

Research Article

Influence of the Static Pre-Stress in Micro-Viscothermoelastic Resonators Based on Dual-Phase-Lagging Heat Conduction

Najat A. Alghamdi ¹ and Hamdy M. Youssef ^{2,3}

¹Mathematics Department, Faculty of Applied Science, Umm Al-Qura University, Makkah, Saudi Arabia

²Department of Mathematics, Faculty of Education, Alexandria University, Alexandria, Egypt

³Mechanical Engineering Department-College of Engineering and Islamic Architecture, Umm Al-Qura University, Makkah, Saudi Arabia

Correspondence should be addressed to Najat A. Alghamdi; najatalghamdi@gmail.com

Received 29 January 2020; Revised 10 April 2020; Accepted 25 June 2020; Published 27 July 2020

Academic Editor: Fumihiko Ashida

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Thermal and mechanical relaxation times play vital roles in the values of the quality factor of micro/nanoresonators. They can control the energy dissipation across the coupling of mechanical and thermal behavior. In this paper, we introduce an analytical model that considers a pre-stress in a micro-viscothermoelastic resonator to modify the thermal and mechanical relaxation times and thus higher the quality factor. The impacts of length scale and static pre-stress on the quality factor have been discussed. The model expects that significant improvement in terms of quality factors is possible by tuning the pre-stress and the thermal and mechanical relaxation times parameters, and the isothermal value of frequency have significant effects on the thermal quality factor of the resonators.

1. Introduction

Many applications based on microelectromechanical resonators are essential in different fields, such as mechanical signal processing, scanning probe microscopes, and ultra-sensitive mass detection. The most critical parameter of micro-viscothermoelastic resonators is the quality factor Q . Higher Q factor indicates less energy. The study of the energy dissipation mechanism is significant for the improvement of the design of micro/nanoelectromechanical resonators [1–4].

Zener [5–7] is the first who introduced the Q -factor in thermoelastic dissipation, and he gave an approximate analytical form of it. He has studied the thermoelastic damping in beams by treating the viscoelastic material. Many research studies have been carried out on the thermoelastic damping in which the classical theory of thermoelasticity based on the Fourier heat law of heat conduction has been used [8]. Lifshitz and Roukes [9] introduced an exact expression for thermoelastic damping. Sun et al. [10] study thermoelastic damping of a beam resonator based on a non-Fourier heat

equation. Sharma and Sharma [11] studied damping in micro-scale circular plate resonators using the Lord-Shulman theory of generalized thermoelasticity theory (L - S).

Tzou [12, 13] proposed a mathematical model to study heat conduction known as dual-phase-lag (DPL). This model established the temperature gradient and heat flux. Many scientists used this model in heat transfer problems [8, 14], physical systems [15–19], and thermoelastic damping vibration [20, 21]. Guo et al. [22, 23] studied the thermoelastic damping theory of micro- and nanomechanical resonators by using the DPL model.

The study of viscoelastic materials has become essential in mechanics. Biot [24, 25] discussed the theory of viscothermoelasticity in thermodynamics. A thermoviscoelasticity model of polymers at finite strains was derived by Drozdov [26]. A new model of thermoviscoelasticity for isotropic media was established by Ezzat and El-Karmany [27].

As the size of a flexural resonator is reduced, its natural frequency increases and thermoelastic damping also

increases in the process. The natural frequency of beams can also be changed by the application of an axial force [3, 28]. A compressive force decreases in the natural frequency, whereas a tensile force increases in the natural frequency [8]. Experimental results have been utilizing the frequency change to tune resonators. Furthermore, these experiments also suggest an increase in the Q -factor with the application of tensile stress [3, 29].

In the present study, we present an analytical model that considers a pre-stress in a micro-viscothermoelastic resonator to quantify the quality factor Q . We also analyze the results with respect to the resonator size and pre-stress and then compare the analytical results with the experimental results available in the literature.

2. Basic Equations and Model Formulation

According to Zener's model of a linear anelastic solid, the stress equation takes the following form [6]:

$$\sigma = \sigma_0 + \sigma_1 e^{i\omega t}, \quad (1)$$

where σ_0 is the static pre-stress, and the second part represents the dynamic-stress due to the ensuing vibration and ω is the natural frequency.

The resultant strain for a thermoelastic solid is given as [6, 8]

$$\varepsilon = \frac{\sigma_0}{E_R} - \frac{\sigma_0}{E_2} e^{(-t/\tau_\sigma)} + \varepsilon_1 e^{i\omega t}. \quad (2)$$

Making derivatives of equations (1) and (2) with respect to time and inserting them into the equation for a thermoelastic solid are given as follows:

$$\sigma + \tau_\varepsilon \dot{\sigma} = E_R (\varepsilon + \tau_\sigma \dot{\varepsilon}), \quad (3)$$

where E_R is relaxed elastic modulus and τ_ε and τ_σ are constants strain and constant stress relaxation time constants; we obtain

$$\begin{aligned} \sigma_0 + \sigma_1 e^{i\omega t} + \tau_\varepsilon (i\omega \sigma_1 e^{i\omega t}) = E_R \left[\frac{\sigma_0}{E_R} - \frac{\sigma_0}{E_2} e^{(-t/\tau_\sigma)} + \varepsilon_1 e^{i\omega t} \right. \\ \left. + \tau_\sigma \left(\frac{1}{\tau_\sigma} \frac{\sigma_0}{E_2} e^{(-t/\tau_\sigma)} + i\omega \varepsilon_1 e^{i\omega t} \right) \right]. \end{aligned} \quad (4)$$

All the terms due to static stress σ_0 get canceled from both sides. That results in static stress independence of Zener's thermoelastic solid model. This model assumes no contribution of a stress field to the thermal relaxation time in solids. Thus, we decided to follow Lifshitz–Roukes' approach to the exact solution for a pre-stressed beam resonator. The constitutive relationship for Euler–Bernoulli's beam with dimensions length L , width b , and depth h along the x , y , and z -axes, respectively, the moment of inertia I , and applied axial force F and flexural displacement $w(x, y, z, t)$ can be written as follows:

$$\sigma_{xx} = \sigma_0 - Ey \frac{\partial^2 w}{\partial x^2} - E\alpha_T \theta, \quad (5)$$

where $\sigma_0 = (F/bh)$ is the stress due to the applied axial force, E is Young's Modulus, and α_T is the coefficient of thermal expansion. The equilibrium temperature of the beam is T_0 , and the ensuing vibration will result in a temperature field $T = T_0 + \theta$.

By deriving the equation of motion for flexural vibration, we obtain

$$EI \frac{\partial^4 w}{\partial x^4} - F \frac{\partial^2 w}{\partial x^2} + E\alpha_T \frac{\partial^2 I_T}{\partial x^2} + \rho A \frac{\partial^2 w}{\partial t^2} = 0, \quad (6)$$

where

$$I = \int_{(-b/2)}^{(b/2)} \int_{(-h/2)}^{(h/2)} y^2 dy dz = \frac{bh^3}{12}, \quad (7)$$

$$I_T = \int_{(-b/2)}^{(b/2)} \int_{(-h/2)}^{(h/2)} y \theta dy dz. \quad (8)$$

The dual-phase-lag heat equation takes the form

$$K \left(1 + \tau_T \frac{\partial}{\partial t} \right) \nabla^2 \theta = \left(\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} \right) \left(\rho C_v \theta + \frac{E\alpha_T T_0}{(1-2\nu)} \varepsilon \right), \quad (9)$$

where K is the thermal conductivity, ν is Poisson's ratio, C_v is the specific heat at constant strain, and τ_q and τ_T are the thermal relaxation of the temperature and its gradient, respectively.

The volumetric deformations are as follows:

$$\varepsilon = \varepsilon_{xx} + \varepsilon_{yy} + \varepsilon_{zz}, \quad (10)$$

where

$$E\varepsilon_{xx} = \sigma_0 - yE \frac{\partial^2 w}{\partial x^2}, \quad (11)$$

$$E\varepsilon_{yy} = E\varepsilon_{zz} = -\nu\sigma_0 + \nu yE \frac{\partial^2 w}{\partial x^2} + (1+\nu)\alpha_T E\theta.$$

Then, we have

$$E\varepsilon = (1-2\nu)\sigma_0 - (1-2\nu)yE \frac{\partial^2 w}{\partial x^2} + 2(1+\nu)\alpha_T E\theta. \quad (12)$$

Substituting from equation (11) into equation (9), we obtain

$$\begin{aligned} \left(1 + \tau_T \frac{\partial}{\partial t} \right) \nabla^2 \theta = \eta \left(\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} \right) \theta + \frac{\alpha_T T_0}{K} \left(\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} \right) \\ \cdot \left[2 \frac{(1+\nu)\alpha_T}{(1-2\nu)} E\theta - yE \frac{\partial^2 w}{\partial x^2} \right], \end{aligned} \quad (13)$$

which gives

$$\begin{aligned} \left(1 + \tau_T \frac{\partial}{\partial t} \right) \nabla^2 \theta + \frac{\alpha_T T_0 E}{K} y \left(\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} \right) \frac{\partial^2 w}{\partial x^2} \\ = \left(\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} \right) \left(\eta + \frac{2ET_0\alpha_T^2(1+\nu)}{K(1-2\nu)} \right) \theta, \end{aligned} \quad (14)$$

where $\eta = (\rho C_v/K)$.

For viscothermoelastic materials, we consider Young's modulus in the form [24, 27]

$$E = E_0 \left(1 + E_1 \frac{\partial}{\partial t} \right), \quad (15)$$

where E_0 is Young's modulus value for the usual case, while E_1 is the mechanical relaxation time (relaxation time means the return of a perturbed system into equilibrium or the time between repeating mechanical waves).

Hence, equation (14) takes the form

$$\begin{aligned} & \left(1 + \tau_T \frac{\partial}{\partial t} \right) \nabla^2 \theta + \frac{\Delta_E}{\alpha_T} \left(1 + E_1 \frac{\partial}{\partial t} \right) \left(\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} \right) y \frac{\partial^2 w}{\partial x^2} \\ & = \left(\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} \right) \left(\eta + 2\Delta_E \frac{(1+\nu)}{(1-2\nu)} \left(1 + E_1 \frac{\partial}{\partial t} \right) \right) \theta, \end{aligned} \quad (16)$$

where $\Delta_E = (T_0 \alpha_T^2 E_0 / K)$.

Because no gradient exists in the z -direction, then $\nabla^2 \theta = (\partial^2 \theta / \partial y^2)$; therefore, we have

$$\begin{aligned} & \left(1 + \tau_T \frac{\partial}{\partial t} \right) \frac{\partial^2 \theta}{\partial y^2} + \frac{\Delta_E}{\alpha_T} y \left(1 + E_1 \frac{\partial}{\partial t} \right) \left(\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} \right) \frac{\partial^2 w}{\partial x^2} \\ & = \left(\frac{\partial}{\partial t} + \tau_q \frac{\partial^2}{\partial t^2} \right) \left(\eta + 2\Delta_E \frac{(1+\nu)}{(1-2\nu)} \left(1 + E_1 \frac{\partial}{\partial t} \right) \right) \theta. \end{aligned} \quad (17)$$

We consider the following functions:

$$\begin{aligned} w(x, t) &= W(x) e^{i\omega t}, \\ \theta(x, y, t) &= \Theta(x, y) e^{i\omega t}. \end{aligned} \quad (18)$$

Hence, equation (17) will be in the form

$$\begin{aligned} & (1 + i\tau_T \omega) \frac{\partial^2 \Theta}{\partial y^2} + \frac{\Delta_E}{\alpha_T} y (1 + iE_1 \omega) (i\omega - \tau_q \omega^2) \frac{d^2 W}{dx^2} \\ & = (i\omega - \tau_q \omega^2) \left(\eta + 2\Delta_E \frac{(1+\nu)}{(1-2\nu)} (1 + iE_1 \omega) \right) \Theta, \end{aligned} \quad (19)$$

which gives

$$\left(\frac{\partial^2}{\partial y^2} - \lambda^2 \right) \Theta(x, y) = -\alpha y \frac{d^2 W(x)}{dx^2}, \quad (20)$$

where

$$\begin{aligned} \lambda &= \lambda(\omega) = \sqrt{\frac{(i\omega - \tau_q \omega^2)}{(1 + i\tau_T \omega)} \left(\eta + 2\Delta_E \frac{(1+\nu)}{(1-2\nu)} (1 + iE_1 \omega) \right)}, \\ \alpha &= \alpha(\omega) = \frac{\Delta_E}{\alpha_T} \frac{(1 + iE_1 \omega)(i\omega - \tau_q \omega^2)}{(1 + i\tau_T \omega)}. \end{aligned} \quad (21)$$

The general solution of the differential equation (20) takes the form

$$\Theta(x, y) = A \cosh(\lambda y) + B \sinh(\lambda y) + \frac{\alpha}{\lambda^2} y \frac{d^2 W(x)}{dx^2}. \quad (22)$$

The boundary conditions are as follows:

$$\frac{\partial \Theta(x, y)}{\partial y} \Big|_{y=0} = \frac{\partial \Theta(x, y)}{\partial y} \Big|_{y=h} = 0. \quad (23)$$

Hence, we obtain

$$\Theta = \left[y - \frac{\sinh(\lambda y)}{\lambda \cosh((\lambda h/2))} \right] \frac{\alpha}{\lambda^2} \frac{d^2 W}{dx^2}. \quad (24)$$

From equations (6), (8), and (18), we obtain

$$\begin{aligned} & \frac{E_0(1 + i\omega E_1)I}{\rho A} \frac{\partial^4 W(x)}{\partial x^4} + \frac{E_0(1 + i\omega E_1)\alpha_T}{\rho A} \frac{\partial^2}{\partial x^2} \\ & \cdot \int_{(-b/2)}^{(b/2)} \int_{(-h/2)}^{(h/2)} y \Theta(x, y) dy dz - \frac{F}{\rho A} \frac{\partial^2 W(x)}{\partial x^2} = \omega^2 W(x). \end{aligned} \quad (25)$$

Substituting from equation (24) in equation (25), we obtain

$$\begin{aligned} & \frac{E_0(1 + i\omega E_1)I}{\rho A} \frac{\partial^4 W(x)}{\partial x^4} + \frac{E_0(1 + i\omega E_1)\alpha_T \alpha}{\rho A \lambda^2} \frac{\partial^4 W(x)}{\partial x^4} \int_{(-b/2)}^{(b/2)} \\ & \cdot \int_{(-h/2)}^{(h/2)} y \left[y - \frac{\sin(\lambda y)}{\lambda \cos(\lambda h/2)} \right] dy dz - \frac{F}{\rho A} \frac{\partial^2 W(x)}{\partial x^2} = \omega^2 W(x). \end{aligned} \quad (26)$$

Finally, we obtain

$$\frac{IE_\omega}{\rho A} \left(\frac{\partial^4 W(x)}{\partial x^4} - \frac{F}{IE_\omega} \frac{\partial^2 W(x)}{\partial x^2} \right) = \omega^2 W(x), \quad (27)$$

where $E_\omega = E_0(1 + i\omega E_1)f(\omega)$ and $f(\omega) = 1 + (\alpha_T b \alpha / \lambda^2 I)((h^3/12) + (h/\lambda^2) - (2/\lambda^3)\tan((\lambda h/2)))$.

For a simply supported beam, the exact analytical solution for the natural frequency is given as [28]

$$\omega = \frac{\pi}{L\sqrt{\rho A}} \sqrt{\frac{\pi^2 IE_0}{L^2} (1 + i\omega_0 E_1) f(\omega_0) + F}, \quad (28)$$

where ω_0 is the isothermal value of fundamental frequency given by [10, 30]

$$\omega_0 = q^2 \frac{b}{L^2} \sqrt{\frac{E_0}{12\rho}}. \quad (29)$$

For beams with both ends clamped, $q = 4.73$, while $q = \pi$ for beams with both ends simply support.

Hence, we obtain

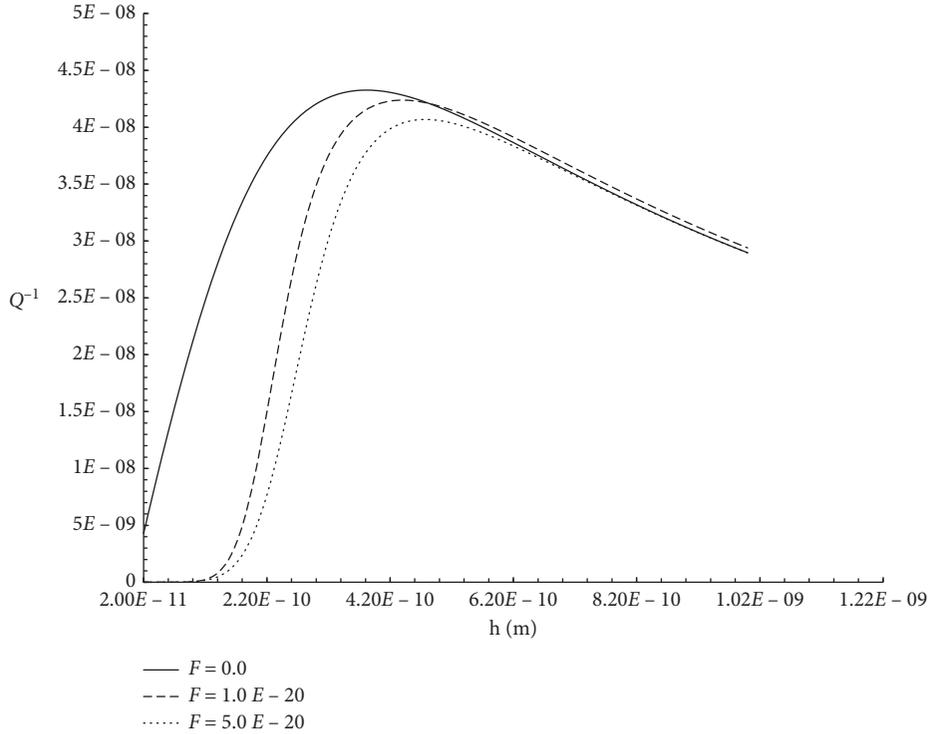


FIGURE 1: The quality factor of the resonator with a different value of static pre-stress.

$$\omega = \frac{\pi^2}{L^2} \sqrt{\frac{IE_0}{\rho A}} \sqrt{\left(1 + iE_1\omega_0\right) \left[1 + \frac{\Delta E b h^3 (1 + iE_1\omega_0)}{12IE_1}\right] + \left(\frac{12\varepsilon_2}{h^2\varepsilon_1} - \frac{24}{h^3} \tanh\left(\frac{\lambda h}{2}\right) \left(\frac{\varepsilon_1}{1 + i\tau_T\omega_0}\right)^{-(3/2)}\right) + \frac{L^2}{\pi^2 I^2 E_0} F}, \quad (30)$$

where $\varepsilon_1 = \eta + (2\Delta E(1 + \nu)(1 + iE_1\omega_0)/1 - 2\nu)$ and $\varepsilon_2 = (1 + i\tau_T\omega_0/i\omega_0 - \tau_q\omega_0^2)$

The quality factor (Q^{-1}) is defined as follows:

$$Q^{-1} = 2 \frac{|\text{Im}(\omega)|}{|\text{Re}(\omega)|}. \quad (31)$$

3. Numerical Results and Discussion

Now, the relationships between the variation of the thermoelastic damping with the beam height h , and the beam length in different values of the applied axial force F on the microbeam resonator, which is made of silicon and clamped at two ends, will be explored.

Material properties of silicon (Si) have been taken as follows [10, 30]:

$T_0 = 300$ (K), $\rho = 2330$ (kgm^{-3}), $K = 141.04$ ($\text{Wm}^{-1}\text{k}^{-1}$), $C_v = 1.64 \times 10^6$ ($\text{Jm}^{-3}\text{K}^{-1}$), $E = 165$ (GPa), and $\alpha = 2.6 \times 10^{-6}$ (K^{-1}). The aspect ratios of the beam are fixed as $(L/h) = 50$ and $b = 5h$. For the micro-scale beam, we will take the beam's thickness h ($10^{-11}\text{m} \leq h \leq 10^{-9}\text{m}$).

Figure 1 represents the quality factor of the resonator with a different value of the force $F = (0.0, 1.0, 5.0) \times 10^{-20}$ Kgms^{-2} of the static pre-stress. It is noted that the force due to the static pre-stress has a significant effect on the quality factor (Q^{-1}). The value of the quality factor (Q^{-1})

decreases when the value of static pre-stress increases until the position $h = 6.0 \times 10^{-10}\text{m}$; then, the three cases are very closed. In Figure 1, for the case $F = 0.0$ Kgms^{-1} , the curve has the same attitude of the curves of Figure 3 in [30], Figure 2 in [31], and Figure 8 in [32].

Figure 2 represents the inverse of the quality factor (Q^{-1}) of the resonator with a different value of the mechanical relaxation time $E_1 = (0.0, 10.0, 20.0) \times 10^{-22}\text{s}$ of the static pre-stress. It is noted that the value of mechanical relaxation time has a significant effect on the quality factor. In essence, the value of the inverse of the quality factor (Q^{-1}) decreases when the value of the mechanical relaxation time increases. In Figure 2, for the case $E_1 = 0.0$ s, the curve agrees with the curves of Figure 3 in [30], Figure 2 in [31], and Figure 8 in [32].

Figure 3 represents the quality factor (Q^{-1}) of the resonator with a different value of the isothermal value of frequency $\omega_0(q)$, where $q = (\pi, 4.73, 7.85)$. It is noted that the value of the isothermal value of frequency has a significant effect on the quality factor (Q^{-1}) of the thermal damping; the value of the quality factor (Q^{-1}) increases when the isothermal value of frequency decreases.

Figure 4 represents the inverse of the quality factor (Q^{-1}) of the resonator with a different value of the ratio of the thermal relaxation times $\tau_q/\tau_T = (20, 25, 30)$, respectively. It is noted that the values of the ratio of the thermal

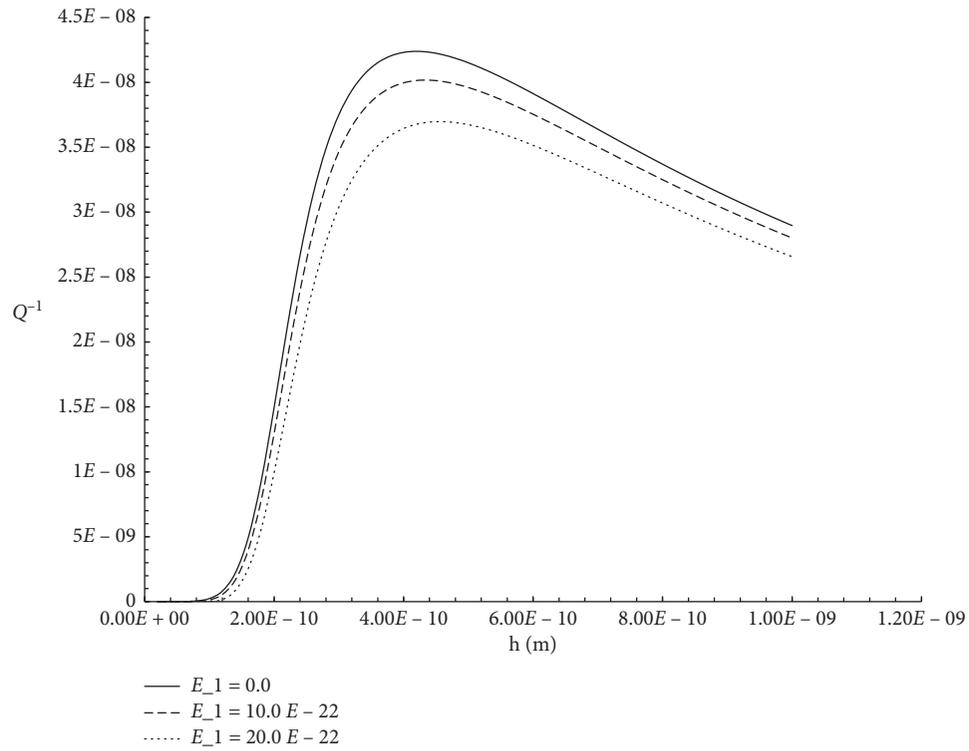


FIGURE 2: The quality factor of the resonator with a different value of mechanical relaxation time parameter.

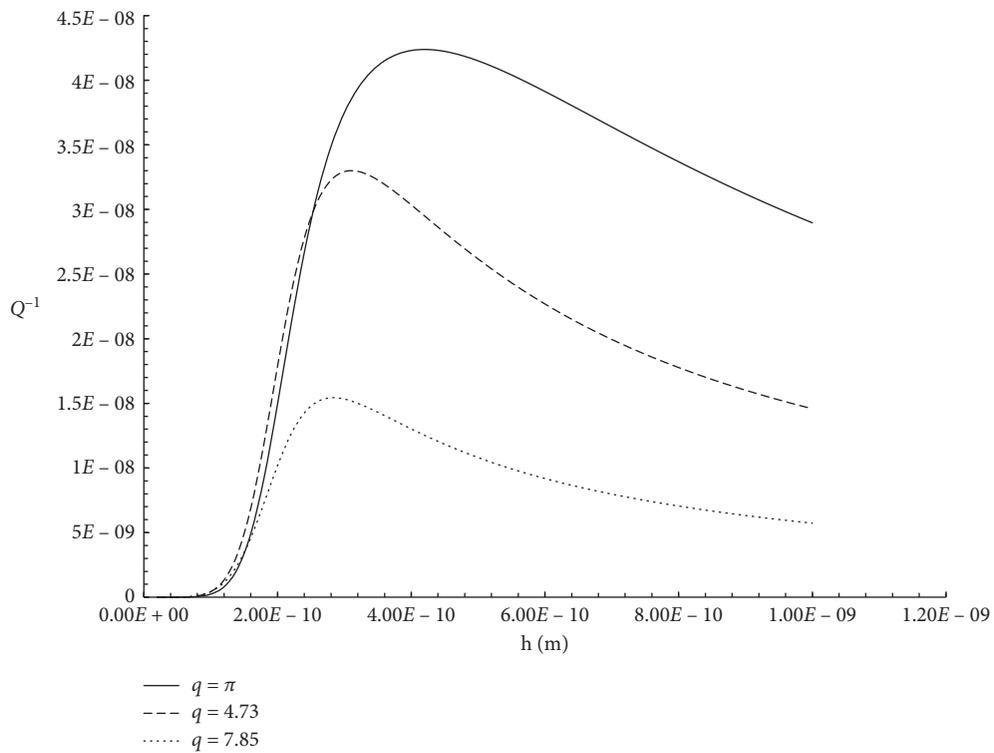


FIGURE 3: The quality factor of the resonator with a different value of the isothermal value of frequency $\omega_0(q_n)$.

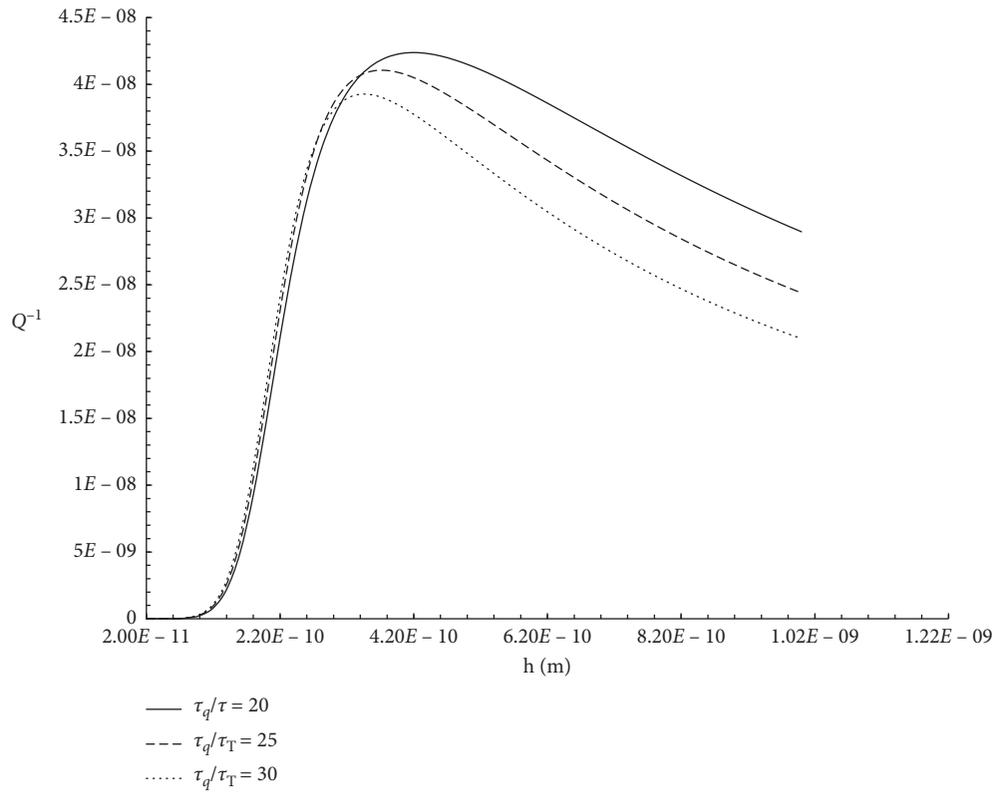


FIGURE 4: The quality factor of the resonator with a different ratio of thermal relaxation times.

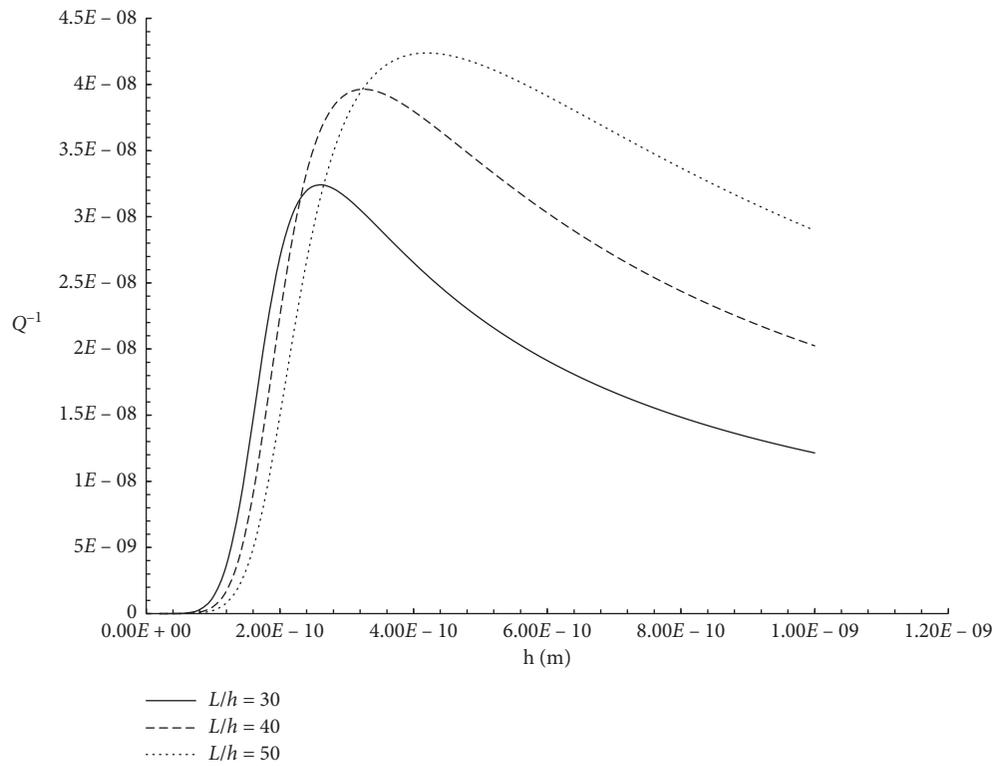


FIGURE 5: The quality factor of the resonator with a different value of size ratio L/h .

relaxation times have a significant effect on the quality factor (Q^{-1}) of the thermal damping. The value of the quality factor (Q^{-1}) decreases when the value of the ratio τ_q/τ_T increases. In Figure 4, the curves agree with the curves of Figure 3 in [30], Figure 2 in [31], and Figure 8 in [32].

Figure 5 represents the quality factor (Q^{-1}) of the resonator with a different value of the size ratio ($L/h = (30, 40, 50)$), respectively. It is noted that the values of the size ratio have a significant effect on the quality factor (Q^{-1}); the value of the quality factor (Q^{-1}) increases when the value of the size ratio L/h increases.

4. Conclusion

An analytical model for viscothermoelastic relaxation times in flexural resonators under axial pre-stress has been developed in the micro-scale. The model confirms that a significant reduction in the quality factor as a tensile axial pre-stress is applied. This reduction in damping can be correlated with a decrease in the relaxation rate as compared to the natural frequency of the resonator. The force due to the static pre-stress, isothermal frequency, thermal relaxation times, mechanical relaxation time, and size ratio have a significant effect on the quality factor of the nanobeams.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest in this work.

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