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Research Article

Dynamic Hybrid Multiple Attribute Decision-Making Problem Based on Reference Point Adaptation

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Individuals' decision-making depends on reference points in prospect theory. This research considers the bounded rationality of decision makers and constructs a dynamic hybrid multiple attribute decision-making (DHMADM) model. Unlike existing models, the DHMADM model focuses on dynamic reference point, which has been proven in prospect theory. This research presents the effects of reference point adaptation on decision-making through model calculation. The optimal choice of decision makers changed with the change of the reference point in the DHMADM model. By experiment, we found that the DHMADM model considering reference point adaptation can more accurately express the final choice of decision makers than models only considering the static reference point.

1. Introduction

We live in a rapidly developing world, and human cognition has limitations and subjectivity. Consequently, individuals experience difficulty in making risky decisions. Multiple attribute decision-making (MADM) is aimed at choosing the best option from a set of possible options [1]. The consideration of multiple attributes helps decision makers (DMs) in choosing a suitable candidate from a number of alternatives [2]. Alternatives, attributes, and weights of these attributes are the main parts of MADM [3]. The weight of an attribute is an important factor in MADM problems [4]. Each alternative has quantitative and qualitative attributes [5]. MADM is widely applied in social and economic areas [6–8].

The world is constantly changing. An increasing number of studies focus on dynamic MADM (DMADM) problems. The core of DMADM is the changeable decision information [9]. The attributes of alternatives [10], attribute weights [11, 12], and attribute values [13] also change as time changes. Xu [13] found that DMADM problems with attribute weights and values occur at different periods. He and Teng [14] investigated DMADM problems in which the attribute values are provided by DMs at different periods. Shen et al. [15] constructed a DMADM model with the weights of attributes in multiple stages and analyzed the competence of private enterprises. Ai and Yang [16] investigated DMADM problems with dynamic attribute values in the form of two-tuple linguistic information. Liu [17] presented a method to solve two-tuple linguistic DMADM problems with entropy weights. Su et al. [18] investigated DMADM problems with attribute values in the form of intuitionistic fuzzy numbers, which are provided by multiple DMs in different periods.

However, the aforementioned studies are based on rationality. Individuals are sometimes irrational [19], and psychology plays an important role in decision-making [20]. Certain studies on MADM are based on bounded rationality. The main theory based on bounded rationality is prospect theory, which is used to model and interpret decision-making in stock trading [21]. Prospect theory is useful in solving MADM problems. Fan et al. [22] proposed a method

with different formats of attribute aspirations to solve MADM problems. Peng et al. [23] proposed a method to solve random fuzzy MADM problems with fuzzy and unknown probability and attribute values on the basis of prospect theory. Li and Chen [24] extended the TOPSIS method for group decision-making on the basis of prospect theory. Liu and Liu [25] proposed a method based on reference points in prospect theory to solve MADM problems with linguistic attributes. Li and Zhang [26] introduced a useful method to solve DMADM problems with three-parameter interval gray numbers on the basis of prospect theory. Zhu et al. [27] considered multiple reference points in MADM problems with the interval numbers of attribute values. Dai et al. [28] introduced a method that considered dynamic information to solve MADM problems with attribute values, which are triangular fuzzy numbers. Liu et al. [29] proposed an MADM method based on prospect theory to solve problems with uncertain linguistic attribute values.

Existing studies have made significant contributions to address MADM problems. When prospect theory is applied to solve MADM problems, reference points are regarded as static. However, reference points have been proven to be changing in the research of reference points in prospect theory [30–34]. Static reference points are unable to reflect the psychological changes of DMs. Hence, the main idea of this paper is to construct a dynamic hybrid multiple attribute decision-making (DHMADM) model on the basis of reference point adaptation. In this model, the attributes and reference points are described in real numbers, interval numbers, and linguistic variables. After the model was established, an experiment was conducted to test the practicality and effectiveness of the model. We find that the optimal choice of DMs changed with the adaptation of the reference point. Furthermore, compared with the hybrid multiattribute decision-making (HMADM) model considering only static reference point, the DHMADM model considering dynamic reference point can more accurately express the final decision-making of the DMs.

The innovations of this work are as follows. First, this research is the application of the dynamic reference point in prospect theory to HMADM problems. The DHMADM model is established on the basis of reference point adaption. Such approach can effectively reflect the real psychological changes of DMs in the decision-making process. The model can also shed light on the combination of the principle of prospect theory and MADM problems. Second, the calculation process of the DHMADM model in this work shows how reference point adaptation influences the decision results of DMs, and it further enriches the existing conclusions of prospect theory.

2. Prospect Theory

Prospect theory is a descriptive theory based on bounded rationality. The manner by which individuals evaluate gains and losses depends on a reference point [35]. The result is seen as a gain when it is above the reference point and a loss when it is below it. Individuals are inclined toward risk aversion in the gain domain. By contrast, individuals are inclined toward risk seeking in the loss domain. The value function model is as follows:

$$V(x) = \begin{cases} x^{a}, & x \geq 0, \\ -\lambda(-x)^{\beta}, & x < 0, \end{cases} \tag{1}$$

where x denotes the evaluation relative to the reference point. When x is greater than zero, the subjective evaluation relative to the reference point is seen as a gain; when x is equal to zero, it is consistent with the reference point; and when x is less than zero, it is seen as a loss. Parameters α and β represent the sensitivity coefficients of the gain and loss areas. The values of α and β are calculated by psychological experiments and $\alpha = \beta = 0.88$. Parameter λ represents the loss aversion coefficient and $\lambda = 2.25$ [36].

3. DHMADM Problems with Reference Point Adaptation

3.1. Reference Point Adaptation. DMs evaluate gains and losses on the basis of reference points and not absolute wealth. Generally, reference points are nonstatic. Reference point adaptation results in changes in evaluation values and decision-making [37]. In reference point adaptation, the shape of the value function is assumed to be constant and moves along the x-axis. In Figure 1, R_0 and R_1 are assumed to be the reference points of A and B, respectively. A and B see P_1 as a loss. Thereafter, B adapts its reference point to R_2 under the information effect [38]. B takes the same P_1 as a gain, and his risk attitude changes correspondingly. Furthermore, B's evaluation and decision-making may change accordingly.

3.2. Description of the Decision-Making Problems. In DHMADM problems with reference point adaptation, m alternatives $A = \{A_1, A_2, \ldots, A_m\}$ and n attributes $C = \{C_1, C_2, \ldots, C_n\}$ are assumed to exist, with $m \geq 2$ and $n \geq 2$. $X = [x_{ij}]_{m\times n}$ is the decision matrix, where x_{ij} denotes attribute j of alternative i. $W = \{w_1, w_2, \ldots, w_n\}$ is the attribute weight vector, with $1 \geq W_j \geq 0$ and $\sum_{j=1}^{n} W_j = 1$. $Q = \{Q_1, Q_2, \ldots, Q_n\}$ is the attribute expectation vector, with Q_j denoting DMs' expectations of attributes C_j, that is, $Q_j = \{q_j^1, q_j^2, \ldots, q_j^k\}$, where q_j^t represents DMs' expectations of attributes C_j in the tth period. Expectations can be used as reference points for DMs to judge losses and benefits [39]. If $q_j^t \geq x_{ij}$, then the evaluation values of alternatives A_i for attributes C_j are losses; otherwise, they are gains. $D(x_{ij}, q_j^t)$ is the distance between the actual situation and the reference point.

C^R, C^I, and C^L are the subsets of real numbers, interval numbers, and linguistic variables, respectively. $C^R = \{C_1, C_2, \ldots, C_j\}$; $C^I = \{C_{j+1}, C_{j+2}, \ldots, C_{j_2}\}$; and $C^L = \{C_{j_2+1}, C_{j_2+2}, \ldots, C_{j_3}\}$. J_1, J_2, and J_3 are the values of the real numbers, interval numbers, and linguistic variables, respectively. $J = J_1 \cup J_2 \cup J_3$ and $C = C^R \cup C^I \cup C^L$.

The real numbers, interval numbers, and linguistic variables are standardized according to different formulas using the previous method [40]. When attribute value

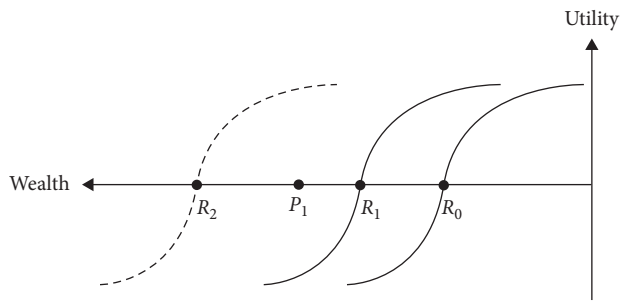

Figure 1: The potential role of reference point adaptation.

$C_j \in C^R$, the real numbers of the attribute values can be expressed as $x_{ij} = y^*_{ij}$. The real numbers of the corresponding reference points can be expressed as $q^t_j = z^{*t}_j$. When attribute value $C_j \in C^I$, the interval numbers of the attribute values can be expressed as $[x^L_{ij}, x^U_{ij}] = [y^{/L}_{ij}, y^{/U}_{ij}]$ and the corresponding reference points can then be expressed as $[q^{Lt}_j, q^{Ut}_j] = [z^{/Lt}_j, z^{/Ut}_j]$. When attribute value $C_j \in C^L$, the linguistic variables of the corresponding reference points can be expressed as follows:

$$s = \left(s^1, s^2, s^3\right) = \left(\max\left\{\frac{(f-1)}{T}, 0\right\}, \frac{f}{T}, \min\left\{\frac{(f+1)}{T}, 1\right\}\right), \tag{2}$$

where $T+1$ is the total number of phrases in a linguistic variable and $f+1$ refers to the geographical location of linguistic variables when describing attribute values and reference points.

The set of linguistic variables S can be inversely calculated with the method in [40] as follows:

$$S = \left\{s_f \,\middle|\, f = 0, 1, \ldots, \left(\frac{T}{2}\right) - 1, \frac{T}{2}, \left(\frac{T}{2}\right) + 1, \ldots, T\right\}, \tag{3}$$

where C_b is the set of attribute values for benefits, C_c is the set of attribute values for cost, and $C = C_b \cup C_c$.

4. Proposed Method

4.1. Data Processing Standardization. Three formats of reference points and attribute values need to be normalized to eliminate the influence of different physical dimensions on decision-making [41]. A reference point's vector $Q_j = \{q^1_j, q^2_j, \ldots, q^k_j\}$ is normalized into $Z^t_j = \{z^1_j, z^2_j, \ldots, z^k_j\}$, and decision matrix $X = [x_{ij}]_{m\times n}$ is normalized into $Y = [y_{ij}]_{m\times n}$ [41].

For $x_{ij} \in C^R$ and $q^t{}_j \in C^R$, the normalized formula for reference points is as follows:

$$z^{*t}_j = \begin{cases} \dfrac{q^t_j - x^-_j}{x^+_j - x^-_j}, & j \in Q_1 \cap Q_b, \\[2ex] \dfrac{x^+_j - q^t_j}{x^+_j - x^-_j}, & j \in Q_1 \cap Q_c. \end{cases} \tag{4}$$

The normalized formula for attribute values is as follows:

$$y^*_{ij} = \begin{cases} \dfrac{x^*_{ij} - x^-_j}{x^+_j - x^-_j}, & i \in M, j \in Q_1 \cap Q_b, \\[2ex] \dfrac{x^+_j - x^*_{ij}}{x^+_j - x^-_j}, & i \in M, j \in Q_1 \cap Q_c, \end{cases} \tag{5}$$

where

$$x^+_j = \max\left\{\max_{1\le i\le m}\left\{x^*_{ij}\right\}, q^t_j\right\}, \quad j \in Q_1, \tag{6}$$

$$x^-_j = \min\left\{\min_{1\le i\le m}\left\{x^*_{ij}\right\}, q^t_j\right\}, \quad j \in Q_1. \tag{7}$$

For $x_{ij} \in C^I$ and $q^t{}_j \in C^I$, the normalized formula for reference points is as follows:

$$\left[z^{/tL}_j, z^{/tU}_j\right] = \begin{cases} \left[\dfrac{q^{/tL}_j - x^{/L}_j}{x^{/U}_j - x^{/L}_j}, \dfrac{q^{/tU}_j - x^{/L}_j}{x^{/U}_j - x^{/L}_j}\right], & j \in Q_2 \cap Q_b, \\[2ex] \left[\dfrac{x^{/U}_j - q^{/tU}_j}{x^{/U}_j - x^{/L}_j}, \dfrac{x^{/U}_j - q^{/tL}_j}{x^{/U}_j - x^{/L}_j}\right], & j \in Q_2 \cap Q_c. \end{cases} \tag{8}$$

And the normalized formula for attribute values is as follows:

$$\left[y^{/L}_{ij}, y^{/U}_{ij}\right] = \begin{cases} \left[\dfrac{x^{/L}_{ij} - x^{/L}_j}{x^{/U}_j - x^{/L}_j}, \dfrac{x^{/U}_{ij} - x^{/L}_j}{x^{/U}_j - x^{/L}_j}\right], & i \in M, j \in Q_2 \cap Q_b, \\[2ex] \left[\dfrac{x^{/U}_{ij} - x^{/U}_j}{x^{/U}_j - x^{/L}_j}, \dfrac{x^{/U}_{ij} - x^{/L}_j}{x^{/U}_j - x^{/L}_j}\right], & i \in M, j \in Q_2 \cap Q_c, \end{cases} \tag{9}$$

where

$$x^{/U}_j = \max\left\{\max_{1\le i\le m}\left\{x^{/U}_{ij}\right\}, q^{/tU}_j\right\}, \quad j \in Q_2, \tag{10}$$

$$x^{/L}_j = \min\left\{\min_{1\le i\le m}\left\{x^{/L}_{ij}\right\}, q^{/tL}_j\right\}, \quad j \in Q_2. \tag{11}$$

For $x_{ij} \in C^L$ and $q^t{}_j \in C^L$, the normalized formula for reference points is as follows:

$$z^t_j = \begin{cases} q^t_j, & j \in Q_3 \cap Q_b, \\ \operatorname{neg}\left(q^t_j\right), & j \in Q_3 \cap Q_c. \end{cases} \tag{12}$$

And the normalized formula for attribute values is as follows:

$$y_{ij} = \begin{cases} x_{ij}, & i \in M, j \in Q_3 \cap Q_b, \\ \operatorname{neg}\left(x_{ij}\right), & i \in M, j \in Q_3 \cap Q_c. \end{cases} \tag{13}$$

According to equation (2), Z^t_j and y_{ij} can be converted into triangular fuzzy numbers $\hat{Z^t_j}$ and $\widehat{y_{ij}}$, respectively, for calculation, that is, $\hat{Z^t_j} = (Z^{1t}_j, Z^{2t}_j, Z^{3t}_j)$, $\widehat{y_{ij}} = (y^1_{ij}, y^2_{ij}, y^3_{ij})$.

4.2. Distance between Attribute Value and Reference Point. The distance between the attribute value and the reference point is expressed as follows:

$$D\left(y_{ij}, z_j^t\right) = \begin{cases} \left|y_{ij}^* - Z_j^{*t}\right|, & i \in M,\ j \in Q_1, \\ \sqrt{\frac{1}{2}\left[\left(y_{ij}^{/L} - Z_j^{/tL}\right)^2 + \left(y_{ij}^{/U} - Z_j^{tU}\right)^2\right]}, & i \in M,\ j \in Q_2, \\ \frac{1}{3}\left[\left(y_{ij}^1 - Z_j^{1t}\right)^2 + \left(y_{ij}^2 - Z_j^{2t}\right)^2 + \left(y_{ij}^3 - Z_j^{3t}\right)^2\right], & i \in M,\ j \in Q_3. \end{cases} \tag{14}$$

(1) For $x_{ij} \in C^R$, $D(z_{ij} - y_j^t) = z_{ij} - y_j^t$; if $z_{ij} \geq y_j^t$, then $D(z_{ij} - y_j^t)$ is a positive number; if $z_{ij} < y_j^t$, then $D(z_{ij} - y_j^t)$ is a negative number

(2) For $x_{ij} \in C^I$, if $M_1 \geq M_2$, then $D(z_{ij} - y_j^t)$ is a positive number; if $M_1 < M_2$, then $D(z_{ij} - y_j^t)$ is a negative number; $M_1 = (y_{ij}^L + y_{ij}^U)/2$ and $M_2 = (z_{ij}^L + z_{ij}^U)/2$

(3) For $x_{ij} \in C^L$, if $f > g$, then $s_f \succ s_g$, and $D(z_{ij} - y_j^t)$ is a positive number, and so on

4.3. Attribute Value. After the distance between the attribute value and the reference point is obtained, the value of each attribute is calculated by the value function of prospect theory. The gain and loss matrix relative to the reference point is obtained:

$$F = \left[F\left(y_{ij}^t\right)\right]_{m \times n \times k}. \tag{15}$$

Then, according to the values of sensitivity coefficients of the gain and loss areas and loss aversion coefficient, the corresponding prospect values are calculated according to equation (1):

$$V\left(F\left(y_{ij}\right)\right) = \begin{cases} \left(F\left(y_{ij}\right)\right)^{\alpha}, & y_{ij} \geq z_j^t, \\ -\lambda\left(F\left(y_{ij}\right)\right)^{\beta}, & y_{ij} < z_j^t. \end{cases} \tag{16}$$

4.4. Attribute Utility. Finally, the utility values of each alternative are calculated and compared according to the weights. The ranking of all alternatives will be determined according to the comprehensive prospects calculated by the following formula. The larger the value, the more likely the DMs will prefer the alternative. The smaller the value, the less likely the DMs will choose the alternative. The numerical value of the comprehensive prospect value may also be negative, indicating that the alternative does not meet the ideal criteria of DMs:

$$U(A_i) = \sum_{j=1}^{n} w_j V\left(F\left(y_{ij}\right)\right). \tag{17}$$

5. Case Study

An experiment on a hypothetical scenario involving a newly opened furniture store is carried out to illustrate the relationship between reference point adaptation and decision-making. At the same time, compare the HMADM model considering only static reference points with the DHMADM model considering dynamic reference points and test which model can more accurately express the DMs' final decision-making.

5.1. Experimental Design. The locations of furniture store directly affect sales, and thus, the most appropriate address must be chosen. The following four factors are considered in the selection of furniture store locations: geographical location C_1, annual rent C_2, decoration cost C_3, and transfer C_4. Geographical location includes the location of shops, the demand around them, and traffic conditions. Annual rent refers to the one-year rent paid by the user of a shop to the owner. Rent is high when the store is near large furniture markets. Decoration cost is regarded as the resources spent in the design and construction activities to make shops beautiful. Transfer means the possibility and acceptance time of other people in the process of transferring the right to use the store.

We present a hypothetical scenario for opening a new store because of business needs and interviewed a furniture brand manager on August 27, 2018. In the hypothetical scenario, the manager needs to select an appropriate store from $A1$, $A2$, and $A3$. The selection was given to the furniture store manager for further evaluation. These three alternatives use four attributes for description: geographical location, annual rent, decoration cost, and transfer. The geographical location and transfer are described as very good (VG), good (G), common (C), poor (P), and very poor (VP). The annual rent and decoration cost are described by real and interval numbers, respectively (Table 1).

Step 1: As in an experiment for a supplier selection problem, DMs themselves can provide the attribute weights [42]. The manager in the experiment was asked to assign weights to the four factors. The weight of a factor is great if it is important. However, the total weight of ownership should not be more than 1.

Step 2: After determining the weights of all factors, the manager was asked to express any expectations toward the four factors. This aspect was set as the first reference point of the manager.

Step 3: The manager was then asked to rank the three alternatives: A1, A2, and A3 (Table 1). This aspect was the first subjective decision-making under the first reference point.

Table 1: Attribute values for three alternatives.

	C_1	C_2 (yuan)	C_3 (yuan)	C_4
$A1$	G	220,000	200,000–250,000	VG
$A2$	VG	250,000	240,000–300,000	C
$A3$	C	150,000	150,000–190,000	G

Step 4: We make use of the effect of unfavorable information on reference point adaptation to promote the manager's reference point adaptation. We informed the manager that the furniture market will go down in the next few years. The manager was asked again regarding his expectations toward the four factors. This aspect was set as the second reference point of the manager.

Step 5: We asked the manager to reorder the three alternatives. This aspect was the second subjective decision-making under the first reference point.

5.2. Experimental Analysis. Table 2 shows that in the subjective assignment of weights, geographical location was assigned the weight of 50%. The manager believed that geographical location is an important factor in opening furniture stores because a good geographical location can increase sales. The weights of the other three factors were 30%, 15%, and 5%. In the experiment, "the market of this industry will go down next year" is regarded as unfavorable information. Before the unfavorable information was provided, the manager's expectation for geographical location was very good. The expectation for annual rent was 240,000 yuan/year, that for decoration cost was between 250,000 and 280,000 yuan, and that for the transfer of the store was common. This aspect was set as the first reference point of the manager. After the unfavorable information was provided, the manager's expectation for geographical location was good. The expectation for annual rent was 200,000 yuan/year, that for decoration cost was between 220,000 and 260,000 yuan, and that for the transfer of the store was good. This aspect was set as the second reference point of the manager.

Under the first reference point, the manager chose the second alternative as the best one. After reference point adaptation, the manager's best choice was the first alternative. We then calculate the weight and reference point data in the DHMADM model and compare the results with the manager's subjective ranking of alternatives.

5.3. Data Analysis. $Q_j = \{q_j^1, q_j^2, \ldots, q_j^k\}$ is normalized into $Z_j^t = \{z_j^1, z_j^2, \ldots, z_j^k\}$. Decision matrix $X = [x_{ij}]_{m\times n}$ is normalized into $Y = [y_{ij}]_{m\times n}$. The normalized matrix with reference point 1 is Z^1 by using equations (2)–(13):

$$Z^1 = \{(0.75, 1.00, 1.00), 0.07, [0.13, 0.33], (0.25, 0.50, 0.75)\}. \tag{18}$$

Similarly, the normalized matrix of reference point 2 with three formats is Z^2. The purpose of doing so is to eliminate the influence of different dimensional units so as to facilitate the calculation and comparison of the comprehensive value of each alternative.

$$Z^2 = \{(0.50, 0.75, 1.00), 0.33, [0.27, 0.53], (0.50, 0.75, 1.00)\}. \tag{19}$$

The decision matrix is normalized by using equations (2)–(13):

$$Y = \left[y_{ij}\right]_{m\times n} = \begin{bmatrix} (0.50, 0.75, 1.00) & 0.20 & [0.33, 0.67] & (0.75, 1.00, 1.00) \\ (0.75, 1.00, 1.00) & 0.00 & [0.00, 0.40] & (0.25, 0.50, 0.75) \\ (0.25, 0.50, 0.75) & 0.67 & [0.73, 1.00] & (0.50, 0.75, 1.00) \end{bmatrix}. \tag{20}$$

The distance matrix for the first reference point is constructed by using equation (14):

$$D_1 = \begin{bmatrix} -0.20 & 0.13 & 0.27 & 0.43 \\ 0.00 & -0.07 & -0.11 & 0.00 \\ -0.43 & 0.60 & 0.63 & 0.25 \end{bmatrix}. \tag{21}$$

And similarly, the relative distance of z_j^2 and y_{ij} is D_2:

$$D_2 = \begin{bmatrix} 0.00 & -0.13 & 0.11 & 0.43 \\ 0.20 & -0.33 & -0.21 & -0.25 \\ -0.25 & 0.33 & 0.47 & 0.00 \end{bmatrix}. \tag{22}$$

The prospect values of each attribute relative to the first reference point can be computed by using equation (15). And the prospect decision matrix is as follows:

$$F_1 = \begin{bmatrix} -0.56 & 0.17 & 0.32 & 0.48 \\ 0.00 & -0.21 & -0.31 & 0.00 \\ -1.08 & 0.64 & 0.67 & 0.30 \end{bmatrix}. \tag{23}$$

Correspondingly, the value of the prospect value function relative to reference point 2 is F_2. And the prospect decision matrix is as follows:

$$F_2 = \begin{bmatrix} 0.00 & -0.38 & 0.13 & 0.48 \\ 0.25 & -0.86 & -0.57 & 0.48 \\ -0.66 & 0.38 & 0.57 & 0.00 \end{bmatrix}. \tag{24}$$

Comprehensive prospect value is the final value judgment combining the attribute of the alternative and the expectation of the DMs. The comprehensive prospect value of each alternative based on reference point adaptation can be calculated according to equations (16) and (17).

Table 2: Subjective weights, reference points, and alternative ranking of the manager.

Questions	Answers
(1) If you are ready to open a new furniture store, with a total of 10 points, how many points would you give to each of the following factors: geographical location C_1, annual rent C_2, decoration cost C_3, and transfer C_4?	C_1: 0.5 C_2: 0.3 C_3: 0.15 C_4: 0.05
(2) What are your expectations for the aforementioned factors?	C_1: VG C_2: 240,000 yuan/year C_3: 250,000–280,000 yuan C_4: C
(3) Please rank the following three alternatives.	$A_2 > A_1 > A_3$
(4) The information shows that the market of this industry will go down next year. When considering opening new stores, what are the expectations for the aforementioned four factors?	C_1: G C_2: 200,000 yuan/year C_3: 220,000–260,000 yuan C_4: G
(5) On the basis of learning this information, please reorder the three alternatives in the table according to your preference.	$A_1 > A_3 > A_2$

Reference point 1: $U(A_1) = -0.15$, $U(A_2) = -0.11$, and $U(A_3) = -0.23$

Reference point 2: $U(A_1) = -0.07$, $U(A_2) = -0.20$, and $U(A_3) = -0.14$

When the data of the first reference point are substituted into the model calculation, the results of the model calculation show that the manager's best choice is the second alternative. Moreover, when we substitute the data of the second reference point into the model for recalculation, the results show that the manager's best choice is the first alternative. This result is consistent with the manager's subjective decision-making. This means that under the influence of information, the reference point of DMs has changed, which affects the final decision-making of DMs. The DHMADM model just obtains the same conclusion, which reflects the final decision-making state of DMs.

5.4. Comparative Analysis. We compare the DHMADM model calculation results with the HMADM model calculation results. The HMADM model only considers a static reference point. In the DHMADM model, not only reference point 1 of the DMs is considered, but also reference point 2 of the DMs is affected by the unfavorable information.

As is shown in Table 3, the HMADM model calculation results show that if only the static reference points are considered, then the optimal choice of DMs is the second alternative, but in experiment result, DMs finally choose the first alternative as the best one. On the contrary, if the dynamic reference point is considered in the DHMADM model, the optimal choice of DMs calculated by the model is the first alternative. And this result is consistent with the real optimal decision of the DMs in the experiment.

Table 3: Comparison of HMADM and DHMADM model calculation results.

	HMADM model	DHMADM model
Model calculation results	$\mathbf{A}_2 > A_1 > A_3$	$\mathbf{A}_1 > A_2 > A_3$
Subjective ranking results	$A_2 > A_1 > A_3$	$\mathbf{A}_1 > A_3 > A_2$

The DMs' subjective attribute ranking and model calculation results show that under the influence of unfavorable information, decision-making changes when DMs' reference points adaptation. Meanwhile, subjective ranking results reflect decision-making results intuitively and truthfully. Comparing subjective ranking results with model calculation results is an important way to test the rationality of the model. Table 3 also shows that the subjective optimal alternative is consistent with the model calculation results in the DHMADM model. From the subjective judgment and calculation of the objective model, we can clearly observe the influence of changes in reference points on the decision-making results by establishing the DHMADM model with reference point adaptation as the core element.

If only reference point 1 is considered, the model in this paper is a HMADM model; if both reference point 1 and reference point 2 are considered, the model in this paper is a DHMADM model; by comparing the calculation results between those models, we found that the changes in reference points lead to changes in decision results. Furthermore, the practicality and effectiveness of the DHMADM model can be obtained by the consistency of the subjective judgment results and the objective calculation results of the model.

6. Conclusions

Making optimum decisions is important in a competitive environment. Prospect theory is a descriptive theory. Such theory is based on bounded rationality. In prospect theory, the reference point decides an individual's feeling of gain and loss [43]. Reference point adaptation causes changes in risk attitude and subsequent decision-making. This work establishes a DHMADM model on the basis of reference point adaptation. The attributes of the alternatives and reference points are described by real numbers, interval numbers, and linguistic variables. Under the effect of unfavorable information, the optimal choice of the DMs changes with the reference point adaptation. The model and method of calculation is also practical and effective.

Unfavorable information affects reference point adaptation and the subsequent decision-making. If only the static reference point is considered, then the optimal choice of DMs is the second alternative in the model calculation, but the DMs finally choose the first alternative as the best one in experiment result. This means that only considering the static reference point cannot accurately reflect the actual decision-making situation of the DM. On the contrary, if the dynamic reference point is considered in the DHMADM model, the optimal choice of DMs calculated by the model is

the first alternative. The results of the DHMADM model are consistent with the subjective representation of DM. Prior to receiving unfavorable information, DMs would choose a very good geographical store location. However, after determining that the industry outlook for the next year is unfavorable, the manager chooses to find a good geographical store location with cheaper rent. The change of decision result accompanied by this psychological change of DM is appropriately expressed through the HDMADM model.

The choice of DMs' subjective optimal alternative is consistent with the model calculation result before and after reference point adaptation. The subjective optimal alternative of DMs before reference point adaptation is the second one. Under the influence of the unfavorable information, the reference point and optimal alternative of DMs change. In the model calculation, before receiving the unfavorable information, the DMs' optimal choice is the second alternative. After receiving the unfavorable information, the DMs' optimal choice is the first alternative. Therefore, the accuracy of the DHMADM model is obtained by comparing the experimental and model data results.

The complexity of the socio-economic environment can be effectively reflected by constructing a DHMADM model on the basis of reference point adaptation. This study has certain limitations. The main limitation of this research is the lack of a reference point adaptation model because the dynamic reference point model varies under different environments. The other limitation is that the reference point is a subjective factor, so the data of the reference point can only be obtained through the subjective expression of DMs. Then, it is hard to compare this model with other models except the HMADM model in this paper. While the consistency of the subjective judgment results and the objective calculation results also can illustrate the practicality of the DHMADM model. The future DHMADM model can involve deviation attributes, such as fuzzy and soft sets.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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