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Research Article

A Modeling Method of Stochastic Parameters' Inverse Gauss Process Considering Measurement Error under Accelerated Degradation Test

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To solve the problem that the individual differences and the measurement errors affect the accuracy of life estimation in accelerated degradation test, the inverse Gauss process with stochastic parameters is applied in the accelerated degradation test with the consideration of the influence of individual differences, and the analysis of measurement uncertainty is carried out. An inverse Gauss accelerated degradation model considering both individual differences and measurement errors is established. In the maximum likelihood estimation of parameters, Genetic Algorithm and Monte Carlo integral are used to solve the problems caused by complex integral and the unobservable measurement errors in the calculation process. Finally, the proposed method is verified by the Monte Carlo simulation under the constant accelerated stress and step accelerated stress and the illustrative example of electrical connectors under the constant acceleration stress, respectively. The results show that the modeling tool is useful for improving the accuracy of the life prediction and the reliability evaluation.

1. Introduction

Prognostics Health Management (PHM) is a new management method of complex engineering system, which integrates fault diagnosis, life prediction, and health management. And it has been widely applied in different fields, such as communications, electronics, military, and aviation. As a key component of PHM, an appropriate life prediction method is significant for the product reliability evaluation, and it contributes to formulate reasonable health management schemes. It also improves the safety of industrial production; especially it saves the maintenance cost [1–3].

With the development of manufacturing technology, massive products with high reliability and long life (e.g., GaAs laser and electrical connector, etc.) have emerged in various fields of industrial production. It is very difficult to obtain the life information which satisfies the practical demands by the traditional life test and the accelerated life test. At present, the reliability evaluation and life prediction methods based on degradation test are widely used; however, under rated conditions, the degradation rate of the products is relatively slow and the degradation information is relatively little. The accelerated degradation test (ADT) nicely solves this problem. Among them, the Constant Stress Accelerated Degradation Test (CSADT) and the Step Stress Accelerated Degradation Test (SSADT) have been favored by many scholars for their advantages of fast acquisition speed and mature test methods in obtaining degradation data.

As the most critical part of the degradation analysis, the degradation models are currently divided into the regression model based on degradation trajectory analysis and the stochastic process model to characterize product performance degradation. Influenced by the changes in the external environment, the manufacturing process and the operation conditions, the degradation process of the products is with some randomness. Therefore, it is reasonable to use the stochastic process to model product degradation, and this is also a widely used modeling method at this stage. Three kinds of commonly used stochastic process models are respectively the Gamma process model, the Wiener process model, and the Inverse Gaussian (IG) process model. However, the degradation trajectories of some products cannot be accurately fitted by the first two models, such as the GaAs laser [4] and the electrical connector [5]. Recently, as a stochastic process with independent increment, the IG process has been increasingly applied in the modeling of degradation. Ye [6] defined the IG process as a composite Poisson process with independent but not necessarily identical distribution in the extreme case, which compensates the physical significance of the IG process. Wang et al. [4] described the degradation characteristics of the products through the IG process and solved the problem of parameter estimation by the maximum likelihood estimation (MLE) method. Peng [7] used the Bayesian method to optimize the parameter estimation in IG process. Then Peng [8] took the individual differences of the similar products into consideration, solved the parameter estimation problem by the EM algorithm, and gave a clear analytical prediction expression of the remaining life. Aiming at handling the impact of catastrophe data on prediction, Xu [9] proposed a self-adaptive IG process which has a great significance on improving the accuracy of prediction.

Many scholars have done a lot of research on the accelerated degradation test using IG process. Wang [10] optimized the generalized IG process under the SSADT. Based on the invariance principle of acceleration coefficient, Wang [11] proposed an accelerated degradation modeling method using IG process with stochastic parameters; Ye [5] represented the individual differences in IG process through the randomization of parameters and optimized the CSADT design; Wu [12] proposed a multiobjective optimization design method and optimized CSADT using IG process. The measurement errors have not been investigated in those studies; however, that affects the fitting degree of the model and the accuracy of life estimation [13]. Therefore, Li [14] integrated the error item into the modeling in SSADT optimization design using IG process; however, the possible influence by the individual differences was neglected in Li's study. Furthermore, in ADT, the individual differences between similar products and measurement uncertainties are ubiquitous and they considerably affect the accuracy of life prediction [15, 16]. Hence, it is necessary to establish an IG process degradation model with the consideration of both measurement errors and individual differences.

The paper is organized as follows. Section 2 introduces the IG process degradation model considering both measurement errors and individual differences. The acceleration models of the constant stress and the step stress using IG process are introduced in Section 3. In Section 4, a MLE method based on Monte Carlo integral and Genetic Algorithm (GA) is proposed, which solves the problem of complex integrals and unobservable measurement errors effectively in parameter estimation process. In Section 5, the efficiency and reasonability of the proposed approach are validated via Monte Carlo simulation study by two examples: the CSADT and the SSADT. And the proposed approach is validated by an illustrative example of electrical connector in Section 6. The conclusions are finally drawn in Section 7.

2. Degradation Modeling

In this section, we adopt the IG model with stochastic parameters to characterize individual differences, with the consideration of the effect of measurement errors through incorporating error terms, and establish a degradation model for products.

As a newly introduced stochastic process model, the IG model shows a good fitting characteristic for the degradation process of many products [4, 5]. According to the previous description, the measurement errors of the degraded data are inevitable due to the interference of various noises during the degradation process; therefore, it is necessary to be considered in the degradation modeling. Set $Y(t)$ to be the observed degradation data and $X(t)$ the real degradation data of product performance. In this paper, the increment of degradation data is used as the modeling object. And the degradation model can be expressed as

$$\Delta Y(t) = \Delta X(t) + \Delta\varepsilon \tag{1}$$

where $\Delta Y(t)$ and $\Delta X(t)$ denote the increment of the degradation inspections and the true degradation of the product's performance characteristic at time t, respectively. $\Delta\varepsilon$ is the increment of measurement errors which follows a normal distribution as $\Delta\varepsilon \sim N(0,\sigma^2)$. The $\Delta X(t)$ follows the IG distribution, and it is expressed as $\Delta X \sim IG(\beta\Delta\Lambda(t), \eta[\Delta\Lambda(t)]^2)$. The probability density function (PDF) of $X(t)$ and $\Delta X(t)$ can be defined as

$$f(x) = \sqrt{\frac{\eta\Lambda(t)^2}{2\pi x^3}} \exp\left[-\frac{\eta}{2x}\left(\frac{x}{\beta} - \Lambda(t)\right)^2\right] \tag{2}$$

$$f(\Delta x) = \sqrt{\frac{\eta\Delta\Lambda(t)^2}{2\pi\Delta x^3}} \exp\left[-\frac{\eta}{2\Delta x}\left(\frac{\Delta x}{\beta} - \Delta\Lambda(t)\right)^2\right] \tag{3}$$

where $\Lambda(t)$ is the time scale function and $\Delta\Lambda(t)$ denotes the increment of time scale function. Under the condition that the failure threshold is ω, it is considered that the first hitting time (FHT) of IG process is product life T:

$$T = \inf\{t : X(t) \geq \omega \mid X(0) < \omega\} \tag{4}$$

Therefore, the cumulative distribution function (CDF) of the life for the IG process can be obtained [4]

$$\begin{aligned} F(t) = {} & \Phi\left[\sqrt{\frac{\eta}{\omega}}\left(\Lambda(t) - \frac{\omega}{\beta}\right)\right] \\ & - \exp\left(\frac{2\eta\Lambda(t)}{\beta}\right)\Phi\left[-\sqrt{\frac{\eta}{\omega}}\left(\Lambda(t) + \frac{\omega}{\beta}\right)\right] \end{aligned} \tag{5}$$

The corresponding PDF is

$$f(t) = \sqrt{\frac{\eta}{\omega}}\phi\left[\sqrt{\frac{\eta}{\omega}}\left(\Lambda(t) - \frac{\omega}{\beta}\right)\right] - \frac{2\eta\Lambda(t)}{\beta}\exp\left(\frac{2\eta\Lambda(t)}{\beta}\right) \times \Phi\left[-\sqrt{\frac{\eta}{\omega}}\left(\Lambda(t) + \frac{\omega}{\beta}\right)\right] + \sqrt{\frac{\eta}{\omega}}\exp\left(\frac{2\eta\Lambda(t)}{\beta}\right)\phi\left[-\sqrt{\frac{\eta}{\omega}}\left(\Lambda(t) + \frac{\omega}{\beta}\right)\right] \quad (6)$$

To reflect the individual differences between the similar products, Wang and Xu [17] proposed a method to incorporate the random effects in the Inverse Gaussian process by letting $\eta \sim Gamma(\delta, \gamma^{-1})$, with PDF

$$g(\eta; \delta, \gamma) = \frac{\gamma^{\delta}\eta^{\delta-1}}{\Gamma(\delta)}\exp(-\gamma\eta), \quad \eta > 0 \quad (7)$$

where $\Gamma(\delta)$ denotes a gamma function, and (3) can be considered as the conditional distribution $f_{\Delta X(t)}(\Delta x \mid \eta)$ under the condition of η being a constant; the expression of $f_{\Delta X(t)}(\Delta x)$ can be calculated by $f_{\Delta X(t)}(\Delta x) = \int_0^{+\infty} f_{\Delta X(t)}(\Delta x \mid \eta) g(\eta; \delta, \gamma) d\eta$, then

$$f_{\Delta X(t)}(\Delta x) = \frac{\Gamma(\delta + 1/2)}{\Gamma(\delta)}\gamma^{\delta}\sqrt{\frac{\Lambda(t)^2}{2\pi(\Delta x)^3}} \times \left(\gamma + \frac{(x/\beta - \Lambda(t))^2}{2\Delta x}\right)^{-\delta-1/2} \quad (8)$$

Based on the concept of FHT, the CDF of IG process with random effects can be obtained [4]

$$F_T = F_{t_{2\delta}}\left(\frac{\delta^{1/2}(\beta\Lambda(t) - \omega)}{\beta\sqrt{\beta\Lambda(t)\gamma}}\right) \quad (9)$$

where $t_{2\delta}$ is the student t-distribution with 2δ degrees of freedom. The corresponding PDF can be obtained as follows:

$$f_T = f_{t_{2\delta}}\left(\frac{\delta^{1/2}(\beta\Lambda(t) - \omega)}{\beta\sqrt{\beta\Lambda(t)\gamma}}\right) \times \left(\frac{\delta^{1/2}\beta^2\sqrt{\beta\Lambda(t)\gamma} - \left[\delta^{1/2}(\beta\Lambda(t) - \omega)\right]\beta\sqrt{\beta\gamma}/2\sqrt{\Lambda(t)}}{\beta^3\Lambda(t)\gamma}\right) \quad (10)$$

From the above exposition, in the case of rated stress, the reliability analysis and the life estimation of the product degradation process described by IG process can be carried out through the analytical solutions.

3. Modeling of Accelerated Degradation

The acceleration model can reflect the effect of the stress change upon the degradation process by establishing the relationship between accelerated stress and model parameters. Considering that the mean of the IG process is $\beta\Lambda(t)$ and the variance is $\beta^3\Lambda(t)/\eta$, we assume that β is a stress dependent function and ignore the effect of stress on the parameter η in order to reflect the effect of stress upon the speed and the volatility of degradation simultaneously. This section takes the temperature stress as an example to illustrate the relationship between the parameters β and the accelerated stress by the Arrhenius model

$$\beta_i = \exp(\alpha_0 + \alpha_i z_i) \quad (11)$$

where α_0 and α_1 are undetermined parameters, stress level S_i, $i = 1, 2, \ldots, m$. z_i is the accelerated stress after standardization, which is defined as

$$z_i = \frac{(1/S_0 - 1/S_i)}{(1/S_0 - 1/S_m)} \quad (12)$$

where S_0 denotes the normal working stress, S_m is the maximum accelerated stress, $z_0 = 0 < z_1 < \cdots < z_m = 1$.

The CSADT and the SSADT have been widely used in all kinds of product tests, such as LED [18], electrical connector [17], and SLD [19]. We suppose that there are n units participating in ADT from the beginning of the experiment $t_0 = 0$ to cut-off time t_m.

In the CSADT, let $S_1 < S_2, \ldots, < S_m$ denote m higher stress levels, $X(t \mid S_i)$ denote the degradation path of the unit under S_i, and $X(t \mid S_i) \sim IG(\exp(\alpha_0 + \alpha_1 z_i)\Lambda(t), \eta\Lambda(t)^2)$, $t_0 \le t \le t_m$.

As for SSADT, assume that there are n units in test, where each of them is tested under a total number of m stress levels $S_1 < S_2, \ldots, < S_m$, which can be expressed as

$$S = \begin{cases} S_1, & \mathrm{t}_0 \le t < t_1 \\ S_2, & \mathrm{t}_1 \le t < t_2 \\ \vdots & \\ S_m, & \mathrm{t}_{m-1} \le t < t_m \end{cases} \quad (13)$$

Under the stress level S_i, each unit has been monitored for l_i times, $i = 1, 2, \ldots, m$. Suppose that the monitoring intervals for each unit is f, and $t_i = f\sum_{k=1}^{i} l_k$, $X_{ss}(t)$ denotes the complete degradation path under SSADT, the relation between $X_{ss}(t)$ and $\{X(t \mid S_i)\}$ can be given as (14).

4. Statistical Inference

In this paper, the generalized IG process is adopted. For the convenience of parameter estimation, set time function $\Lambda(t) = t^{\rho}$ and the model parameters $\theta = (\alpha_0, \alpha_1, \delta, \gamma, \rho, \sigma)$. Since the measurement errors are integrated into model, the solution of the MLE becomes complicated. On the basis of considering individual difference and measurement errors simultaneously, a MLE method based on combination of Monte Carlo integral and GA for the IG process in ADT is proposed. The SSADT is taken as an example to make statistical inference, and the CSADT can draw corresponding calculation results with reference to it.

$$X_{ss}(t) = \begin{cases} X(t \mid S_1) \sim IG\left(\beta_1 \Lambda(t), \eta \Lambda(t)^2\right), & t_0 \le t \le t_1; \\ X(t_1 \mid S_1) + X((t-t_1) \mid S_2) \sim IG\left(\beta_1 \Lambda(t_1) + \beta_2 (\Lambda(t) - \Lambda(t_1)), \eta \Lambda(t)^2\right), & t_1 \le t \le t_2; \\ \vdots & \\ \sum_{i=2}^{m} X((t_{i-1} - t_{i-2}) \mid S_{i-1}) + X((t-t_{m-1}) \mid S_m) \sim IG\left(\sum_{i=2}^{m} \beta_{i-1} \Lambda(t_{i-1}) + \beta_m (\Lambda(t) - \Lambda(t_{m-1})), \eta \Lambda(t)^2\right), & t_{m-1} < t < t_m. \end{cases} \tag{14}$$

Let ΔY_{ijk} denote the increment of degradation inspections of the unit j at time t_k under the stress S_i and ΔX_{ijk} denote the increment of the true degradation, and $\Delta\varepsilon_{ijk}$ is the increment of measurement errors which is independent and identically distributed. By (1) we can obtain

$$\Delta Y_{ijk} = \Delta X_{ijk} + \Delta\varepsilon_{ijk} \tag{15}$$

For the SSADT, $\{X_{ss}^{(j)}(t_k)\}_{k=1}^{\xi_m}$ $(j = 1, 2, \ldots, n)$ denote the complete degradation path under SSADT of units j and ξ_m denote the total measurement times of a single unit, $\xi_m = \sum_{i=1}^{m} l_i$. And the expression of ΔX_{ijk} can be obtained as follows:

$$\Delta X_{ijk} = X_{ss}^{(j)}(t_k) - X_{ss}^{(j)}(t_{k-1}) \tag{16}$$

ΔY_{ijk} contains the observation error, which makes ΔY_{ijk} and ΔX_{ijk} follow different distributions. By (15), let $\Delta Y_{ijk} - \Delta\varepsilon_{ijk}$ denote ΔX_{ijk} and ΔX_{ijk} follow IG distribution with random effects. The expression of the maximum likelihood function can be obtained by the full probability formula

$$\begin{aligned} L(\theta) &= \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=\xi_{i-1}+1}^{\xi_i} L_{ijk}\left(\alpha_0, \alpha_1, \delta, \gamma, \rho, \sigma \mid \Delta y_{ijk}\right) = \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=\xi_{i-1}+1}^{\xi_i} \int_{-\Delta y_{ijk}}^{\Delta y_{ijk}} f_{\Delta X_{ijk}}\left(\Delta y_{ijk} - \Delta\varepsilon_{ijk}\right) \times f_{\Delta\varepsilon_{ijk}}\left(\Delta\varepsilon_{ijk}\right) d\Delta\varepsilon_{ijk} \\ &= \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=\xi_{i-1}+1}^{\xi_i} \int_{-\Delta y_{ijk}}^{\Delta y_{ijk}} \frac{\Gamma(\delta + 1/2)}{\Gamma(\delta)} \gamma^{\delta} \left[\frac{\left(\Delta t_{ijk}^{\rho}\right)^2}{2\pi\left(\Delta y_{ijk} - \Delta\varepsilon_{ijk}\right)^3}\right]^{1/2} \times \left[\gamma + \frac{\left(\left(\Delta y_{ijk} - \Delta\varepsilon_{ijk}\right)/\exp\left(\alpha_0 + \alpha_1 z_i\right) - \Delta t_{ijk}^{\rho}\right)^2}{2\left(\Delta y_{ijk} - \Delta\varepsilon_{ijk}\right)}\right]^{-\delta-1/2} \times \left(\frac{1}{2\pi\sigma^2}\right)^2 \exp\left(-\frac{\left(\Delta\varepsilon_{ijk}\right)^2}{2\sigma^2}\right) d\Delta\varepsilon_{ijk} \end{aligned} \tag{17}$$

During each measurement, the absolute value of each measurement error should be less than the degradation inspection. Thus, the integral bounds of $\Delta\varepsilon_{ijk}$ in (17) is $\pm\Delta y_{ijk}$. Considering that the direct integral of (17) is complicated, we adopt a simplified method referring to [14] as follows.

Define a two-variable function

$$h\left(\Delta y_{ijk}, \Delta\varepsilon_{ijk}\right) = \begin{cases} f_{\Delta X_{ijk}}\left(\Delta y_{ijk} - \Delta\varepsilon_{ijk}\right), & \Delta\varepsilon_{ijk} < \Delta y_{ijk} \\ 0, & \text{others} \end{cases} \tag{18}$$

$L_{ijk}(\alpha_0, \alpha_1, \delta, \gamma, \rho, \sigma \mid \Delta y_{ijk})$ can be simplified as

$$\begin{aligned} L_{ijk} &= \int_{-\Delta y_{ijk}}^{\Delta y_{ijk}} f_{\Delta X_{ijk}}\left(\Delta y_{ijk} - \Delta\varepsilon_{ijk}\right) \times f_{\Delta\varepsilon_{ijk}}\left(\Delta\varepsilon_{ijk}\right) d\Delta\varepsilon_{ijk} \\ &= \int_{-\Delta y_{ijk}}^{\Delta y_{ijk}} h\left(\Delta y_{ijk}, \Delta\varepsilon_{ijk}\right) \times f_{\Delta\varepsilon_{ijk}}\left(\Delta\varepsilon_{ijk}\right) d\Delta\varepsilon_{ijk} \\ &= E_{\Delta\varepsilon_{ijk}}\left[h\left(\Delta y_{ijk}, \Delta\varepsilon_{ijk}\right)\right] \end{aligned} \tag{19}$$

Formula (19) converts L_{ijk} into the expectation of the two-variables function $h(\Delta y_{ijk}, \Delta\varepsilon_{ijk})$ to $\Delta\varepsilon_{ijk}$. Then, an approximate calculation is conducted as follows:

$$E_{\Delta\varepsilon_{ijk}}\left[h\left(\Delta y_{ijk}, \Delta\varepsilon_{ijk}\right)\right] \approx \frac{1}{N_s}\sum_{q=1}^{N_s} h\left(\Delta y_{ijk}, \Delta\varepsilon_{ijk}^{q}\right) \tag{20}$$

In order to calculate the parameter configuration in the case of minimum $E_{\Delta\varepsilon_{ijk}}[h(\Delta y_{ijk}, \Delta\varepsilon_{ijk})]$, the MLE of unknown parameters is proposed based on Monte Carlo integral and Genetic Algorithm:

(1) The upper limit of σ is calculated by formulation (21) and setting the step length to be a_σ, a series of $\sigma \in [0, \sqrt{D(\Delta y_{ijk})}]$ can be obtained:

$$\sqrt{D\left(\Delta y_{ijk}\right)} = \sqrt{\frac{\sum_{i=1}^{m}\left(\Delta y_{ijk} - \left(\sum_{i=1}^{m}\Delta y_{ijk}\right)/m\right)}{m}} \tag{21}$$

(2) For each σ, simulate N_s times by normal distribution random number. By substituting $\Delta\varepsilon_{ijk}^{q} \sim N(0, \sigma)$, $q = 1, 2, \ldots, N_s$ into (20), the approximate maximum

likelihood function expression can be obtained as follows:

$$\begin{aligned} L(\theta) &= \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=\xi_{i-1}+1}^{\xi_i} L_{ijk}\left(\alpha_0,\alpha_1,\delta,\gamma,\rho,\sigma \mid \Delta y_{ijk}\right) \\ &= \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=\xi_{i-1}+1}^{\xi_i} \frac{1}{N_s}\sum_{q=1}^{N_s} h\left(\Delta y_{ijk},\Delta\varepsilon_{ijk}^{q}\right) \end{aligned} \tag{22}$$

(3) In the maximum likelihood function, there are no other unknown parameters except the remaining unknown parameters $\theta_1 = (\alpha_0,\alpha_1,\delta,\gamma,\rho)$. In this case, the parameter estimation is regarded as an unconstrained minimum optimization problem. Since the maximum likelihood estimation requires $L(\theta_1)$ to be the maximum, here $-L(\theta_1)$ is regarded as an objective function and $\theta_1 = (\alpha_0,\alpha_1,\delta,\gamma,\rho)$ as the variable. By calling the gatool in Matlab, the minimum $-L(\theta_1)$ and the corresponding $\theta_1 = (\alpha_0,\alpha_1,\delta,\gamma,\rho)$ can be obtained by setting parameters ranges, population sizes, and iteration times in the corresponding item columns. A series of $-L(\theta_1)$ and its corresponding $\theta_1 = (\alpha_0,\alpha_1,\delta,\gamma,\rho)$ can be obtained by performing the operations above under all values of σ. According to the definition of maximum likelihood estimation, the minimum $-L(\theta_1)$ can be obtained by comparison. The corresponding $\theta_1 = (\alpha_0,\alpha_1,\delta,\gamma,\rho)$ and σ are the optimal parameters for the estimation. The flow chart of parameter estimation is shown in Figure 1.

The approximated value of $\Delta\varepsilon_{ijk}$ is given by Monte Carlo method, and it is substituted into the expression of maximum likelihood function, which solves the unknown $\Delta\varepsilon_{ijk}$. The optimal values of other parameters are obtained through GA, and the parameter θ including σ is selected by comparing the value of a series of $L(\theta)$. Finally, the unknown parameters estimation of model is realized. Here are three points for explanation.

(i) In step (1), for all the units within two adjacent intervals, the variance of $\Delta\varepsilon_{ijk}$ is less than variance of Δy_{ijk} because the fluctuation of Δy_{ijk} includes the random effect, individual difference of Δx_{ijk} and the effects of $\Delta\varepsilon_{ijk}$.

(ii) In step (3), as the objective function, the maximum likelihood function $L(\theta)$ is searched, and its numerical value is simplified, so there is a certain deviation from the true value, which is only as the basis for the selection of the optimal parameter θ. The actual value is calculated by substituting the optimal parameter into (17).

(iii) In the searching process, since $L(\theta)$ is carried out under the condition that σ is determined and the variable is only θ_1, $L(\theta)$ is denoted as $L(\theta_1)$ in the description above. However, since σ is also a variable in the whole process of parameter estimation, we finally express as $L(\theta)$. There is no difference in numerical values between $L(\theta)$ and $L(\theta_1)$.

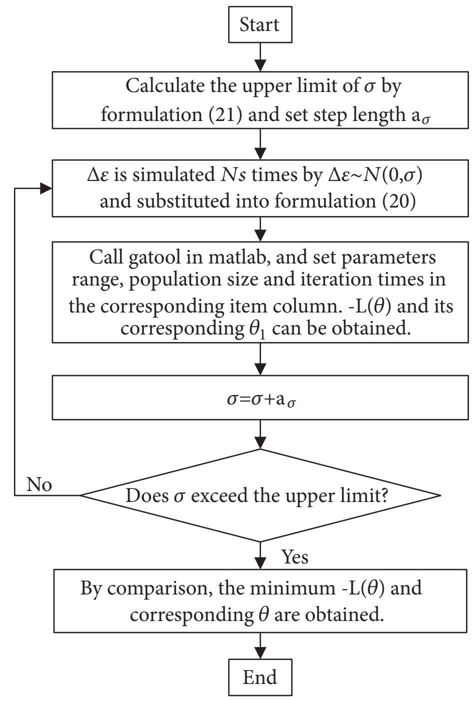

Figure 1: The flow chart of parameter estimation.

5. Simulation Study

In order to verify the correctness of the proposed modeling method and the effectiveness of the parameter estimation method, the simulation and verification of the IG process under CSADT and SSADT are carried out, respectively. For the convenience of following description, M0 is used as the IG model with the consideration of both individual differences and measurement errors; M1 is used as the IG model with the consideration of individual differences proposed by Ye in [5], and M2 as the IG model with the consideration of measurement errors proposed by Ye in [14].

5.1. CSADT. In this section a simulation study for the CSADT is conducted. Assume that the observation times of each unit are equal under each accelerated stress, and the time intervals are also equal. 15 groups of degradation data are simulated under three accelerated stresses of 60°C, 80°C, and 100°C, and the working stress is 40°C. The failure threshold $\omega = 35$ and the individual differences are simulated by parameter randomization. The interval is 100h within 12 measurements. The degradation data and the measurement time are shown in Table 1.

As for the three IG models mentioned above, the estimated parameters and real values are separately given in Table 2. Introducing the Akaike Information Criterion (AIC) as the standard to evaluate model fitting degree, and combining with the maximum value of log likelihood function (Log-LF), the three IG models mentioned above can be evaluated. The calculation formula of AIC is given as

$$AIC = 2\left(p - \ln L(\theta)\right) \tag{23}$$

Table 1: Simulation data of CSADT.

Stress	Time/h											
	100	200	300	400	500	600	700	800	900	1000	1100	1200
60°C	4.241	6.918	8.893	10.505	11.822	13.143	14.238	15.626	16.968	18.094	19.168	20.404
	4.527	6.785	8.602	9.979	11.501	12.690	13.956	15.364	16.541	17.671	18.742	19.750
	4.491	7.814	9.805	11.338	12.469	13.713	14.502	15.258	16.637	18.604	19.327	20.177
	4.218	6.574	8.296	10.041	11.380	12.536	13.866	14.833	16.276	17.569	18.831	19.642
	4.329	6.459	8.355	9.767	10.968	12.275	13.572	14.529	15.689	16.943	17.899	19.035
80°C	6.276	9.669	11.771	14.489	16.406	18.583	20.625	22.646	23.863	25.493	26.976	28.476
	6.046	9.142	11.719	14.331	16.688	18.725	20.720	22.634	24.066	25.664	27.432	28.888
	6.625	10.434	13.049	15.200	17.016	18.922	21.207	23.144	24.970	26.357	27.922	29.478
	6.138	9.736	11.984	14.003	16.379	18.483	20.179	21.423	23.084	24.742	26.415	27.524
	4.314	9.829	13.640	16.848	19.404	21.811	23.550	25.507	26.828	28.579	29.444	30.861
100°C	7.618	11.411	14.403	17.291	19.591	22.277	24.973	26.862	28.936	31.513	33.204	35.119
	7.820	11.702	14.649	17.664	20.021	21.813	23.861	25.671	26.745	29.113	30.272	35.294
	7.574	11.425	15.152	19.381	22.349	24.883	27.815	29.779	31.376	33.675	35.321	37.083
	6.830	10.088	12.242	15.564	18.013	21.419	23.404	25.524	27.016	29.843	31.435	32.145
	7.806	11.775	15.782	18.738	20.897	23.960	26.297	27.949	29.920	31.837	33.655	35.421

Table 2: Comparison of three models in CSADT.

Model	α_0	α_1	δ	γ	ρ	σ	Log-LF	AIC	MTTF/h
Real	-2.172	1.342	1.415	2.716	0.623	0.050	-185.719	383.438	10362.514
M0	-2.284	1.456	1.225	2.515	0.517	0.042	-195.432	402.864	10312.417
M1	-1.912	1.216	1.514	2.351	0.549	—	-200.241	410.482	10192.347
M2	-2.341	1.160	—	—	0.721	0.054	-205.329	420.658	10476.214

Table 3: The TMSE under multiple simulations.

Numbers	α_0	α_1	δ	γ	ρ	σ
500	0.092	0.024	0.035	0.049	0.103	0.063
1000	0.053	0.009	0.012	0.031	0.062	0.026

where p denotes the number of undetermined parameters in model and the AIC values of the three IG models are shown in Table 2. To reflect the accuracy of the proposed method in life estimation, the mean time to failure (MTTF) of the models is also listed in Table 2. For proving the correctness of the parameter estimation method proposed above, the total mean square errors (TMSE) of parameter estimated values to real values tested in 500 and 1000 times simulation are respectively shown in Table 3, and the calculation formula of TMSE is given as

$$TMSE = \frac{1}{M_s} \sum_{v=1}^{M_s} \left(\widetilde{\theta} - \widehat{\theta} \right)^2 \tag{24}$$

where $\widetilde{\theta}$ denotes the true values of parameters, $\widehat{\theta}$ denotes the estimated values of parameters, and M_s denotes the numbers of simulation.

Table 2 shows that the AIC value of M0 is smaller than the others, which indicates a good fitness. The MTTF is closer to the real value, reflecting its high precision in life estimation. From Table 3, it can be observed that the proposed parameter estimation method has high precision under the results of multiple simulations and can be used to estimate the parameters of proposed model reliably. The reliability curve of each model is shown in Figure 2. It can be observed that the fitting degree of M0 to real model is the highest, and this proves the correctness of proposed method in CSADT.

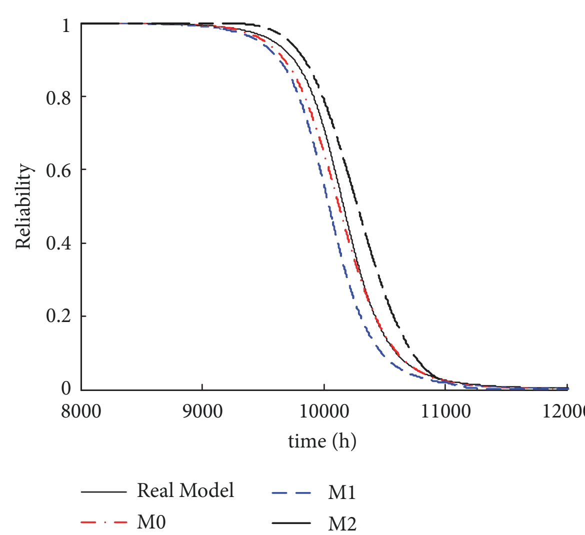

Figure 2: Comparison of reliability of CSADT.

Table 4: Simulation data of SSADT.

Number	Time/h											
	100	200	300	400	500	600	700	800	900	1000	1100	1200
1	3.810	6.114	8.102	9.399	13.245	15.609	17.547	18.807	22.559	24.889	26.919	28.205
2	4.806	7.028	8.936	10.471	15.218	17.370	19.231	20.684	25.538	27.801	29.664	31.129
3	4.437	6.312	7.985	9.758	14.175	16.139	17.856	19.712	24.078	25.995	27.625	29.443
4	4.223	6.170	8.851	10.568	14.750	16.608	19.131	20.794	24.923	26.851	29.522	31.214
5	4.053	6.824	9.631	10.831	14.880	17.652	20.323	21.475	25.584	28.393	31.043	32.201
6	4.675	6.689	8.557	10.116	14.826	16.888	18.799	20.312	24.962	26.937	28.719	30.228
7	3.938	6.082	8.075	9.642	13.606	15.811	17.792	19.352	23.227	25.462	27.431	29.011
8	4.133	6.349	8.330	10.001	14.131	16.215	18.135	19.838	23.896	26.120	28.052	29.788

Table 5: Comparison of three models in SSADT.

Model	α_0	α_1	δ	γ	ρ	σ	Log-LF	AIC	MTTF/h
Real	-2.172	1.342	1.415	2.716	0.623	0.050	-184.951	381.902	10362.514
M0	-2.041	1.423	1.191	2.842	0.584	0.046	-190.514-	393.028	10410.325
M1	-2.304	1.418	1.251	2.638	0.715	—	-198.230	406.460	10223.514
M2	-2.410	1.806	—	—	0.493	0.068	-204.244	416.488	10537.241

5.2. SSADT. The simulation study for SSADT is conducted in this section. Assume that units are tested in three accelerated stresses at 60°C, 80°C, and 100°C in sequence and the working stress is 40°C. The failure threshold $\omega = 35$ and the measurement times of each unit are equal at each accelerated stress. The measurement interval is the same, 100h. 8 groups of the degenerate data are simulated in SSADT, and the measurement times are set to be 12 times, 4 times for each stress. The degradation data and the measurement times are shown in Table 4.

By adopting the parameter estimation method proposed above, the results of the parameter estimation of simulation data in SSADT are obtained and shown in Table 5, and compared with the other two models. According to the analysis of Log-LF, AIC, and MTTF, it can be observed that the parameter estimation precision of model M0 is higher and the value of AIC is lower, which reflects a better fitness under the consideration of individual difference and measurement error. For M1, since the measurement errors cannot be considered, the parameter estimation precision of it is reduced and the goodness of fit is lower than that of model M0. Since in the model M2 only the measurement errors are considered and it cannot randomize the parameters, its accuracy of life estimation and fitness are reduced. This is the same as the results from the simulation study of CSADT. It can be observed from the MTTF of the three models that the life estimation of M0 is closer to the one from the real model compared to M1 and M2. The same results can be observed from the reliability curves of three models, as shown in Figure 3.

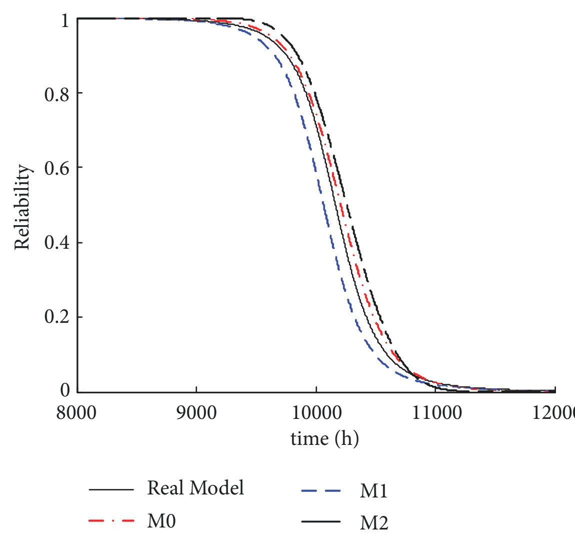

Figure 3: Comparison of reliability of SSADT.

6. Illustrative Example

Stress relaxation is the phenomenon that the strain on a component remains unchanged while the stress gradually decreases. With the increase of working temperature, the contact of electrical connectors often fails due to stress relaxation. Yang [20] verified that the IG process has a good fitting effect in the degradation test of electrical connectors. In this section, the case of stress relaxation test [20] is used for modeling analysis. Set the time function $\Lambda(t) = t^{\rho}$ and the failure threshold $\omega = 30$. The normal working temperature stress is 40°C, and three accelerated temperature stresses are 65°C, 85°C, and 100°C, respectively. The experimental data and the measurement time are shown in Tables 6 and 7. The degradation trend of stress relaxation is shown in Figure 4.

For validating the superiority of the proposed method in the modeling of stress relaxation of electrical connectors, the three proposed models are evaluated by a combination of Log-LF and AIC, as shown in Table 8. And to validate goodness of fit of IG model for the stress relaxation of

Table 6: The stress relaxation data.

Temperature	ID	Stress Relaxation/%										
65°C	1	2.12	2.7	3.52	4.25	5.55	6.12	6.75	7.22	7.68	8.46	9.46
	2	2.29	3.24	4.16	4.86	5.74	6.85	*	7.4	8.14	9.25	10.55
	3	2.4	3.61	4.35	5.09	5.5	7.03	8.24	8.81	9.629	10.27	11.11
	4	2.31	3.48	5.51	6.2	7.31	7.96	8.57	9.07	10.46	11.48	12.31
	5	3.14	4.33	5.92	7.22	8.14	9.07	9.44	10.09	11.2	12.77	13.51
	6	3.59	5.55	5.92	7.68	8.61	10.37	11.11	12.22	13.51	14.16	15
85°C	7	2.77	4.62	5.83	6.66	8.05	10.61	11.2	11.98	13.33	15.64	
	8	3.88	4.37	6.29	7.77	9.16	9.9	10.37	12.77	14.72	16.8	
	9	3.18	4.53	6.94	8.14	8.79	10.09	11.11	14.72	16.47	18.66	
	10	3.61	4.37	6.29	7.87	9.35	11.48	12.4	13.7	15.37	18.51	
	11	3.42	4.25	7.31	8.61	10.18	12.03	13.7	15.27	17.22	19.25	
	12	5.27	5.92	8.05	9.81	12.4	13.24	15.83	17.59	20.09	23.51	
100°C	13	4.25	5.18	8.33	9.53	11.48	13.14	15.15	16.94	18.05	19.44	
	14	4.81	6.16	7.68	9.25	10.37	12.4	15	16.2	18.24	20.09	
	15	5.09	7.03	8.33	10.37	12.22	14.35	16.11	18.7	19.72	21.66	
	16	4.81	7.5	9.16	10.55	13.51	15.55	16.57	19.07	20.27	22.4	
	17	5.64	6.57	8.61	12.5	14.44	16.57	18.7	21.2	22.59	24.07	
	18	4.72	8.14	10.18	12.4	15.09	17.22	19.16	21.57	24.35	26.2	

Table 7: The measurement time under three temperatures.

Temperature	Measurement time epochs/h										
65°C	108	241	534	839	1074	1350	1637	1890	2178	2513	2810
85°C	46	108	212	408	632	764	1011	1333	1517	2586	
100°C	46	108	212	344	446	626	729	927	1005	1218	

Table 8: Comparison of three models.

Model	α_0	α_1	δ	γ	ρ	σ	Log-LF	AIC
M0	-2.261	1.715	2.152	3.816	0.449	0.083	-170..373	352.746
M1	-2.291	1.868	2.022	3.761	0.492	—	-200.041	410.082
M2	-2.518	2.272	—	—	0.459	0.085	-180.725	371.45

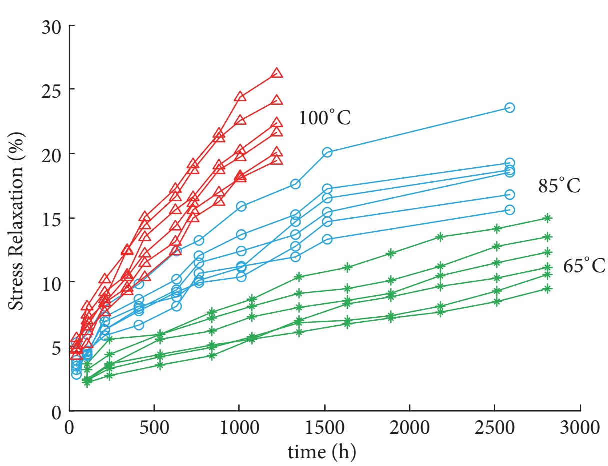

Figure 4: Degradation trend of stress relaxation.

electrical connectors, the quantile-quantile plot (Q-Q plot) is shown in Figures 5–7.

From Table 8, it can be observed that comparing with the models M1 and M2, M0 has a higher Log-LF and a smaller AIC in the three stress level CSADT, which indicates that the model considering measurement errors and individual differences have better fitting performance than the other two under three stress levels. Through the comparison of Q-Q plot, it can also be observed that the quantile of model M0 displays a good linearity to test data, which reflects a good fitness compared with the other two models under three stress levels. Thus, it is meaningful to consider individual differences and measurement errors simultaneously.

Through the reliability curve of three IG models, the advantage of the proposed method in reliability analysis can be seen more intuitively, as shown in Figure 8. It can be observed that the model M1 evaluates reliability conservatively compared with M0, and the maintenance time is estimated early, which leads to lower utilization of product. The model M2 has a high reliability but may lead to the lag of maintenance and increase the risk of early failure of

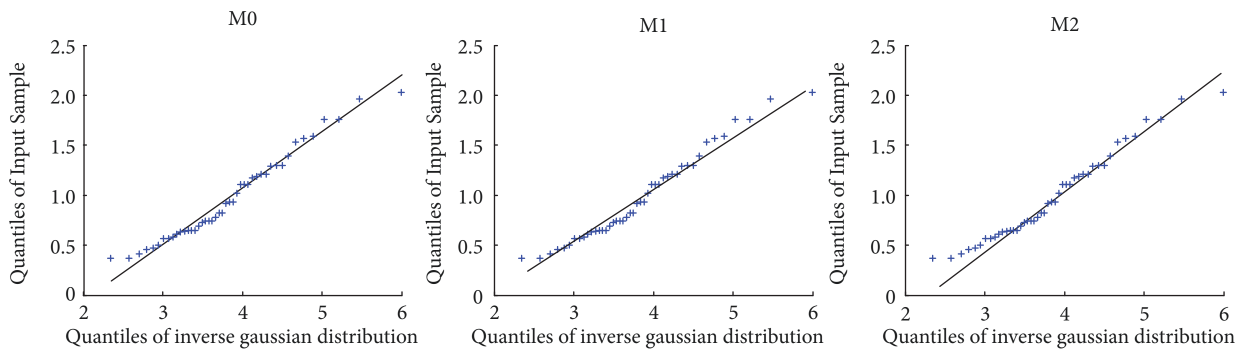

Figure 5: Q-Q plot (65°C).

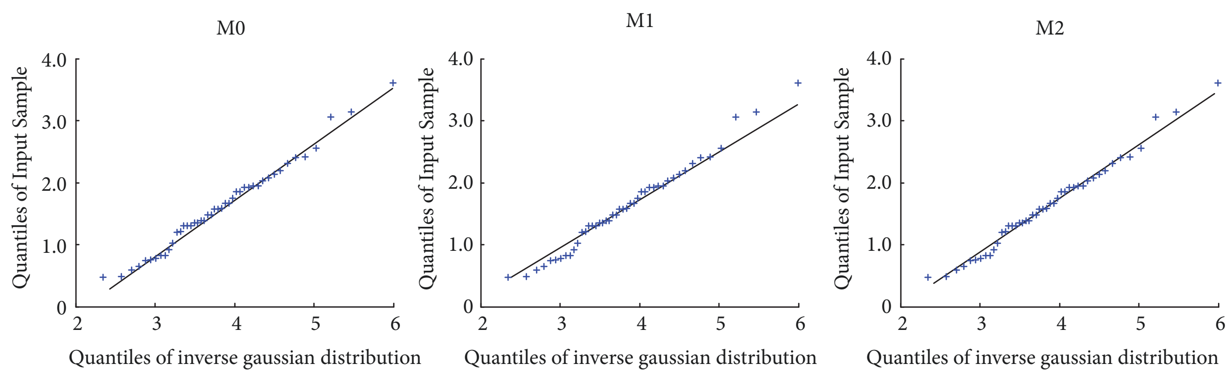

Figure 6: Q-Q plot (85°C).

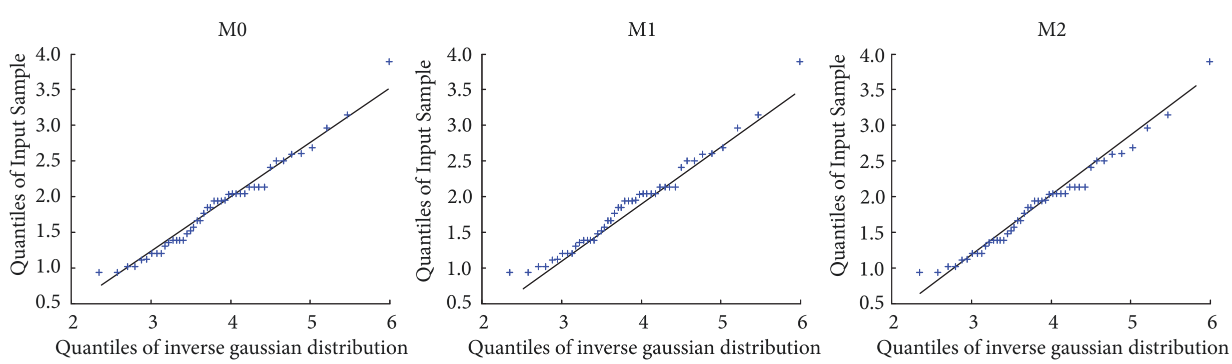

Figure 7: Q-Q plot (100°C).

product. Therefore, it is reasonable to evaluate the reliability of products via the model M0.

7. Conclusion

In this paper, considering the influence of individual differences and measurement errors of life estimation in ADT, the errors are incorporated into the IG process with stochastic parameters. The IG degradation model considering individual differences and measurement errors simultaneously is proposed in ADT. Considering the characteristics of the unobservable measurement errors and the complex integral problem in the process of MLE, a MLE method is proposed by combining the Monte Carlo integral with GA, and the method has been verified to be accurate and reliable in the simulation study. The simulations study towards CSADT and SSADT have verified the correctness of the proposed model considering individual differences and measurement errors in the fitting degree of the degradation trajectory and the life estimation. And the illustrative example of electrical connector in CSADT has also demonstrated the efficiency and reliability of the proposed model.

Based on the research in this paper, the following points should be further studied: (1) it is worth investigating the analytical expression of life estimation using IG process considering individual differences and measurement errors simultaneously, based on the FHT concept; more details are given in [21]; (2) it may be of interest to reduce the

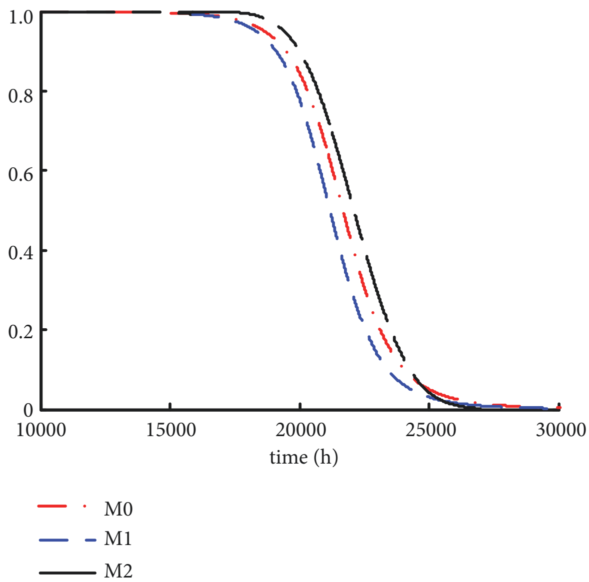

Figure 8: Comparison of reliability.

effect of measurement errors by the filtering method; more details can be found in [13]; (3) it is significant to establish an effective maintenance plan based on proposed model in actual engineering; (4) a further study should focus on accelerated degradation tests considering measurement errors when both the mean and scale parameters are stress dependent; (5) considering that there are still some deviations between the estimated parameters and the real parameters of the model, it may be of interest to consider the interval estimation of parameters.

Data Availability

The simulation study and illustrative example data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare no conflicts of interest.

Acknowledgments

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