

Research Article

Exponential Synchronization for Neutral-Type Neural Network with Stochastic Perturbation and Markovian Jumping Parameters

Xue Lv,¹ Xueqing Yang ,² Yaoqing Xi,¹ Jinyong Yu,¹ and Wuneng Zhou³

¹Guangxi City College, Chongzuo 532100, China

²Educational Technology Center, Donghua University, Shanghai 200051, China

³College of Information Sciences and Technology, Donghua University, Shanghai 200051, China

Correspondence should be addressed to Xueqing Yang; etdaqing@163.com

Received 18 November 2018; Revised 27 January 2019; Accepted 12 February 2019; Published 14 March 2019

Academic Editor: Leonid Shaikhet

Copyright © 2019 Xue Lv et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The problem of exponential synchronization for neutral-type neural network with stochastic perturbation and Markovian switching parameters is considered in this article. Based on Lyapunov functional method and the theory of stochastic process, the exponential synchronism of the neural network is analyzed. By designing an adaptive state feedback controller, the exponential synchronization criterion of the neural network is obtained which can be represented as linear matrix inequality. Furthermore, the update rule for the adaptive controller is obtained. Finally, a simulation example is proposed to explain the availability of the results and method obtained in this article.

1. Introduction

The neutral-type system not only characterises the dynamic property of the system state, but also describes the dynamic varying rule of the delay state of the system. There are many practical applications in difference areas, such as machinery and communication (see [1–5]).

Reference [2] studied the problem of global robust stability for neutral-type interval stochastic neural system by means of Lyapunov functional technology. And some novel stability criteria which are represented by linear matrix inequality (LMI) were proposed in [2]. For the neutral neural network, Park et al. in [3] gave a concise and effective stability criterion expressed as LMIs. And a dynamic feedback controller was designed for this system to ensure the response system stochastic synchronizes to the drive system in [3]. Reference [4] studied the problem of global robust exponential synchronization for neutral complex network with coupling time-delay by Lyapunov functional and Luoneike methods, and some synchronization criteria were obtained.

In [5], Kolmanovskii et al. not only set up the basic stability criteria for neutral-type stochastic differential equation

with Markovian jumping, but also obtained some valuable results on boundedness and stability.

The analysis method of system stability in above works is Lyapunov functional method and linear matrix inequality. About this method, [6] can be also referred.

It is noted that neural network usually suffered from components failure, subsystem changes, and environmental disturbance, which results in the change of the system structure and parameters. The parameters of this kind of neural network may jump among finite state spaces. This kind of neural network is called neural network with Markovian jumping parameters. There exist many research results on this kind of system (see, e.g., [7–11]).

For the adaptive synchronization of neural network, the system parameters need adjustment online, and the control law also needs update timely. In recent years, many researching results of adaptive synchronization control problem for this kind of neural network were achieved (see, e.g., [12–15]).

However, there is a little of research on the exponential synchronization for the neutral-type neural network with Markovian jumping parameters and stochastic disturbance. Based on this point, this paper considers the above problem.

Firstly, we describe the problem of exponential synchronization for the neutral-type neural network with stochastic disturbance and Markovian jumping parameters. Next, we analyze the condition of exponential stability of the error system by use of Lyapunov functional method, stochastic differential equation theory, and LMI technique and design the adaptive controller assuring the drive system is exponentially synchronized by the response system. Finally, a simulation example is proposed to explain the availability of the results and method obtained in this article. Because the speed of the exponential synchronization of the system is faster than asymptotic synchronization, the research of the exponential synchronization for neutral neural network with stochastic disturbance and Markovian jumping parameters possesses particular theoretical significance and application value.

2. Modeling and Preliminaries

Let $\{r(t)\}_{t \geq 0}$ be a right-continuous Markovian chain with a finite state space $S = \{1, 2, \dots, N\}$ whose generator $\Gamma = (\gamma_{ij})_{N \times N}$ possesses the following property:

$$\begin{aligned} \mathbb{P}\{r(t + \delta) = j \mid r(t) = i\} \\ = \begin{cases} \gamma_{ij}\delta + o(\delta) & \text{if } i \neq j, \\ 1 + \gamma_{ij}\delta + o(\delta) & \text{if } i = j, \end{cases} \end{aligned} \quad (1)$$

where $\delta > 0, \gamma_{ij} \geq 0$ is the transition rate from i to j ($i \neq j$) and

$$\gamma_{ii} = - \sum_{j=1, j \neq i}^N \gamma_{ij}. \quad (2)$$

Consider the drive neural network with discrete and distribute time-delay and Markov switching parameters whose dynamic model is as follows:

$$\begin{aligned} d[x(t) - D(r(t))x(t - \tau_1)] &= \left[-C(r(t))x(t) \right. \\ &+ A(r(t))f(x(t)) + B(r(t))f(x(t - \tau_1)) \\ &\left. + E\left(r(t) \int_{t-\tau_2}^t h(x(s)) ds + J(r(t))\right) \right] dt, \end{aligned} \quad (3)$$

where $x(t) = [x_1(t), x_2(t), \dots, x_n(t)]^T \in \mathbb{R}^n$, d is the state vector with n -neurons, $f(\cdot)$ represents the neuron activation function, $h(\cdot)$ denotes the function of distribute time-delay term, τ_1 denotes the state time-delay, and τ_2 denotes the distribute time-delay. Mark $\tau = \max\{\tau_1, \tau_2\}$. Denote $r(t) = i$, $A(r(t)) = A^i$, $B(r(t)) = B^i$, $C(r(t)) = C^i$, $D(r(t)) = D^i$, and $E(r(t)) = E^i$. In neural system (3), $\forall i \in S$, $A^i = (a_{jk}^i)_{n \times n}$ and $B^i = (b_{jk}^i)_{n \times n}$ represent the connection weight matrix and time-delay connection weight matrix, respectively. $C^i = \text{diag}\{c_1^i, c_2^i, \dots, c_n^i\}$ is a diagonal matrix which has positive elements $c_j^i > 0$. D^i is called the parameters matrix of neutral term. $E^i = [E_1^i, E_2^i, \dots, E_n^i]^T \in \mathbb{R}^n$ represents the intensity

matrix of distribute time-delay term, and $J(r(t))$ is a constant external input vector.

Give the initial data for the drive system (3) as follows:

$$\begin{aligned} x(s) &= \xi_x(s), \quad s \in [-\tau, 0], \\ r(0) &= i_0 \end{aligned} \quad (4)$$

for every $\xi_x \in \mathbb{L}_{\mathcal{F}_0}^2([-\tau, 0]; \mathbb{R}^n)$.

For the drive neural system (3), consider the following response neural system:

$$\begin{aligned} d[y(t) - D(r(t))y(t - \tau_1)] &= \left[-C(r(t))y(t) \right. \\ &+ A(r(t))f(y(t)) + B(r(t))f(y(t - \tau_1)) \\ &+ E(r(t)) \int_{t-\tau_2}^t h(y(s)) ds + J(r(t)) + U(r(t)) \\ &\left. + \sigma(t, r(t), y(t) - x(t), y(t - \tau_1)) \right. \\ &\left. - x(t - \tau_1) \right] d\omega(t), \end{aligned} \quad (5)$$

where $y(t) = [y_1(t), y_2(t), \dots, y_n(t)]^T \in \mathbb{R}^n$ is the state vector of (5) and

$$U^i = U(r(t)) = [u_1^i(t), u_2^i(t), \dots, u_n^i(t)]^T \in \mathbb{R}^n \quad (6)$$

is the vector of control input. $\omega(t) = [\omega_1(t), \omega_2(t), \dots, \omega_n(t)]^T$ is the n -dimension Brownian motion which is defined on complete probability space (Ω, \mathcal{F}, P) with the filtration $\{\mathcal{F}_t\}_{t \geq 0}$; i.e., $\mathcal{F}_t = \sigma\{\omega(s) : 0 \leq s \leq t\}$ is the σ -algebra which is independent with $\{r(t)\}_{t \geq 0}$. Moreover, $\sigma : \mathbb{R}_+ \times S \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}^{n \times n}$ is the noise intensity matrix. In general, external stochastic fluctuate and the other probability cause may usually arouse this disturbance.

We also give the initial data for the response network (5) as follows:

$$\begin{aligned} y(s) &= \xi_y(s), \quad s \in [-\tau, 0], \\ r(0) &= i_0 \end{aligned} \quad (7)$$

for every $\xi_y \in \mathbb{L}_{\mathcal{F}_0}^2([-\tau, 0]; \mathbb{R}^n)$.

Let the state error vector of the drive network and the response network be $e(t) = y(t) - x(t)$. From the dynamic equations of drive network (3) and the response system (5), we can obtain the error system as follows:

$$\begin{aligned} d[e(t) - D(r(t))e_{\tau_1}(t)] &= \left[-C(r(t))e(t) \right. \\ &+ A(r(t))g(e(t)) + B(r(t))g(e_{\tau_1}(t)) \\ &+ E(r(t)) \int_{t-\tau_2}^t (h(y(s)) - h(x(s))) ds \\ &\left. + U(r(t)) \right] dt + \sigma(t, r(t), e(t), e_{\tau_1}(t)) d\omega(t), \end{aligned} \quad (8)$$

where $e_{\tau_1}(t) = e(t - \tau_1)$ and $g(e(t)) = f(x(t) + e(t)) - f(x(t))$.

The initial data for the error system (8) is

$$\begin{aligned} e(s) &= \xi(s) = \xi_y(s) - \xi_x(s), \quad s \in [-\tau, 0], \\ r(0) &= i_0, \end{aligned} \quad (9)$$

with $e(0) = 0$.

Now we give the following assumptions for systems (3), (5), and (8).

Assumption 1. The activation function $f(\cdot)$ of neuron and the function of distribute time-delay term $h(\cdot)$ satisfy Lipschitz condition; namely, there exist two constants $L > 0, L_1 > 0$, such that

$$\begin{aligned} L_1 \|x - y\| &\leq \|f(x) - f(y)\| \leq L \|x - y\|, \\ L_1 \|x - y\| &\leq \|h(x) - h(y)\| \leq L \|x - y\|. \end{aligned} \quad (10)$$

for every $x, y \in \mathbb{R}^n$ and $h(0) \equiv 0; f(0) \equiv 0$.

Assumption 2. For noise matrix $\sigma(t, i, u(i), v(i))$, there are two positive numbers H_1 and H_2 such that

$$\begin{aligned} \text{trace} \left[\sigma^T(t, r(t), u(t), v(t)) \sigma(t, r(t), u(t), v(t)) \right] \\ \leq H_1 |u(t)|^2 + H_2 |v(t)|^2 \end{aligned} \quad (11)$$

for every $(t, r(t), u(t), v(t)) \in \mathbb{R}_+ \times S \times \mathbb{R}^n \times \mathbb{R}^n$, and $\sigma(t, r_0, 0, 0) \equiv 0$.

Assumption 3. For the parameter matrix $D^i (i = 1, 2, \dots, N)$ of neutral term, there is a positive $\kappa_i \in (0, 1)$, such that

$$\rho(D^i) = \kappa_i \leq \kappa, \quad (12)$$

where $\kappa = \max_{i \in S} \kappa_i \rho(D^i)$ is the spectral radius of the matrix D^i .

Next, we give the conception of exponential stability of the error system (8).

Definition 4 (see [16]). We say $e(t; \xi_e, i_0)$ (the trivial solution of (8)) is exponential stable, if there exist numbers $\alpha > 0, \mu > 0$, such that

$$\mathbb{E} \|e(t; \xi_e, i_0)\|^2 \leq \mu e^{\alpha t} \sup_{-\tau \leq s \leq 0} \mathbb{E} \|\xi_e(s)\|^2, \quad \forall t > 0, \quad (13)$$

for every initial data $\xi_e \in \mathbb{L}_{\mathcal{F}_0}^p([-\tau, 0]; \mathbb{R}^n)$.

Target Description. For neutral neural network with stochastic noise and Markovian jumping parameters, the problem of exponential stability will be studied. By means of Lyapunov stability theory and stochastic process theory, the exponential synchronization will be analyzed. The adaptive state feedback controller will be designed. The criterion of exponential synchronization will be obtained which is represented as linear matrix inequality. The update method of adaptive control law will be obtained yet. Finally, a simulation example

will be introduced to explain the availability of the results and method obtained in this article..

To obtain the main results, some useful lemmas are given as follows.

Lemma 5 (see [17]). Let $x, y \in \mathbb{R}^n$. Then for every $\epsilon > 0$ the following inequality holds:

$$x^T y + y^T x \leq \epsilon x^T x + \epsilon^{-1} y^T y. \quad (14)$$

Lemma 6 (see [18]). For every positive definite matrix $M \in \mathfrak{R}^{n \times n}$, scalar $\gamma > 0$ and vector function $\omega : [0, \gamma] \rightarrow \mathfrak{R}^n$, the relative integral is defined. Then

$$\begin{aligned} \left(\int_0^\gamma \omega(s) ds \right)^T M \left(\int_0^\gamma \omega(s) ds \right) \\ \leq \gamma \int_0^\gamma \omega^T(s) M \omega(s) ds. \end{aligned} \quad (15)$$

3. Main Results

In this section, some main results and their proofs are given.

Theorem 7. For the drive neural network (3) and the response network (5), suppose Assumptions 1–3 hold, if there exist symmetric positive definite matrices $Q_1 > 0, P^i > 0 (i = 1, 2, \dots, N)$, and positive scalars $\rho_1, \rho_2, \mu_1, \mu_2, \eta, \beta, \epsilon_i (i = 1, 2, 3)$, such that

$$\rho_1 I \leq P^i \leq \rho_2 I, \quad (16)$$

$$\mu_1 I \leq Q_1 \leq \mu_2 I, \quad (17)$$

$$\begin{bmatrix} \Phi_{11} & C^i P^i D^{iT} \\ * & \epsilon_2 L^2 I + \rho_2 H_2 I - Q_1 \end{bmatrix} < - \begin{bmatrix} \eta & 0 \\ 0 & \eta \end{bmatrix} I, \quad (18)$$

where $\Phi_{11} = -2P^i C^i + \epsilon_1 L^2 I + \rho_2 H_1 I + Q_1 + \tau_1^2 Q_1 + \tau_2^3 \epsilon_3 L^4 I + \tau_2^2 \epsilon L^2 I$,

$$\begin{bmatrix} -2\zeta P^i + \sum_{k=1}^N \gamma_{ik} P^k & P^i A^i & P^i B^i & P^i E^i \\ * & -\epsilon_1 I & 0 & 0 \\ * & 0 & -\epsilon_2 I & 0 \\ * & 0 & 0 & -\epsilon_3 I \end{bmatrix} < 0, \quad (19)$$

$$\beta \rho_2 - \eta < 0, \quad (20)$$

$$\beta (1 + \tau_1^2) \mu_2 - \tau_1 \mu_1 < 0, \quad (21)$$

$$\beta (\tau_2^2 L^2 + \tau_2) (\epsilon_3 \tau_2 L^2) - \tau_2 (\epsilon_3 \tau_2 L^2) L_1^2 < 0, \quad (22)$$

$i = 1, 2, \dots, N$.

Moreover, choose state feedback controller U^i as

$$U^i = (\text{diag} \{k_1, k_2, \dots, k_n\} - \zeta I) (e - D^i e_{\tau_1}) \quad (23)$$

which satisfies

$$\dot{k}_j = -\alpha_j p_j^i (e - D^i e_{\tau_1})_j^2, \quad (24)$$

where $\alpha_j > 0$ ($j = 1, 2, \dots, n$) are arbitrary constants, p_j^i is the j th diagonal element of P^i , and $(e - D^i e_{\tau_1})_j$ is the j th component of $e - D^i e_{\tau_1}$.

Then the error system (8) is exponential stable. So the drive neural network (3) exponential synchronizes with the response neural network system (5).

Proof. Since Assumptions 1–3 hold, $e(t; \xi_e, i_0)$ exists. For every $i \in S$, take following Lyapunov function:

$$V(t, i, x) = V_1 + V_2 + V_3 + V_4 + V_5 + V_6, \quad (25)$$

where

$$\begin{aligned} V_1 &= x^T P^i x, \\ V_2 &= \int_{t-\tau_1}^t e^T(s) Q_1 e(s) ds, \\ V_3 &= \tau_1 \int_{-\tau_1}^0 \int_{t+s}^t e^T(\beta) Q_1 e(\beta) d\beta ds, \\ V_4 &= \tau_2 \int_{\tau_2}^0 \int_{t+s}^t g^T(e(\beta)) Q_2 g(e(\beta)) d\beta ds, \\ V_5 &= \int_{-\tau_2}^0 \int_{t+s}^t e^T(\beta) Q_2 e(\beta) d\beta ds, \\ V_6 &= \sum_{j=1}^n \frac{1}{\alpha_j} k_j^2, \end{aligned} \quad (26)$$

and

$$Q_2 = \epsilon_3^{-1} \tau_2 L^2 I. \quad (27)$$

For system

$$\begin{aligned} d[x(t) - \bar{D}(x(t-\tau), r(t))] \\ = \bar{F}(t, r(t), x(t), x(t-\tau)) dt \\ + \bar{G}(t, r(t), x(t), x(t-\tau)) d\omega(t), \end{aligned} \quad (28)$$

definite infinitesimal operator $\mathcal{L}V: \mathbb{R}_+ \times S \times \mathbb{R}^n \times \mathbb{R}^n \rightarrow \mathbb{R}$ as follows:

$$\begin{aligned} \mathcal{L}V(t, i, x, y) &= V_t(t, i, x - \bar{D}(y, i)) + V_x(t, i, x \\ &\quad - \bar{D}(y, i)) \bar{F}(t, i, x, y) + \frac{1}{2} \text{trace} \left[\bar{G}^T(t, i, x, y) \right. \\ &\quad \cdot V_{xx}(t, i, x - \bar{D}(y, i)) \bar{G}(t, i, x, y) \left. \right] \\ &\quad + \sum_{j=1}^N \gamma_{ij} V(t, j, x - \bar{D}(y, i)), \end{aligned} \quad (29)$$

where

$$\begin{aligned} V_t(t, i, x(t)) &= \frac{\partial V(t, i, x(t))}{\partial t}, \\ V_x(t, i, x(t)) &= \left(\frac{\partial V(t, i, x(t))}{\partial x_1}, \frac{\partial V(t, i, x(t))}{\partial x_2}, \dots, \right. \\ &\quad \left. \frac{\partial V(t, i, x(t))}{\partial x_n} \right), \\ V_{xx}(t, i, x(t)) &= \left(\frac{\partial^2 V(t, i, x(t))}{\partial x_j \partial x_k} \right)_{n \times n}. \end{aligned} \quad (30)$$

Then for the error system (8), computing $\mathcal{L}V(t, i, e, e_\tau)$ obtains

$$\begin{aligned} \mathcal{L}V_1 &= 2(e - D^i e_{\tau_1})^T P^i \left[-C^i e(t) + A^i g(e) \right. \\ &\quad \left. + B^i g(e_{\tau_1}) + E^i \int_{t-\tau_2}^t (h(y(s)) - h(x(s))) ds \right. \\ &\quad \left. + U^i \right] + \left(\frac{1}{2} \right) \text{trace}(\sigma^T (2P^i) \sigma) \\ &\quad + \sum_{k=1}^N \gamma_{ik} (e - D^i e_{\tau_1})^T P^k (e - D^i e_{\tau_1}), \\ \mathcal{L}V_2 &= e^T(t) Q_1 e(t) - e^T(t - \tau_1) Q_1 e(t - \tau_1), \\ \mathcal{L}V_3 &= \tau_1^2 e^T(t) Q_1 e(t) - \tau_1 \int_{t-\tau_1}^t e^T(s) Q_1 e(s) ds, \\ \mathcal{L}V_4 &= \tau_2^2 g^T(e(t)) Q_2 g(e(t)) \\ &\quad - \tau_2 \int_{t-\tau_2}^t g^T(e(s)) Q_2 g(e(s)) ds \\ \mathcal{L}V_5 &= \tau_2 e^T(t) Q_2 e(t) - \int_{t-\tau_2}^t e^T(s) Q_2 e(s) ds, \\ \mathcal{L}V_6 &= 2 \sum_{j=1}^n \left(\frac{1}{\alpha_j} \right) k_j \dot{k}_j = -2 \sum_{j=1}^n k_j p_j^i (e - D^i e_{\tau_1})_j^2. \end{aligned} \quad (31)$$

While

$$\begin{aligned} 2(e - D^i e_{\tau_1})^T P^i A^i g(e) &\leq \epsilon_1^{-1} (e - D^i e_{\tau_1})^T \\ &\quad \cdot P^i A^i A^{iT} P^i (e - D^i e_{\tau_1}) + \epsilon_1 L^2 e^T e, \end{aligned} \quad (32)$$

$$\begin{aligned} 2(e - D^i e_{\tau_1})^T P^i B^i g(e_{\tau_1}) &\leq \epsilon_2^{-1} (e - D^i e_{\tau_1})^T \\ &\quad \cdot P^i B^i B^{iT} P^i (e - D^i e_{\tau_1}) + \epsilon_2 L^2 e_{\tau_1}^T e_{\tau_1}, \end{aligned} \quad (33)$$

$$\begin{aligned} 2(e - D^i e_{\tau_1})^T P^i E^i \int_{t-\tau_2}^T (h(y(s)) - h(x(s))) ds \\ \leq \epsilon_3^{-1} (e - D^i e_{\tau_1})^T P^i E^i E^{iT} P^i (e - D^i e_{\tau_1}) \\ + \epsilon_3 \left(\int_{t-\tau_2}^T (h(y(s)) - h(x(s))) ds \right)^T \\ \times \left(\int_{t-\tau_2}^T (h(y(s)) - h(x(s))) ds \right) \leq \epsilon_3^{-1} (e - D^i e_{\tau_1})^T \\ P^i E^i E^{iT} P^i (e - D^i e_{\tau_1}) \\ + \epsilon_3 \tau_2 \int_{t-\tau_2}^T (h(y(s)) - h(x(s)))^T \\ \cdot (h(y(s)) - h(x(s))) ds \leq \epsilon_3^{-1} (e - D^i e_{\tau_1})^T \\ \cdot P^i E^i E^{iT} P^i (e - D^i e_{\tau_1}) \\ + \epsilon_3 \tau_2 L^2 \int_{t-\tau_2}^T e^T(s) e(s) ds, \end{aligned} \quad (34)$$

$$\left(\frac{1}{2}\right) \text{trace}(\sigma^T (2P^i) \sigma) \leq \rho_2 (H_1 e^T e + H_2 e_{\tau_1}^T e_{\tau_1}), \quad (35)$$

$$\begin{aligned} \tau_2^2 g^T(e(t)) Q_2 g(e(t)) &\leq \tau_2^3 \epsilon_3 L^2 g^T(e(t)) g(e(t)) \\ &\leq \tau_2^3 \epsilon_3 L^4 e^T e. \end{aligned} \quad (36)$$

Therefore

$$\begin{aligned} \mathcal{L}V &= \mathcal{L}V_1 + \dots + \mathcal{L}V_6 \leq -2(e - D^i e_{\tau_1})^T P^i C^i e \\ &+ \epsilon_1^{-1} (e - D^i e_{\tau_1})^T P^i A^i A^{iT} P^i (e - D^i e_{\tau_1}) \\ &+ \epsilon_1 L^2 e^T e + \epsilon_2^{-1} (e - D^i e_{\tau_1})^T P^i B^i B^{iT} P^i (e - D^i e_{\tau_1}) \\ &+ \epsilon_2 L^2 e_{\tau_1}^T e_{\tau_1} + \epsilon_3^{-1} (e - D^i e_{\tau_1})^T \\ &\cdot P^i E^i E^{iT} P^i (e - D^i e_{\tau_1}) \\ &+ \epsilon_3 \tau_2 L^2 \int_{t-\tau_2}^T e^T(s) e(s) ds + 2(e - D^i e_{\tau_1})^T \\ &\cdot P^i (\text{diag}\{k_1, \dots, k_n\} - \zeta I) (e - D^i e_{\tau_1}) \\ &+ \rho_2 (H_1 e^T e + H_2 e_{\tau_1}^T e_{\tau_1}) \\ &+ \sum_{k=1}^N \gamma_{ik} (e - D^i e_{\tau_1})^T P^k (e - D^i e_{\tau_1}) + e^T(t) Q_1 e(t) \\ &- e^T(t - \tau_1) Q_1 e(t - \tau_1) + \tau_1^2 e^T(t) Q_1 e(t) \\ &- \tau_1 \int_{t-\tau_1}^t e^T(s) Q_1 e(s) ds + \tau_2^3 \epsilon_3 L^4 e^T e \\ &- \tau_2 \int_{t-\tau_2}^t g^T(e(s)) Q_2 g(e(s)) ds + \tau_2^2 \epsilon_3 L^2 e^T e \\ &- \int_{t-\tau_2}^t e^T(s) Q_2 e(s) ds - 2 \sum_{j=1}^n k_j p_j^i (e - D^i e_{\tau_1})_j^2. \end{aligned} \quad (37)$$

Summarizing the above process, we obtain

$$\begin{aligned} \mathcal{L}V &\leq \begin{bmatrix} e^T & e_{\tau_1}^T \end{bmatrix} \begin{bmatrix} \Phi_{11} & C^i P^i D^{iT} \\ * & \epsilon_2 L^2 I + \rho H_2 I - Q_1 \end{bmatrix} \begin{bmatrix} e \\ e_{\tau_1} \end{bmatrix} \\ &+ (e - D^i e_{\tau_1})^T \left(\epsilon_1^{-1} P^i A^i A^{iT} P^i + \epsilon_2^{-1} P^i B^i B^{iT} P^i \right. \\ &+ \epsilon_3^{-1} P^i E^i E^{iT} P^i - 2\zeta P^i + \sum_{k=1}^n \gamma_{ik} P^k \left. \right) (e - D^i e_{\tau_1}) \\ &- \tau_1 \int_{t-\tau_1}^t e^T(s) Q_1 e(s) ds \\ &- \tau_2 \int_{t-\tau_2}^t g^T(e(s)) Q_2 g(e(s)) ds. \end{aligned} \quad (38)$$

From condition (19) and Schur complement lemma, we know

$$\begin{aligned} &\left(\epsilon_1^{-1} P^i A^i A^{iT} P^i + \epsilon_2^{-1} P^i B^i B^{iT} P^i + \epsilon_3^{-1} P^i E^i E^{iT} P^i \right. \\ &\left. - 2\zeta P^i + \sum_{k=1}^n \gamma_{ik} P^k \right) < 0. \end{aligned} \quad (39)$$

So, from condition (18), we obtain

$$\begin{aligned} \mathcal{L}V &\leq -\eta e^T e - \eta e_{\tau_1}^T e_{\tau_1} - \tau_1 \int_{t-\tau_1}^t e^T(s) Q_1 e(s) ds \\ &- \tau_2 \int_{t-\tau_2}^t g^T(e(s)) Q_2 g(e(s)) ds. \end{aligned} \quad (40)$$

Thus

$$\mathcal{L}V \leq -\eta e^T e. \quad (41)$$

Namely,

$$\frac{dEV}{dt} \leq -\eta E \|e(t)\|^2. \quad (42)$$

This shows that the neutral-type error system (8) is global mean square stale. Hereinafter, we will prove the neutral-type error system (8) is global exponential stable.

Firstly, from Lyapunov function (25), we know

$$\begin{aligned} V(t, i, e(t)) &\leq \rho_2 e^T e + (1 + \tau_1^2) \lambda_{\max}(Q_1) \int_{t-\tau_1}^t \|e(s)\|^2 ds \\ &+ (\tau_2^2 L^2 + \tau_2) \lambda_{\max}(Q_2) \int_{t-\tau_2}^t \|e(s)\|^2 ds \\ &+ \sum_{j=1}^n \frac{1}{\alpha_j} k_j^2. \end{aligned} \quad (43)$$

Next, let $V^*(t, i, x(t)) = e^{\beta t} V(t, i, x(t))$. Then

$$\begin{aligned} dV^*(t, i, e(t)) &= e^{\beta t} [\beta V(t, i, e(t)) + dV(t, i, e(t))] \\ &= e^{\beta t} [\beta V(t, i, e(t)) + \mathcal{L}V(t, i, e(t)) + V_x^T(t, i, e(t))] \end{aligned}$$

$$\begin{aligned}
& -D^i e_{\tau_1}) \sigma d\omega(t) \leq e^{\beta t} \left\{ \beta \left[\rho_2 e^T e \right. \right. \\
& + (1 + \tau_1^2) \lambda_{\max}(Q_1) \int_{t-\tau_1}^t \|e(s)\|^2 ds \\
& + (\tau_2^2 L^2 + \tau_2) (\epsilon_3 \tau_2 L^2) \int_{t-\tau_2}^t \|e(s)\|^2 ds \\
& + \left. \left. \sum_{j=1}^n \frac{1}{\alpha_j} k_j^2 \right] + \left[-\eta e^T e - \eta e_{\tau_1}^T e_{\tau_1} \right. \right. \\
& - \tau_1 \lambda_{\min}(Q_1) \int_{t-\tau_1}^t e^T(s) e(s) ds \\
& - \left. \left. \tau_2 (\epsilon_3 \tau_2 L^2) L_1^2 \int_{t-\tau_2}^t e^T(s) e(s) ds \right] \right\} + V_x^T(t, i, e \\
& - D^i e_{\tau_1}) \sigma d\omega(t). \tag{44}
\end{aligned}$$

From condition (20)-(22), we obtain

$$dV^*(t, i, e(t)) \leq V_x^T(t, i, e - D^i e_{\tau_1}) \sigma d\omega(t). \tag{45}$$

Taking the mathematical expectation on both of the above formula, one can obtain

$$\frac{d\mathbb{E}V^*(t, i, e(t))}{dt} \leq 0. \tag{46}$$

Thus

$$\begin{aligned}
& e^{\beta t} \mathbb{E}V(t, i, e(t)) \leq \mathbb{E}V(0, r(0), e(0)) \leq \rho_2 e^T(0) e(0) \\
& + (1 + \tau_1^2) \lambda_{\max}(Q_1) \mathbb{E} \int_{-\tau_1}^0 \|e(s)\|^2 ds + (\tau_2^2 L^2 \\
& + \tau_2) (\epsilon_3 \tau_2 L^2) \mathbb{E} \int_{-\tau_2}^0 \|e(s)\|^2 ds \\
& + \mathbb{E} \left(\sum_{j=1}^n \frac{1}{\alpha_j} k_j^2 \right) \Big|_{t=0} \leq \rho_2 e^T(0) e(0) + (1 + \tau_1^2) \\
& \cdot \lambda_{\max}(Q_1) \tau_1 \sup_{-\tau \leq t \leq 0} \mathbb{E} \|\xi_e(t)\|^2 + (\tau_2^2 L^2 + \tau_2) \\
& \cdot (\epsilon_3 \tau_2 L^2) \tau_2 \sup_{-\tau \leq t \leq 0} \mathbb{E} \|\xi_e(t)\|^2 + \eta \mathbb{E} e^T(-\tau_1) e(-\tau_1) \\
& \leq [\rho_2 + (1 + \tau_1^2) \lambda_{\max}(Q_1) \tau_1 \\
& + (\tau_2^2 L^2 + \tau_2) (\epsilon_3 \tau_2 L^2) \tau_2 + \eta] \sup_{-\tau \leq t \leq 0} \mathbb{E} \|\xi_e(t)\|^2.
\end{aligned} \tag{47}$$

Therefore

$$\begin{aligned}
& \mathbb{E} \|e(t; \xi_e, i_0)\|^2 \leq \left(\frac{1}{\rho_1} \right) \mathbb{E}V(t, i, e(t)) \leq e^{-\beta t} \left(\frac{1}{\rho_1} \right) \\
& \cdot \mathbb{E}V(0, r(0), e(0)) \leq e^{-\beta t} \left(\frac{1}{\rho_1} \right) [\rho \\
& + (1 + \tau_1^2) \lambda_{\max}(Q_1) \tau_1 + (\tau_2^2 L^2 + \tau_2) (\epsilon_3 \tau_2 L^2) \tau_2 \\
& + \eta] \sup_{-\tau \leq t \leq 0} \mathbb{E} \|\xi_e(t)\|^2. \tag{48}
\end{aligned}$$

According to Definition 4, the error system (8) is global exponential stable. So the drive neural network (3) synchronizes exponentially the response neural network (5). This completes the proof. \square

Remark. In system models (3) and (5), if we change constant time-delay τ_1 and τ_2 into time-varying delay $\tau_1(t)$ and $\tau_2(t)$ and assume that the time-varying delay is bounded and the derivative of time-varying delay is also bounded and less than 1, then the we can obtain the relative result as Theorem 7.

For three special cases of neural network, we give relative three corollaries, respectively.

Special Case 1. There is no parameters switching in drive neural network (3), response neural network (5), and error system (8)

For this case of system, according to the proof of Theorem 7, we have the following result about exponential synchronization.

Corollary 8. For the response neural network (5) and the drive neural network (3) with no parameter switching, suppose Assumptions 1-3 hold, if there exist symmetric positive definite matrices $Q_1 > 0$, $P > 0$, and positive scalars $\rho_1, \rho_2, \mu_1, \mu_2, \eta, \beta, \epsilon_i$ ($i = 1, 2, 3$), such that

$$\rho_1 I \leq P \leq \rho_2 I, \tag{49}$$

$$\mu_1 I \leq Q_1 \leq \mu_2 I, \tag{50}$$

$$\begin{bmatrix} \Phi_{11} & CPD^T \\ * & \epsilon_2 L^2 I + \rho_2 H_2 I - Q_1 \end{bmatrix} < - \begin{bmatrix} \eta & 0 \\ 0 & \eta \end{bmatrix} I, \tag{51}$$

where

$$\begin{aligned}
& \Phi_{11} \\
& = -2PC + \epsilon_1 L^2 I + \rho_2 H_1 I + Q_1 + \tau_1^2 Q_1 + \tau_2^3 \epsilon_3 L^4 I \\
& + \tau_2^2 \epsilon L^2 I, \tag{52}
\end{aligned}$$

$$\begin{bmatrix} -2cP & PA & PB & PE \\ * & -\epsilon_1 I & 0 & 0 \\ * & 0 & -\epsilon_2 I & 0 \\ * & 0 & 0 & -\epsilon_3 I \end{bmatrix} < 0, \tag{53}$$

$$\beta \rho_2 - \eta < 0, \tag{54}$$

$$\beta(1 + \tau_1^2)\mu_2 - \tau_1\mu_1 < 0, \quad (55)$$

$$\beta(\tau_2^2L^2 + \tau_2)(\epsilon_3\tau_2L^2) - \tau_2(\epsilon_3\tau_2L^2)L_1^2 < 0. \quad (56)$$

Moreover, choose state feedback controller U^i as

$$U = (\text{diag}\{k_1, k_2, \dots, k_n\} - \zeta I)(e - De_{\tau_1}) \quad (57)$$

which satisfies

$$\dot{k}_j = -\alpha_j p_j (e - De_{\tau_1})_j^2, \quad (58)$$

where $\alpha_j > 0$ ($j = 1, 2, \dots, n$) are arbitrary constants, p_j is the j th diagonal element of P , and $(e - De_{\tau_1})_j$ is the j th component of $e - De_{\tau_1}$.

Then the error network (8) is exponential stable. So the drive neural network (3) exponential synchronizes with the response neural network (5).

Special Case 2. There is no neutral term in drive neural network (3), response neural network (5), and error network (8)

For this case of system, according to the proof of Theorem 7, we have the following result about exponential synchronization.

Corollary 9. For the response neural network (5) and the drive neural network (3) with no neutral term, suppose Assumptions 1–3 hold, if there exist symmetric positive definite matrices $Q_1 > 0$, $P^i > 0$ ($i = 1, 2, \dots, N$), and positive scalars $\rho_1, \rho_2, \mu_1, \mu_2, \eta, \beta, \epsilon_i$ ($i = 1, 2, 3$), such that

$$\rho_1 I \leq P^i \leq \rho_2 I, \quad (59)$$

$$\mu_1 I \leq Q_1 \leq \mu_2 I, \quad (60)$$

$$\begin{bmatrix} \Phi_{11}^* & C^i P^i D^{iT} \\ * & \epsilon_2 L^2 I + \rho_2 H_2 I - Q_1 \end{bmatrix} < - \begin{bmatrix} \eta & 0 \\ 0 & \eta \end{bmatrix} I, \quad (61)$$

where

$$\begin{aligned} \Phi_{11}^* = & -2P^i C^i + \epsilon_1 L^2 I + \rho_2 H_1 I + Q_1 + \tau_1^2 Q_1 \\ & + \tau_2^3 \epsilon_3 L^4 I + \tau_2^2 \epsilon L^2 I + \left(\epsilon_1^{-1} P^i A^i A^{iT} P^i \right. \\ & + \epsilon_2^{-1} P^i B^i B^{iT} P^i + \epsilon_3^{-1} P^i E^i E^{iT} P^i - 2\zeta P^i \\ & \left. + \sum_{k=1}^n \gamma_{ik} P^k \right). \end{aligned} \quad (62)$$

$$\beta\rho_2 - \eta < 0, \quad (63)$$

$$\beta(1 + \tau_1^2)\mu_2 - \tau_1\mu_1 < 0, \quad (64)$$

$$\beta(\tau_2^2L^2 + \tau_2)(\epsilon_3\tau_2L^2) - \tau_2(\epsilon_3\tau_2L^2)L_1^2 < 0, \quad (65)$$

$i = 1, 2, \dots, N$.

Moreover, choose state feedback controller U^i as

$$U^i = (\text{diag}\{k_1, k_2, \dots, k_n\} - \zeta I)e \quad (66)$$

which satisfies

$$\dot{k}_j = -\alpha_j p_j^i e_j^2, \quad (67)$$

where $\alpha_j > 0$ ($j = 1, 2, \dots, n$) are arbitrary constants, p_j^i is the j th diagonal element of P^i , and e_j is the j th component of e . Then the error network (8) is exponential stable. So the drive neural network (3) exponential synchronizes with the response neural network (5).

Special Case 3. There are no both parameter switching and neutral term in drive neural network (3), response neural network (5), and error network (8)

For this case of system, according to the proof of Theorem 7, we have the following result about exponential synchronization.

Corollary 10. For the response neural network (5) and the drive neural network (3) with no both parameter switching and neutral term, suppose Assumptions 1–3 hold, if there exist symmetric positive definite matrices $Q_1 > 0$, $P > 0$, and positive scalars $\rho_1, \rho_2, \mu_1, \mu_2, \eta, \beta, \epsilon_i$ ($i = 1, 2, 3$), such that

$$\rho_1 I \leq P \leq \rho_2 I, \quad (68)$$

$$\mu_1 I \leq Q_1 \leq \mu_2 I, \quad (69)$$

$$\begin{bmatrix} \overline{\Phi}_{11} & CPD^T \\ * & \epsilon_2 L^2 I + \rho_2 H_2 I - Q_1 \end{bmatrix} < - \begin{bmatrix} \eta & 0 \\ 0 & \eta \end{bmatrix} I, \quad (70)$$

where

$$\begin{aligned} \overline{\Phi}_{11} = & -2PC + \epsilon_1 L^2 I + \rho_2 H_1 I + Q_1 + \tau_1^2 Q_1 + \tau_2^3 \epsilon_3 L^4 I \\ & + \tau_2^2 \epsilon L^2 I + (\epsilon_1^{-1} PAA^T P + \epsilon_2^{-1} PBB^T P \\ & + \epsilon_3^{-1} PEE^T P - 2\zeta P). \end{aligned} \quad (71)$$

$$\beta\rho_2 - \eta < 0, \quad (72)$$

$$\beta(1 + \tau_1^2)\mu_2 - \tau_1\mu_1 < 0, \quad (73)$$

$$\beta(\tau_2^2L^2 + \tau_2)(\epsilon_3\tau_2L^2) - \tau_2(\epsilon_3\tau_2L^2)L_1^2 < 0. \quad (74)$$

Moreover, choose state feedback controller U^i as

$$U = (\text{diag}\{k_1, k_2, \dots, k_n\} - \zeta I)e \quad (75)$$

satisfies

$$\dot{k}_j = -\alpha_j p_j e_j^2, \quad (76)$$

where $\alpha_j > 0$ ($j = 1, 2, \dots, n$) are arbitrary constants, p_j is the j th diagonal element of P , and e_j is the j th component of e . Then the error system (8) is exponential stable. So the drive neural network (3) exponential synchronizes with the response neural network (5).

4. Numerical Example

In this section, we will give a numerical example to explain the availability of the results and method obtained in this article.

Consider the response neural network (5) and the drive neural network (3). Choose the Markovian chain with two states (i.e., $N = 2$) whose generator is

$$\Gamma = \begin{bmatrix} -2 & 2 \\ 1 & -1 \end{bmatrix}. \quad (77)$$

Let the parameters of the system be as follows:

$$\begin{aligned} D(1) &= \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix}, \\ D(2) &= \begin{bmatrix} 0.1 & 0 \\ 0 & 0.3 \end{bmatrix}, \\ C(1) &= \begin{bmatrix} 3.5 & 0 \\ 0 & 4.2 \end{bmatrix}, \\ C(2) &= \begin{bmatrix} 6 & 0 \\ 0 & 2 \end{bmatrix}, \\ A(1) &= \begin{bmatrix} 0.2 & 0.4 \\ 0.8 & 1.5 \end{bmatrix}, \\ A(2) &= \begin{bmatrix} -3 & 1 \\ 0.5 & -2 \end{bmatrix}, \\ B(1) &= \begin{bmatrix} -0.4 & 0.7 \\ -0.8 & -1 \end{bmatrix}, \\ B(2) &= \begin{bmatrix} -3 & 2 \\ 1 & -2 \end{bmatrix}, \\ E(1) &= \begin{bmatrix} -4 & 2 \\ 1 & -4 \end{bmatrix}, \\ E(2) &= \begin{bmatrix} -3 & 2 \\ 1 & -4 \end{bmatrix}, \\ J(1) &= \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \\ J(2) &= \begin{bmatrix} 1 \\ -1 \end{bmatrix}. \end{aligned} \quad (78)$$

Take the discrete and distributive time-delay as $\tau_1 = \tau_2 = \tau = 1$, $\varsigma = 8$. The neuron activation function and the function of distribute time-delay term are taken as $f(\cdot) = \tanh(\cdot) = h(\cdot)$. The noise intensity matrix is a diagonal matrix whose elements are the components of $e(t) + e_\tau(t)$. By computing, we can obtain $L_1 = 0.65$, $L = 1$, $H_1 = H_2 = 2$, and $\kappa_1 = \kappa_2 = 0.3$. Thus Assumptions 1–3 hold. Moreover, by

making use of LMI toolbox in Matlab, the parameters which satisfy the conditions (16)–(22) of Theorem 7 can be computed as follows:

$$\begin{aligned} Q_1 &= \begin{bmatrix} 1.2881 & -0.0868 \\ -0.0868 & 1.0590 \end{bmatrix}, \\ P_1 &= \begin{bmatrix} 2.7548 & -0.6270 \\ -0.6270 & 3.0455 \end{bmatrix}, \\ P_2 &= \begin{bmatrix} 2.7664 & -0.2007 \\ -0.2007 & 2.1875 \end{bmatrix}, \\ \mu_1 &= 1.0298, \\ \mu_2 &= 1.3173, \\ \rho_1 &= 2.1247, \\ \rho_2 &= 3.5438, \\ \eta &= 4.5, \\ \beta &= 0.3, \\ \epsilon_1 &= 2.6567, \\ \epsilon_2 &= 5.7793, \\ \epsilon_3 &= 28.7657. \end{aligned} \quad (79)$$

According to Theorem 7, the error network (8) is global exponential stable. So the drive neural network (3) global exponential synchronizes with the response neural network (5).

To explain the availability of the results and method obtained in this article, we plot the evolution figures of Markovian jumping process, Gauss noise, state variable of the error system, and the gain of the controller as Figures 1–3, respectively. Figure 2 shows us that the error system is stable. Figure 3 shows us that the update law of the controller does not update after a period of adjustment.

5. Conclusions

For neutral-type response neural network and drive neural network with discrete and distributive time-delay, Markovian switching parameters, and stochastic disturbance, the problem of global exponential stability has been studied in this article. By taking use of Lyapunov functional method and Itô differential formula in stochastic analysis theory, the asymptotic stability and exponential stability of the error network have been analyzed. The criterion of exponential stabilization has been obtained which ensures the drive neural network synchronizing the response neural network. Meanwhile, the update law of gain of the adaptive controller has been also obtained. Finally, the availability of the results and method obtained in this article has been explained by a numerical example.

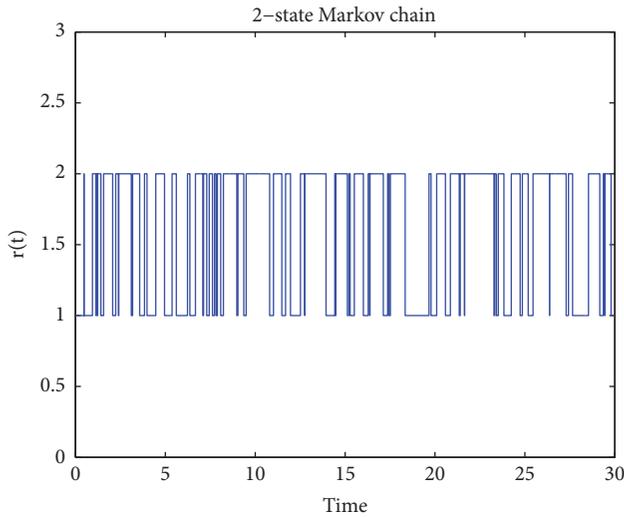


FIGURE 1: The varying curve of Markovian chain with 2-states.

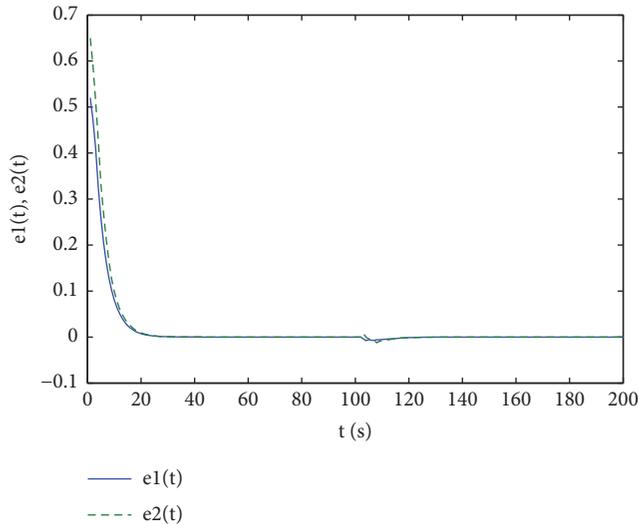


FIGURE 2: The state trajectory of the error system.

It is note that the speed of exponential synchronization for the neutral-type neural network is faster than the asymptotic synchronization. Furthermore, the control strategy adopted in this paper is the adaptive state feedback control whose feedback gain depends on not only some adaptive update parameters but also an adjustable parameter.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

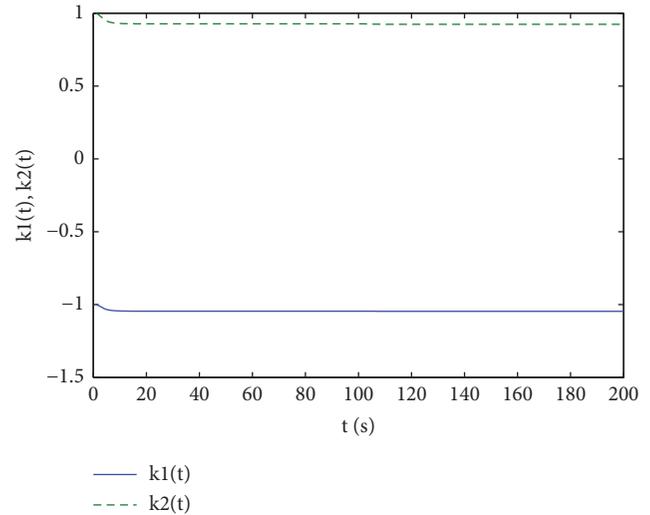


FIGURE 3: The dynamic curve of the update law of the gain of controller $K(t)$.

Acknowledgments

This work is partially supported by the National Natural Science Foundation of China (61573095).

References

- [1] V. P. Rubanik, *Oscillations of Qasilinear Systems with Retardation*, Nauka, Moscow, 1969.
- [2] X. Li, "Global robust stability for stochastic interval neural networks with continuously distributed delays of neutral type," *Applied Mathematics and Computation*, vol. 215, no. 12, pp. 4370–4384, 2010.
- [3] J. H. Park, "Synchronization of cellular neural networks of neutral type via dynamic feedback controller," *Chaos, Solitons & Fractals*, vol. 42, no. 3, pp. 1299–1304, 2009.
- [4] X. Mao, Y. Shen, and C. Yuan, "Almost surely asymptotic stability of neutral stochastic differential delay equations with Markovian switching," *Stochastic Processes and Their Applications*, vol. 118, no. 8, pp. 1385–1406, 2008.
- [5] V. Kolmanovskii, N. Koroleva, T. Maizenberg, X. Mao, and A. Matasov, "Neutral stochastic differential delay equations with Markovian switching," *Stochastic Analysis and Applications*, vol. 21, no. 4, pp. 819–847, 2003.
- [6] W. Zhou, X. Zhou, J. Yang, J. Zhou, and D. Tong, "Stability analysis and application for delayed neural networks driven by fractional Brownian noise," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 29, no. 5, pp. 1491–1502, 2018.
- [7] Z. G. Wu, P. Shi, H. Su, and J. Chu, "Stochastic synchronization of markovian jump neural networks with time-varying delay using sampled-data," *IEEE Transactions on Cybernetics*, vol. 43, no. 6, pp. 1796–1806, 2013.
- [8] Z. Wu, P. Shi, H. Su, and J. Chu, "Passivity analysis for discrete-time stochastic markovian jump neural networks with mixed time delays," *IEEE Transactions on Neural Networks and Learning Systems*, vol. 22, no. 10, pp. 1566–1575, 2011.
- [9] W. Zhou, D. Tong, Y. Gao, C. Ji, and H. Su, "Mode and delay-dependent adaptive exponential synchronization in pth

- moment for stochastic delayed neural networks with markovian switching,” *IEEE Transactions on Neural Networks & Learning Systems*, vol. 23, no. 4, pp. 662–668, 2012.
- [10] J. Zhou, T. Cai, W. Zhou, and D. Tong, “Master-slave synchronization for coupled neural networks with Markovian switching topologies and stochastic perturbation,” *International Journal of Robust and Nonlinear Control*, vol. 28, no. 6, pp. 2249–2263, 2018.
- [11] J. Zhou, X. Ding, L. Zhou, W. Zhou, J. Yang, and D. Tong, “Almost sure adaptive asymptotically synchronization for neutral-type multi-slave neural networks with Markovian jumping parameters and stochastic perturbation,” *Neurocomputing*, vol. 214, pp. 44–52, 2016.
- [12] Y. Sun and J. Cao, “Adaptive lag synchronization of unknown chaotic delayed neural networks with noise perturbation,” *Physics Letters A*, vol. 364, no. 3-4, pp. 277–285, 2007.
- [13] H. R. Karimi, M. Zapateiro, and N. Luo, “Adaptive synchronization of master-slave systems with mixed neutral and discrete time-delays and nonlinear perturbations,” *Asian Journal of Control*, vol. 14, no. 1, pp. 251–257, 2012.
- [14] L. Zhou, Q. Zhu, Z. Wang, W. Zhou, and H. Su, “Adaptive exponential synchronization of multislave time-delayed recurrent neural networks with Lévy noise and regime switching,” *IEEE Transactions on Neural Networks and Learning Systems*, vol. 28, no. 12, pp. 2885–2898, 2017.
- [15] Y. Sun, Y. Zhang, W. Zhou, J. Zhou, and X. Zhang, “Adaptive exponential stabilization of neutral-type neural network with Lévy noise and Markovian switching parameters,” *Neurocomputing*, vol. 284, pp. 160–170, 2018.
- [16] Z. Wang, S. Lauria, J. Fang, and X. Liu, “Exponential stability of uncertain stochastic neural networks with mixed time-delays,” *Chaos, Solitons & Fractals*, vol. 32, no. 1, pp. 62–72, 2007.
- [17] X. Luan, F. Liu, and P. Shi, “Neural network based stochastic optimal control for nonlinear Markovian jump systems,” *International Journal of Innovative Computing, Information and Control*, vol. 6, no. 8, pp. 3715–3723, 2010.
- [18] K. Gu, “An integral inequality in the stability problem of time-delay systems,” in *Proceedings of the IEEE Conference on Decision and Control*, pp. 2805–2810, Sydney, Australia, December 2000.

