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H_{∞} Control for Discrete-Time Networked T-S Fuzzy Systems with Packet Loss Based on Event-Triggered Mechanism

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The H_{∞} state feedback control problem for a class of nonlinear networked control systems with data packet loss is studied using an event-triggered scheme. The data packet loss is described as an independent and homogeneous Bernoulli process. Under an event-triggered scheme, the nonlinear networked control system with packet loss is modeled as a Takagi-Sugeno (T-S) fuzzy system, based on which sufficient conditions on the existence of event-triggered state feedback controllers are derived such that the closed-loop system is mean-square stable with a desired H_{∞} performance index. The simulation results show that the presented event-triggered scheme can not only ensure the closed-loop performance but also effectively reduce the data transmission rate.

1. Introduction

Networked control systems (NCSs) are control systems in which the control loop is closed via a wired or wireless communication network [1–3]. However, since the bandwidth of the communication network is limited, only a finite number of executions can be performed over the communication network, so that the actuator cannot communicate with the controller timely during the transmission of the control signal due to uncertain external disturbances and packet loss phenomenon [4–7]. Therefore, studying NCSs with packet loss and resource constraints is of much significance both in theory and in practice.

As is known, a Takagi-Sugeno (T-S) fuzzy model can generate more complex nonlinear functions with a small number of fuzzy rules. Therefore, it is a very useful tool for dealing with nonlinear systems. In [8], the problem of an event-triggered nonparallel distribution compensation (PDC) control is considered for the networked T-S fuzzy systems with the limited data transmission bandwidth and the imperfect premise matching membership functions. In [9–11], the H_{∞} control problem for a class of T-S fuzzy Markov jump systems with time-varying delay under unreliable communication relatives has been studied. In [12], it is

assumed that the data transmissions between the plant and the controller are subject matter to random packet loss which satisfies Bernoulli distribution. In [13, 14], the controller is designed by the PDC technique in which the controller shares the same membership functions and fuzzy premise variables with the T-S fuzzy model. An event-triggered fuzzy controller design method for a class of discrete-time nonlinear NCSs with time-varying communication delays has been studied in [15].

Up to now, event-triggered control has gained increasing interest in the last several years, which can effectively shrink feedback data flow while retaining a certain level of control performance compared with a time-triggered scheme. In [16], a class of continuous linear NCSs are considered, where some event-triggered schemes have been devised based on the system states. Concerning distributed energy management and control issues of both generators and loads, papers [17, 18] aim to maximize the total social welfare that balances generation-side expanses, user-side payments, and transmission line costs and provide an overview of recent advances on event-triggered consensus of MASs. A pulse system method has been proposed in [19], which simulates and analyzes the scattered control system with an eventtriggered scheme. In [20–22], a time delay system method



FIGURE 1: Networked control system with packet loss and transmission data generator.

has been proposed, which can be used to analyze the stability of a continuous NCS under an event-triggered transmission scheme. In [23], a codesign method of corresponding event-triggered transmission with quantizer and controller has been proposed. The state observation has been used to describe the system model. Then the NCSs have been modeled as a discrete-time switching system with uncertain parameters.

It is well known that packet loss often arises in the network transmission. To solve this problem, in [24], the data measurements processes from the plant to the filter are subject to random packet loss which satisfies Bernoulli distribution with bounded nonlinearity. In [25, 26], the network-based output tracking control is investigated for a T-S fuzzy system and it is concluded that the system cannot be stabilized by a nondelayed fuzzy static output feedback controller but can be stabilized by a delayed fuzzy static output feedback controller. In [27], jump neural NCSs with transmission delay and packet loss have been considered. However, there is no further consideration for the nonlinear problem. The resource constraints and packet loss problems in [28] also have similar discussions, using the channel constraints to reduce resource consumption, but there is only one actuator connected to the communication network at a time. In [29], the packet loss and channel constraints processes have been modeled as $u(k) = \Theta(k)M_o(k)\hat{u}(k)$, from which it can be seen that the study is for single-packet transmission strategy, and the input strategy is applicable to different NCSs. Because the above-mentioned packet loss model is a probability event and sometimes does not match the real situation. In order to address two issues mentioned above, in [30, 31], an event-triggered mechanism is proposed, which can effectively reduce feedback data flow and reduce the broadband occupancy rate such that network resources can be saved. In [32], packet loss in both sensor-controller (S/C) and controller-actuator (C/A) channels is considered. The Markovian chain principle is used in modeling the packet loss in S/C and C/A channels. The time scale adopted in these two independent homogeneous Markov chains is linear with the physical time. However, most of the above studies are based on linear systems. In fact, practical systems are usually nonlinear, while few results have focused on nonlinear NCSs by taking event-triggered mechanisms and data packet loss into account, which motivates the study of this paper.

In this paper, in order to deal with the resource constraints and packet loss, an event-triggered mechanism strategy based on a relative error is proposed. Then the H_{∞} state feedback control problem is investigated for a class of nonlinear NCSs using an event-triggered mechanism. First, the packet loss under multichannel strategy is described as an independent and homogeneous Bernoulli process. Then a new model of nonlinear NCSs is established by employing T-S fuzzy model method. Based on this model and by employing the Lyapunov stability theory and linear matrix inequality technique, a sufficient condition for the existence of controller is presented to ensure the meansquare stability of NCSs and an improved H_{∞} performance. Moreover, the corresponding controller can be designed using the PDC technique. Finally, a simulation example is given to illustrate the effectiveness of the proposed method.

The remaining of this paper is organized as follows. Section 1 introduces the background and significance of the research. Section 2 formulates the problem consideration. H_{∞} performance analysis and state feedback controller design are presented in Section 3. An illustrative example is given in Section 4 to demonstrate the effectiveness of the presented method. The conclusion is drawn in Section 5.

2. Problem Description and Modeling

In this paper, the NCS with packet loss and transmission data generator is shown in Figure 1. The sensors in the communication network are time-triggered with a constant sampling period h, while the controller and actuators are event-triggered. The TD generator in Figure 2 is event-triggered, which is used to generate sampled data and transmission data. The zero-order holder (ZOH) is used to hold



FIGURE 2: The transmission data generator (TDG) with eventtriggered mechanism (ETM).

the sampling signal until the new sampling signal reaches the controller. Consider the nonlinear plant model as follows:

$$\dot{x}(t) = f_1(x(t)) + g_1(x(t))u(t) + g_2(x(t))w(t)$$

$$\dot{z}(t) = f_2(x(t)) + g_3(x(t))u(t)$$
(1)

where $x(t) \in \mathbb{R}^n$ is the state vector of plant; $u(t) \in \mathbb{R}^m$ is the input vector and w(t) is the external interference signal satisfying $w(k) \in l_2[0,\infty)$. $f_1(x)$, $f_2(x)$, $g_1(x)$, $g_2(x)$, and $g_3(x)$ are continuous functions of x, and $f_i(0) = 0$, $g_j(0) =$ 0 (i = 1, 2; j = 1, 2, 3). Since the transmission process belongs to the discrete process, the nonlinear system on the compact set Ω is considered to the following discrete-time T-S model with plant rules:

Plant rule *i*: IF $\zeta_1(k)$ is $\Gamma_{i1} \cdots$ and $\zeta_s(k)$ is Γ_{is} , THEN

$$x (k + 1) = A_{i} x (k) + B_{i} u (k) + B_{wi} w (k)$$

$$z (k) = C_{i} (x (k)) + D_{i} u (k)$$
(2)

where $i \in S_r$ (r is the number of IF-THEN rules); $x(k) \in R^n$, $u(k) \in R^m$, and $z(k) \in R^m$ are the state vector, input vector, and regulated output, respectively. A_i, B_i, B_{wi}, C_i , and $D_i(i \in S_r)$ are system matrices with appropriate dimensions. $\Gamma_{ij}(i \in S_r, j \in S_s)$ are fuzzy sets, via the membership function $\Gamma_{ij}(\zeta_j(k))$ to Ω_{ζ_j} , and $\zeta_k = [\zeta_1(k), \zeta_2(k), \dots, \zeta_s(k)]^T$ are the premise variables, which are defined on $\Gamma_{\zeta} = \Gamma_{\zeta_1} \times \Gamma_{\zeta_2} \times \dots \times \Gamma_{\zeta_s}$, and S_r indicates the number of rules on the S plane, and S_s indicates the number of states on the S plane. Then the T-S fuzzy system can be described as

$$x (k + 1)$$

$$= \sum_{i=1}^{r} \mu_{i} (\zeta (k)) \{A_{i} (x (k)) + B_{i} u (k) + B_{wi} w (k)\}$$
(3)

$$z(k) = \sum_{i=1}^{k} \mu_i(\zeta(k)) \{C_i(x(k)) + D_i(x(k))u(k)\}$$

where $\mu_i(\zeta(k)) = \Gamma_i(\zeta(k)) / \sum_{i=1}^r \Gamma_i(\zeta(k)) \ge 0$; $\Gamma_i(\zeta(k)) = \prod_{j=1}^s \Gamma_{ij}(\zeta_j(k))$; $\Gamma_{ij}(\zeta_j(k))$ $(i \in S_r, j \in S_S)$ is the grade of membership of $\zeta_j(k)$ in Γ_{ij} , and $\sum_{i=1}^r \mu_i(\zeta(k)) = 1$.

For each transmission of *n* measurement data and control signal, due to the uncertainty of the communication network,

a packet loss usually occurs; these n measurement data and control signal packets may not be able to reach the actuator. Suppose that the two-valued function $v_i(k) : R \longrightarrow \{0, 1\}$ and $\zeta_i(k) : R \longrightarrow \{0, 1\}$ express the packet loss of the *i*th control signal. Then the corresponding control signal will be transferred to the actuator when the *i*th actuator accesses the channel. $v_i(k) = 0$ and $\zeta_i(k) = 0$ express that the *i*th control signal is lost during transmission; $v_i(k) = 0$ and $\zeta_i(k) = 1$ express that the *i*th control signal is successfully transmitted. Notice that data packet loss process often occurs randomly. Thus, in this paper, it is described as Bernoulli process. Assume that the channel packet loss processes are independent of each other, and the measurement data packet loss rate is v; that is, $prob\{v_i(k) = 1\} = E\{v_i(k) = 1\} = v;$ $prob\{v_i(k) = 0\} = 1 - E\{v_i(k) = 1\} = 1 - v$. Similarly, the loss of the control signal has the same process; the packet loss rate is ζ ; that is, $prob\{\zeta_i(k) = 1\} = 1 - E\{\zeta_i(k) = 1\} = 1 - \zeta$; $prob\{\zeta_i(k) = 0\} = E\{\zeta_i(k) = 1\} = \zeta.$

In this paper, the design of the event-triggered transmission is based on a relative error threshold:

$$\widehat{x}_{i}(k)$$

$$=\begin{cases} x_{i}(k), & |\hat{x}_{i}(k-1) - \hat{x}_{i}(k)| > \sigma_{i} |x_{i}(k)| & (4) \\ \hat{x}_{i}(k-1), & |\hat{x}_{i}(k-1) - \hat{x}_{i}(k)| \le \sigma_{i} |x_{i}(k)| \end{cases}$$

where $\sigma_i \in [0, 1]$ $(i = 1, 2, \dots, n)$ are error threshold parameters and $\hat{x}_i(k-1)$, $\hat{x}_i(k)$, and $x_i(k)$ are the last transmission signal, output signal, and input signal, respectively.

Set $\Phi = \text{diag}\{\sigma_1, \sigma_2, \dots, \sigma_n\}$ and $F(k) = \text{diag}\{f_1(k), f_2(k), \dots, f_n(k)\}, f_i(k) \in [-1, 1]$. Then, from the event-triggered transmission scheme (4), the input-output relationship of ETM can be transformed to

$$\widehat{x}(k) = x(k) + \Phi F(k) x(k)$$
(5)

where $F^T(k)F(k) \leq I$.

From the above analysis, we can see that, in the NCS with an event-triggered mechanism, since each sampling period signal does not need to be transmitted, the purpose of reducing the data transmission rate can be achieved. Suppose that there are *n* channels in the event trigger to send the data. We set $\tilde{x}(k) = [\tilde{x}_1^T(k), \dots, \tilde{x}_n^T(k)]^T$; $N_v(k) = \text{diag}\{v_1(k), \dots, v_n(k)\}$

Considering packet loss, the final measurement output is given as

$$\overline{x}(k) = (I - N_v(k)) \widetilde{x}(k) + N_v(k) \widetilde{x}(k-1)$$
(6)

Similarly, setting $u(k) = [u_1^T(k), \dots, u_m^T(k)]^T$ and $N_{\zeta}(k) = \text{diag}\{\zeta_1(k), \dots, \zeta_m(k)\}$, the transmission of the control signal can be described by

$$\overline{u}(k) = \left(I - N_{\zeta}(k)\right)u(k) + N_{\zeta}(k)u(k-1)$$
(7)

With formulas (5), (6), and (7) we can get the following closed-loop model of NNCSs:

$$\beta (k+1) = \overline{\overline{A}} \beta (k) + \overline{\overline{B}} u (k) + \overline{\overline{B}}_{w} \omega (k)$$

$$z (k) = \overline{\overline{C}} \beta (k) + \overline{\overline{D}} u (k)$$
(8)

where

$$\begin{split} \overline{\overline{A}} &= \begin{bmatrix} \overline{A} & N_{\zeta}(k) \overline{B} & 0 \\ 0 & N_{\zeta}(k) & 0 \\ (I - N_{v}(k)) (I + \Phi F(k)) & 0 & N_{v}(k) \end{bmatrix}, \\ \overline{\overline{B}} &= \begin{bmatrix} (I - N_{\zeta}(k)) \overline{B} \\ (I - N_{\zeta}(k)) \\ 0 \end{bmatrix}, \\ \overline{\overline{B}}_{w} &= \begin{bmatrix} \overline{B}_{w} \\ 0 \\ 0 \\ 0 \end{bmatrix}, \\ \overline{\overline{C}} &= \begin{bmatrix} \overline{C} & N_{\zeta}(k) \overline{B} & 0 \\ 0 & N_{\zeta}(k) & 0 \\ (I - N_{v}(k)) (I + \Phi F(k)) & 0 & N_{v}(k) \end{bmatrix} \\ \overline{\overline{D}} &= \begin{bmatrix} (I - N_{\zeta}(k)) \overline{D} \\ (I - N_{\zeta}(k)) \\ 0 \end{bmatrix}, \\ \beta(k) &= \begin{bmatrix} x^{T}(k) & u^{T}(k - 1) & \widetilde{x}^{T}(k - 1) \end{bmatrix}^{T}, \\ \overline{A} &= \sum_{i=1}^{r} \mu_{i}(\zeta(k)) A_{i}, \\ \overline{B} &= \sum_{i=1}^{r} \mu_{i}(\zeta(k)) B_{i}, \end{split}$$

$$\overline{B}_{w} = \sum_{i=1}^{r} \mu_{i} \left(\zeta \left(k \right) \right) B_{wi}$$

$$\overline{C} = \sum_{i=1}^{r} \mu_{i} \left(\zeta \left(k \right) \right) C_{i},$$

$$\overline{D} = \sum_{i=1}^{r} \mu_{i} \left(\zeta \left(k \right) \right) D_{i}.$$
(9)

In this section, a T-S fuzzy-model controller will be designed via PDC technique to stabilize the T-S fuzzy system (1) with *i*th controller rule:

Controller rule *i*: IF $\zeta_1(k)$ is Γ_{i1} ... and $\zeta_s(k)$ is Γ_{is} , THEN

$$u\left(k\right) = K_i \hat{x}\left(k\right) \tag{10}$$

where $i \in s_r$ (*r* is the number of IF-THEN rules); u(k) and $\hat{x}(k)$ are the controller output and input data, and $K_i \in \mathbb{R}^{m \times n}$ is the controller gain to be designed. Thus, the T-S fuzzy controller reads as

$$u(k) = \sum_{i=1}^{r} \mu_i(\zeta(k)) K_i \overline{x}(k)$$
(11)

From (5), (6), (7), and (8), the closed-loop system can be obtained as follows:

$$\beta (k+1) = \widetilde{A}\beta (k) + \widetilde{B}_{w}\omega (k)$$

$$z (k) = \widetilde{C}\beta (k)$$
(12)

where

$$\widetilde{A} = \begin{bmatrix} \overline{A} + (I - N_{\zeta}(k)) \overline{B} \overline{K} N_{\nu}(k) (I + \Phi F(k)) & N_{\zeta}(k) \overline{B} & 0 \\ I - N_{\zeta}(k) & N_{\zeta}(k) & 0 \\ (I - N_{\nu}(k)) (I + \Phi F(k)) & 0 & N_{\nu}(k) \end{bmatrix},$$

$$\widetilde{C} = \begin{bmatrix} \overline{C} + (I - N_{\zeta}(k)) \overline{D} \overline{K} N_{\nu}(k) (I + \Phi F(k)) & N_{\zeta}(k) \overline{D} & 0 \\ I - N_{\zeta}(k) & N_{\zeta}(k) & 0 \\ (I - N_{\nu}(k)) (I + \Phi F(k)) & 0 & N_{\nu}(k) \end{bmatrix}, \quad \overline{K} = \sum_{i=1}^{r} \mu_{i} (\zeta(k)) K_{i}.$$

$$(13)$$

Definition 1. If there exist scalars $0 < a \le 1, k_1 \ge 0$, and $k_2 \ge 0$ such that

$$E\left\{\left\|x\left(k\right)\right\|^{2}\right\} \le k_{1} + k_{2}\left(1 - a\right)^{k},$$
(14)

then the stochastic state of the discrete-time stochastic system is mean-square stable.

The goal of our research is to design the gain matrix of the state feedback controller (10) based on the event-triggered strategy (4) within a tolerable threshold such that system (12) is mean-square stable and satisfies a given H_{∞} performance parameter γ .

(1) In the case of external disturbances w(k) = 0, the nonlinear system (12) is mean-square stable.

(2) Under zero initial conditions, for any nonzero $w(k) \in l_2$, the closed-loop system (12) satisfies the following H_{∞} performance index, where $\gamma > 0$ is a given constant.

$$E\left\{\sum_{k=0}^{\infty} z^{T}(k) z(k)\right\} < \gamma^{2} E\left\{\sum_{k=0}^{\infty} w^{T}(k) w(k)\right\},$$
(15)

$$\forall w(k) \neq 0$$

3. Stability Analysis and Controller Design

In the beginning, we introduce two lemmas.

Lemma 2 (see [33]). *If there exist scalars* $\mu > 0$, $\nu > 0$, $\varepsilon > 0$, *and* $0 < \varphi < 1$ *such that*

$$\mu \|x(k)\|^{2} \leq V(k)$$

$$\leq v \|x(k)\|^{2} E\{V(k+1) | V(k)\}$$
(16)
$$- E\{V(k)\} \leq \varepsilon - \varphi V(k)$$

then sequence x(k) satisfies

$$E\left\{\|x(k)\|^{2}\right\} \leq \frac{\nu}{\mu} E\left\{\|x(0)\|^{2}\right\} \left(1-\phi\right)^{k} + \frac{\varepsilon}{\mu\phi}$$
(17)

Lemma 3 (see [34]). Let Y, M, N, and F(k) be real matrices of appropriate dimensions with $F^{T}(k)F(k) \leq I$ and $Y = Y^{T}$; then

$$Y + MF(k)N + N^{T}F(k)^{T}M^{T} < 0$$
(18)

holds, if and only if there exists a real scalar $\varepsilon > 0$, satisfying

$$Y + \varepsilon M M^T + \varepsilon^{-1} N^T N < 0.$$
⁽¹⁹⁾

Regarding the random sequence $\{v_i(k), \varsigma_i(k)\}$, we suppose that

$$E \left\{ v_i(k) \varsigma_i(k) - v\varsigma \right\} = 0,$$

$$E \left\{ v_i(k) \varsigma_i(k) - v\varsigma \right\}^2 = v\varsigma (1 - v\varsigma).$$
(20)

Theorem 4. For given scalars $\varepsilon > 0$ and $\eta > 0$, under the communication scheme (4), the nonlinear closed-loop system (12) is mean-square stable with H_{∞} performance index γ if there exist symmetric positive-definite matrices $P = P^T > 0$, U > 0, and $W = W^T > 0$ with appropriate dimensions such that

$$\begin{bmatrix} \Pi_{11} & \Pi_{12} & \Pi_{13} & \tilde{B}_{\omega} & \Pi_{15} \\ * & -P + \Pi_{22}^{T} \Pi_{22} & 0 & 0 & 0 \\ * & * & -\varepsilon I & 0 & 0 \\ * & * & * & -\gamma^{2} & 0 \\ * & * & * & * & -W \end{bmatrix} < 0$$
(21)
$$\begin{bmatrix} -U & P \\ P & -\delta P \end{bmatrix} < 0$$
(22)

where

$$\Pi_{11} = -P^{-1} + \varepsilon II^{T},$$

$$\Pi_{12} = \begin{bmatrix} \overline{A} + (I - N_{\zeta}) \overline{B} \overline{K} N_{\nu} I & N_{\zeta} \overline{B}_{i} & 0 \\ I - N_{\zeta} & N_{\zeta} & 0 \\ I - N_{\nu} & 0 & N_{\nu} \end{bmatrix},$$

$$\Pi_{13} = \begin{bmatrix} (I - N_{\zeta}) \overline{B} \overline{K} N_{\nu} \Phi \\ 0 \\ (I - N_{\nu}) \Phi \end{bmatrix},$$

$$\Pi_{15} = \begin{bmatrix} (I - N_{\zeta}) \overline{D} \overline{K} N_{\nu} \Phi \\ 0 \\ (I - N_{\nu}) \Phi \end{bmatrix},$$

$$\Pi_{22} = \begin{bmatrix} \overline{C} + (I - N_{\zeta}) \overline{D} \overline{K} N_{\nu} I & N_{\zeta} \overline{D} & 0 \\ I - N_{\zeta} & N_{\zeta} & 0 \\ I - N_{\nu} & 0 & N_{\nu} \end{bmatrix},$$

$$N_{\nu} = \operatorname{diag} \{\nu, \dots, \nu\}, N_{\zeta} = \operatorname{diag} \{\zeta, \dots, \zeta\}.$$

$$(23)$$

Proof. For the nonlinear closed-loop system (12), we construct the Lyapunov function

$$V(k) = \beta^{T}(k) P\beta(k)$$
(24)

where P>0 is a symmetric positive definite matrix. Then we have

$$E \{V (k + 1) | V (k)\} - E \{V (k)\}$$

$$= E \{\beta^{T} (k + 1) P\beta (k + 1)\} - E \{\beta^{T} (k) P\beta (k)\}$$

$$\leq E \{\beta^{T} (k) \{\widetilde{A}^{T} P \ \widetilde{A} - P\} \beta (k)$$

$$+ 2\beta^{T} (k) \widetilde{A}^{T} P \widetilde{B}_{\omega} \omega (k) + \omega^{T} (k) \widetilde{B}_{\omega} P \widetilde{B}_{\omega} \omega (k)\}$$

$$= \beta^{T} (k) \{\widehat{A}^{T} P \ \widehat{A} - P\} \beta (k) + 2\beta^{T} (k) \widehat{A}^{T} P \widetilde{B}_{\omega} \omega (k)$$

$$+ \omega^{T} (k) \widetilde{B}_{\omega} P \widetilde{B}_{\omega} \omega (k)$$
(25)

where

 \widehat{A}

$$= \begin{bmatrix} \overline{A} + (I - N_{\zeta}) \overline{B} \overline{K} (N_{\nu} ((I + \Phi F (k))) & N_{\zeta} \overline{B} & 0 \\ I - N_{\zeta} & N_{\zeta} & 0 \\ (I - N_{\nu}) ((I + \Phi F (k))) & 0 & N_{\nu} \end{bmatrix}$$
(26)

When w(k) = 0, we have

$$E\{V(k+1) | V(k)\} - E\{V(k)\} = \beta^{T}(k) \Lambda \beta(k)$$
(27)

From (25), (27), and the Schur complement, we can get that

$$\Lambda = \begin{bmatrix} -P^{-1} & \widehat{A} \\ \\ * & -P \end{bmatrix} < 0 \tag{28}$$

To derive the expression for the controller parameters, set P and P^{-1} as follows:

$$P = \begin{bmatrix} R & X_{12} & X_{13} \\ * & X_{22} & X_{23} \\ * & * & X_{33} \end{bmatrix},$$

$$P^{-1} = \begin{bmatrix} S & Y_{12} & Y_{13} \\ * & Y_{22} & Y_{23} \\ * & * & Y_{33} \end{bmatrix}$$
(29)

where $R \in \mathbb{R}^{(n+m)(n+m)}$, $S \in \mathbb{R}^{(n+m)(n+m)}$, $X_{ij} \in \mathbb{R}^{(n+m)(n+m)}$, and $Y_{ij} \in \mathbb{R}^{(n+m)(n+m)}$ (i, j = 1, 2, 3)

From (28), \widehat{A} , *P*, and P^{-1} , one has

$$\begin{bmatrix} -S & -Y_{12} & -Y_{13} & \overline{A} + (I - N_{\zeta}) \overline{B} \overline{K} N_{\nu} \left((I + \Phi F(k)) \right) & N_{\zeta} \overline{B} & 0 \\ * & -Y_{22} & -Y_{23} & I - N_{\zeta} & N_{\zeta} & 0 \\ * & * & -Y_{33} & (I - N_{\nu}) \left(I + \Phi F(k) \right) & 0 & N_{\nu} \\ * & * & * & -R & -X_{12} & -X_{13} \\ * & * & * & * & -X_{22} & -X_{23} \\ * & * & * & * & * & -X_{33} \end{bmatrix} < 0$$
(30)

Set

$$= \begin{bmatrix} \overline{A} + (I - N_{\zeta}) \overline{B} \overline{K} N_{\nu} ((I + \Phi F(k)) & N_{\zeta} \overline{B} & 0 \\ (I - N_{\zeta}) & N_{\zeta} & 0 \\ (I - N_{\nu}) (I + \Phi F(k)) & 0 & N_{\nu} \end{bmatrix}$$
(31)
$$< 0$$

Applying Lemma 3, it is clear that (30) is equivalent to

$$\begin{bmatrix} -P^{-1} \begin{bmatrix} \overline{A} + (I - N_{\varsigma}) \overline{B} \overline{K} N_{\gamma} I & N_{\varsigma} \overline{B} & 0 \\ I - N_{\varsigma} & N_{\varsigma} & 0 \\ I - N_{\gamma} & 0 & N_{\gamma} \end{bmatrix} \\ * & -P \end{bmatrix} + \begin{bmatrix} (I - N_{\varsigma}) \overline{B} \overline{K} N_{\gamma} \Phi \\ 0 \\ (I - N_{\gamma}) \Phi \\ 0 \end{bmatrix} F(k) \begin{bmatrix} 0 & I & 0 \end{bmatrix} (32) \\ + \begin{bmatrix} 0 & 0 & I & 0 \end{bmatrix}^{T} F^{T}(k) \begin{bmatrix} (I - N_{\varsigma}) \overline{B} \overline{K} N_{\gamma} \Phi \\ 0 \\ (I - N_{\gamma}) \Phi \\ 0 \end{bmatrix} \end{bmatrix}^{T}$$

Employing Lemma 3, we can get that

$$\widehat{\Lambda} = \begin{bmatrix} -P^{-1} \begin{bmatrix} \overline{A} + (I - N_{\zeta}) \overline{B} \overline{K} N_{\nu} I & N_{\zeta} \overline{B} & 0 \\ I - N_{\zeta} & N_{\zeta} & 0 \\ (I - N_{\nu}) & 0 & N_{\nu} \end{bmatrix} \\ + \begin{bmatrix} \varepsilon^{-1} \begin{bmatrix} (I - N_{\zeta}) \overline{B} \overline{K} N_{\nu} \Phi \Phi^{T} N_{\nu}^{T} \overline{K}^{T} \overline{B}^{T} (I - N_{\zeta})^{T} \\ 0 \\ (I - N_{\nu}) \Phi \Phi^{T} (I - N_{\nu})^{T} \end{bmatrix} \begin{bmatrix} 0 \\ \varepsilon III^{T} \end{bmatrix}$$
(33)
$$= \begin{bmatrix} -P^{-1} + \varepsilon II^{T} & \Pi_{12} & \Pi_{13} \\ * & -P & 0 \\ * & * & -\varepsilon I \end{bmatrix} < 0$$

Applying Lemma 2, (27) is equivalent to

$$E \{V(k+1) \mid V(k)\} - E \{V(k)\}$$

$$\leq \lambda_{min} \left(-\widehat{\Lambda}\right) x^{T}(k) x(k) < \frac{-aV(k)}{\lambda_{max}(P)} = -\phi V(k)$$
(34)

where

$$0 < a < \min\left\{\lambda_{\min}\left(-\widehat{\Lambda}\right), \lambda_{\max}\left(P\right)\right\} 0 < \phi$$

$$< a/\lambda_{\max}\left(P\right) < 1$$
(35)

When $\varepsilon = 0$ and w(k) = 0, from (34), we can get that the nonlinear system (12) is mean-square stable. When $w(k) \neq 0$, we have that

$$E \{V (k + 1) | V (k)\} - E \{V (k)\} + E \{z^{T} (k) z (k)\}$$
$$- \gamma^{2} E \{\omega^{T} (k) \omega (k)\} \leq \beta^{T} (k) \Lambda \beta (k)$$
$$+ 2\beta^{T} (k) \widehat{A}^{T} P \widetilde{B}_{\omega} \omega (k) + \omega^{T} (k) \widetilde{B}_{\omega}^{T} P \widetilde{B}_{\omega} \omega (k) \qquad (36)$$
$$+ \beta^{T} (k) \widehat{C}^{T} \widehat{C} \beta (k) - \gamma^{2} \omega^{T} (k) \omega (k)$$
$$= \alpha^{T} (k) \Psi \alpha (k)$$

where

$$\begin{split} \widehat{C} \\ &= \begin{bmatrix} \overline{C} + (I - N_{\zeta}) \overline{D} \overline{K} (N_{\nu} (I + \Phi F (k))) & N_{\zeta} \overline{D} & 0 \\ & I - N_{\zeta} & N_{\zeta} & 0 \\ & (I - N_{\nu}) (I + \Phi F (k)) & 0 & N_{\nu} \end{bmatrix} \\ \alpha &= \begin{bmatrix} \beta (k) \\ \omega (k) \end{bmatrix}, \end{split}$$
(37)
$$\Psi &= \begin{bmatrix} \widehat{\Lambda} + \widehat{C}^T \widehat{C} & \widehat{A} P \overline{B}_w \\ &* & \widetilde{B}_w^T P \widetilde{B}_w - \gamma^2 I \end{bmatrix}. \end{split}$$

When $\Psi < 0$, by Schur complement, we can get that

$$\Psi = \begin{bmatrix} -P^{-1} + \varepsilon II^{T} & \Pi_{12} & \Pi_{13} & \tilde{B}_{\omega} \\ * & -P + \hat{C}^{T}\hat{C} & 0 & 0 \\ * & * & -\varepsilon I & 0 \\ * & * & * & -\gamma^{2}I \end{bmatrix} < 0$$
(38)

Similarly, substituting \widehat{C} into (38) gives

$$\begin{bmatrix} -P^{-1} + \varepsilon II^{T} & \Pi_{12} & \Pi_{13} & \tilde{B}_{w} \\ * & -P + \Pi_{22}^{T}\Pi_{22} & 0 & 0 \\ * & * & -\varepsilon I & 0 \\ * & * & * & -\gamma^{2}I \end{bmatrix} + \begin{bmatrix} 0 \\ \left[\begin{pmatrix} (I - N_{\zeta}) \overline{D} \overline{K} N_{v} \Phi \Phi^{T} N_{v}^{T} \overline{K}^{T} \overline{D}^{T} (I - N_{\zeta})^{T} \\ 0 \\ (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \end{bmatrix} \right] F(k)$$

$$\begin{bmatrix} 0 \\ (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ 0 \\ 0 \end{bmatrix}$$
(39)

 $\cdot \begin{bmatrix} 0 & 0 & 0 & I & 0 & 0 \end{bmatrix} + \begin{bmatrix} 0 & 0 & 0 & I & 0 & 0 \end{bmatrix}^T F^T(k)$

$$\cdot \begin{bmatrix} 0 \\ (I - N_{\zeta})\overline{D}\overline{K}N_{\nu}\Phi\Phi^{T}N_{\nu}^{T}\overline{K}^{T}\overline{D}^{T}(I - N_{\zeta})^{T} \\ 0 \\ (I - N_{\zeta})\Phi\Phi^{T}(I - N_{\zeta})^{T} \\ 0 \\ 0 \end{bmatrix}^{T} < 0.$$

Likewise, employing Lemma 3, we have that

$$\begin{bmatrix} -P^{-1} + \varepsilon II^{T} & \Pi_{12} & \Pi_{13} & \tilde{B}_{w} \\ * & -P + \Pi_{22}^{T} \Pi_{22} & 0 & 0 \\ * & * & -\varepsilon I & 0 \\ * & * & * & -\gamma^{2}I \end{bmatrix}^{T} \\ + \begin{bmatrix} 0 & & \\ \left[(I - N_{\zeta}) \overline{D} \overline{K} N_{v} \Phi \Phi^{T} N_{v}^{T} \overline{K}^{T} \overline{D}^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ & (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ & 0 & \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} 0 & & \\ \left[(I - N_{\zeta}) \overline{D} \overline{K} N_{v} \Phi \Phi^{T} N_{v}^{T} \overline{K}^{T} \overline{D}^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ & 0 & \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \overline{D} \overline{K} N_{v} \Phi \Phi^{T} N_{v}^{T} \overline{K}^{T} \overline{D}^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ & 0 & \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \overline{D} \overline{K} N_{v} \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta})^{T} \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta}) \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta}) \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta}) \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta}) \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta}) \\ & 0 & \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta}) \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta}) \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta}) \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_{\zeta}) \\ \end{bmatrix} \end{bmatrix}_{W^{-1}} \begin{bmatrix} (I - N_{\zeta}) \Phi \Phi^{T} (I - N_$$

 $+ \begin{bmatrix} 0 & 0 & 0 & I & 0 & 0 \end{bmatrix}^T W \begin{bmatrix} 0 & 0 & 0 & I & 0 & 0 \end{bmatrix}$

(46)

 $\begin{bmatrix} -\overline{U} & X \\ X & -\delta X \end{bmatrix} < 0$

which is equivalent to (21). Furthermore, by the use of Schur complement, we can get that $\Psi < 0$ and (15) is obtained. Hence, it is clear that

$$\sum_{0}^{\infty} \left\{ E\left\{ V\left(k+1\right) \mid V\left(k\right)\right\} - E\left\{ V\left(k\right)\right\} - E\left\{ z^{T}\left(k\right)z\left(k\right)\right\} - \gamma^{2}E\left\{ w^{T}\left(k\right)w\left(k\right)\right\} < 0 \right\}$$
(41)

which leads to

$$E\left\{\sum_{0}^{\infty} z^{T}(k) z(k)\right\} < \gamma^{2} E\left\{\sum_{0}^{\infty} w^{T}(k) w(k)\right\} + E\left\{V(0)\right\}, \quad \forall w(k) \neq 0$$
(42)

Due to the initial state x(0) = 0, we get $E\{V(0)\} = 0$, where γ is a given positive constant. Thus

$$E\left\{\sum_{0}^{\infty} z^{T}(k) z(k)\right\} < \gamma^{2} E\left\{\sum_{0}^{\infty} w^{T}(k) w(k)\right\}$$
(43)

holds for any nonzero $\omega(k) \in l_2$. So the nonlinear system (12) is mean-square stable and has an H_{∞} performance level $\gamma > 0$. This completes the proof.

Remark 5. If the initial state $\eta(0) \in \Omega = \{\eta : E\{\eta^T U\eta\} < 1, U > 0\}, E\{V(0)\} < \delta$, and inequalities (21) and (22) hold, then the nonlinear system (12) also has an H_{∞} performance level $\gamma > 0$ such that

$$E\left\{\sum_{0}^{\infty} z^{T}(k) z(k)\right\} < \gamma^{2} E\left\{\sum_{0}^{\infty} w^{T}(k) w(k)\right\} + \eta \qquad (44)$$

where γ and δ are given constants.

The following theorem provides an algorithm to design suitable controller gains.

Theorem 6. For given scalars $\varepsilon > 0$ and $\eta > 0$, the nonlinear system described by (12) under the event-triggered condition (4) is mean-square stable with H_{∞} performance index γ , if there exist real symmetric positive definite matrices $X = X^T > 0$, $\overline{U} > 0$, and $\overline{W} = \overline{W}^T > 0$ and $Y_j \in {}^{m \times n} R(i, j \in S)$ with appropriate dimensions such that

$$\begin{bmatrix} \overline{\Pi}_{11} & \overline{\Pi}_{12} & \overline{\Pi}_{13} & \widetilde{B}_{\omega} & \overline{\Pi}_{15} \\ * & \overline{\Pi}_{22} & 0 & 0 & 0 \\ * & * & -\varepsilon I & 0 & 0 \\ * & * & * & -\gamma^2 & 0 \\ * & * & * & * & -\overline{W} \end{bmatrix} < 0$$
(45)

where

$$\begin{split} \overline{\Pi}_{11} &= -X + \varepsilon X I I^T X^T + I \overline{W} I^T, \\ \overline{W} &= X W X^T, \\ \overline{U} &= X U X^T \\ \overline{\Pi}_{12} &= \begin{bmatrix} A_i + (I - N_{\varsigma}) B_i Y_j N_{\nu} I & N_{\varsigma} B_i & 0 \\ I - N_{\varsigma} & N_{\varsigma} & 0 \\ I - N_{\nu} & 0 & N_{\nu} \end{bmatrix}, \\ \overline{\Pi}_{13} &= \begin{bmatrix} (I - N_{\varsigma}) B_i Y_j N_{\nu} \Phi \\ 0 \\ (I - N_{\nu}) \Phi \end{bmatrix}, \\ (47) \\ \overline{\Pi}_{15} &= \begin{bmatrix} (I - N_{\varsigma}) D_i Y_j N_{\nu} \Phi \\ 0 \\ (I - N_{\nu}) \Phi \end{bmatrix}, \\ \overline{\Pi}_{22} &= \begin{bmatrix} C_i + (I - N_{\varsigma}) D_i Y_j N_{\nu} I & N_{\varsigma} D_i & 0 \\ I - N_{\nu} & 0 & N_{\nu} \end{bmatrix}. \end{split}$$

Moreover, the desired H_{∞} controller gains are given by

$$K_j = Y_j X^{-1} \quad (j \in S_r) \tag{48}$$

Proof. From Theorem 4, if (21) and (22) hold, the nonlinear closed-loop system (12) is mean-square stable with H_{∞} performance index γ . Firstly, the matrix inequality (21) is multiplied by diag{X, X, I, I, X} and its transpose, respectively. Setting $X = P^{-1}$ and performing the congruent transformation on the left side of inequality (21), we can get (45). Similarly, the matrix inequality (22) is multiplied by diag{X, X} and its transpose, respectively. Performing the congruent transformation on the left side of inequality (22), we can get (46). This completes the proof.

Remark 7. Although the conditions in Theorem 6 are not convex, using the cone complementarity linearization (CCL) method [35], we can change it to the nonlinear minimization problem with LMI constraints which are equivalent to the

nonlinear minimization problem with linear matrix inequality constraints as follows:

Minimize Trace (*PX*)
Subject to
$$P > 0$$
,
 $X > 0$, (49)
 $W > 0$,
 $\varepsilon > 0$
 $\lceil \Pi_{11} \ \Pi_{12} \ \Pi_{13} \ \tilde{B}_w \ \Pi_{15} \rceil$

$$\left. \begin{array}{cccc} * & \overline{\Pi}_{22}^{T} & 0 & 0 & 0 \\ * & * & -\varepsilon I & 0 & 0 \\ * & * & * & -\gamma^{2}I & 0 \\ * & * & * & * & -\overline{W} \end{array} \right| < 0,$$

$$\left[\begin{array}{c} -U & X \\ X & -\delta X \end{array} \right] < 0,$$

$$\left[\begin{array}{c} P & I \\ I & X \end{array} \right] \ge 0$$

$$(51)$$

The above nonlinear minimization problem can be solved by an iterative algorithm as presented as follows.

Algorithm 8.

Step 1. Find a set of feasible solutions $\Xi_0 = \{P_0, X_0, W_0, \varepsilon_0, K_0\}$ such that (51), and set the iterative number l = 0.

Step 2. Use LMI toolbox of mincx solver to solve the following linear objective function minimization problem:

Minimize Trace
$$(P_l X + P X_l)$$

Subject to (50) and (51) (52)

The solution is set to be $\Xi^+ = \{P^+, X^+, W^+, \varepsilon^+, K^+\}.$

Step 3. Substitute the solution into the matrix inequalities (45) and (46) in Theorem 6. If (45) and (46) are satisfied, then K^+ is the state feedback gain matrix and stop; otherwise, go to Step 4.

Step 4. If the iterative number satisfies $l \leq L$ (L is a predetermined iterative number upper bound), set $\Xi_{l+1} = \Xi^+$ and l = l+1 and go to Step 2 for the next iteration. Otherwise, go to Step 1 and reselect a set of feasible solutions Ξ_0 for calculation.

4. Simulation Example

In this section, we will give a simulation example to demonstrate the effectiveness of the proposed approach.

The discrete-time T-S model fuzzy systems are expressed as follows. Plant rules: IF x_k^1 is Γ_1 , THEN

$$x_{k+1} = A_1 x_k + B_1 w_k + B_{w1} u_k$$
(53)

$$z_k = C_1 x_k + D_1 u_k \tag{54}$$

IF x_k^l is Γ_2 , THEN

$$x_{k+1} = A_2 x_k + B_2 w_k + B_{w2} u_k \tag{55}$$

$$z_k = C_2 x_k + D_2 u_k \tag{56}$$

where

$$A_{1} = \begin{bmatrix} 1.2 & 0.5 & 0.5 \\ 0 & 0.7 & -0.5 \\ 0.1 & 0 & -0.8 \end{bmatrix},$$

$$B_{1} = \begin{bmatrix} 0.1 \\ 0.2 \\ 0.8 \end{bmatrix},$$

$$B_{w1} = \begin{bmatrix} 0.5 \\ 1 \\ 1 \end{bmatrix},$$

$$C_{1} = \begin{bmatrix} 0 & 0.5 & 1 \end{bmatrix},$$

$$D_{1} = 0.5,$$

$$A_{2} = \begin{bmatrix} 0.5 & 0 & -0.8 \\ 0.5 & 1.8 & -0.6 \\ 0 & 0.9 & 0.2 \end{bmatrix},$$

$$B_{2} = \begin{bmatrix} 0.5 \\ 0.8 \\ 0.2 \\ 0.2 \end{bmatrix},$$

$$B_{w2} = \begin{bmatrix} 0.5 \\ 0.2 \\ 0.2 \\ 0.2 \end{bmatrix},$$

$$C_{2} = \begin{bmatrix} 1 & 0.6 & 0 \end{bmatrix},$$

$$D_{3} = 0.2.$$
(57)

We suppose that the sampling period T = 0.1s and the initial condition $x_0 = \begin{bmatrix} 1 & -1 & -1 \end{bmatrix}^T$ and the external disturbance is

$$w(k) = \begin{cases} -0.5, & 0 \le k \le 5\\ -0.5, & 6 \le k \le 15\\ 0, & 16 \le k \le 20 \end{cases}$$
(58)



FIGURE 3: State response of the controlled system (with disturbance and packet dropouts).



FIGURE 4: State response of the controlled system.

The membership functions h_1 and h_2 are described, respectively, as

$$h_1\left(x_k^{(l)}\right) = 1 - \frac{\left(x_k^{(l)}\right)^2}{2.25},$$

$$h_2\left(x_k^{(l)}\right) = 1 - h_1\left(x_k^{(l)}\right).$$
(59)

Let the event-triggered parameter $\sigma = 0.05$, the packet loss rate $\alpha(k) = 0.05$ and $\beta(k) = 0.05$, and H_{∞} performance



FIGURE 5: Bernoulli communication sequence.



FIGURE 6: Release time distribution.

level $\gamma^2 = 0.6325$. Applying Theorem 6, we can obtain controller feedback gains as

$$K_1 = \begin{bmatrix} -0.6723 & -0.5064 & -0.7142 \end{bmatrix},$$

$$K_2 = \begin{bmatrix} -0.3212 & -0.8434 & -1.0216 \end{bmatrix}.$$
(60)

Figures 3 and 4 illustrate that the state responses of the system are mean-square stable. Compared with these two pictures, packet loss may have some influence on the stability of the system at the same transmission conditions and states. Figure 5 shows the Bernoulli communication sequence about packet loss. Figure 6 shows the intervals of different times between event-triggered transmitters. IAE denotes the control performance of the system. Under four different thresholds σ , the performance of transfer rate and IAE = $\sum ||e||_2 \cdot h$ is shown for the system with the above three error threshold values in Figure 7, where case 1 is $\sigma = 0.03$, case 2 is $\sigma = 0.05$, and case 3 is $\sigma = 0.07$.

It can be seen that although only a small section is triggered to be transmitted to the controller, the networked control system is mean-square stable with the prescribed H_{∞} performance level. Comparing Table 1 with Figure 7, we



TABLE 1: The data transfer rate.

	Data transfer rate
This paper ($\sigma = 0.07$)	0.193
This paper ($\sigma = 0.05$)	0.208
This paper ($\sigma = 0.03$)	0.236
Reference [31]	0.225
Reference [32]	0.336

find that when the value of σ gets larger, the transmission rate is lower, but the error increases, so the appropriate σ value can be determined. Only 20.8% of the measurement data is transmitted if $\sigma = 0.05$, which means that the required transmission can save limited network resources. From Figure 6, we find that the event-triggered mechanism can not only relieve the problem of resource constraints but also make the data in the transmission process faster and more stable, so the method proposed in this paper is effective.

5. Conclusion

This paper focuses on the packet loss and limited resources problems for the nonlinear NCSs. A discrete-time model of nonlinear NCSs is established by the T-S fuzzy approach. The packet loss process of each channel is described as an independent and distributed Bernoulli process. An eventtriggered scheme is put forward for the nonlinear NCSs based on a relative error-triggered mechanism. Using the Lyapunov stability theory and linear matrix inequality method, the feedback controller is given via PDC technique, which guarantees the mean-square stability of nonlinear NCSs with the desired performance. Since the principles and algorithms of the trigger in this paper are relatively simple, the smart sensor is easy to implement. In addition, the simulation results show that the designed communication scheme can effectively decrease the data transmission rate, so it is very suitable for application to NCSs with limited bandwidth resources.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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