \frac{r_p}{r_0} \quad (22)$$

On the upstroke ($0 - t_1$), according to (19) and (21), the viscous energy consumption along the oil flow can be given by

$$\begin{aligned} W_{d1} &= \int_0^L \int_0^{t_1} N dt dx \\ &= \left[\frac{2\pi\mu}{\ln m} + 2\pi\mu \frac{(m^2 - 1 - 2 \ln m)^2}{(m^4 - 1) \ln^2 m - (m^2 - 1)^2 \ln m} \right] \\ &\cdot \int_0^L \int_0^{t_1} v^2 dt dx + 8\pi\mu \frac{(m_2^2 - 1)^2 \ln m}{(m^4 - 1) \ln m - (m^2 - 1)^2} \\ &\cdot \int_0^L \int_0^{t_1} v_{x=L}^2 dt dx - 8\pi\mu \\ &\cdot \frac{(m^2 - 1 - 2 \ln m)(m_2^2 - 1)}{(m^4 - 1) \ln m - (m^2 - 1)^2} \int_0^L \int_0^{t_1} v_{x=L} u dt dx \end{aligned} \quad (23)$$

4.3. Viscous Energy Consumption of the Downstroke. The inlet valve of defueling pump is closed and the outlet valve is opened during downstroke, at the same time the sucker rod

enters the tubing and discharges corresponding volume of oil. Therefore, without considering leakage, the actual output flow of the pumping system is equal to the volume of the sucker rod entering the tubing per time.

The volume of the sucker rod entering the tubing in per time can be expressed as

$$q = -\pi r_0^2 v_{x=L} \quad (24)$$

Combine (17) and (24). P can be given by

$$P = \frac{4\mu}{r_0^2} \frac{(m^2 - 1 - 2 \ln m) v + 2v_{x=L} \ln m}{(m^4 - 1) \ln m - (m^2 - 1)^2} \quad (25)$$

On the downstroke ($t_1 - T$), according to (18) and (25), the viscous energy consumption along the tubing can be given by

$$\begin{aligned} W_{d2} &= \int_0^L \int_{t_1}^T N dt dx = 2\pi\mu \\ &\cdot \frac{(m^2 - 1 - 2 \ln m)^2 + (m^4 - 1) \ln m - (m^2 - 1)^2}{(m^4 - 1) \ln^2 m - (m^2 - 1)^2 \ln m} \\ &\cdot \int_0^L \int_{t_1}^T v^2 dt dx + 8\pi\mu \frac{\ln m}{(m^4 - 1) \ln m - (m^2 - 1)^2} \\ &\cdot \int_0^L \int_{t_1}^T v_{x=L}^2 dt dx + 8\pi\mu \\ &\cdot \frac{(m^2 - 1 - 2 \ln m)}{(m^4 - 1) \ln m - (m^2 - 1)^2} \int_0^L \int_{t_1}^T v_{x=L} v dt dx \end{aligned} \quad (26)$$

5. Local Viscous Energy Consumption of Oil

Without considering the eccentricity of the sucker rod, the annulus area between sucker rod and tubing is small at the coupling since diameter of the coupling is larger than the sucker rod. After the oil passing through the coupling, the flow regime is changed and local resistance is lost. Similarly, local resistance loss will also occur at position of rod guides.

5.1. Local Viscous Energy Consumption at Couplings. Calculation formula of local damping force at coupling can be given by [25]

$$F_h = \frac{1}{2} \pi \lambda \rho w_h^2 (r_h^2 - r_0^2) \text{sgn}(v_h) \quad (27)$$

where r_h is the radius of coupling; w_h is the average velocity of oil passing through coupling; λ is the dimensionless coefficient which is defined as

$$\lambda = \frac{5.2 \times 10^4 \times (r_h/r_1 - 0.381)^{2.57} [2.77 - 1.69 (N_{Re}'/N_{Re}) \text{sgn}(v_h)]}{N_{Re}} \quad (28)$$

While on the upstroke, $\text{sgn}(v_h)$ is equal to 1 when v_h is larger than 0; on the downstroke, $\text{sgn}(v_h)$ is equal to -1 when v_h is less than 0.

$$N_{\text{Re}} = \frac{2w_h\rho(r_1 - r_0)}{\mu} \quad (29)$$

$$N_{\text{Re}'} = \frac{2v_h\rho(r_1 - r_0)}{\mu} \quad (30)$$

Local resistance pressure drop can be defined as shown below:

$$\Delta P_h = \frac{F_h}{\pi(r_h^2 - r_0^2)} = \frac{1}{2}\lambda\rho w_h^2 \text{sgn}(v_h) \quad (31)$$

$$W_{h1} = \frac{1.3\pi\mu \times 10^4 \times (r_h/r_1 - 0.381)^{2.57} (2.77m^2 - 1.69m^2 - 1.08)(m^2 - 1)r_0}{(m - 1)(m^2 - 1)} \sum_1^n \int_0^{t_1} v_h v_{x=L} dt \quad (34)$$

On the downstroke, the relationship between the velocity of oil (w_h) and the rod speed (v_h) can be obtained:

$$w_h = \frac{1}{1 - m^2} v_h \quad (35)$$

According to (28)-(32) and (35), the local viscous energy consumption of downstroke at coupling can be given by

$$W_{h2} = \frac{1.3\pi\mu \times 10^4 \times (r_h/r_1 - 0.381)^{2.57} (1.08 - 1.69m^2)r_0}{(m - 1)(1 - m^2)} \cdot \sum_1^n \int_{t_1}^T v_h v_{x=L} dt \quad (36)$$

5.2. Local Viscous Energy Consumption at Rod Guides. Friction caused by the resistance along rod guides can be expressed as [26]

$$F_{g1} = 2\pi\mu\chi l_g n_g w_g \quad (37)$$

Friction caused by local hydraulic loss at rod guides can be expressed as

$$F_{g2} = 2\pi\mu\chi l_{ge} n_g w_g \quad (38)$$

According to (37) and (38), the calculation formula of local damping force at rod guides can be given by

$$F_g = 2\pi\mu\chi n_g w_g (l_g + l_{ge}) \quad (39)$$

where l_g is the length of rod guides; n_g is the number of rod guides; w_g is the average velocity of oil passing through rod guides; χ is the dimensionless coefficient which is defined as

$$\chi = \frac{1}{\ln(2r_1/d_{ge})} \quad (40)$$

Local viscous energy consumption at coupling:

$$W_h = \sum_1^n \int_0^t \Delta P_h Q dt \quad (32)$$

On the upstroke, according to the continuity equation, the relationship between the velocity of oil (w_h) and the rod speed (v_h) can be obtained:

$$w_h = \frac{r_p^2 - r_0^2}{r_1^2 - r_0^2} v_h \quad (33)$$

According to (28)-(33), the local viscous energy consumption of upstroke at coupling can be given by

l_{ge} is the equivalent length of rod guides.

$$l_{ge} = \frac{\varepsilon_g}{\lambda_g} d_{ge} \quad (41)$$

ε_g is the local resistance coefficient of rod guides; λ_g is the resistance coefficient along the rod guides; d_{ge} is the equivalent diameter of rod guides.

$$d_{ge} = \frac{-2A_{gt} + 2\sqrt{A_{gt}^2 + (C_{gt}r_1^2 - 2A_{gt}r_1)C_{gt}}}{C_{gt}} \quad (42)$$

where A_{gt} is the flow surface between rod guides and tubing; C_{gt} is the wet circumference of flow surface between rod guides and tubing.

Local resistance pressure drop can be defined as shown below:

$$\Delta P_g = \frac{F_g}{A_{gt}} \quad (43)$$

Local viscous energy consumption at rod guides is as follows:

$$W_g = \int_0^t \Delta P_g Q dt \quad (44)$$

On the upstroke, according to the continuity equation, the relationship between the velocity of oil (w_g) and the rod speed (v_h) can be obtained:

$$w_g = \frac{\pi(r_p^2 - r_0^2)}{A_{gt}} v_h \quad (45)$$

According to (37)-(45), the local viscous energy consumption of upstroke at rod guides can be given by

$$W_{g1} = \frac{2\pi^3 \mu \chi (l_g + l_{ge}) n_g (r_p^2 - r_0^2)^2 \int_0^{t_1} v_h^2 dt}{A_{gt}} \quad (46)$$

On the downstroke, the relationship between the velocity of oil (w_g) and the rod speed (v_h) can be obtained:

$$w_g = -\frac{\pi r_0^2}{A_{gt}} v_h \quad (47)$$

According to (37)-(44) and (47), the local viscous energy consumption of downstroke at rod guides can be given by

$$W_{g2} = \frac{2\pi^3 \mu \chi (l_g + l_{ge}) n_g r_0^4 \int_{t_1}^T v_h^2 dt}{A_{gt}} \quad (48)$$

6. Equivalent Viscous Damping Coefficient

6.1. Theoretical Calculation. During the actual movement of the sucker rod, the energy consumption includes the resistance consumption along the sucker rod and the local energy consumption, which can be, respectively, expressed by (23), (26), (34), (36), (46), and (48). These energy consumption equations all contain the velocity of sucker rod. When calculating the equivalent viscous damping coefficient, it is necessary to simplify the term of velocity in the energy consumption equation at the final calculation of the equivalent viscous damping coefficient.

According to (1)-(4), the velocity equation of pumping rod can be obtained by Fourier series expansion:

$$v = v_{x=0} + \sum_{n=1}^{\bar{n}} (a_{nx} \cos n\omega t + b_{nx} \sin n\omega t) \quad (49)$$

The first half of (49) represents the suspension velocity; the second half represents the wave velocity; a_{nx} and b_{nx} are the Fourier coefficient.

The wave velocity is a periodic function and is far less than the suspension velocity. Therefore, the influence of wave velocity can be neglected in calculating energy consumption.

$$v \approx v_{x=0} \quad (50)$$

$$\begin{aligned} \int_0^L \int_0^T v^2 dt dx &\approx \int_0^L \int_0^T v_{x=L} v dt dx \approx \int_0^L \int_0^T v_{x=L}^2 dt dx \\ &\approx \int_0^L \int_0^T v_{x=0}^2 dt dx \end{aligned} \quad (51)$$

In the motion period of the sucker rod, the upstroke speed and downstroke speed can be regarded as equal, and (51) can be simplified to

$$\begin{aligned} \int_0^L \int_0^{t_1} v^2 dt dx &\approx \int_0^L \int_{t_1}^T v_{x=L} v dt dx \\ &\approx \frac{1}{2} \int_0^L \int_0^T v_{x=L}^2 dt dx \end{aligned} \quad (52)$$

The velocity of integral terms in (34) and (36) can be approximated to

$$\sum_1^n \int_0^{t_1} v_h^2 dt \approx \sum_1^n \int_{t_1}^T v_h^2 dt \approx \frac{1}{2L_1} \int_0^L \int_0^T v^2 dt dx \quad (53)$$

The velocity of integral terms in (46) and (48) can be approximated to

$$\int_0^{t_1} v_h^2 dt \approx \int_{t_1}^T v_h^2 dt \approx \frac{1}{2L} \int_0^L \int_0^T v^2 dt dx \quad (54)$$

where L_1 is the length of per sucker rod.

Substituting (23), (26), (34), (36), (46), (48), and (50)-(54) into (7), the viscous damping coefficient can be given by

$$\begin{aligned} c = & \frac{\pi \mu (m^2 - 1 - 2 \ln m) (2m^2 - 4m^2 \ln m + 4 \ln m - 2)}{\pi \rho_r r_0^2 [(m^4 - 1) \ln m - (m^2 - 1)^2 \ln m]} \\ & + \frac{2\pi \mu [(m^4 - 1) \ln m - (m^2 - 1)^2 + 2 \ln^2 m (m^2 - 1)^2 + 2 \ln^2 m]}{\pi \rho_r r_0^2 [(m^4 - 1) \ln m - (m^2 - 1)^2 \ln m]} \\ & + \frac{1.3\pi \mu \times 10^4 \times (r_h/r_1 - 0.381)^{2.57} (2.77m^2 - 1.69m^2 - 1.08) (m^2 - 1)}{2\pi \rho_r r_0 (m - 1) (m^2 - 1) L_1} \\ & + \frac{1.3\pi \mu \times 10^4 \times (r_h/r_1 - 0.381)^{2.57} (1.69m^2 - 1.08)}{2\pi \rho_r r_0 (m^2 - 1) (m - 1) L_1} + \frac{\pi^2 \mu \chi ((r_p^2 - r_0^2)^2 + r_0^4) (l_g + l_{ge}) n_g}{A_{gt} \rho_r r_0^2 L} \end{aligned} \quad (55)$$

Equation (55) can be decomposed into

$$c = c_1 + c_2 + c_3 \quad (56)$$

$$c_1 = \frac{\pi\mu(m^2 - 1 - 2\ln m)(2m^2 - 4m^2 \ln m + 4\ln m - 2)}{\pi\rho_r r_0^2 [(m^4 - 1)\ln m - (m^2 - 1)^2 \ln m]} \quad (57)$$

$$+ \frac{2\pi\mu [(m^4 - 1)\ln m - (m^2 - 1)^2 + 2\ln^2 m (m^2 - 1)^2 + 2\ln^2 m]}{\pi\rho_r r_0^2 [(m^4 - 1)\ln m - (m^2 - 1)^2 \ln m]}$$

$$c_2 = \frac{1.3\pi\mu \times 10^4 \times (r_h/r_1 - 0.381)^{2.57} (2.77m^2 - 1.69m^2 - 1.08)(m^2 - 1)}{2\pi\rho_r r_0 (m - 1)(m^2 - 1)L_1} \quad (58)$$

$$+ \frac{1.3\pi\mu \times 10^4 \times (r_h/r_1 - 0.381)^{2.57} (1.69m^2 - 1.08)}{2\pi\rho_r r_0 (m^2 - 1)(m - 1)L_1}$$

$$c_3 = \frac{\pi^2 \mu \chi ((r_p^2 - r_0^2)^2 + r_0^4)(l_g + l_{ge})n_g}{A_{gt} \rho_r r_0^2 L} \quad (59)$$

c_1 is the equivalent damping coefficient caused by the viscous energy consumption in the process of sucker rod movement, c_2 is the equivalent viscous damping coefficient caused by the coupling energy consumption in the process of sucker rod movement, and c_3 is the equivalent viscous damping coefficient caused by the rod guides energy consumption in the process of sucker rod movement. The equivalent damping coefficient of the sucker rod system is equal to the linear superposition of the three parts.

The equivalent damping coefficient of the sucker rod can be calculated by (55). According to the actual measure of downhole pump dynamometer card and the surface dynamometer card, the actual equivalent damping coefficient of the sucker rod can be estimated. Next, the relevant parameters of oil wells in Xinjiang Karamay Oilfield are taken into (55) and Zhang Qi formula to estimate the equivalent damping coefficient of sucker rods. Compared with the measured equivalent damping coefficient of sucker rods, the results are shown in Table 1.

In this paper, the energy consumption of coupling and rod guides is considered. The predicted value of the equivalent damping coefficient derived from this method is larger than that predicted by Zhang Qi formula and is closer to the calculated value of the measured parameters (Table 1).

6.2. Application and Regulation Analysis. The theoretical formula of equivalent damping coefficient is showed in (55). The main factors affecting the equivalent damping coefficient are the radius of sucker rod, the radius of tubing, and oil viscosity. When calculating the equivalent damping coefficient, the oil well parameters (Table 2) of Xinjiang Karamay Oilfield are referred. The radius of sucker rod is regarded as the independent variable under different tubing radius. The

equivalent damping coefficient caused by the viscous energy consumption along with the sucker rod movement, the local energy consumption, and the sucker rod system is calculated, respectively.

In Figure 4, the equivalent damping coefficient of the sucker rod system is between 0.01 and 0.281. When the tubing size is fixed, the equivalent damping coefficient of the sucker rod system and the equivalent damping coefficient caused by the viscous energy consumption along the sucker rod system decrease with the increasing of the radius of the sucker rod. Subsequently, the reduction rate of equivalent damping coefficient decreases gradually. The equivalent damping coefficient caused by coupling energy consumption raises with the increasing of sucker rod radius, and the rate is gradually raised. The equivalent damping coefficient caused by rod guides energy consumption is between 0.00042 and 0.0060. The rod guides have little effect on the equivalent damping coefficient. The larger the radius of the sucker rod, the smaller the influence of the rod guide. In practical application, this phenomenon is usually manifested as follows: due to the large downhole load in deep well and heavy oil well, it is necessary to select large size sucker rod in order to meet the strength, which results in the decrease of annulus area between tubing and sucker rod. The velocity of oil is quickened when the oil is passing through the coupling and the rod guide; the sudden change of the sectional area in coupling and rod guides increases the influence on flow state of oil and the energy consumption of coupling increases rapidly under certain conditions of oil well production. Therefore, it is necessary to ensure that there is sufficient annulus area between sucker rod and tubing and select sucker rod with smaller size when strength is sufficient in the design and matching of sucker rod and tubing.

TABLE 1: Main parameters and calculated results of c .

r_0 /mm	r_p /mm	r_1 /mm	$\mu/(N\cdot s/m^2)$	Pump setting depth/m	well depth/m	c_a	c_b	c_c
8	16	25.3	0.033	1148	1239	0.3845	0.4085	0.3477
9.5	16	25.3	0.028	1415	1504	0.3604	0.3807	0.2984
9.5	16	25.3	0.022	1836	1963	0.3221	0.3689	0.2345
9.5	19	31	0.029	1168	1253	0.3708	0.4605	0.3091
9.5	19	31	0.023	1724	1804	0.3369	0.3653	0.2451

Notes: c_a is the value calculated by measured parameters; c_b is the value calculated by (55); c_c is the value calculated by Zhang Qi formula.

TABLE 2: Parameters in field application.

r_0 / mm	r_1 / mm	r_p / mm	$\rho/(kg/m^3)$	$\mu/(N\cdot s/m^2)$
6.5-9.5	20.5	16	7850	0.020
6.5-11.5	25.3	19	7850	0.020
6.5-11.5	31	22	7850	0.020
8-14.5	38	28.5	7850	0.020
9.5-14.5	44.2	35	7850	0.020
11-14.5	50.3	41.5	7850	0.020

Compared with Figures 4(a)–4(f), the ratio of equivalent damping coefficient caused by coupling energy consumption to equivalent damping coefficient of sucker rod system decreases gradually with the increasing of radius of the tubing. The equivalent damping coefficient of sucker rod system is mainly produced by viscous energy consumption along the tubing. When the radius of the tubing increases to a value, the equivalent damping coefficient caused by the energy consumption of the coupling can be neglected. With the increase of the radius of tubing, the effect of rod guides on equivalent damping coefficient increases gradually, but the increase is not obvious. The proportion of equivalent damping coefficient caused by rod guides to the equivalent damping coefficient of sucker rod system is less than 4%. At the scene of the oil field, the main reason is that the annulus area between the tubing and the sucker rod amplifies, and the sudden change of the sectional area in sucker rod coupling weakens the influence on flow state of oil with the increasing of the tubing radius. Therefore, during the design and manufacture of sucker rod, reducing the sudden change of cross-section area at sucker rod coupling has remarkable effect on reducing damping force on the basis of satisfying the strength of sucker rod coupling.

In order to more accurately analyze the proportion of equivalent damping coefficient caused by viscous energy consumption in the total equivalent damping coefficient of sucker rod system, the proportional coefficient K is introduced; K represents the ratio of area of cross-section in the sucker rod and tubing.

$$K = \frac{r_0^2}{r_1^2} \tag{60}$$

The smaller the ratio k of sectional area of tubing to sucker rod is (Figure 5), the larger the damping coefficient caused by viscous energy consumption in the sucker rod system is.

For $K < 0.095$, the damping coefficient caused by viscous energy consumption in the sucker rod system accounts for more than 90% of the equivalent damping coefficient. In the pumping field, the larger the annulus area between the tubing and the sucker rod is, the smaller the damping of the sucker rod system by coupling energy consumption is. In slim hole oil well, the tubing size is small, and coupling energy consumption is an important part of energy consumption of sucker rod system. Therefore, to calculate the equivalent damping coefficient of sucker rod system, the local damping at the coupling should be considered. However, for the larger size of oil tube and smaller size of sucker rod, the damping of sucker rod system is mainly caused by viscous energy consumption. So, the influence of coupling and rod guides can be neglected for the sake of simplified calculation.

7. Conclusions

(1) The function of energy consumption along the sucker rod stroke is derived when taking the fluid in the tubing as Couette flow, and the viscous energy consumption along the sucker rod downstroke and upstroke is deduced. Considering the influence of sucker rod coupling and rod guides, the function of the local energy consumption is derived under the condition of oil passing through the coupling and rod guides.

(2) Based on the principle of equal friction loss, the theoretical formula of equivalent damping coefficient of sucker rod system is deduced by considering the viscous energy consumption along the sucker rod and the local energy consumption of the coupling and rod guides. The effect of equivalent damping coefficient on sucker rod system is analyzed combined with the parameters in field application. The results show that, with the increase of the radius of rod, the annulus area of sucker rod and tubing, the equivalent damping coefficient, the equivalent damping coefficient caused by viscous energy consumption, and the

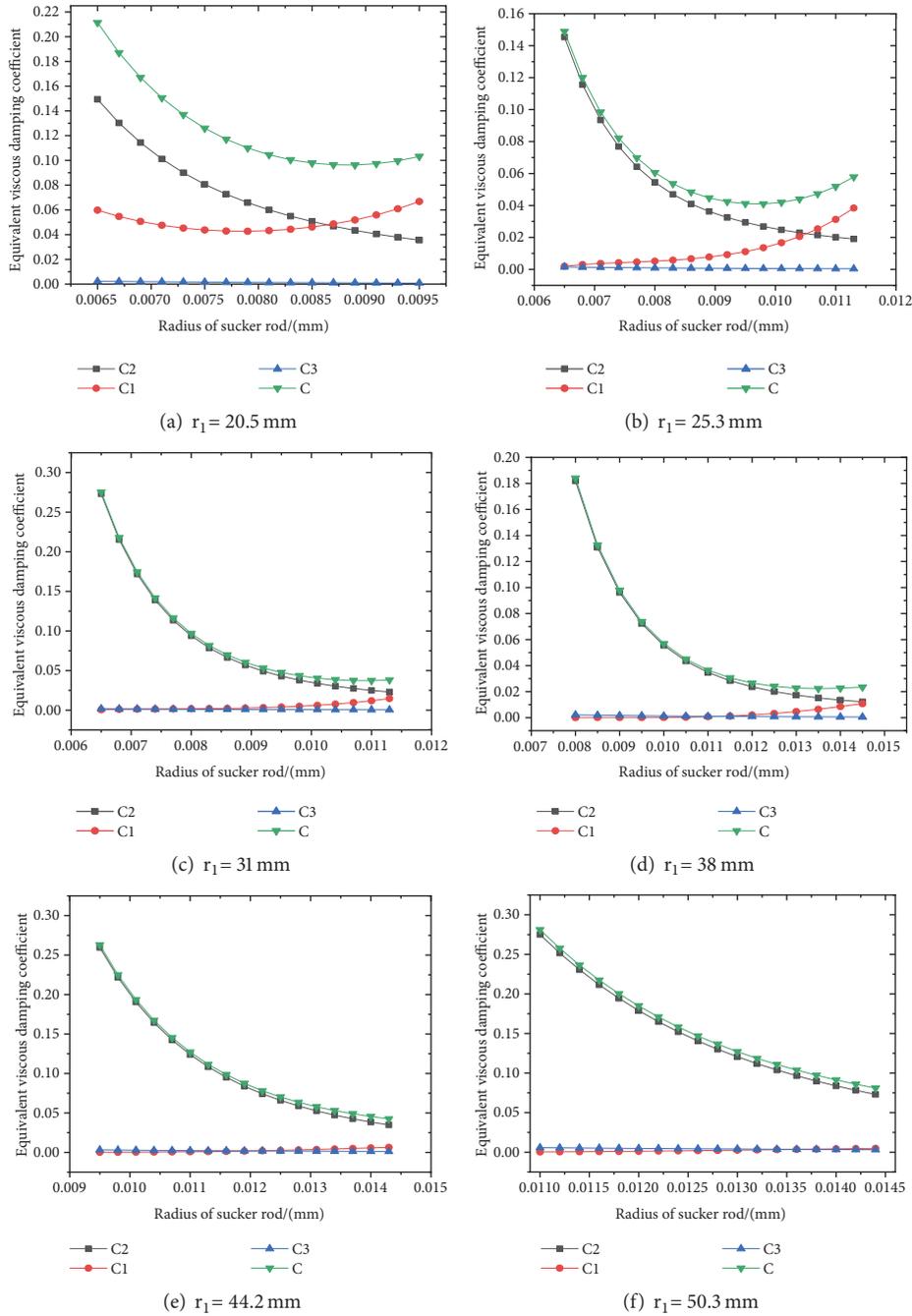


FIGURE 4: Relationship between damping coefficient and radius of sucker rod. (a) $r_1 = 20.5$ mm, (b) $r_1 = 25.3$ mm, (c) $r_1 = 31$ mm, (d) $r_1 = 38$ mm, (e) $r_1 = 44.2$ mm, and (f) $r_1 = 50.3$ mm.

equivalent damping coefficient caused by rod guides energy consumption all decrease; on the contrary, the equivalent damping coefficient caused by coupling energy consumption increases. During the design and matching of sucker rod and tubing, it is helpful to reduce the energy consumption of coupling by selecting the smaller sucker rod with sufficient strength.

(3) With the increase of the radius of tubing, the ratio of the equivalent damping coefficient caused by local viscous

energy consumption to the equivalent damping coefficient of the sucker rod system decreases gradually. The equivalent damping coefficient of the sucker rod system is mainly produced by the viscous energy consumption along the sucker rod. The smaller the ratio k of sectional area of tubing to sucker rod is, the larger the damping coefficient caused by viscous energy consumption in the sucker rod system is. For $K < 0.095$, the damping coefficient caused by viscous energy consumption in the sucker rod system accounts for more than

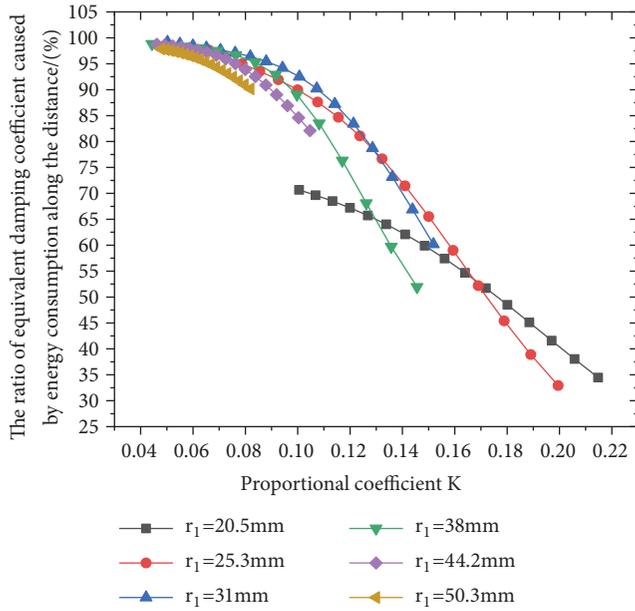


FIGURE 5: Relationship between equivalent damping coefficient along the tubing and proportional coefficient K.

90% of the equivalent damping coefficient. In the design and manufacture of sucker rod, reducing the sudden change of cross-section area of sucker rod coupling has a remarkable effect on reducing damping force.

Nomenclature

- u : Displacement of the sucker rod node (m)
- x : Distance between the node and the wellhead (m)
- c : Equivalent viscous damping coefficient
- w_g : Average flow velocity of oil before and after rod guides (m/s)
- d_{ge} : Equivalent diameter of rod guides (m)
- C_{gt} : Wet circumference of flow surface between rod guides and tubing (m)
- W_g : Local viscous energy consumption at rod guides (N·m)
- W_d : Total energy consumption (N·m)
- L : Vertical depth (m)
- μ : Viscous coefficient of oil (Pa·s)
- L_1 : Length of per sucker rod
- m : Ratio of radius of tubing to sucker rod
- W_{d1} : Viscous energy consumption along upstroke (N·m)
- W_{d2} : Viscous energy consumption along downstroke (N·m)
- W_{h1} : Coupling energy consumption along upstroke (N·m)
- t_1 : Upstroke time (s)
- r_0 : Radius of sucker rod (m)
- g : Gravity acceleration (m/s²)
- r_p : Radius of the pump plunger (m)

- K : Ratio of sectional area of tubing to sucker rod
- l_g : Length of rod guides (m)
- A_{gt} : Flow surface between rod guides and tubing (m²)
- n_g : Number of rod guides
- ΔP_g : Local resistance pressure drop at rod guides (Pa)
- W_h : Frictional work of sucker rod string (N·m)
- w_h : Velocity of oil at coupling (m/s)
- Q : Cross-section flow (m³/s)
- T : Period of the sucker rod motion (s)
- v_h : Speed of sucker rod (m/s)
- t : Time (s)
- W_{g1} : Local viscous energy consumption of upstroke at rod guides (N·m)
- W_{g2} : Local viscous energy consumption of downstroke at rod guides (N·m)
- W_{h2} : Coupling energy consumption along downstroke (N·m)
- ρ_r : Density of the rod string (Kg/m³)
- r_1 : Radius of tubing (m)
- ρ : Density of the fluid (Kg/m³).

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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