Hindawi
Mathematical Problems in Engineering
Volume 2020, Article ID 1369348, 15 pages
https://doi.org/10.1155/2020/1369348

Research Article

Analytical Model and Back-Analysis for Pile-Soil System Behavior under Axial Loading

Hong-Fa Xu, Ji-Xiang Zhang, Xin Liu, Han-Sheng Geng, Ke-Liang Li, and Yin-Hao Yang

State Key Laboratory of Disaster Prevention & Mitigation of Explosion & Impact, National Defense Engineering College, Army Engineering University of PLA, Nanjing 210007, China

Correspondence should be addressed to Ji-Xiang Zhang; zjixiang@hotmail.com

Received 5 October 2019; Revised 9 February 2020; Accepted 21 February 2020; Published 19 March 2020

Academic Editor: Giovanni Lancioni

The interaction mechanism between piles and soils is very complicated. The load transfer function is generally nonlinear and is affected by factors such as pile side roughness, soil characteristics, section depth, and displacement. Therefore, it is difficult to solve the pile-soil system based on load transfer function. This paper presents a new method to study the soil-pile interaction problem with respect to axial loads. First, the shapes of the axial force-displacement curves at different depths and the displacement distribution curves along pile axis at different pile-top displacements were analyzed. A simple exponential function was taken as relationship model to express the relationship curves between two distribution functions of axial force and displacement along pile shaft obtained by using the geometric drawing method. Second, a new analytical model of the pile-soil system was established based on the basic differential equations for pile-soil load transfer theory and the relationship model and was used to derive the mathematical expressions on the distribution functions of the axial force, the lateral friction, and the displacement along pile shaft and the load transfer function of pile-side. We wrote the MATLAB program for the analytical model to analyze the influence laws of the parameters u and m on the pile-soil system characteristics. Third, the back-analysis method and steps of the pile-soil system characteristics were proposed according to the analytical model. The back-analysis results were in good agreement with the experimental results for the examples. The analysis model provides an effective way for the accurate design of piles under axial loading.

1. Introduction

The pile foundation has the characteristics of high bearing capacity, good stability, small settlement, simple construction, and consumption of low material consumption. It has been widely used in construction engineering, marine engineering, industrial engineering, and other fields. In the study of pile-soil system under vertical load, there are actually two major problems: (a) to estimate the load-displacement curve of pile top by pile-soil parameters and (b) to investigate the system characteristics of pile-soil from pile-top load-displacement curve and soil parameters. The former is a forward-analysis problem, and traditional theories such as Poulos elastic theory method [1], Seed and Reese load transfer method [2], and shear displacement method [3, 4] are typical methods to solve this problem. The latter is a back-analysis problem and the researchers are fewer.

The load transfer method was first proposed by Seed and Reese in 1955 [2]. Since the method is clear in concept and easy to understand, it has been immediately responded by scholars and engineers and has been rapidly developed [5–14]. The basic concept of the method is to assume that the pile-side and the pile-bottom are connected with the soil by springs. The constitutive relationships of the springs represent the load transfer relationship between piles and soils, and it is called the piles-soil load transfer functions. An element body is intercepted at a certain section along the pile axis (see Figure 1). According to the static balance and deformation compatibility of the element body, the corresponding differential equations can be established. Solving

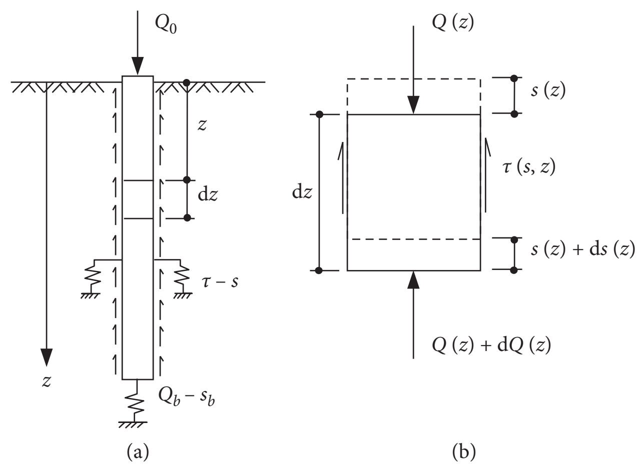

Figure 1: Pile-soil system model. (a) Simple model. (b) A pile element.

the differential equations depends on the pile-soil load transfer functions. Therefore, many scholars have conducted extensive research on transfer functions and proposed various pile-side load function forms. Seed and Reese proposed hyperbolic model as the transfer function [2]. Kezdi proposed an exponential function relating skin friction to strain for the case of piles embedded in sand based on the model tests carried out in the laboratory or in the field [6]. Vijayvergiya [7] used the parabolic model as pile-soil load transfer function to analyze the axial behavior of piles. Armaleh and Desai [8] used the Ramberg–Osgood model to describe the pile side-soil transfer curve and the model provides a general method and includes other special functions, such as the hyperbola. According to the shear displacement theory that Randolph and Wroth proposed, the pile-soil load transfer curve in uniform soil is a linear function [4]. Sato [13] used a linear elastic perfect plastic model to calculate the bearing capacity of pile in multilayered soil. Currently, bilinear model [9] and trilinear model [9, 11] were used to describe the pile side-soil load transfer curve and soften [9, 11] and harden [9] phenomenon. The methods of solving the differential equation vary with the transfer function forms. Generally, for linear or exponential transfer functions, the results of elementary function expressions [9, 11, 13] or series form expressions [6] can be obtained by the analytic method solving the differential equations. For complex nonlinear pile-soil load transfer function, it is very difficult to solve differential equations using analytic method, which is generally solved by numerical methods, such as displacement coordination method [5, 12, 14, 15], finite element method [8], and finite difference method [16].

The back-analysis method of pile and soil characteristics is often used to determine the geotechnical parameters. Danziger et al. [17] carried out plenty of back analyses for field data of offshore piles driven in calcareous sands using an improved soil model developed by Simons and the back analyses applied in a one-dimensional finite element to calculate the relative displacements between pile and soil at various points along the pile. Xiao and Yang [18] used Chin's plot method to back-analyze a large number of driven pile test data to obtain the two key design parameters, skin friction coefficient, and bearing coefficient. McCarron [19] executed the back-analysis of pile tests using the empirical one-dimensional load transfer method and used the model to research the cyclic capacity of the pile under the same conditions.

The complex pile-soil lateral load transfer function is not only related to the nature of the soil, the nature of the pile, the lateral conditions of the pile, the pile forming method, the groundwater, the soil pressure on the pile-side, the distribution of the soil layer, and other factors, but also the function of two independent variables, the depth, and relative displacement between pile and soil at a cross section.

In pile foundation engineering, the designers pay close attention to the moments, shear forces, or axial force and deformation of piles in different depth piles so that it is properly reinforced. It must be obtained through special experiments or complex numerical analysis. None of the above forward- or back-analysis can accurately grasp the characteristics of the pile-soil system, which is quite different from the actual engineering properties. This paper focuses on the vertical forces of pile foundation such as bridge foundations or foundations of bracing systems.

Hence, this paper constructs a new analytical model of the pile-soil system based on the pile-soil load transfer theory and the back-analysis methods. It is a fact that the pile foundation is a concealed project, and most of the available test data which can be directly obtained are load-displacement curve and stratum survey data. It is beneficial to the pile foundation design and will be welcomed by engineering design technicians if the load-displacement curve of the pile top can be used to find the characteristics of the pile-soil system, such as the axial force distribution curve, the side resistance distribution curves, and the load transfer curves.

2. Differential Equation of Pile-Soil Load Transfer

The load transfer method was proposed by Seed and Reese [2]. The pile-soil system can be simplified as shown in

Figure 1(a). The pile-side springs indicate the relationship between the pile-side resistance τ (z) and the shear displacement s (z), which are the pile-side transfer function $(\tau - s)$; the pile-bottom spring indicates the relationship between the pile-bottom resistance Q_b and the pile-bottom displacement s_b, which is the pile-bottom load transfer function $(Q_b - s_b)$.

Figure 1(b) is a microelement body of pile and regardless of the weight of the pile. According to the static balance condition of the element body, we can obtain

$$\mathrm{d}Q(z) = -C\tau(z)\mathrm{d}z = -C\tau(z)\mathrm{d}z, \tag{1}$$

where z is the depth of the element body, Q (z) is the force distribution function along the pile axis, τ (z) is the skin friction of the pile-soil contact surface, and C is the perimeter of the pile section.

By calculating the amount of elastic elongation produced by the element body, it is obtained that

$$\frac{\mathrm{d}s(z)}{\mathrm{d}z} = -\frac{1}{AE_p}Q(z), \tag{2}$$

where E_p is the elastic modulus of the pile, s (z) is the displacement distribution function along the pile axis, and A is the cross-sectional area of the pile.

By taking the derivative of equation (2) and substituting equation (1) into it, the basic differential equation of the load transfer method can be obtained:

$$\frac{\mathrm{d}^2 s(z)}{\mathrm{d}z^2} = \frac{C}{AE_p}\tau(s, z). \tag{3}$$

According to equation (3), the displacement distribution function can be solved only if the pile transfer function τ (s, z) is correctly determined. Since the transfer function is very complicated, it is difficult to express it with a mathematical function. In this paper, through the relationship between the distribution function of axial force and the distribution function of displacement along the pile axis, the differential equations are further solved. Finally, a new mathematical model of the pile-soil system is obtained, which provides a new way for the accurate design of the pile.

3. The Relationship between Two Distribution Functions of Force and Displacement along Pile Axis

Xu et al. [20] used the geometric drawing method to analyze the geometric relationship curves, Q $(z) - s$ (z), between the axial force distribution function along pile shaft, Q (z), and the displacement distribution function along pile shaft, s (z), for the vertical pile buried into soft soil and subjected to axial load. The test has confirmed that the displacement distribution function, s (z), decreases gradually along pile shaft [21]. The three curves in the s-z coordinate system of Figure 2 expressed the law and their corresponding pile-top displacements, s_0, are s_{01}, s_{02}, s_{03}, respectively, and the pile-bottom displacements, s_b, are s_{b1}, s_{b2}, s_{b3}, respectively.

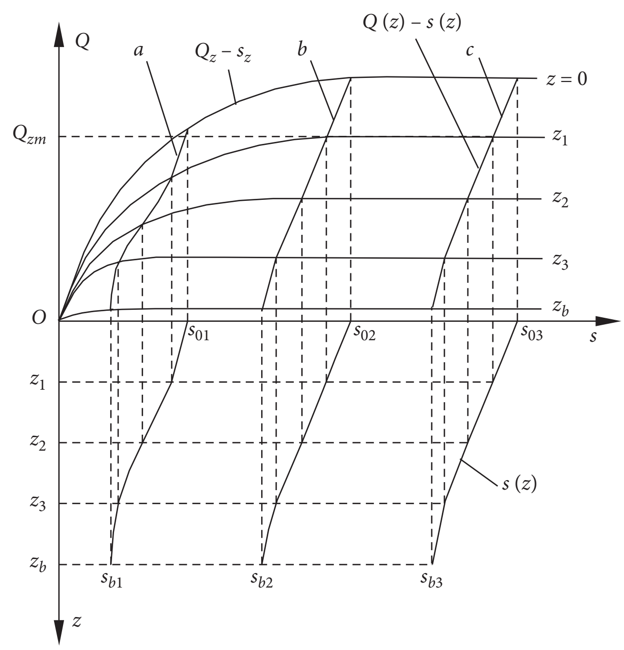

Figure 2: Q $(z) - s$ (z) curve shapes [20].

The test found that the pile-top load-displacement curves, $Q_0 - s_0$, are similar to the models of the hyperbolic or exponential function [23, 24]. Xu et al. synthesized the characteristics of two models and proposed a universally applicable power function model [22]. The relationship between the axial force, Q_z, of the pile section at depth z and the displacement, s_z, is similar to the curve of $Q_0 - s_0$ [6, 24]. The initial stiffness and ultimate axial force of the $Q_z - s_z$ curves decrease gradually with depth increasing. The five curves in the $s - Q$ coordinate system of Figure 2 reflect the rules and their corresponding depths are 0, z_1, z_2, z_3, z_b, where 0 is the pile-top, z_b is the pile-bottom, and $z_1 > z_2 > z_3$ are the pile sections of different depth. The $Q_z - s_z$ curve at $z = 0$ is actually the pile-top load-displacement curve, $Q_0 - s_0$. The $Q_z - s_z$ curve at $z = z_b$ is actually the load transfer function $Q_b - s_b$ of pile-bottom. The model of $Q_z - s_z$ curve was proposed by Xu et al. [22] as follows:

$$Q_z = Q_{zm}\left[1 - \left(1 + \frac{(n_z - 1)K_{zi}}{Q_{zm}}s_z\right)^{(1/(1-n_z))}\right], \tag{4}$$

where K_{zi} is the initial stiffness of the axial force-displacement curve at the depth z, Q_{zm} is the asymptotic value of the ultimate axial force at the depth z, and n_z is the exponent.

Afterwards, we draw vertical lines upwards from any point of displacement distribution curve $s(z)$ and intersect with the corresponding $Q_z - s_z$ curves at depth z corresponding to the same depth. Connect these intersections to get the rough shape of the corresponding $Q(z) - s(z)$ curve corresponding to the same pile-top displacement. The curves a, b, and c in Figure 1 are $Q(z) - s(z)$ curves when the displacement of pile top is s_{01}, s_{02}, and s_{03}. It can be seen that the functions $Q(z)$ and $s(z)$ are related to the displacement of pile-top, s_0.

Xu et al. gave the mathematical expression of $Q(z)-s(z)$ curve [20]. When the pile-top displacement is s_0,

$$Q(z)=\eta[s(z)-b]^u\phi(z), \tag{5}$$

where η and b are parameters related to the displacements of the pile-top and the pile-bottom, $\eta>0$, $0\le b\le s_b$; u is the exponent, $u>0$; and $\phi\ (z)$ is the influence function controlling the curve shape related to pile-top displacement s_0, generally not less than 1 and monotone increasing.

According to the boundary conditions of the pile-top and pile-bottom, it can be obtained from equation (5) that

$$\begin{cases} Q_b=\eta[s_b-b]^u\phi(L),\\ Q_0=\eta[s_0-b]^u\phi(0). \end{cases} \tag{6}$$

Solve equation (6) to obtain the parameters η and b.

$$\begin{cases} b=\dfrac{s_b[Q_0\phi(L)]^{(1/u)}-s_0[Q_b\phi(0)]^{(1/u)}}{Q[U_0\phi(L)]^{(1/u)}-[Q_b\phi(0)]^{(1/u)}},\\ \\ \eta=\dfrac{\left\{[Q_0\phi(L)]^{(1/u)}-[Q_b\phi(0)]^{(1/u)}\right\}^u}{(s_0-s_b)^u\phi(L)\phi(0)}. \end{cases} \tag{7}$$

4. Mathematical Model of Pile-Soil System and Its Analysis

Substituting equation (5) into equation (2),

$$\frac{\mathrm{d}s(z)}{[s(z)-b]^u}=-\frac{\eta}{AE_p}\phi(z)\mathrm{d}z. \tag{8}$$

According to the displacement boundary condition of pile top, we obtain through solving the upper differential equation:

$$[s(z)-b]^{1-u}=\frac{\eta(1-u)}{AE_p}[\Phi(0)-\Phi(z)]+(s_0-b)^{1-u}. \tag{9}$$

where Φ is the original function of the influence function.

Let

$$G(z)=\frac{\eta(1-u)}{AE_p}[\Phi(0)-\Phi(z)]+(s_0-b)^{1-u}. \tag{10}$$

Obviously, $G\ (z)\ge 0$.

From equations (9) and (10),

$$s(z)=G^{(1/(1-u))}(z)+b. \tag{11}$$

The first and second derivatives of equation (11) can be obtained:

$$\frac{\mathrm{d}s(z)}{\mathrm{d}z}=-\frac{\eta}{AE_p}G^{(u/(1-u))}(z)\phi(z), \tag{12}$$

$$\frac{\mathrm{d}^2s(z)}{\mathrm{d}z^2}=\frac{\eta}{AE_p}\left[\frac{u\eta}{AE_p}G^{(2u-1)/(1-u)}(z)\phi^2(z)-G^{u/(1-u)}\phi'(z)\right], \tag{13}$$

where $\phi'(z)$ is the first derivative of the influence function ϕ (z).

By substituting equations (12) and (9) into equations (2) and (3), respectively, the axial force distribution function, Q, and pile skin friction distribution function, τ, can be obtained as follows:

$$Q(z)=\eta G^{(u/(1-u))}(z)\phi(z), \tag{14}$$

$$\tau(z)=\frac{\eta}{C}\left[\frac{u\eta}{AE_p}G^{((2u-1)/(1-u))}\phi^2(z)-G^{(u/(1-u))}\phi'(z)\right]. \tag{15}$$

Xu et al. suggested a specific form of the influence function [20]:

$$\phi(z,s_0)=\left(1+r(s_0)\frac{z}{L}\right)^m, \tag{16}$$

where L is the pile length, r is the influence factor which is related to the displacement of the pile top, and m is the exponent. According to the condition that the influence function is an increasing function, it is easy to obtain a parameter range of ($m<0$ & $-1<r<0$) or ($m>0$ & $r>0$). These two values have the same effect, so the final recommended parameter range is $m>0$ & $r>0$.

According to equation (16), the original function of the influence function on z is

$$\Phi(z,s_0)=\frac{L}{(m+1)r(s_0)}\left(1+r(s_0)\frac{z}{L}\right)^{m+1}. \tag{17}$$

Substituting the influence function (16) into equation (7), the specific expression of parameters b and η are obtained as follows:

$$b(s_0)=\frac{s_bQ_0^{(1/u)}(1+r)^{(m/u)}-s_0Q_b^{(1/u)}}{Q_0^{(1/u)}(1+r)^{(m/u)}-Q_b^{(1/u)}}, \tag{18}$$

$$\eta(s_0)=\frac{\left[Q_0^{(1/u)}(1+r)^{(m/u)}-Q_b^{(1/u)}\right]^u}{(s_0-s_b)^u(1+r)^m}. \tag{19}$$

Substituting the original function (17) into equation (10), the specific expression of the G function is

$$G(z,s_0)=\frac{\eta(1-u)L}{AE_p(m+1)r}\left[1-\left(1+r\frac{z}{L}\right)^{m+1}\right]+(s_0-b)^{1-u}. \tag{20}$$

Substituting equations (16) and (20) into equations (11), (14), and (15), the displacement distribution function and the axial force distribution function along the pile axis and the frictional resistance distribution function are

$$s(z,s_0)=G^{(1/(1-u))}(z,s_0)+b(s_0), \tag{21}$$

$$Q(z,s_0)=\eta(s_0)G^{(u/(1-u))}(z,s_0)\left(1+\frac{r(s_0)z}{L}\right)^m, \tag{22}$$

$$\tau(z,s_0)=\frac{\eta}{C}\left[\frac{u\eta}{AE_p}G^{(2u-1)/(1-u)}\left(1+\frac{rz}{L}\right)^{2m} - G^{(u/(1-u))}\frac{mr}{L}\left(1+\frac{rz}{L}\right)^{m-1}\right]. \quad (23)$$

From equations (16) to (23), it can be seen that Φ, ϕ, parameters b, η, and functions G, s, Q, and τ are all related to pile-top displacement s_0.

Because the pile-top is the position with the largest displacement along pile axis and there is always the shear-slip state between pile and soil, the skin friction at pile-top can be approximated as a constant, τ_0. If the pile-top is at the same level as the ground and not loaded on ground, the lateral pressure of the pile-top can be approximated as 0. It can be considered that the lateral friction at pile-top is approximately equal to the cohesion between the pile and soil. Therefore, when pile-top displacement, s_0, is sufficiently large, the influence factor can be obtained from equation (23) according to the boundary conditions $\tau(z,s_0)|_{z=0}=\tau_0$, as shown in the following equation:

$$r(s_0)=\frac{Lu\eta^2(s_0-b)^{2u-1}-AE_pLC\tau_0}{AE_p\eta(s_0-b)^u m}. \quad (24)$$

The above analysis shows that u and m are pile-soil characteristic parameters related to pile-top displacement. To find the distribution functions of the displacement, axial force, and skin friction, the parameters b, η, and r must be obtained at first. In fact, by solving the equations of equations (18), (19), and (24), the values of b, η, and r can be obtained. Due to the complexity of above equations, the iterative algorithm can be used to obtain approximate values.

According to the force boundary conditions of the pile-top, it is obtained from equation (5) that

$$\eta=\frac{Q_0}{(s_0-b)^u}. \quad (25)$$

Substituting the above equation into equation (24) and simplifying, we can obtain that

$$r(s_0)=\frac{LuQ_0^2-AE_pLC\tau_0(s_0-b)}{AE_pQ_0m(s_0-b)}. \quad (26)$$

Thus, the following iterative equations can be established to obtain approximations of r and b, according to equations (18) and (26):

$$\begin{cases} r_j=r(s_0,b_{j-1}),\\ b_j=b(s_0,r_j), \end{cases} \quad (27)$$

where j is the number of iterations, r_j represents the value of r at the jth iteration, b_j represents the value of b at the jth iteration, and the initial value of the iteration $b_0=0$. The iteration continues until $|b_j-b_{j-1}|\le\delta$; δ is a small positive real number, preferably 10^{-8}.

At depth z_k, the pile-side load transfer function τ_z-s_z can be expressed by the parametric equation composed of equations (21) and (23) taking the displacement of the pile-top as the parameter:

$$\begin{cases} s_z=s(z_k,s_0),\\ \tau_z=\tau(z_k,s_0). \end{cases} \quad (28)$$

When the pile-top displacement is small, or the pile is exceptionally long or flexible, where the displacement zero point is yet to be transmitted to the pile-bottom, it is easy to obtain $b(s_0)=0$, $\eta(s_0)=(Q_0/s_0^u)$ by equations (18) and (19). The depth of the displacement zero point (z_0) along the pile axis meets that $s(z,s_0)|_{z=z_0}=0$. From equation (21), we can obtain the depth of the displacement zero point as follows:

$$z_0=\frac{L}{r}\left\{\left[1+\frac{AE_p(m+1)rs_0}{Q_0(1-u)L}\right]^{(1/(m+1))}-1\right\}. \quad (29)$$

Because $z_0>0$, then $1-u>0$. Combined with the condition of equation (5), $0<u<1$.

When the displacement is transmitted to the pile-bottom and the displacement s_b is generated, the axial force of the pile-bottom is determined by the load transfer function of the pile-bottom, Q_b-s_b. Q_b-s_b can be assumed to be the same model as the pile-top load-displacement curve model. According to equation (4), we obtain

$$Q_0=Q_{0m}\left[1-\left(1+\frac{(n_0-1)K_{0i}}{Q_{0m}}s_0\right)^{(1/(1-n_0))}\right], \quad (30)$$

$$Q_b=Q_{bm}\left[1-\left(1+\frac{(n_b-1)K_{bi}}{Q_{bm}}s_b\right)^{(1/(1-n_b))}\right], \quad (31)$$

where K_{0i} and K_{bi} are the initial stiffness of the load-displacement curve at the pile-top and the pile-bottom, respectively; Q_{0m} and Q_{bm} are the asymptotic values of the ultimate load at the pile-top and the pile-bottom; and n_0 and n_b are exponents. These parameters are generally obtained by a field test data with fitting method.

If the pile-bottom displacement is s_b, the whole pile has a rigid body displacement s_b. The pile-bottom displacement s_b can be obtained by an iterative method, and the iteration equation is

$$s_b^{(n+1)}=\frac{1}{2}\left[s_b^{(n-1)}+s_b^{(n)}\right], \quad (32)$$

where $s_b^{(n)}$ is the pile-bottom displacement at the nth iteration. Generally, the iteration initial value $s_b^{(0)}=0$. The iteration continues until $|s_b^{(n-1)}-s_b^{(n)}|\le\delta$, where δ is a small positive real number, preferably 10^{-8}.

5. Influence of Parameters *u* and *m* on Mechanical Properties of Pile-Soil System

5.1. Forward Analysis Program (FAP). The question of analysis is knowing the geometric, physical, and mechanical parameters of pile and soil, the pile-top load-displacement curve, the pile-bottom load transfer function, and the values u and m, seeking the distribution curves of the displacement, the axial force, and the skin friction along pile axis.

The basic steps of FAP are shown as follows.

(1) Input the geometric, physical, and mechanical parameters of pile and soil, u and m values, and pile-top displacement s_0.

(2) Calculate the pile-top load Q_0 according to the pile-top load-displacement model of equation (30).

(3) Input the initial displacement value of the pile-bottom $s_b^{(0)} = 0$, and calculate the axial force of the pile-bottom according to the load transfer function of the pile-bottom in equation (31).

(4) Solve equations (27) with the iterative method; find b and r, and then find η from equation (25).

(5) Calculate the depth of the displacement zero point z_0 according to equation (29) to determine whether the displacement have to be transmitted to the pile-bottom or not. If $z_0 > L$, it indicates that the pile-bottom has displacement.

(6) Calculate the displacement distribution curve along the pile axis s (z) from equation (21).

(7) If the pile-bottom displacement is 0, skip this step; else if$|s_b^{(n-1)} - s_b^{(n)}| > \delta$, give a new pile-bottom displacement according to equation (32) and go to step (2).

(8) Finally, the distribution function of pile axial displacement, axial force, pile lateral friction, and pile lateral load transfer function are calculated, and the curves are drawn.

The authors wrote the MATLAB program of FPA, and the block diagram is shown in Figure 3.

5.2. Influence of Parameters u and m on Mechanical Properties of Pile-Soil System

5.2.1. Parameters u and m Range of Values. Combining the values of equations (5), (16), and (29), the domain of parameters u and m are

$$\begin{cases} 0 < u < 1, \\ m > 0. \end{cases} \quad (33)$$

5.2.2. Influence of the Parameter u on Pile-Soil Characteristics. For the super-long pile with equal section, let the pile diameter $d = 0.35$ m, the pile-top displacement $s_0 = 5$ mm, the pile-top load $Q_0 = 521.94$ kN, the fixed parameter $m = 5$, the parameter u vary from 0 to 1. Through calculations, the influence of the parameter u on the characteristics of the pile soil is shown in Figures 4 and 5. It can be seen from the figures that the influence of the parameter u on the pile-soil characteristics is as follows.

(a) The depth of displacement zero point (or axial force zero point) increases as u increases.

(b) When $u \le 0.5$, the pile end friction has a concentrated phenomenon. The pile end resistance is relatively large, and the upper side resistance is relatively small, representing the soil above soft and below hard. When u is small, it means that the pile-bottom is in the base layer, and pile-bottom axial force is mainly borne by pile end resistance.

(c) When $u > 0.5$, the lateral frictional resistance is distributed in an arch shape. When $u = 0.6$~0.7, the distribution of lateral resistance is similar to that of equal section piles in uniform soil. When $u = 0.7$~0.9, it forms an asymmetrical normal distribution and shows that the soil has certain characteristics of upper hard and lower soft.

5.2.3. Influence of the Parameter m on Pile-Soil Characteristics. The same as the pile-soil conditions analyzed above, with the fixed parameter $u = 0.6$ and the parameter m varies from 0 to 100. The calculation results show that the influence of the parameter m on the characteristics of the pile soil is shown in Figures 6 and 7. It can be seen from the figure that the influence of the parameter m on the characteristics of the pile soil is as follows.

(a) The depth of displacement zero point (or axial force zero point) decreases as m increases.

(b) From the distribution pattern of the side frictional resistance of the pile, when m is small, the soil appears to be above hard and below soft. When m gradually increases, the degree of above hardness and below softness gradually decreases, until the pile-soil characteristics are mainly controlled by the parameter u.

(c) The greater the value of m, the smaller the effect on the pile-soil characteristics. When $m = 0$~5, the effect is strong; when $m = 5$~50, the effect is small; when $m > 50$, there is almost no influence. Therefore, the value of m does not have to be too large, generally less than 10 is fine.

(d) When other u values are fixed, the influence of the parameter m on the pile-soil characteristics has a similar law.

5.2.4. Combined Control of Pile-Soil Characteristics with Parameters u and m. The above analysis shows that u and m are closely related to the physical and mechanical properties of pile and soil. The parameters u and m always jointly control the pile-soil characteristics. When m is small (e.g., less than 0.5), u and m are equally important in the degree of control; when m is large (e.g., greater than 1), the pile-soil characteristics are mainly controlled by u, and that m only plays a minor role in the adjustment of the distribution pattern.

6. Back-Analysis of Pile-Soil System

The above analysis indicated that parameters u and m determine the distribution patterns of pile axis displacement, axial force, and pile side frictional resistance. If the pile-soil characteristic parameters (u, m) can be obtained using the

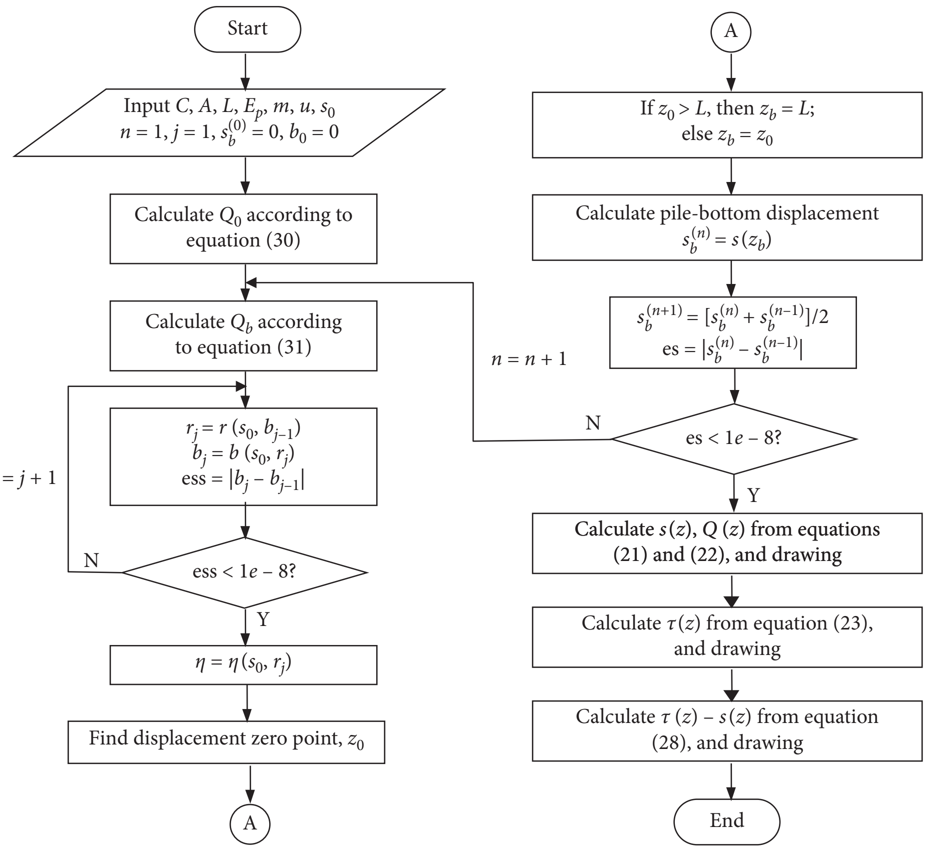

Figure 3: Block chart of FAP.

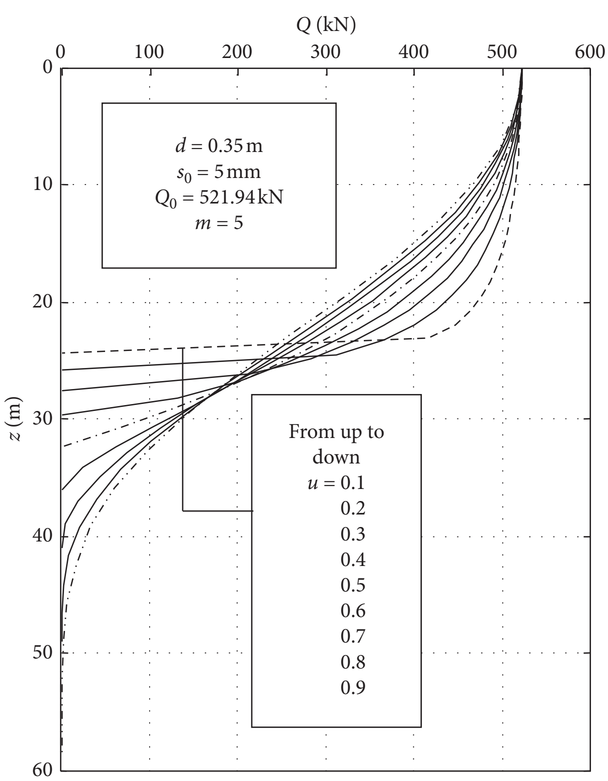

Figure 4: Effect of the parameter u on axial force distribution.

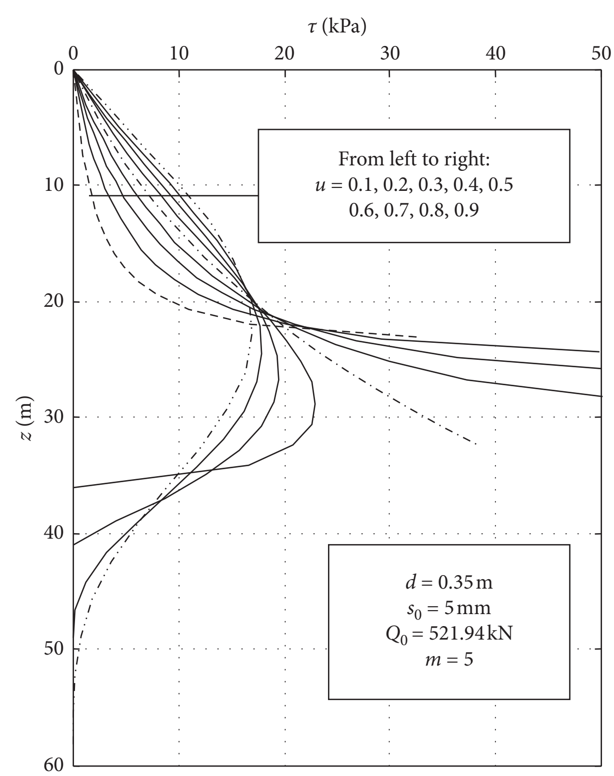

Figure 5: Effect of the parameter u on shear distribution of pile side.

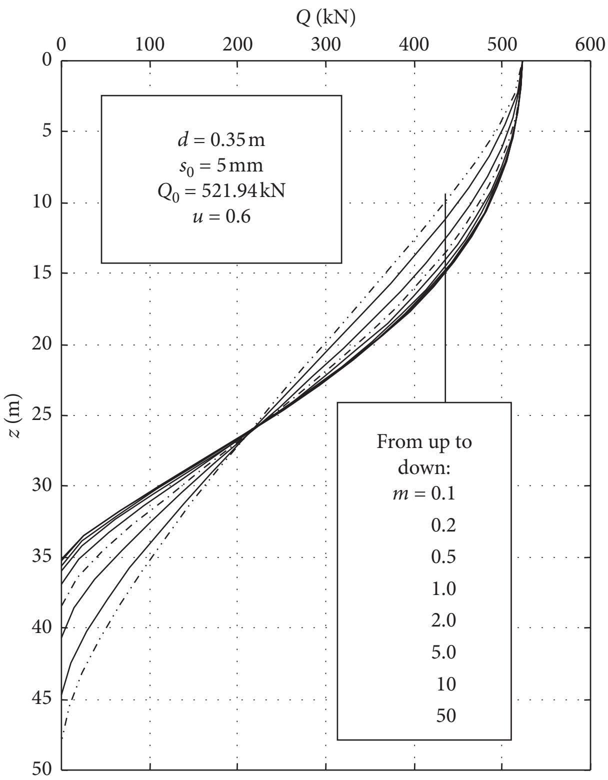

Figure 6: Effect of the parameter m on axial force distribution.

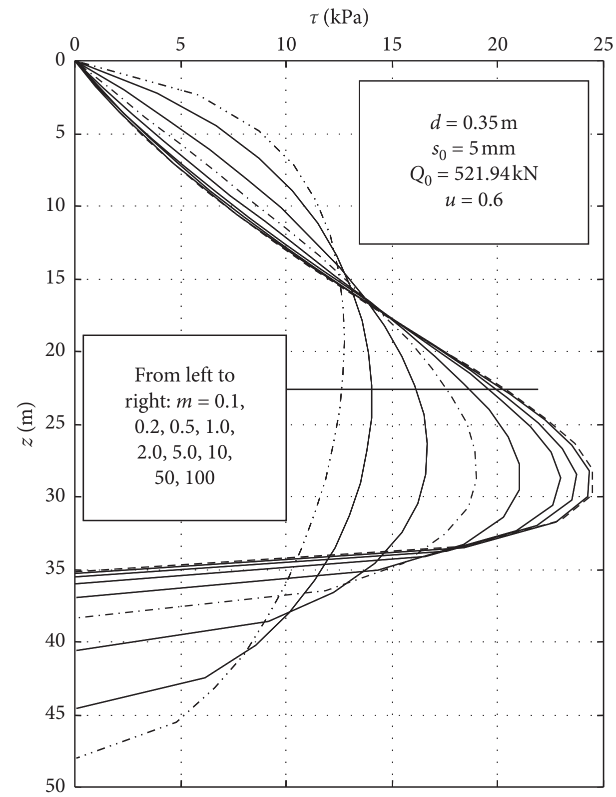

Figure 7: Effect of the parameter m on shear distribution of pile side.

back-analysis method through known test data, such as load-displacement curve of pile-top and pile-bottom and survey data of soil, we can solve the distribution curve of axial displacement, axial force, and pile side frictional resistance according to the analytical model established above, which is very beneficial to the design of pile.

6.1. Diversity of Back-Analysis Results. In general, the forward-analysis results are unique; that is, for a particular pile-soil system we can obtained only the pile-top $Q_0 - s_0$ curve. Otherwise, there can be several pile-soil systems corresponding to a pile-top $Q_0 - s_0$ curve. In other words, the results of the back-analysis are not unique.

The correspondence between the parameters (u, m) and the pile-soil system is complex and difficult to express with mathematical formulas, so it is impossible to find a unique result through mathematical relationships. In order to obtain the unique pile-soil system, other known information in the pile-soil system may be added, such as the distribution curve of frictional resistance, the distribution curve of axial force, the distribution curve of displacement, and the pile-soil load transfer curve of a certain soil layer. The optimal pile-soil system can be found by means of optimization methods through the known information.

6.2. Optimization Objective Function. The best parameter set (u, m) can be found through the optimization method. Assume that the distribution curve of the displacement, the axial force, or the shear force along the pile axis has been measured by test under the pile-top displacement s_0, and the test value (y_j) on the j^{th} section depth (z_j) is $\{(z_j, y_j) : (j = 1, 2, \ldots, k)\}$, where k is pile section all number. Set a parameter group (u, m), to calculate the theoretical distribution curve of the displacement, the axial force, or the shear force along the pile axis under the pile-top displacement s_0 through mathematical model of pile-soil system, and the theory value $(\widehat{y}_j)$ on the j^{th} section depth (z_j) is $\{(z_j\, \widehat{y}_j) : (j = 1, 2, \ldots, k)\}$. In order to verify whether the setting parameter group (u, m) is optimal, the theoretical distribution curve must be the most consistent with the experimental distribution curve. The coefficient of determination is used as the optimization objective function [25] as follows:

$$\begin{cases} R^2 = 1 - \dfrac{\text{SSE}}{\text{SST}}, \\[2ex] \text{SSE} = \displaystyle\sum_{j=1}^{k} \left(y_j - \widehat{y}_j \right)^2, \\[2ex] \text{SST} = \displaystyle\sum_{j=1}^{k} \left(y_j - \overline{y}_j \right)^2, \end{cases} \tag{34}$$

where R^2 is the coefficient of determination, representing the degree of conformity between the test curve and the theoretical curve, and the closer R^2 to 1, the better the degree of conformity between the two curves; SST stands for the total sum of squared deviations; SSE represents the sum of squared residuals (errors); $\overline{y}$ is mean of test values, $\overline{y} = (1/k)\sum_{j=1}^{k} y_j$.

6.3. Back-Analysis and Optimization Steps. In the back-analysis, first fix an m value, such as $m = 2$, then let u go from 0 to 1 by increasing step size Δu, and calculate the objective function R^2 for each increment step. When the value of R^2 is the closest to 1, the value of u is the optimal value. Then fix u as the optimal value, increase m from 0 to 10, and find the optimal value of m when R^2 is the closest to 1. The precision of optimal parameter set (u, m) is related to increasing step size. Generally, the small step length corresponds to high precision but slow calculation speed. The calculation steps are as follows.

(a) Selection of optimizable object: one of the distribution curves of the frictional resistance, the axial force and the displacement distribution curve, or the pile-soil load transfer curve can be selected as the optimization object. The back-analysis results are compared with the measured results.

(b) Optimization of the parameter u: fixing m value, u is incremented from 0 to 1, and the incremental step is Δu. The theoretical value of optimization object can be calculated using FAP. By comparing the theoretical value with the experimental value, the variation rule of the objective function R^2 with the parameter u can obtain, and the optimal parameter u_1 is found.

(c) Optimization of the parameter m: fixing $u = u_1$, m is incremented from 0 to 10, and the incremental step is Δm. The theoretical value of optimization object can be calculated using FAP. By comparing the theoretical value with the experimental value, the variation rule of the objective function R^2 with the parameter m can obtain, and the optimal parameter m_1 is found.

(d) Optimization of the parameter group (u, m) again: fix $m = m_1$ and repeat from step (b) to get the better parameter u_2. Then fix $u = u_2$ and repeat from step (c) to get the better parameter m_2. This process can be repeated until the best parameter group (u, m) is obtained.

6.4. Example of Back-Analysis for Tension Pile

Example 1. Knowing that a cross-section pile diameter $d = 500$ mm and length $L = 10$ m, the mechanical parameters of the pile are elastic mode $E_p = 2.7e7$ kPa and Poisson's ratio $\mu_p = 0.18$. The basic mechanical parameters of the cohesive soil are elastic modulus $Es = 30000$ kPa, Poisson's ratio $\mu_s = 0.38$, cohesion $c = 10$ kPa, internal friction angle $\varphi = 20^\circ$, and pile-soil friction coefficient $f = 0.4$. The uplift load-displacement curve of pile-top is obtained by finite element calculation, as shown in Figure 8 [26].

The pile-top load-displacement curve is fitted by using equation (30), and the values of the parameters in the model are obtained that $Q_{0m} = 348.7$ kN, $n_0 = 2$, $K_{0i} = 96.5$ kN/mm, and the model equation of $Q_0 - s_0$ is $Q_0 = 348.7[1-(1 + 0.277s_0)^{-1}]$.

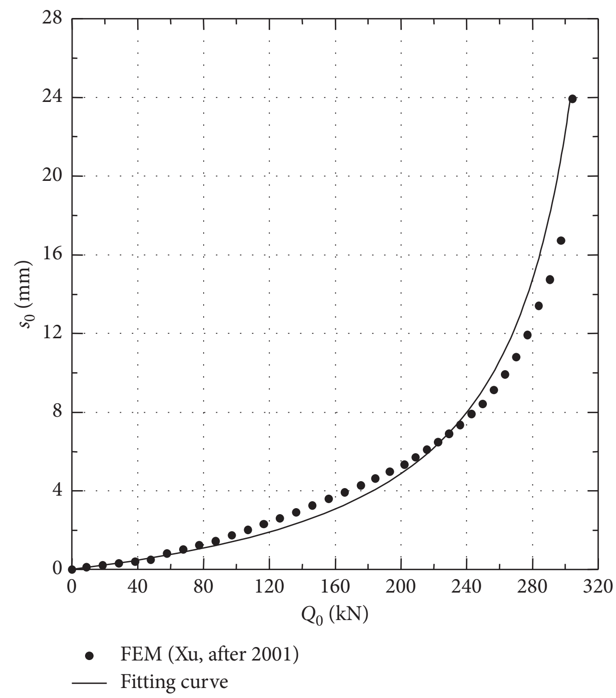

Figure 8: Pile-top uplift load-displacement curve.

According to the above back-analysis steps, the parameter group (u, m) is optimized as follows.

(a) When the pile-top load is 243.4 kN, the axial force distribution is selected as the optimization object. The theory results of the back-analysis are compared with the axial force obtained by the finite element calculation along pile axis (see "o" in Figure 9).

(b) Preliminary optimization of u. Make $m = 2$, u increase from 0 to 1, the incremental step is 0.1, and the change of the objective function R^2 with u is shown in Figure 10(a). It can be seen that $u = 0.7$ is the optimal value.

(c) Fine step optimization of u. Make $m = 2$, u increase from 0.6 to 0.8, the incremental step is 0.01. We can obtain optimal values that $u = 0.67$ and $R^2 = 0.999382$.

(d) Preliminary optimization of m. Make $u = 0.67$, m increase from 0 to 10, the incremental step is 0.5, and the change of the objective function R^2 with m is shown in Figure 10(b). It can be seen that $m = 2$ is the optimal value.

(e) Fine step optimization of m. Make $u = 0.68$, m increase from 1.5 to 2.5, and the incremental step is 0.01. We can obtain optimal values that $m = 2.02$ and $R^2 = 0.999383$.

Finally, the optimal parameter group of the pile-soil system is $(u, m) = (0.67, 2.02)$. Figure 9 shows comparison between two results of back-analysis and FEM.

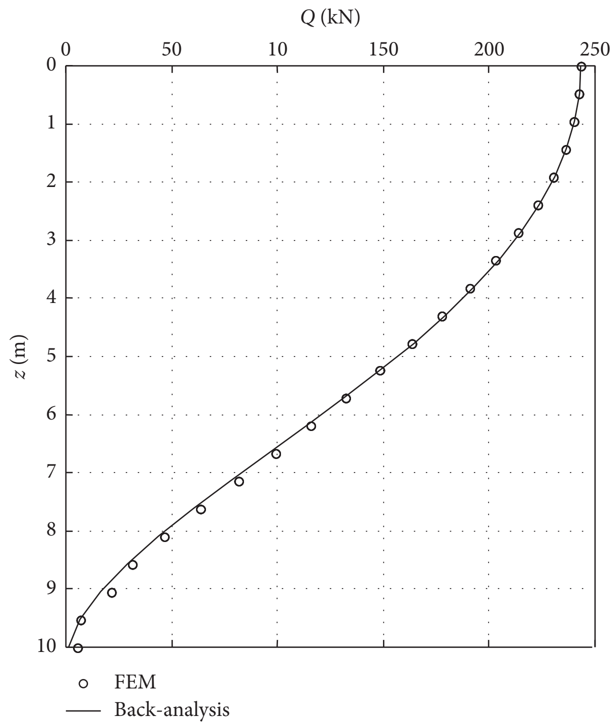

Figure 9: Comparison of axial force distribution curves and back-analysis results ($s_0 = 8.35$ mm).

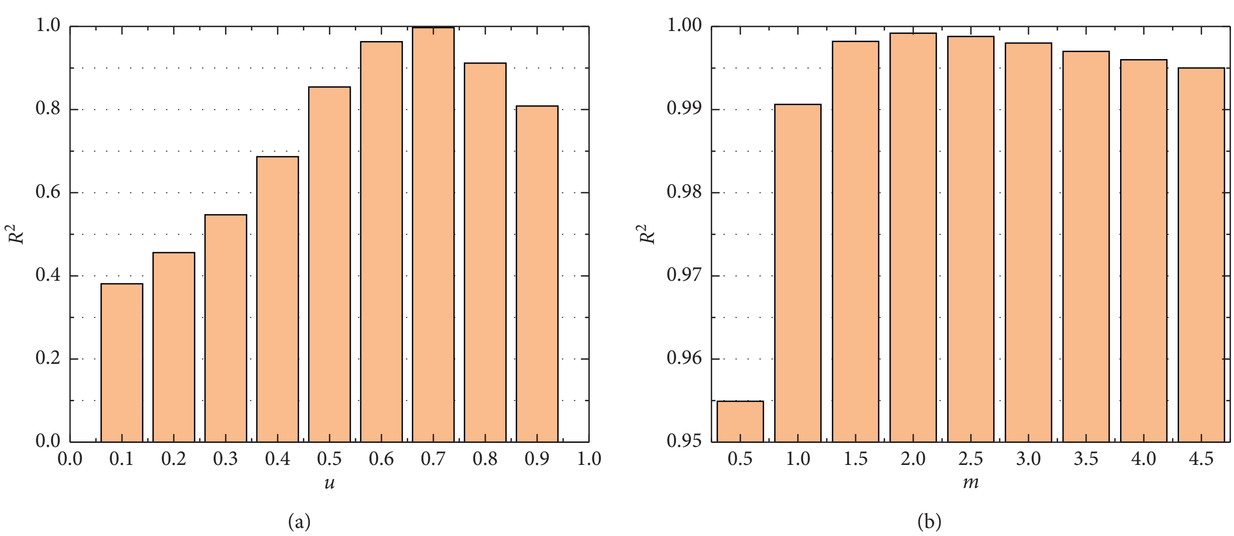

Figure 10: Change of the value of the optimization objective function with u and m. (a) $m = 2$. (b) $u = 0.67$.

As the distribution pattern of pile lateral friction changes dynamically during the uplifting process, the parameter group (u, m) also changes with the pile-top uplift displacement. According to the influence of the parameter group (u, m) on the pile-soil characteristics introduced in Section 5.2, it can be concluded for tension piles that u is a decreasing function and m is an increasing function. Based on a lot of trial compute and analysis, a coincidental result showed that the variation law of the parameter group (u, m) with pile-top displacement is related to the model of pile-top load-displacement as follows:

$$u = 1 - u_t \left\{ 1 - \left[1 + \left(\frac{(n_0 + 1) K_{0i}}{Q_{0m}} s_0 \right)^{(1/(1-n_0))} \right] \right\} \text{(tension pile)}, \tag{35}$$

$$m = m_t \left\{ 1 - \left[1 + \left(\frac{(n_0 + 1) K_{0i}}{Q_{0m}} s_0 \right)^{(1/(1-n_0))} \right] \right\} \text{(tension pile)}, \tag{36}$$

where u_t and m_t are constants that are related to the physical and mechanical properties of the pile-soil and are independent of the pile-top displacement.

Substituting the parameter group $(u, m) = (0.67, 2.02)$ and $s_0 = 8.35$ mm into equations (35) and (36), we can obtain that $u_t = 0.4744$, $m_t = 2.9039$. Let the pile-top displacements equal 0.75, 1.89, 3.85, 7.98, and 22.2 mm, respectively, and the corresponding pile-top load is 60, 120, 180, 240, and 300 kN, respectively. From equations (35) and (36), the corresponding parameter groups $(u, m) = (0.9148, 0.4991)$, $(0.8342, 0.9973)$, $(0.7532, 1.4980)$, $(0.6721, 1.9988)$, and $(0.5914, 2.4974)$ can be obtained. By substituting above parameter groups into the FAP, the axial force distribution curves and the frictional resistance distribution curves can be obtained (see Figures 11 and 12 respectively).

According to the parametric equation (28), the curves of pile-soil load transfer can be obtained at the different depth (see Figure 13). It can be seen that the maximum friction increases with the increase of the section depth except the pile-end and softening appears in the shallow layer. These laws are in good agreement with the experimental results of Zhao et al. [27]. It shows that the results of the back-analysis are reliable.

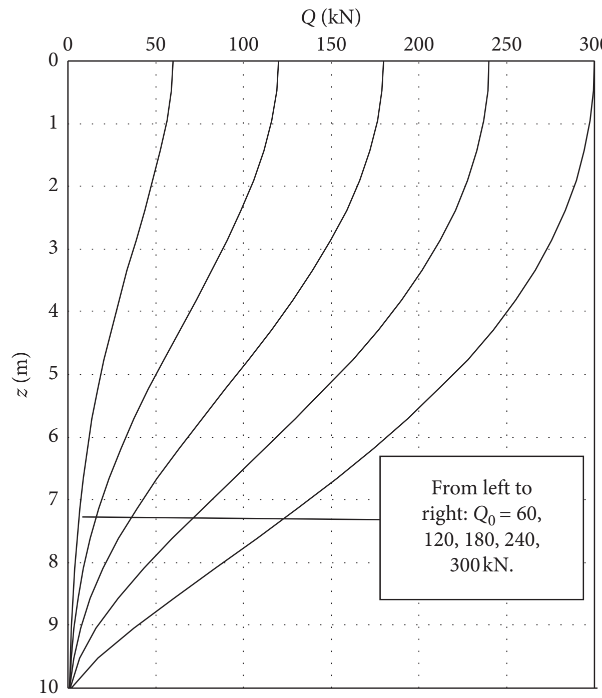

Figure 11: Axial force distribution curves for Exa.1.

6.5. Back-Analysis Example for Compression Pile

Example 2. (see [28]). Two engineering test piles for the pile foundation of coal storage bunker were designed that the diameter is 1 m and depth is 25 to 27 m. The no. 2 test pile is selected as the back-analysis calculation. The test pile no. 2 uses cast in situ bored forming method, and its average diameter $d = 1.1$ m and the length $L = 27$ m. The top to down soil layers are filled soil, clayey soil, clay soil, clayey soil, coarse sand, clayey soil, coarse sand, clayey soil, coarse sand, clay soil, coarse sand, and hard clay. The pile-bottom passes through a coarse sand layer and is supported on a hard clay layer. The test load-displacement curves of pile-top and pile-bottom are shown in Figure 14. The measured value of ultimate frictional resistance τ_m of pile side is shown in Figure 15.

(a) The pile-top test load-displacement $(Q_0 - s_0)$ is fitted using the power function model of equation (30), and the model parameters are obtained that $Q_{0m} = 8098.3$ kN, $n_0 = 1.429$, and $K_{0i} = 1580.2$ kN/mm. The pile-top $Q_0 - s_0$ curve is shown in Figure 14, and the model formula is $Q_0 = 8098.3[1 - (1 + 0.0837s_0)^{-2.331}]$.

(b) Selection of the optimization object: take the pile-side ultimate friction as the optimization object when $s_0 = 50$ mm. Due to the heterogeneous soil layer, the ultimate friction distribution is very discrete and it may cause the optimization distortion. Therefore, the polymorphic fitting is first performed for the test ultimate friction τ_m, the fitting curve is shown in Figure 15, and the expression is as follows (correlation coefficient 0.98783):

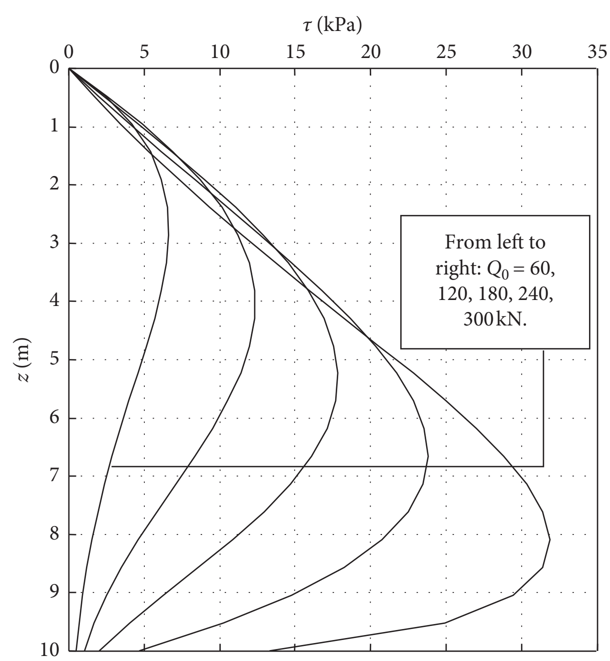

Figure 12: Friction distribution curves for Exa.1.

$$\tau_m = 1.0359 + 8.43405z - 0.55205z^2 + 0.02251z^3 - 0.0002449z^4. \quad (37)$$

The fitting curve is taken as optimization object to be compared with the back-analysis results.

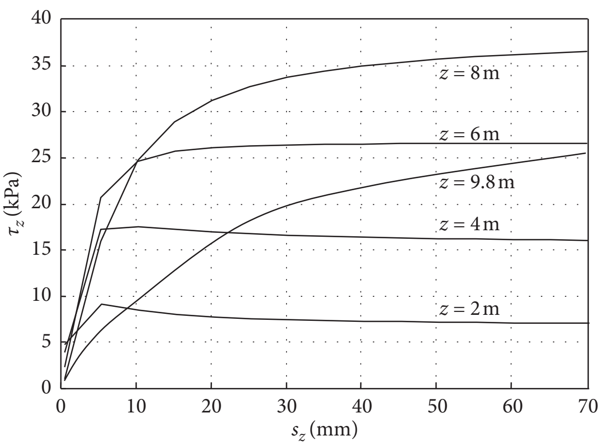

Figure 13: Tension pile load transfer curves for Example 1.

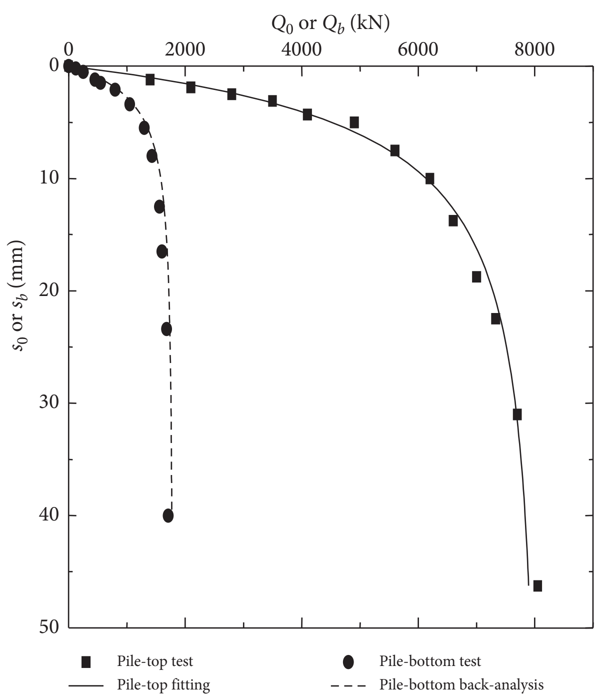

Figure 14: $Q-s$ curves of pile-top and pile-bottom for Example 2.

(c) Optimization: first, the initial values are estimated that $Q_{bm} = 1600$ kN and $\tau_0 = 0$. Second, with the method of Example 1 for optimization, $u = 0.41$ and $m = 0.13$ are obtained. Third, Q_{bm} is optimized to get $Q_{bm} = 1786$ kN. Fourthly, τ_0 is optimized to get $\tau_0 = 0$. The above optimization process can be repeated 2 to 3 times, and the final objective function value of $R^2 = 0.994946$. The maximum resistance of the pile-bottom is 1710 kN by Yukang and Qiang-hua [28]. The dashed line in Figure 14 is the load-displacement curve of pile-bottom which was obtained by back-analysis. It can be seen that the error between the back-analysis result and the measured value is small.

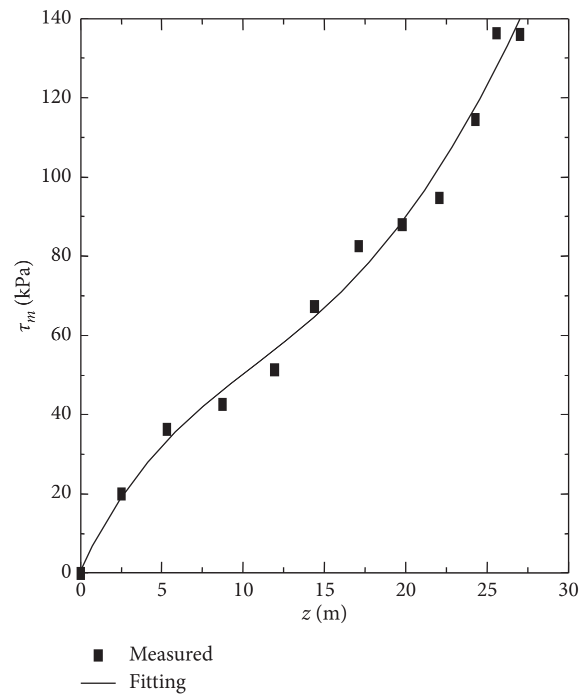

Figure 15: Pile-side ultimate friction curves for Example 2.

(d) Considering the dynamic variation law of the lateral resistance, which is along with the displacement of the pile top, is different between the compression pile and the uplift pile

Considering that the dynamic change of lateral resistance of the compressive pile with the pile-top displacement is different from that of the tension pile [26]; therefore, the variation rule of parameters (u, m) with the pile-top displacement is also different from that of drawing pile.

According to the influence rule of parameters (u, m) on the pile-soil characteristics described in Section 5.2, it can be concluded that for the compression pile, u is a decreasing function, but the change magnitude is smaller than that of equation (35), and m basically stays the same. The functions of the parameters (u, m) are determined as follows after a lot of trial calculation:

$$u = (1 - u_c)\left\{1 - u_c + u_c\left[1 + \left(\frac{(n_0 + 1)K_{0i}}{Q_{0m}} s_0\right)^{(1/(1-n_0))}\right]\right\} \text{(compression pile)}, \quad (38)$$

$$m = m_c \text{(compression pile)}, \quad (39)$$

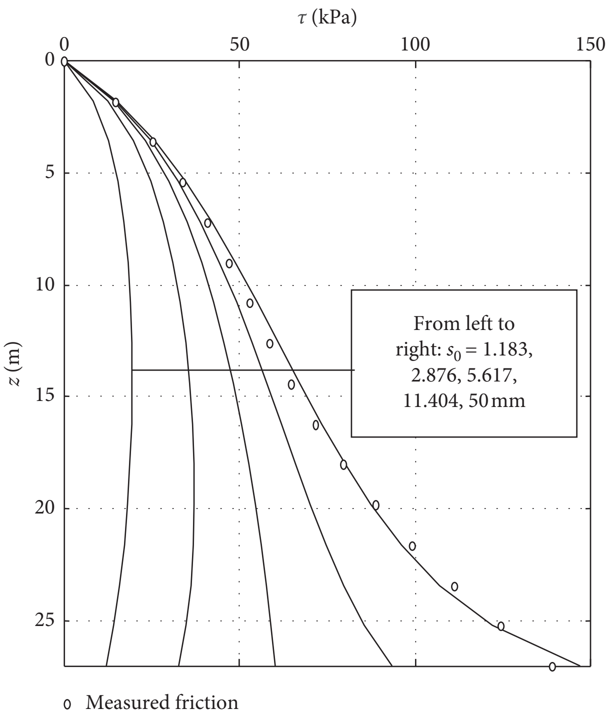

Figure 16: Pile-side friction distribution curves for Example 2.

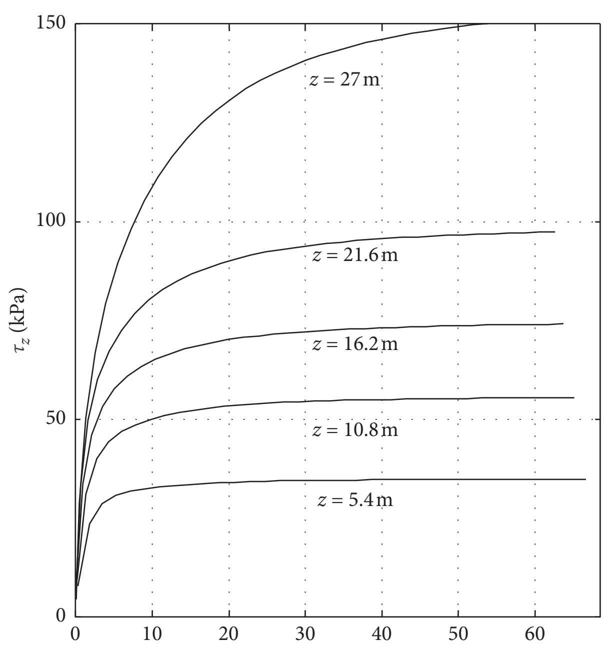

Figure 17: Load transfer curves for Example 2.

where u_c and m_c are constants, which are related to the physical and mechanical properties of the pile and soil and are independent of the pile-top displacement.

The optimizational parameters $u = 0.41$ and $m = 0.13$ are substituted into equations (38) and (39) to obtain $u_c = 0.36$ and $m_c = 0.13$. Then we calculate the axial force distribution, the friction distribution, and the load transfer curves at the different pile-top displacement. Figure 16 is the friction distribution curves along pile axis. The pile-top displacement and pressure forces corresponding to every curve from the left to the right are $(s_0, Q_0) = (1.183, 1600)$, (2.876, 3200.3), (5.617, 4800.4), (11.404, 6400.3), and (50, 7923.6) (mm, kPa), respectively. Symbol "○" in Figure 16 is the measured ultimate friction. Figure 17 is the load transfer curves $(\tau_z - s_z)$ at different depth. It can be seen that the back-analysis results reflect the basic law of load transfer correctly.

7. Conclusion

The piles acting as axial force are common in foundation engineering, such as high-rise building foundation, bridge foundation, and antifloating pile. The pile-soil load transfer theory is widely used in pile foundation analysis. At present, there are two main algorithms of the load transfer analysis based on the theory of the analytical method and deformation coordination method [1, 4, 6]. These two algorithms need to establish the load transfer function $(\tau_z - s_z)$ of the pile-side and the load-displacement function $(Q_b - s_b)$ of the pile-bottom. Due to the complexity of pile-soil interaction, load transfer functions are usually nonlinear and vary with depth. Therefore, it is difficult to solve the basic differential equations based on load transfer theory. Therefore, the analytical method is generally difficult to implement for solving basic differential equations based on load transfer theory. In order to overcome this problem, a new algorithm is proposed in this paper, that is, the basic differential equations of load transfer are solved by constructing a relationship model between the distribution functions of axial force and displacement along the pile axis, and the analytical results of the force and the displacement on pile-soil system are obtained.

Establishing the relationship model is the key in the new algorithm. After analyzing the shapes of the axial load-displacement curves at different depth and the displacement distribution curves along the pile axis under different pile-top displacements, the relationship curves between the distribution functions of axial force and displacement along pile shaft are obtained by using geometric mapping method. The relationship curves are simulated by a simple exponential function, that is, the relationship model between two distribution functions of axial force and displacement. The relational model was applied for the first time to solve the differential equations of load transfer, and then the analytical model of the force and deformation analysis of the pile-soil system was obtained. Through the analytical model, the axis displacement distribution function, axial force distribution function, pile side friction distribution function, and pile side load transfer function can be obtained.

The MATLAB program of the analytical model is written. The program is used to analyze the influence of parameters u and m on the characteristics of pile-soil system. The analysis shows that the parameters u and m are closely related to the physical and mechanical properties of the pile and soil and control the shape of lateral friction distribution curves. When m is small (e.g., less than 0.5), u and m are equally important in controlling the shapes of the distribution curves; when m is large (e.g., greater than 1), the shapes of the distribution curves are mainly controlled by u, and m only plays an auxiliary effect.

If the parameter m is unchanged ($m = 5$), when $u \leq 0.5$, the soil layer shows the characteristics of upper-soft and lower-hard, and the smaller u is, the softer the upper is and the harder the lower is; when $u = 0.6\sim0.7$, the soil layer showed uniform characteristics; when $u = 0.7\sim0.9$, the soil layer exhibits upper-hard and lower-soft characteristics. If the parameter u is fixed ($u = 0.6$), when m is small, the soil layer exhibits upper-hard and lower-soft characteristics; with the gradual increase of m, the influence degree of m on the upper-hard and lower-soft characteristics gradually decreases until it is mainly controlled by parameter u.

According to the analytical model, a back-analysis method for the characteristics of pile and soil system is proposed. The distribution curves of the axial force and the displacement along pile axis and the pile-soil load transfer curve can be obtained from the pile-top load-displacement curve and the stratum survey data using the back-analysis method. The examples illustrate that the back-analysis results are consistent with the experimental results. The analysis finds that the parameters u and m are related to the pile-top load-displacement curve, and then the empirical formulas for the variation of the parameters u and m with the pile-top displacement under tension and compression are given.

When using the analytical model in this paper, it is important to determine the values of parameters u and m. In engineering design, the values of parameters u and m can be obtained by back-analysis method. In order to facilitate the use of this analytical model by engineers, the authors will further study the correlations between the parameters u and m and the mechanical parameters of the soil and pile.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest regarding the publication of this paper.

Acknowledgments

The authors would like to thank Natural Science Foundation of Jiangsu Province of China (BK20141067) for their financial support.

References

[1] H. G. Poulos and E. H. Davis, *Pile Foundation Analysis and Design*, John Wiley & Sons, Hoboken, NJ, USA, 1980.

[2] H. B. Seed and L. C. Reese, "The action of soft clay along friction piles," *Transactions of the American Society of Civil Engineers*, vol. 122, pp. 731–754, 1955.

[3] R. W. Cooke and G. Price, "Strains and displacements around friction piles," in *Proceedings of the Eighth International Conference on Soil Mechanics and Foundation Engineering*, Moscow, Russia, 1973.

[4] M. F. Randolph and C. P. Wroth, "Analysis of deformation of vertically loaded piles," *Journal of the Geotechnical Engineering Division*, vol. 104, no. 12, pp. 1465–1488, 1978.

[5] H. M. Coyle and L. C. Reese, "Load transfer for axially loaded pile in clay," *Journal of the Soil Mechanics and Foundations Division*, vol. 92, pp. 1–26, 1966.

[6] A. Kezdi, "Bearing capacity of piles and pile groups," in *Proceeding of the Fourth International Conference on Soil Mechanics and Foundation Engineering*, vol. 2, Butterworths Scientific Publications, London, UK, pp. 46–51, August 1957.

[7] V. N. Vijayvergiya, "Load-movement characteristics of piles," in *Proceedings of the 4th Symposium of Waterway, Port Coastal and Ocean Division*, vol. 2, ASCE, Long Beach, CA, USA, March 1977.

[8] S. Armaleh and C. S. Desai, "Load-deformation response of axially loaded piles," *Journal of Geotechnical Engineering*, vol. 113, no. 12, pp. 1483–1500, 1987.

[9] M.-H. Zhao, M.-H. Yang, and X.-J. Zou, "Vertical bearing capacity of pile based on load transfer model," *Journal of Central South University of Technology*, vol. 12, no. 4, pp. 488–493, 2005.

[10] J. R. Omer, R. Delpak, and R. B. Robinson, "An empirical method for analysis of load transfer and settlement of single piles," *Geotechnical and Geological Engineering*, vol. 28, no. 4, pp. 483–501, 2010.

[11] J. Li, Y. Tan, and F. Liang, "A modified analysis method for the nonlinear load transfer behaviour of axially loaded piles," *KSCE Journal of Civil Engineering*, vol. 16, no. 3, pp. 325–333, 2012.

[12] P. Ni, L. Song, G. Mei et al., "Generalized nonlinear softening load-transfer model for axially loaded piles," *International Journal of Geomechanics*, vol. 17, no. 8, Article ID 04017019, 2017.

[13] S. Sato, "Bearing mechanism of piles (3)," *Civil Engineering*, vol. 20, no. 3, pp. 97–111, 1965, in Japanese.

[14] J. Zhang, S.-M. Zhang, Q.-Q. Zhang, S.-W. Liu, and R.-F. Feng, "Analysis of the response of an axially loaded pile considering softening behavior of pile-soil system," *Soil Mechanics and Foundation Engineering*, vol. 55, no. 1, pp. 25–32, 2018.

[15] S. Nanda and N. R. Patra, "Theoretical load-transfer curves along piles considering soil nonlinearity," *Journal of Geotechnical and Geoenvironmental Engineering*, vol. 140, no. 1, pp. 91–101, 2014.

[16] M. E. Suryatriyastuti, H. Mroueh, and S. Burlon, "A load transfer approach for studying the cyclic behavior of thermo-active piles," *Computers and Geotechnics*, vol. 55, no. 1, pp. 378–391, 2014.

[17] B. R. Danziger, A. M. Costa, F. R. Lopes, and M. P. Pacheco, "Back analysis of offshore pile driving with an improved soil model," *GeÂotechnique*, vol. 49, no. 6, pp. 777–799, 1999.

[18] D. Xiao and H. Yang, "Back analysis of static pile load test for SPT-based pile design: a Singapore experience,"*In Advances in Pile Foundations, Geosynthetics, Geoinvestigations, and Foundation Failure Analysis and Repairs,* pp. p144–152, ASCE, Reston, VA, USA, 2011.

[19] W. O. McCarron, "Plasticity model for cyclic axial pile response with back-analyses of west delta pile tests," *International Journal of Geomechanics*, vol. 17, no. 4, 2017.

[20] H.-F. Xu, C. Xue-xin, X. Shui-qing et al., "A relationship model between axial force distribution function and

displacement distribution function on uplift pile," *Rock and Soil Mechanics*, vol. 28, pp. 882–885, 2007, in Chinese.

[21] F. Liang, F. Yu, and M. Huang, "A simplified analysis method for an axially loaded single pile subjected to lateral soil movement," *Journal of Tongji University (Natural Science)*, vol. 39, no. 6, pp. 807–813, 2011, in Chinese.

[22] H.-F. Xu, Q. Qi-hu, and F.-N. Jin, "Power function model to describe load-displacement curve of tension pile," *Chinese Journal of Geotechnical Engineering*, vol. 22, no. 5, pp. 622–624, 2000, in Chinese.

[23] Q.-Q. Zhang and Z.-M. Zhang, "Complete load transfer behavior of base-grouted bored piles," *Journal of Central South University*, vol. 19, no. 7, pp. 2037–2046, 2012.

[24] K. Adam and W. Mateusz, "Static load test on instrumented pile: field data and numerical simulations," *Studia Geotechnica et Mechanica*, vol. 39, no. 3, pp. 17–25, 2017.

[25] C. Samprit and S. Ali, *HADI. Regression Analysis by Example*, Wiley, Hoboken, NJ, USA, 4th edition, 2006.

[26] H.-F. Xu, Study on uplift ultimate capacity and nonlinear calculation theory of piles, Ph.D. thesis, PLA University of Science and Technology, Nanjing, China, 2001, in Chinese.

[27] Z. Chunfeng, L Jia, S. Qichao et al., "Experimental study of load transmission property of large-diameter bored cast-in-situ deep and long pile in different soil layers," *Chinese Journal of Rock Mechanics and Engineering*, vol. 28, no. 5, pp. 1020–1026, 2009, in Chinese.

[28] H. Yukang and C. Qiang-hua, "Load transfer behavior of bored piles," *Chinese Journal of Geotechnical Engineering*, vol. 7, no. 5, pp. 22–34, 1985, in Chinese.