Research Article

Integrated Fault Estimation and Fault-Tolerant Control for Dynamic Positioning of Ships

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Received 31 March 2019; Revised 1 September 2019; Accepted 28 September 2019; Published 20 October 2019

1. Introduction

Dynamic positioning systems (DPS) can only rely on their own propulsion to counteract the interference of the external environment and maintain the ship’s position and heading at the fixed location or along the predetermined track [1]. With the increasing focus on the ocean exploitation, dynamic positioning system has become an attractive research topic and has drawn great attention from control communities during the past two decades, with many results reported in the literature, such as hybrid control [2], fuzzy control [3, 4], backstepping [5, 6], and model predictive control [7]. The highly disturbed marine environment will lead to aging of the components of ships, which causes inevitable malfunction in actuators. Once the actuator faults occur, it is not possible that the faulty thrusters can be repaired or replaced by the backup one in time. Therefore, how to design a fault-tolerant controller (FTC) for dynamic positioning (DP) ships is a critical problem.

Recently, to increase the safety and reliability of DPS, many researches on fault-tolerant control for DPS have been carried out. The authors in [5] constructed an iterative learning observer to estimate the fault signal and combined the pseudoinverse method to generate a fault-tolerant controller for the dynamic positioning of ships. Andrea and Tor utilized an unknown input observer (UIO) technique to produce a fault detection and isolation mechanism for an overactuated marine vessel [8]. Fault detection and diagnosis mechanism, based on two techniques: the parity space approach and the Luenberger observer, was proposed to guarantee a fault-tolerant robust control for the dynamic positioning of ships [9]. A fault-tolerant supervisory controller was designed based on the certainty equivalence principle, which could solve both sensor and actuator faults of ships [10]. Other remarkable results on FTC for dynamic positioning of ships can be found in [11, 12]. However, most of the current studies of FTC for DPS are carried out as two
separate entities, fault estimation and fault-tolerant control, such that it is difficult to determine whether or not fault estimation satisfies high requirement for the fault-tolerant controller. Moreover, there exists the time delay problem in the fault-tolerant control strategies in [8, 9, 12] due to using the fault detection and the isolation (FDI) module.

Motivated by the aforementioned considerations, an integrated fault estimation and fault-tolerant control scheme for DPS with actuator faults are presented in this paper. Comparing with the adaptive law, the updating law in this paper is an algebraic equation, which requires less on-line computing power and is easy to implement in practice. Moreover, the novelty of the approach with respect to existing results consists of the following key points:

(1) In [5, 11], the authors made a strict assumption that the velocities of ships are measurable. In fact, the velocities of ships cannot be deduced from the measured signals through differentiation because such a scheme tends to significantly magnify the noise level in velocities [1]. Instead, in the proposed work, such assumption is not needed.

(2) Unlike in [8, 9, 12] where fault detection and isolation and fault-tolerant control are designed separately, in this paper, the fault estimation and the FTC are simultaneously considered, and their coupling problem can be effectively solved.

(3) This paper is concerned with the fault-tolerant control design problem with a general actuator fault mode. Compared with the supervisory control method in [10], the proposed scheme does not need to know the specific fault mode information which is indispensable for the supervisory fault-tolerant controller.

The rest of this paper is organized as follows. Several preliminaries and problem formulation are presented in Section 2. Section 3 addresses the fault-tolerant controller for DPS based on the iterative learning observer with theoretical stability analysis of the closed-loop system. In Section 4, we illustrate the effectiveness of the proposed method via simulations on the dynamic positioning of ships. Finally, some concluding remarks are provided in Section 5.

Throughout the paper, \( \lambda_{\text{min}}(Q) \) and \( \lambda_{\text{max}}(Q) \) are the minimum and maximum eigenvalues of \( Q \), respectively; \( Q > 0 \) \((Q < 0)\) denotes that \( Q \) is a positive (negative) definite matrix, \( \| \cdot \| \) represents Euclidean norm of the vector or the matrix, and \( * \) means symmetric term.

2. Preliminaries and Problem Formulation

2.1. Modeling of Ships. For the horizontal motion of a surface vessel, let the Earth-fixed position \((x, y)\) and orientation \( \psi \) of the vessel relative to an Earth-fixed frame \( X_E Y_E Z_E \) be expressed in vector form by \( \eta = [x \ y \ \psi]^T \), and let the velocities decomposed in a body-fixed reference frame XYZ be represented by the vector \( v = [u \ v \ r]^T \). These three modes are referred to as the surge, sway, and yaw of a vessel [1]. The Earth-fixed inertial frame and body-fixed frame are depicted by Figure 1.

The kinematics and dynamics of ships in 3-DOF can be described as follows:

\[
\dot{\eta} = J(\psi)v, \quad (1a)
\]

\[
M\dot{v} = \tau - Dv + d + \tau_f, \quad (1b)
\]

where \( J(\psi) = \begin{bmatrix} \cos(\psi) & -\sin(\psi) & 0 \\ \sin(\psi) & \cos(\psi) & 0 \\ 0 & 0 & 1 \end{bmatrix} \) is the rotation matrix from the body-fixed frame to the Earth-fixed inertial frame; \( M \in \mathbb{R}^{3\times3} \) and \( D \in \mathbb{R}^{3\times3} \) denote the inertia matrix and damping matrix, respectively; \( \tau, \tau_f \) represents the control input vector and uncertain fault vector, respectively; and \( d \) denotes the time-varying external disturbance caused by winds, waves, and ocean currents.

Remark 1. In practice, most faults are nonlinear functions of the state vector and/or input vector. The formulation given by \( \tau_f = \tau_f(\eta, v, \tau) \) can capture such practical fault models [13].

By defining the new vessel parallel coordinate position as \( \eta_p = J^T(\psi)\eta \), for low-speed assumptions, we can deduce that \( \dot{\eta}_p = v [1] \). Then, the linear time-invariant state-space model of ships can be written in the compact form as

\[
\dot{x} = Ax + Br + Ef, \quad (2a)
\]

\[
y = [y_1 \ y_2 \ y_3]^T = Cx, \quad (2b)
\]

\[
y_i = C_i x, \quad (2c)
\]

where \( x = [\eta^T \ v^T]^T \), \( f = d + \tau_f \), \( A = \begin{bmatrix} 0_{3\times3} & I_{3\times3} \\ 0_{3\times3} & -M^{-1}D \end{bmatrix} \), \( B = E \in \mathbb{R}^{3\times3} \), and \( C = [I \ 0] \).

The control objective of this paper is to design a fault-tolerant control law \( \tau \) for dynamic positioning of ships in the absence of velocity measurements and subject to actuator faults, for the purpose that the DP ships can maintain the desired position \( \eta_d \).

2.2. Assumptions and Lemmas. Throughout this paper, the following assumptions are made.

Assumption 1. For low-speed applications, it can also be assumed that \( M^T = M > 0 \) and \( D > 0 \).

Remark 2. Assumption 1 is true if starboard and port symmetries and low speed are assumed [14]. Actually, the nonlinear damping term can be neglected since the linear
term dominates at lower speeds. This is a good assumption for dynamic positioning of ships [1].

Assumption 2. The following inequality holds 
\[ \| f(t) - K_1 f(t - T) \| = \| f_d(t) \| \leq T_d, \] where \( T \) is a short time delay and we call it as a learning interval later.

Definition 1 (relative degree [15]). The relative degree \( \gamma_i \) of the systems (1a) and (1b) with respect to the \( i \)-th output \( y_i \) is defined as follows:
\[
\begin{align*}
C_iA^kE &= 0, & \text{for all } k < \gamma_i - 1, \\
C_iA^{\gamma_i-1}E &\neq 0.
\end{align*}
\]

Definition 2 (Invariant zeros). For a MIMO state-space model, the invariant zeros are the complex values of \( s \) for which the rank of Rosenbrock’s system matrix
\[
\begin{bmatrix}
sI - A & E \\ C & 0
\end{bmatrix}
\]
drops from its norm value.

Lemma 1 (Schur complement lemma [16]). Suppose \( S = \begin{bmatrix} S_1 & S_2 \\ * & S_4 \end{bmatrix} \) is a given symmetric matrix, where \( S_1 \in \mathbb{R}^{m \times m} \). Then, the following three conditions are equivalent:
\[
\begin{align*}
S &< 0, \\
S_1 < 0, S_4 - S_2 S_1^{-1} S_2 &< 0, \\
S_4 < 0, S_1 - S_2 S_4^{-1} S_2 &< 0.
\end{align*}
\]

Lemma 2 (Young’s inequality [17]). Let \( L, R, \) and \( F(t) \) be real matrices of appropriate dimensions with \( F(t) \) being a matrix function. Then, for any \( \varepsilon > 0 \) and \( F^T(t)F(t) \leq I \), we have
\[
LF(t)R + R^T F^T(t) L^T \leq \frac{1}{\varepsilon} LL^T + \varepsilon R^T R. \tag{5}
\]

Lemma 3 (UIO existence conditions [18]). There exist a matrix \( L, G \) and a symmetric positive-definite matrix \( P \) such that
\[
\begin{align*}
&\begin{bmatrix}
P(A + LC) + (A + LC)^T P < 0, \\
E^T P = GC,
\end{bmatrix}
\end{align*}
\]
if and only if the rank \( (CE) \neq \text{rank}(E) \) and the invariant zeros of \( \{A, E, C\} \) lie in the open left-hand complex plane.

3. Main Results

3.1. Construction of Auxiliary Derivative Outputs and the High-Gain Observer. In this subsection, the auxiliary derivative outputs (ADOs) are generated using a high-gain observer (HGO) in order to make the new auxiliary system satisfy the UIO existence condition which is defined in Lemma 3. The ADO estimation error is shown to be uniformly ultimately bounded with respect to a ball whose radius is a function of design parameters.

After calculation and analysis, we get that \( \text{rank}(CE) \neq \text{rank}(E) \), and the relative degree of the dynamic positioning of ships is 2. Then, by the definition of the relative degree, we can construct the auxiliary derivative output matrix
\[
C_A \triangleq \begin{bmatrix}
C_1 \\
C_1 A \\
C_2 \\
C_2 A \\
C_3 \\
C_3 A
\end{bmatrix} = \begin{bmatrix}
1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 1 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1
\end{bmatrix}, \tag{7}
\]
(such that \( \text{rank}(C_AE) = \text{rank}(E) = 3 \). Then, we construct the HGO to obtain accurately the ADOs:
\[
y_a = C_a x
\]
\[
= \begin{bmatrix}
y_{a_{11}} \\
y_{a_{12}} \\
y_{a_{21}} \\
y_{a_{22}} \\
y_{a_{31}} \\
y_{a_{32}}
\end{bmatrix}^T, \tag{8}
\]
just based on the measurable outputs \( y = Cx \). In order to simplify the analysis process and to highlight important concepts, we only consider the \( i \)-th pair of ADOs (8) which is given by

\[
y_{ai} = \begin{bmatrix} y_{a_i} \\ y_{si} \\ y_{oi} \end{bmatrix}.
\]

Then, by taking the first-time derivative of the ADOs (9) along \( \dot{x} = Ax + Br + Ef \), one can achieve that

\[
\begin{cases}
y_{ai} = A_i y_{ai} + B_i r + E_i \phi, \\
y_{oi} = C_i y_{ai},
\end{cases}
\]

(10)

where \( \phi = C_i Af + C_i A^2 x \), \( \tau_i = [1 \ 0] \), \( A_i = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \),

\[
B_i = \begin{bmatrix} 0 \ 1 \\ C_i AB \end{bmatrix}, \text{ and } B_i = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.
\]

We construct the HGO for ADOs (10) as follows:

\[
\begin{aligned}
\dot{y}_{hi} &= \tilde{A_i} y_{hi} + \tilde{B_i} r + \tilde{I_i} \tau_i (y_{ai} - y_{hi}), \\
y_{hi} &= \tau_i y_{hi},
\end{aligned}
\]

(11)

where \( \tilde{I_i} = [\chi_{i1}/\mu \ \chi_{i2}/\mu^2]^{\top} \) is a design parameter vector with \( \mu > 0 \). According to (10) and (11), the error dynamics can be described as

\[
\dot{\tilde{y}}_{ai} = \begin{bmatrix} -I_{i1} & 1 \\ -I_{i2} & 0 \end{bmatrix} \tilde{y}_{ai} + \tilde{B}_i \phi,
\]

(12)

where \( \tilde{y}_{ai} = y_{ai} - y_{hi} \), \( I_{i1} = \chi_{i1}/\mu \), and \( I_{i2} = \chi_{i2}/\mu^2 \). Define the new states as \( \xi_{ai} = D_{\mu}^{-1} \tilde{y}_{ai} \), where \( D_{\mu} = \begin{bmatrix} \mu & 0 \\ 0 & 1 \end{bmatrix} \). Then, we have

\[
\dot{\xi}_{ai} = A_i \xi_{ai} + \mu \tilde{B}_i \phi,
\]

(13)

where

\[
A_i = \begin{bmatrix} -\chi_{i1} & 1 \\ -\chi_{i2} & 0 \end{bmatrix}.
\]

\[
\lambda_i = \begin{bmatrix} -\chi_{i1} \\ -\chi_{i2} \end{bmatrix}.
\]

Lemma 4. For the high-gain observer (11), there exist a positive constant \( \beta \), arbitrarily small positive number \( \overline{\mu} \) and a finite time \( T(\overline{\mu}) \) such that \( \|\tilde{y}_{hi}\| \leq \beta \mu \) for \( t \geq t_0 + T(\overline{\mu}) \), where \( y_h = \begin{bmatrix} y_{h1}^T \\ y_{h2}^T \\ y_{h3}^T \end{bmatrix} \).

3.2. Integrated Design of Fault Estimation and FTC via Iterative Learning Observers. It is proved in [15] that the invariant zeros of the triples \( \{A, E, C\} \) and \( \{A, E, C_a\} \) are identical. Moreover, the system (2a)–(2c) has no invariant zeros. By Lemma 3, there must exist matrices \( L, G \) and a symmetric positive-definite matrix \( P \) such that

\[
\begin{aligned}
P(A - LC_a) + (A - LC_a)^T P &< 0, \\
GC_a &= E^T P.
\end{aligned}
\]

A fault-tolerant controller based on the iterative learning observer and high-gain observer for dynamic positioning of ships with actuator faults is proposed as follows:

\[
\begin{aligned}
\tau(t) &= -K_c \hat{x}(t) - \tilde{f}(t), \\
\hat{x}(t) &= Ax(t) + Br(t) + EF(t) + L(y_h(t) - C_a \hat{x}(t)), \\
y_h(t) &= Ay_h(t) + B r(t) + L(x(t) - y_h(t)), \\
\tilde{f}(t) &= K_1 \tilde{f}(t - T) + K_2 (y_h(t) - C_a \hat{x}(t)),
\end{aligned}
\]

(16a)–(16d)

where \( \hat{x} = \text{diag}(\hat{A}_1, \hat{A}_2, \hat{A}_3) \), \( B = \begin{bmatrix} B_1 & B_2 & B_3 \end{bmatrix}^T \), \( L = \begin{bmatrix} L_1 & L_2 & L_3 \end{bmatrix}^T \), and \( \tau = [\tau_1, \tau_2, \tau_3] \); \( K_c \) is the control gain matrix to be designed; \( \tilde{f} \) is the reconstructed signal of the fault and is used as feed-forward compensation; and \( K_1 > 0 \) and \( K_2 > 0 \) are the learning rates.

Remark 3. From the formulas (16a)–(16d), it can be observed that the high-gain observer (16c) can be designed independently of the whole fault-tolerant controller, and the function of the high-gain observer is to generate a set of auxiliary signals \( y_h \) to update the state of the iterative learning observer (16b) and fault estimation (16d). Parameter \( T \) in equation (16d) is called the learning interval that can be adjusted to guarantee fault-reconstruction accuracy. It should be selected large if a fault is constant or slow-changing; otherwise, it should be chosen small. Specifically, in sampled-data control systems, it can be taken as the sampling interval or as an integer multiple of the sampling interval. Parameters \( K_1, K_2 \) are the gains of the updating law (16d). Generally speaking, \( K_1 > 0 \) should be chosen to be 1 or close to 1 such that \( f_s(t) = f(t) - K_1 f(t - T) \) is small enough.

Theorem 1. The dynamics of dynamic positioning ships described in (2a)–(2c) is considered, and Assumptions 1 and 2 are supposed. If the fault-tolerant controller is designed as (16a)–(16d) with the parameters \( K_c = N_1 N_2^{-1} \) and \( L = P^{-1} N_3 \), where \( N_1, N_2, N_3, P \) are the solutions of the following LMIs:
\[
\begin{align*}
\text{min } \theta, \\
0 < \theta, \\
0 < N_1, \\
0 < P,
\end{align*}
\]

\[
\begin{bmatrix}
-\theta I & (E^T P - \alpha_1 K_2 C_a)^T \\
* & -I
\end{bmatrix} \leq 0,
\]

\[
\begin{bmatrix}
\Omega_1 & BN_2 & 0 & 0 & E \\
* & -2\gamma N_1 & \gamma I_2 & 0 & 0 \\
* & * & \Omega_21 & N_3 & 0 \\
* & * & * & -\frac{1}{\epsilon_2 \tilde{u}} & 0 \\
* & * & * & * & \frac{1}{\epsilon_1} \tilde{T}
\end{bmatrix} < 0,
\]

\[
-Q + (1 + \epsilon_4 \alpha_1) \alpha_1 K_1^T K_1 + \epsilon_3 \alpha_1 \mu K_1^T K_2^T K_1 < 0,
\]

where \( \tilde{u} \leq \lambda_{\text{min}} \left( -Q_1 / \epsilon_2 \lambda_{\text{max}} (PL (PL)^T), Q_1 = P (A - LC_a) + (A - LC_a)^T P, \right. \)

\[
\Omega_1 = AN_1 - BN_2 + N_3 A^T - N_3^T B^T, \quad \text{and} \quad \Omega_21 = PA - N_3 C_a + A^T P - C_a N_3^T \]

then, the state-estimation error, fault-estimation error, and the state vector of the closed-loop system are uniformly ultimately bounded.

**Proof.** Denote \( \bar{x} = x - \bar{x} \) and \( \bar{f} = f - \tilde{f} \). Consider the Lyapunov–Krasovskii function candidate \( V \) for \( t \geq t_0 + T (\bar{P}) \) as

\[
V = \bar{x}^T P \bar{x} + \bar{x}^T P_1 \bar{x} + \int_{t-T}^{t} \bar{f}^T Q \bar{f},
\]

The derivative of the Lyapunov–Krasovskii function (18) with respect to time can be derived as

\[
\dot{V} = \bar{x}^T (2P (A - LC_a)) \bar{x} + 2 \bar{x}^T P E \bar{f} - 2 \bar{x}^T PL (y_h - y_a) \]

\[
- \epsilon_0 \tilde{f}^T Q \tilde{f} + \alpha_1 \tilde{f}^T \tilde{f} - \tilde{f}^T (t-T) Q \tilde{f} (t-T)
\]

\[
+ x^T (2P_1 (A - BK_a)) x + 2x^T P_1 BK_a \bar{x} + 2x^T P_1 E \tilde{f},
\]

where \( \epsilon_0, \epsilon_3, \epsilon_4 > 0 \). Fault-estimation error can be calculated as follows:

\[
\tilde{f} = f_d (t) + K_1 \tilde{f} (t-T) - K_2 \bar{y}_h - K_3 C_a \bar{x},
\]

\[
f_d = f (t) - K_1 f (t-T). \]

Following Lemma 2, we have

\[
2x^T P_1 E \tilde{f} \leq \epsilon_1 x^T P_1 E (P_1 E)^T x + \frac{1}{\epsilon_1} \tilde{f}^T \tilde{f},
\]

where \( \epsilon_1 > 0 \). Then, the derivative of the Lyapunov–Krasovskii function (19) can be rewritten as

\[
\dot{V} \leq \bar{x}^T (2P (A - LC_a)) \bar{x} + 2 \bar{x}^T P E \tilde{f} - 2 \bar{x}^T PL \bar{y}_h
\]

\[
- \epsilon_0 \tilde{f}^T Q \tilde{f} + \alpha_1 \tilde{f}^T \tilde{f} - \tilde{f}^T (t-T) Q \tilde{f} (t-T)
\]

\[
+ x^T (2P_1 (A - BK_a)) x + \epsilon_1 x^T P_1 E (P_1 E)^T x
\]

\[
+ 2 \bar{x}^T P_1BK_a \bar{x} \leq \bar{x}^T (2P (A - LC_a)) \bar{x} + 2 \bar{x}^T (PE - \alpha_1 (K_2 C_a)^T)
\]

\[
K_1 \tilde{f} (t-T) + 2 \bar{x}^T (PE - \alpha_1 (K_2 C_a)^T) f_d (t)
\]

\[
+ 2x^T \left( \frac{\alpha_1}{2} (K_2 C_a)^T - PE \right) K_2 C_a \bar{x} - \tilde{f}^T (t-T) \tilde{f} (t-T)
\]

\[
+ \alpha_1 f_d^T f_d + \alpha_1 \tilde{f}^T (t-T) K_1^T K_1 \tilde{f} (t-T)
\]

\[
+ 2 \alpha_1 f_d^T K_1 \tilde{f} (t-T) - \tilde{f}^T (t-T) Q \tilde{f} (t-T)
\]

\[
\bar{x}^T (2P_1 (A - BK_a)) x + \epsilon_1 P_1 E (P_1 E)^T x
\]

\[
+ 2x^T P_1 BK_a \bar{x} + \epsilon_1 \tilde{f}^T K_1^T K_2 \tilde{y}_h - 2 \alpha_1 f_d^T K_2 \tilde{y}_h
\]

\[
- 2 \alpha_1 \tilde{f}^T (t-T) K_1^T K_2 \tilde{y}_h - 2 \bar{x}^T (PE - \alpha_1 (K_2 C_a)^T) K_2 \tilde{y}_h,
\]

\[
(22)
\]

where \( \alpha_1 = ((1 + \epsilon_0) \lambda_{\text{max}} (Q) + 1/\epsilon_1). \) By Lemma 3, we can always choose \( K_1 = (1/\alpha_1)G \) such that

\[
\alpha_1 K_2 C_a = E^T P.
\]

Then, equation (22) can be rewritten as

\[
\dot{V} \leq \bar{x}^T (2P (A - LC_a)) \bar{x} - \epsilon_0 \tilde{f}^T Q \tilde{f} + 2 \alpha_1 f_d^T K_1 \tilde{f} (t-T)
\]

\[
+ \alpha_1 \tilde{f}^T (t-T) K_1^T K_1 \tilde{f} (t-T)
\]

\[
+ 2 \bar{x}^T P_1 BK_a \bar{x} + x^T (2P_1 (A - BK_a)) x + \epsilon_1 P_1 E (P_1 E)^T x
\]

\[
+ 2x^T P_1 BK_a \bar{x} - 2 \alpha_1 \tilde{f}^T (t-T) K_1^T K_2 \tilde{y}_h - \bar{x}^T (PE - \alpha_1 (K_2 C_a)^T) K_2 \tilde{y}_h,
\]

\[
(24)
\]
By using Lemma 2 and Lemma 4, we get
\[
-2\tilde{x}^T P L \tilde{y}_h + \alpha_1 \tilde{y}_h^T K_1^T K_2 \tilde{y}_h - 2\alpha_1 f^T \tilde{y}_h - 2\alpha_1 \tilde{f}^T (t - T) K_1^T K_2 \tilde{y}_h
\]
\[
= -2\tilde{x}^T P L \sqrt{\beta} \frac{1}{\sqrt{\beta}} \tilde{y}_h + \alpha_1 \tilde{y}_h^T K_1^T K_2 \tilde{y}_h - 2\alpha_1 f^T \tilde{y}_h - 2\alpha_1 \tilde{f}^T (t - T) K_1^T K_2 \sqrt{\beta} \frac{1}{\sqrt{\beta}} \tilde{y}_h
\]
\[
\leq \varepsilon_2 \pi x^T P L (PL)^T \tilde{x} + \frac{1}{\varepsilon_2} \mu^2 + \alpha_1 \lambda_{\text{max}}(K_2^T K_2) \beta^2 \mu^2 + 2\alpha_1 \beta \pi \|K_2\| \|f_d\| \\
+ \varepsilon_3 \alpha_1^2 \mu^2 \tilde{f}^T (t - T) K_1^T K_2 K_1 \tilde{f} (t - T) + \frac{1}{\varepsilon_3} \mu^2,
\]
\[
2\alpha_1 f^T \tilde{y}_h - 2\alpha_1 \tilde{f}^T (t - T) \leq \varepsilon_4 \alpha_1^2 \tilde{f}^T (t - T) K_1^T K_1 \tilde{f} (t - T) + \frac{1}{\varepsilon_4} \|f_d\|^2.
\]

Therefore, the derivative of the Lyapunov–Krasovskii function (24) can be further transformed into the compact form as follows:

\[
\dot{V} \leq \begin{bmatrix}
\bar{x}(t) \\
\bar{f}
\end{bmatrix}^T \begin{bmatrix}
\Pi_1 & \Pi_2 & 0 \\
\Pi_2^T & \Pi_2^T \Pi_1 \Pi_2 & \Pi_2^T \Pi_2 \\
0 & \Pi_2^T \Pi_2 & -Q_h
\end{bmatrix} \begin{bmatrix}
\bar{x}(t) \\
\bar{f}
\end{bmatrix}
\]
\[
= \begin{bmatrix}
\bar{x}(t) \\
\bar{f}
\end{bmatrix}^T \begin{bmatrix}
\Pi_1 & \Pi_2 & 0 \\
\Pi_2^T & \Pi_2^T \Pi_1 \Pi_2 & \Pi_2^T \Pi_2 \\
0 & \Pi_2^T \Pi_2 & -Q_h
\end{bmatrix} \begin{bmatrix}
\bar{x}(t) \\
\bar{f}
\end{bmatrix}
\]
\[
+ \left( \alpha_1 + \frac{1}{\varepsilon_4} \right) \|f_d\|^2 + \left( \frac{1}{\varepsilon_2} + \frac{1}{\varepsilon_3} + \alpha_1 \lambda_{\text{max}}(K_2^T K_2) \beta^2 \mu^2 + 2\alpha_1 \beta \pi \|K_2\| \|f_d\| \right) \frac{r}{\kappa_2}.
\]
If the proper parameters have been chosen such that
\[
\Gamma = -Q + (1 + \epsilon_4 \alpha_1) \alpha_1 K_1^T K_1 + \epsilon_3 \alpha_1^2 \beta_1 \kappa K_2^T K_2 + \epsilon_2 \alpha_1 \lambda_{\max}(K_2^T K_2) < 0,
\]
(26)

then (26) can be transformed into
\[
\dot{V} \leq -\kappa_1 \|X\|^2 + \kappa_2,
\]
where
\[
X = \begin{bmatrix} x \times \ddot{x} \end{bmatrix}^T, \quad \kappa_1 = \lambda_{\min}(Q_0), \quad \text{and} \quad \kappa_2 = (\alpha_1 + 1/\epsilon_3) \bar{f}^2 + ((1/\epsilon_3) + (1/\epsilon_2) + \alpha_1 \lambda_{\max}(K_2^T K_2) \beta_2 \bar{f}^2 + 2 \alpha_1 \beta_1 \epsilon_2 \epsilon_2 + 2 \alpha_1 \lambda_{\max}(K_2^T K_2) \beta_2 \bar{f}).
\]

It follows that \( \|X\| > \kappa_2/\kappa_1 \) renders \( \dot{V} < 0 \). It is obviously observed that \( V \) is uniformly ultimately bounded. According to the definition of \( V \), the signals of \( X \) are bounded. Theorem 1 is thus proved.

Next, we will analyze how to select parameters to satisfy the aforementioned formula. For simplicity of analysis, rewrite \( -Q_0 \) as:

\[
\begin{bmatrix}
\Pi_1 & P_1 BK_x \\
* & \Pi_2
\end{bmatrix}
\begin{bmatrix}
0 \\
0
\end{bmatrix} < 0,
\]
(27)

then (26) can be transformed into
\[
\dot{V} \leq -\kappa_1 \|X\|^2 + \kappa_2,
\]
where
\[
\Pi_1 = P_1 (A - BK_x) + (A - BK_x)^T P_1 + \epsilon_1 P_1 E(P_1 E)^T,
\]
\[
\Pi_2 = P (A - LC_0) + (A - LC_0)^T P + \epsilon_2 B P L (P L)^T,
\]
and
\[
-1 - \frac{1}{\alpha_1} PE(PE)^T.
\]
(29)

Furthermore, by using Lemma 1, (28) is equivalent to
\[
\begin{bmatrix}
\Pi_1 & P_1 BK_x \\
* & \Pi_2
\end{bmatrix}
\begin{bmatrix}
P_1 & \Pi_2
\end{bmatrix} < 0.
\]
(30)

Postmultiply and premultiply (30) by \( P_1^{-1} \), it is obtained that
\[
\begin{bmatrix}
P_1^{-1} \Pi_1 P_1^{-1} & BK_x P_1^{-1} \\
P_1^{-1} \Pi_2 P_1^{-1}
\end{bmatrix} < 0.
\]
(31)

Since
\[
PE(PE)^T = P \begin{bmatrix} 0 & 0 \\ M^{-1} & 0 \end{bmatrix} \begin{bmatrix} 0 \\ (M^{-1})^T \end{bmatrix} P T \geq 0, \tag{32}
\]

one has
\[
P_1^{-1} \Pi_2 P_1^{-1} \leq P_1^{-1} \Omega_2 P_1^{-1}, \tag{33}
\]

where
\[
\Omega_2 = \Pi_2 + \frac{1}{\alpha_1} PE(PE)^T
\]

\[= P(A - L C_a) + (A - L C_a)^T P + \varepsilon_2 R P L(P L)^T. \tag{34}\]

Thus, sufficient conditions for (31) are obtained as follows:

From Lemma 3, we conclude that \( L \) can always be chosen such that

\[
P(A - L C_a) + (A - L C_a)^T P = -Q_1 < 0. \tag{36}\]

Then, we choose \( \nu \leq \lambda_{\text{min}}(-Q_1)/(|v_2 \lambda_{\text{max}}(P L(P L)^T)|) \) such that \( \Omega_2 < 0 \). Since \( P_1^{-1}, \Omega_2 \) are symmetric matrices, so we obtain

\[
\begin{bmatrix} P_1^{-1} \Pi_1 P_1^{-1} & BK_\tau P_1^{-1} \\ P_1^{-1} \Omega_2 P_1^{-1} & - \end{bmatrix} < 0. \tag{35}
\]

Thus, we obtain the following sufficient condition for (35):

\[
\begin{bmatrix} P_1^{-1} \Pi_1 P_1^{-1} & BK_\tau P_1^{-1} \\ P_1^{-1} \Omega_2 P_1^{-1} & - \end{bmatrix} < 0. \tag{35}
\]
By using Lemma 1 repeatedly, (38) can be converted into equivalent condition as follows:

$$
\begin{bmatrix}
\Phi_1^{-1}\Pi_1\Phi_1^{-1} & BK_\tau\Phi_1^{-1} \\
* & -2\gamma\Phi_1^{-1} - \gamma^2\Omega_2^{-1}
\end{bmatrix} < 0.
$$

(38)

Let $P_1 = N_1$, $K_\tau N_1 = N_2$, and $PL = N_3$; then, we can obtain the LMIs in Theorem 1. Besides, using the conclusions in [21], the equation $\alpha_1 K_2 C_n = E^T P$ can be converted into LMI equivalently as follows:

$$
\begin{bmatrix}
-\theta I & (E^T P - \alpha_1 K_2 C_n) \\
* & -I
\end{bmatrix} \leq 0,
$$

(40)

where $\theta$ is a small positive scalar.

4. Simulation Results

In this section, a simulation based on the MATLAB-Simulink platform is presented to confirm the effectiveness of the proposed fault-tolerant control scheme. We consider a DP ship equipped with three azimuth thrusters $T_1, T_2$ and...
and two transverse tunnel thrusters $T_4$ and $T_5$. A schematic diagram of the actuator distribution is shown in Figure 2.

For the DP ships, the actual actuator force is related to the control input through the equation $\tau = Gu$ with

$$G = \begin{bmatrix}
\cdots & \cos(\alpha_i) & \cdots \\
\cdots & \sin(\alpha_i) & \cdots \\
\cdots & l_{xi} \sin(\alpha_i) - l_{yi} \cos(\alpha_i) & \cdots \\
\end{bmatrix},$$

(41)

where $\alpha_i, i = 1, \ldots, m$ denotes the angle between the force of the $i$-th actuator and surge direction, $(l_{xi}, l_{yi})$ is the location of the $i$-th actuator in the body-fixed frame, and $u \in \mathbb{R}^m$ is the actual actuator commands. However, the orientation angles of the thrusters $T_1, T_2$ and $T_3$ are time-varying, which increases the complexity of the control design. To reduce the complexity of the simulation, the thruster commands are decomposed into the surge force and sway force [8, 11]. Then, the configuration matrix $G$ can be transformed into the linear form. According to [8], the matrix $G$ is chosen as

$$G = \begin{bmatrix}
1 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 1 & 1 & 1 \\
-5.91 & -19.1 & 5.91 & -19.1 & 0 & 18.5 & 30 & 35
\end{bmatrix},$$

(42)

and the numerical values for the involved matrices are

$$M = 10^7 \begin{bmatrix}
0.027521 & 0 & 0 \\
0 & 0.076348 & -0.073803 \\
0 & -0.073803 & 6.690963
\end{bmatrix},$$

(43)

$$D = 10^6 \begin{bmatrix}
0.000025 & 0 & 0 \\
0 & 0.009865 & 0.001375 \\
0 & 0.000711 & 2.813355
\end{bmatrix}.$$  

The expected position and heading vectors are taken as $\eta_d = [0 \text{ m} \ 0 \text{ m} \ 0 \text{ rad}]^T$, and the initial states are taken as $\eta(t_0) = [10 \text{ m} \ 10 \text{ m} \ 1 \text{ rad}]^T$. The parameters of the fault-tolerant control system are selected as $\bar{\mu} = 10^{-5}$, $l_i = [50 \ 500]^T$, $\forall i \in \{1, 2, 3\}$, $\epsilon_0 = 0.1$, $\epsilon_1 = 100$, $\epsilon_2 = 2$, $\epsilon_3 = 10$.
\[ \varepsilon_3 = 10^{-6}, \quad \varepsilon_4 = 0.1, \quad \text{and} \quad \gamma = 1. \]

Furthermore, the gain matrices can be calculated by MATLAB LMI toolbox as follows:

\[ \theta = 4.4860 \times 10^{-8}, \]

\[ K_1 = \begin{bmatrix} 0.9 & 0 & 0 \\ 0 & 0.9 & 0 \\ 0 & 0 & 0.9 \end{bmatrix}, \]

\[ K_2 = \begin{bmatrix} \begin{pmatrix} -0.0004 & 0 & 0 \\ 0 & -0.002 & -0.000 \\ 0 & -0.000 & -0.000 \end{pmatrix} \\ \begin{pmatrix} 0.0033 \\ 0 \\ 0 \end{pmatrix} \end{bmatrix}, \]

\[ K_r = 10^8 \begin{bmatrix} 0 & 0.0091 & -0.0088 & 0 & 0.0153 & -0.0148 \\ 0 & -0.0070 & 0.8087 & 0 & -0.0124 & 1.3454 \end{bmatrix}, \]

\[ L = \begin{bmatrix} 0.5000 & 0 & 0 & 0.5000 & 0 & 0 \\ 0 & 0.5000 & -0.0000 & 0 & 0.5000 & -0.0000 \\ 0 & -0.000 & 0.5000 & 0 & -0.000 & 0.5000 \\ 0.5000 & 0 & 0 & 0.4999 & 0 & 0 \\ 0 & 0.5000 & 0 & 0 & 0.4870 & -0.0215 \\ 0 & -0.000 & 0.5000 & 0 & -0.0215 & 0.4575 \end{bmatrix}. \]

\[ (44) \]

Two fault scenarios of all thrusters are summarized in Figures 3 and 4, in which Case 1 and Case 2 represent the single faults and multiple faults, respectively. In addition, the UIO-based control allocation (CA) method in [8] is taken to show the advantages of the proposed scheme.

Simulation results are depicted in Figures 5–8. It is shown from Figure 5 that the proposed scheme and the UIO-based CA method both can force the ship to the desired target position and heading in Case 1. Due to the time delay effect caused by FDI, the CA method will deviate slightly when the actuator faults occur, but it can still ensure the stability of the system in Case 1. Furthermore, since the fault signal is estimated and compensated by feedforward in the proposed scheme, the deviations from the equilibrium point are also much smaller than those in the CA method. Also, it can be observed from Figure 7 that the velocity estimation errors can converge to a sufficiently small neighborhood of zero. It is worth noting that the proposed approach can be applied to systems with multiple faults (i.e., Case 2). To verify this point, simulation results on the case of multiple actuator faults are given in Figure 6, demonstrating that the proposed scheme is also effective for the cases of multiple faults.
actuator faults. As a comparative method, the CA method is not able to control the system well in Case 2, and we can see that the states of the system have a large deviation in around 110 seconds and 350 seconds whenever multiple faults occur simultaneously.

In order to verify the effectiveness of the proposed fault reconstruction method in this paper, we will compare it with the adaptive fault observer (AFO) in reference [22]. First, we consider constant fault situations for actuators in dynamic positioning ships, which are created as

\[
 f(t) = \begin{cases} 
 [0, 0, 0]^T, & 0 \leq t \leq 100, \\
 [10^5, 10^5, 4 \times 10^3]^T, & \text{otherwise}. 
\end{cases} 
\]

(45)

Simulation results for the constant actuator faults are depicted in Figure 9. Furthermore, it is assumed that a time-varying fault occurs in the actuator, i.e.,

\[
 f(t) = \begin{cases} 
 [0, 0, 0]^T, & 0 \leq t \leq 100, \\
 10^5 \times \sin(0.02t)\cos^2(0.02t), & 105 \leq t \leq 100, \\
 10^5 \times \cos(0.02t)\sin^2(0.02t), & \text{otherwise}. 
\end{cases} 
\]

(46)

Simulation results for the time-varying faults are depicted in Figure 10.
It can be observed from the aforementioned simulation results that for a constant fault in Figure 9, the convergence of the fault-estimation error can be achieved using the two methods, but the proposed scheme can improve the rapidity of fault reconstruction evidently. As for the time-varying fault in Figure 10, the estimation accuracy of the proposed scheme is significantly better than that of the AFO method. This is because the estimation performance of AFO depends on the first time derivative of the fault signal. By the way, it is worth noting that from Figure 9, although the boundedness of errors can be only guaranteed of the proposed scheme, the upper bound can be made small enough by choosing appropriate parameters.

In practical application, the marine environment is complex and the disturbance forces caused by wind, wave, and current cannot be ignored. A frequently used disturbance model for marine control applications is the first-order Markov process

\[ \dot{d} = -A_d^{-1}d + B_d \omega_d, \]  

(47)

where \( d \in \mathbb{R}^3 \) is the vector representing the changing disturbance forces and moment, \( \omega_d \in \mathbb{R}^3 \) is the vector of the
Gaussian white noise with zero mean value, and $A_d$ and $B_d$ are the diagonal matrix of positive time constants and the diagonal matrix scaling the amplitude of Gaussian white noise. For the simulation purposes, the parameters are selected as $A_d = \text{diag}(3000, 3000, 3000)$ and $B_d = \text{diag}(1000, 1000, 1000)$. Figure 11 displays the total fault estimation when the aforementioned disturbance (47) is taken into account and shows that the proposed method, compared with the AFO method, provides much better performances. Therefore, it is concluded that the proposed scheme is effective in fault reconstruction and fault-tolerant control for the dynamic positioning of ships.

**5. Conclusion**

The paper presented a fault-tolerant control method for dynamic positioning of ships with actuator faults based on iterative learning observers and high-gain observers. The proposed fault-tolerant control scheme does not need the fault detection and isolation mechanism which may cause time delay problem. It has been proved that, by constructing the auxiliary derivative outputs, the UIO existence condition is guaranteed for dynamic positioning ships. Simulation results demonstrate the effectiveness of the proposed scheme in fault-tolerant control and fault reconstruction. It is worth
noting that the proposed approach can be applied to systems with multiple faults. In the future, we will consider how to isolate faults from disturbance signals. Furthermore, we will study fault-tolerant control methods for dynamic positioning ships with nonlinear dynamics and other constraints.

Data Availability

All the data supporting the conclusions of the study have been provided in Simulations and readers can access these data in [23].

Conflicts of Interest

The authors declare that there are no conflicts of interest related to this paper.

Acknowledgments

This research was partially supported by the National Science Technology Support Program of China (Project no. 51609046).

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