Research Article

A Novel Compound Nonlinear State Error Feedback Super-Twisting Fractional-Order Sliding Mode Control of PMSM Speed Regulation System Based on Extended State Observer

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In this article, a novel compound nonlinear state error feedback super-twisting fractional-order sliding mode control (NLSEF-STFOSMC) is proposed for the control of the permanent magnet synchronous motor (PMSM) speed regulation system. Firstly, a novel fractional-order proportion integration differentiation (FOPID) switching manifold is designed. A modified sliding mode control (SMC) is constructed by a super-twisting reaching law and the novel FOPID sliding surface. Secondly, the nonlinear state error feedback control law (NLSEF) has been widely used because of high control accuracy, fast convergence, and flexible operation. Therefore, combining the modified SMC with the NLSEF, the compound NLSEF-STFOSMC is proposed, which has an excellent performance. At the same time, the external disturbance of the system is observed by a novel extended state observer. Finally, the performance of the corresponding control law to the speed operation of the PMSM is fully investigated compared with other related algorithms to demonstrate the effectiveness. The comparison results show that the proposed compound control strategy has excellent dynamic and static performance and strong robustness.

1. Introduction

The permanent magnet synchronous motor (PMSM) with the advantages of low rotor inertia, simple structure, high efficiency, and high power density has been widely used in many fields [1, 2]. At present, the traditional linear control scheme still occupies the active position in the control of PMSM in the industrial field, but because of the disturbance and time-varying and strong coupling of PMSM, it is difficult to obtain the satisfactory speed regulation performance of the traditional linear control scheme [3, 4]. Nowadays, many scholars have proposed many different nonlinear control strategies to solve the problems in traditional linear control scheme, such as backstepping control [5, 6], finite time control [7, 8], model predictive control [9], fuzzy logic control [10], iterative learning control [11], and active disturbance rejection control [12]. The sliding mode control (SMC) has been widely used for many decades, because of its simple implementation, disturbance rejection, strong robustness, and fast dynamic responses [13, 14]. Till now, approaches proposed by many researchers were the combination of SMC with the different methods for the control of the PMSM speed regulation system [15–19].

Combined with the SMC of classical proportion integration differentiation (PID) control, the control performance can be improved by adjusting the parameters of PID control. Fractional theory has been studied by scholars as pure mathematics theory for nearly 300 years. The traditional PID control theory is integrated with the fractional-order theory to come out the fractional-order PID. Compared with the integer-order controller, the extra degree of freedom of the fractional-order integrator and differentiator can further improve the control performance of the system [20]. Recently, SMC strategies based on the fractional-order PID (FOPID) sliding surface have been widely applied to various systems [21–23]. The SMC strategies based on the
fractional-order PID (FOPID) sliding surface also have been designed for the speed control of PMSM [24]. However, the conventional FOPID also has the disadvantages of simple and rough signal processing [25–27]. In this study, a nonlinear saturation function is introduced into the integral term and differential term of the traditional FOPID sliding surface, thus forming a novel FOPID sliding surface, which can effectively settle the abovementioned issues.

The derivative information of the super-twisting (ST) scheme is not required as it is a high-order SMC scheme with many advantages [28]. The ST scheme has been widely used in various control systems [13, 29–34]. In this paper, a super-twisting fractional-order sliding mode control (STFOSMC) is constructed by the ST scheme and the novel FOPID sliding surface. Han proposed a nonlinear state error feedback control law (NLSEF) in [35]. The NLSEF has been widely used because of high control accuracy, fast convergence, and performance of flexibility [36–39]. Considering the robustness of SMC and the high precision and fast convergence of NLSEF and combining the STFOSMC and the NLSEF, a novel compound nonlinear state error feedback super-twisting fractional-order sliding mode control (NLSEF-STFOSMC) is proposed.

Since the extended state observer (ESO) rarely relies on the mathematical model of the system, it is widely used to estimate the total disturbance of the system [35, 40–42]. However, the nonsmooth and nonderivable characteristics of the conventional ESO can easily cause flutter phenomenon in the response of the system. In this paper, a smooth and derivable ESO which reduces the flutter phenomenon is built.

Motivated by the above discussion and our previous research [43], the main contributions of the proposed control method, which are novel in comparison to previous studies, are listed as follows:

1. A novel compound NLSEF-STFOSMC combining the advantages of the modified SMC and the NLSEF is proposed. The modified SMC uses a novel FOPID sliding surface to improve the control performance of the sliding mode stage. The proposed compound controller is analyzed and used for the PMSM speed regulation system.

2. The external disturbance of the system is observed by one smooth and derivable ESO. The modified ESO based on a smooth and derivable nonlinear saturation function is introduced to the compound NLSEF-STFOSMC to estimate the external disturbance.

3. Lyapunov approach and fractional calculus are used to show the stability of the novel compound NLSEF-STFOSMC.

Based on our previous research [43], the rest of this paper is organized as follows: the model of PMSM and some definitions of fractional-order calculus are given in Section 2. In Section 3, the control strategies and the stability analysis are given. The comparative simulations and the analysis of simulation results are given in Section 4, which demonstrate that the proposed compound control strategy has excellent dynamic, static performance, and strong robustness. Main conclusions and future work are given in Section 5.

2. Mathematical Preliminaries

The model of PMSM and some definitions of fractional-order calculus are given in this section.

For surface-mount PMSM, under a mechanical load torque, the equation of mechanical torque is expressed in (1) and the dynamics of PMSM is expressed in (2):

\[
\begin{align*}
T_c - T_L &= f \dot{\omega}(t) + B \omega(t), \\
T_c &= 1.5 p_n \phi_p i_q(t),
\end{align*}
\]

\[
\dot{\omega}(t) = \frac{3 p_n \phi_p}{2J} i_q(t) - \frac{B}{J} \omega(t) - \frac{1}{J} T_L,
\]

where \(T_c\), \(\phi_p\), \(p_n\), \(T_L\), \(B\), \(\omega(t)\), and \(i_q(t)\) represent the electromagnetic torque, flux linkage, number of pole pairs, load torque, rotational inertia, friction coefficient, mechanical rotor angular speed, and armature currents, respectively.

**Definition 1** (see [44, 45]). The Riemann–Liouville fractional derivative and integral of function \(f(t)\) are given by

\[
\begin{align*}
D_t^\alpha f(t) &= \frac{1}{\Gamma(1 - \alpha)} \frac{d}{d t} \int_{t_0}^t f(\tau) (t - \tau)^{-\alpha - 1} d\tau, \\
D_t^{\alpha-u} f(t) &= \frac{1}{\Gamma(u)} \int_{t_0}^t f(\tau) (t - \tau)^{1-u} d\tau, \\
\Gamma(z) &= \int_0^\infty e^{-t} t^{z-1} d\tau,
\end{align*}
\]

where \(z \in \mathbb{R}; \Gamma(\cdot)\) is Euler gamma function; and \(\alpha\) and \(u\) are the fractional derivative order and integral order, respectively; \(0 < \alpha, u < 1\).

**Lemma 1** (see [44, 45]). The fractional-order integration and differentiation have the following properties:

\[
\frac{d^u}{d t^u} (D_t^\alpha f(t)) = D_t^{\alpha+u} f(t),
\]

\[
\frac{d^u}{d t^u} (D_t^{\alpha-u} f(t)) = D_t^{\alpha-u} f(t).
\]

**Lemma 2** (see [44, 45]). The fractional-order differentiation has the property of linear operation:

\[
D_t^\alpha (a f(t) + b f(t)) = a D_t^\alpha f(t) + b D_t^\alpha f(t).
\]

3. Control Strategies and Stability Analysis

In this section, the control strategies and the stability analysis are given.
3.1. Design of Controllers

**Definition 2** (see [21–24]). The conventional FOPID sliding surface was defined as

\[ s(t) = K_p e(t) + K_d D_1^{-
u} e(t) + K_d D_1^{1+
u} e(t), \]

where \( e(t) = \omega_r(t) - \omega(t) \), \( \omega_r(t) \) is desired speed, and \( K_p, K_d, K_d \in R^* \) and \( K_d \in R^* \).

According to Lemmas 1 and 2 and the exponential reaching law (ERL), it can be obtained that

\[ K_p \dot{e}(t) + K_d D_1^{1-
u} e(t) + K_d D_1^{1+
u} e(t) = -k s(t) - \eta \text{sign}(s(t)), \]

where

\[ \begin{align*}
    k & \in R^+, \\
    \eta & \in R^+,
\end{align*} \]

\[ \text{sign}(s(t)) = \begin{cases} 
    1, & s(t) > 0, \\
    0, & s(t) = 0, \\
    -1, & s(t) < 0.
\end{cases} \]

Substituting (2) into (8), the following can be derived:

\[ K_p \left( \dot{\omega}_r(t) + \frac{B}{J} \omega(t) - \frac{3p \phi f}{2} \dot{i}_q(t) + \frac{T_i}{J} \right) + K_d D_1^{1-
u} e(t) + K_d D_1^{1+
u} e(t) = -k s(t) - \eta \text{sign}(s(t)). \]  

Afterwards, the conventional SMC can be given as follows:

\[ i_q(t) = \frac{2J}{3p \phi f} K_p \left( K_p \left( \dot{\omega}_r(t) + \frac{T_i}{J} + \frac{B}{J} \omega(t) \right) + K_d D_1^{1-
u} e(t) + K_d D_1^{1+
u} e(t) + k s(t) + \eta \text{sign}(s(t)) \right). \]

**Definition 3.** The proposed FOPID sliding surface is given as

\[ s(t) = K_p e(t) + K_d D_1^{-\nu} \Phi(e(t), \tau) + K_d D_1^{1+\nu} \Phi(e(t), \tau), \]

where

\[ \Phi(e(t), \tau) = \begin{cases} 
    \tau, & e(t) > \tau, \\
    -\frac{1}{\tau} \frac{\tau}{\tau} e(t)^2 + 2e(t), & \tau \geq e(t) \geq 0, \\
    -\frac{1}{\tau} e(t)^2 + 2e(t), & 0 \geq e(t) \geq -\tau, \\
    -\tau, & e(t) < -\tau,
\end{cases} \]

where \( K_p > 0, K_d > 0, \) and \( K_d > 0 \) and \( \tau \) is a positive number.

Figure 1 shows the corresponding function curve of the \( \Phi(\cdot) \) with \( \tau = 1 \). \( \Phi(\cdot) \) has the following characteristics [46]; it can be seen that, when \( e(t) > \tau \) and \( e(t) < -\tau \), the error is limited to the value of \( \pm \tau \) and the function has the characteristic of the saturated error; when \( \tau \geq e(t) \geq 0 \), the minimum value of the function \( y = \Phi(e(t), \tau) - e(t) \) is zero, that is, \( \Phi(\cdot) \) is always satisfied; when \( 0 \geq e(t) \geq -\tau \), the maximum value of the function \( y = \Phi(e(t), \tau) - e(t) \) is zero, that is, \( \Phi(\cdot) \) is always satisfied and the nonlinear function can amplify the error and make the system have better performance even if there is a small error.

The structure diagram of the sliding surface (12) is shown in Figure 2.

**Remark 1.** Compared with conventional FOPID sliding surface, the proposed FOPID sliding mode surface with a nonlinear integral term and a nonlinear differential term makes it to effectively meet the control requirements of high quality.

According to Lemmas 1 and 2, equations (2) and (12), and ERL, it can be obtained that

\[ K_p \dot{e}(t) + K_d D_1^{1-\nu} \Phi(e(t), \tau) + K_d D_1^{1+\nu} \Phi(e(t), \tau) = -k s(t) - \eta \text{sign}(s(t)). \]

Afterwards, the novel fractional-order sliding mode control (FOSMC) based on the proposed FOPID sliding surface and the ERL can be given as follows:

\[ i_q(t) = \frac{2J}{3p \phi f} \left( K_p \left( \dot{\omega}_r(t) + \frac{B}{J} \omega(t) + \frac{T_i}{J} \right) + K_d D_1^{1-\nu} \Phi(e(t), \tau) + K_d D_1^{1+\nu} \Phi(e(t), \tau) + k s(t) + \eta \text{sign}(s(t)) \right). \]
However, the ERL may produce the chattering phe-
omena in the system. In order to effectively solve the
herent chattering phenomenon, the following ST reach-
ing law (STRL) is chosen in this paper [28]:

\[
\dot{s}(t) = -\eta_1 |s(t)|^{(1/2)} \text{sign}(s(t)) - \eta_2 \int \text{sign}(s(t)) \, dt. 
\]  

(16)

where \( \eta_1 \) and \( \eta_2 \) are the positive coefficients.

According to Lemmas 1 and 2 and equations (12) and (16), the following equation can be obtained:

\[
K_p \dot{e}(t) + K_i D^{\tau} \Phi(e(t), \tau) + K_d D^{\nu+1} \Phi(e(t), \nu+1) + \eta_1 |s(t)|^{(1/2)} \text{sign}(s(t)) - \eta_2 \int \text{sign}(s(t)) \, dt. 
\]  

(17)

Afterwards, the proposed STFOSMC can be given as

\[
\tilde{u}(t) = \tilde{k}_p f_{\text{new}}(e(t), \alpha, \delta) + \tilde{k}_d f_{\text{new}}(\dot{e}(t), \alpha, \delta),
\]  

(19)

An improved NLSEF is adopted in this paper. The improved NLSEF expression is established as [47]

\[
f_{\text{new}}(x, \alpha, \delta) = \begin{cases} 
|x|^{\alpha} \text{sign}(x) & |x| > \delta, \\
\delta^{-\alpha-1} \alpha - \frac{(1 - \alpha) \delta^\alpha \sin \delta}{1 - \cos \delta - \delta \sin \delta} x + \frac{(1 - \alpha) \delta \alpha}{1 - \cos \delta - \delta \sin \delta} (\text{versin}(|x|) \text{sign}(x)) |x| \leq \delta,
\end{cases}
\]  

(20)
where \(0 < \alpha, \delta < 1\).

The function \(f_{new}(x, a, \delta)\) is a smooth and derivable function. A detailed description of the smooth and derivable function (20) can be found in the literature [47].

Integrating the improved NLSEF and the STFOSMC with respect to their effect, the proposed compound NLSEF-STFOSMC can be defined as

\[
\tilde{u}(t) = i_{q1}(t) + \tilde{u}(t).
\] (21)

### 3.2. Design of Disturbance Observer

According to (2), define \(x_1 = \omega(t)\) and \(x_2 = -T_i/f\). Equation (2) can be rewritten as

\[
\begin{cases}
\dot{x}_1 = x_2 - B/J x_1 + \frac{3p_n \phi_f}{2J} i_{q1}(t), \\
\dot{x}_2 = \vartheta(t).
\end{cases}
\] (22)

where \(\vartheta(t)\) can be regarded as a disturbance of \(x_2\).

The expression of conventional nonlinear extended state observer (ESO) is as follows [35]:

\[
\begin{align*}
e(t) &= Z_{21} - x_1, \\
\dot{Z}_{21} &= -\beta_1 \text{fal}(e(t), \tilde{\omega}, \tilde{\alpha}) + b_y i_{q1}(t), \\
\dot{Z}_{22} &= -\beta_2 \text{fal}(e(t), \tilde{\omega}, \tilde{\alpha}),
\end{align*}
\] (23)

where

\[
\text{fal}(x, \tilde{\alpha}, \tilde{\omega}) = \begin{cases} 
|x|^\alpha \text{sign}(x), & |x| > \tilde{\omega} \\
\frac{x}{\tilde{\omega}^{1-\alpha}}, & |x| \leq \tilde{\omega},
\end{cases}
\] (24)

\[
i_{q1}(t) = \frac{2J}{3p_n \phi_f} \left(K_p \left(\dot{\omega}_r(t) - Z_{22} + \frac{B}{J} \omega(t)\right) + K_d D^1_{1-\alpha} e(t) + K_d D^1_{1-\alpha} e(t) + K_t \Delta s(t) + \eta \text{sign}(s(t))\right).
\] (26)

The proposed FOSMC method based on the novel ESO can be given as

\[
i_{q2}(t) = \frac{2J}{3p_n \phi_f} \left(K_p \left(\dot{\omega}_r(t) - Z_{22} + \frac{B}{J} \omega(t)\right) + K_d D^1_{1-\alpha} \Phi(e(t), \tau) + K_d D^1_{1-\alpha} \Phi(e(t), \tau) + k_s(t) + \eta \text{sign}(s(t))\right).
\] (27)

The proposed STFOSMC method based on the novel ESO can be given as

\[
i_{q3}(t) = \frac{2J}{3p_n \phi_f} \left(K_p \left(\dot{\omega}_r(t) - Z_{22} + \frac{B}{J} \omega(t)\right) + K_d D^1_{1-\alpha} \Phi(e(t), \tau) + K_d D^1_{1-\alpha} \Phi(e(t), \tau) + k_s(t) + \eta \text{sign}(s(t)) + \eta \int \text{sign}(s(t)) dt\right).
\] (28)
Based on equations (19) and (28), the proposed compound NLSEF-STFOSMC method based on the novel ESO can be given as

\[
\dot{\eta}_{q}(t) = \dot{i}_{q}(t) + \bar{u}(t). \tag{29}
\]

**Remark 3.** Compared with other control strategies, the proposed compound NLSEF-STFOSMC method has the robustness of SMC and the high precision and fast convergence of NLSEF.

A novel PMSM speed regulation system based on the proposed compound NLSEF-STFOSMC (29) with the novel ESO (25) is shown in Figure 3.

The structure diagrams of the proposed FOSMC, the proposed STFOSMC, and the proposed compound NLSEF-STFOSMC are shown in Figures 4–6, respectively.

### 3.3. Stability Analysis

The stability of the proposed FOSMC system is discussed.

**Proof.** Step 1: the Lyapunov function for the proposed FOSMC is defined as

\[
V = \frac{1}{2} \varepsilon(t)^2 > 0. \tag{30}
\]

Taking derivative of (30), we get

\[
\dot{V} = s(t) \cdot \dot{s}(t)
\]

\[
= s(t) \left( K_{p} \dot{e}(t) + K_{d} D_{1}^{-1} \Phi(e(t), \tau) + K_{d} D_{1}^{-1} \Phi(e(t), \tau) \right)
\]

\[
= s(t) \left( K_{p} \left( \dot{\omega}_{1}(t) + B f(t) - \frac{3P_{m} \Phi}{2J} i_{q}(t) + \frac{T_{m}}{J} \right) + K_{d} D_{1}^{-1} \Phi(e(t), \tau) + K_{d} D_{1}^{-1} \Phi(e(t), \tau) \right)
\]

\[
= s(t) \left( -K_{d} \dot{e}(t) - \eta \text{sign}(s(t)) + K_{p} \Delta d(t) \right)
\]

\[
= -K_{d} \dot{e}(t)^2 - \eta \text{sign}(s(t)) \overset{35}{\underset{34}{\leq}} \dot{s}(t)^2 - |s(t)|K_{p} \Delta d(t)
\]

\[
\leq -K_{d} \dot{e}(t)^2 - \eta |s(t)| + |s(t)|K_{p} \Psi < 0. \tag{32}
\]

**Assumption 1.** The value of \( \Delta d(t) \) has a boundary layer, which satisfies the following condition:

\[
|\Delta d(t)| < \Psi, \tag{32}
\]

where \( \Psi \in R^{+} \).

If \( \eta > K_{p} \Psi \) is satisfied, it can be obtained that

\[
\dot{V} \leq -K_{d} \dot{e}(t)^2 - \eta |s(t)| + |s(t)|K_{p} \Psi < 0. \tag{33}
\]

Accordingly, we can easily conclude that the sliding surface \( s(t) \) will be bounded as \( |s(t)| \leq \ell \). \( \ell \) is a small neighborhood of zero.

Step 2: according to \( |s(t)| \leq \ell \), the following formula can be obtained:

\[
\begin{align*}
\{ s(t) = K_{p} \dot{e}(t) + K_{d} D_{1}^{-1} \Phi(e(t), \tau) + K_{d} D_{1}^{-1} \Phi(e(t), \tau), \\
|s(t)| \leq \ell.
\end{align*}
\]

Using (34), it can be obtained that

\[
b(t) + \frac{K_{d}}{K_{p}} D_{1}^{-1} \Phi(e(t), \tau) + \left( \frac{K_{d}}{K_{p}} - \frac{1}{K_{p}} \right) s(D_{1}^{-1} \Phi(e(t), \tau))^{-1} D_{1}^{-1} \Phi(e(t), \tau) = 0. \tag{35}
\]

When \( K_{d} - s(D_{1}^{-1} \Phi(e(t), \tau))^{-1} > 0 \) holds, (35) will still keep the proposed FOPID sliding surface form as (12). Therefore, According to (33) and (34), the trajectory of the system will continue to converge to the proposed FOPID sliding surface until it reaches \( |D_{1}^{-1} \Phi(e(t), \tau)| \leq (1/K_{d}) \ell \).

Using (34) and (36), we get:
\[ e(t) + \left( \frac{K_i}{K_p} - \frac{1}{K_p} s(t) \left( D^{-\alpha}_1 \Phi(e(t), \tau) \right)^{-1} \right) D^{-\alpha}_1 \Phi(e(t), \tau) + \frac{K_d}{K_p} D^\alpha_1 \Phi(e(t), \tau) = 0. \]  
\[ (36) \]

So, \(|D^{-\alpha}_1 \Phi(e(t), \tau)| \leq (1/K) \ell \) also can be obtained. Sequentially, it can be obtained that

\[
e(t) = \frac{1}{K_p} s(t) - \frac{K_i}{K_p} D^{-\alpha}_1 \Phi(e(t), \tau) - \frac{K_d}{K_p} D^\alpha_1 \Phi(e(t), \tau)
\]

\[
\leq \left| \frac{1}{K_p} s(t) \right| + \left| \frac{K_i}{K_p} D^{-\alpha}_1 \Phi(e(t), \tau) \right| + \left| \frac{K_d}{K_p} D^\alpha_1 \Phi(e(t), \tau) \right| \leq \frac{3 \ell}{K_p}
\]

\[ (37) \]

According to (37), the control error of the proposed FOSMC can converge to a small neighborhood of zero, and the stable condition has been achieved.

The stability of the proposed STFOSMC system is discussed.

Proof. Step 1: the Lyapunov function for STFOSMC is defined as

\[
V = \frac{1}{2} s(t)^2 > 0.
\]

Taking derivative of (38), it can be obtained that
\[
\dot{V} = \dot{s}(t) \cdot \ddot{s}(t)
\]
\[
= s(t) \left( K_p \dot{e}(t) + K_i \int_{\tau}^{t} \dot{e}(\tau) \, d\tau + K_d D_t^{\varepsilon} \dot{e}(t) \right)
\]
\[
= s(t) \left( K_p \omega(t) + \frac{B}{J} \omega(t) - \frac{3 p_J \varphi_f}{2 J} i_{q3}(t) + T_E \right) + K_i D_t^{\varepsilon} \dot{e}(t) + K_d D_t^{\varepsilon} \dot{e}(t)
\]
\[
= s(t) \left( -\eta_1 |s(t)|^{1/2} \text{sign}(s(t)) - \int \eta_2 \text{sign}(s(t)) \, dt + K_p \Delta d(t) \right)
\]
\[
\leq - \eta_1 |s(t)||s(t)|^{1/2} - |s(t)| \int \eta_2 \, dt + K_p |s(t)| \int |\Delta d(t)| \, dt.
\]
Assumption 2. The derivative of the value of $\Delta d(t)$ has an upper limit, which satisfies
\[
|\Delta \dot{d}(t)| < \Theta, \quad (40)
\]
where $\Theta$ is the upper limit of the derivative of the value of $\Delta d(t)$, and it is a constant.

According to Assumption 2 and if $K_p \Theta < \eta_2$ is satisfied, it can be obtained that
\[
\dot{V} < -\eta_1 |s(t)||s(t)|^{1/2} - |s(t)| \left[ \eta_2 dt + K_p |s(t)| \right] \Theta dt
\]
\[
= -\eta_1 |s(t)||s(t)|^{1/2} - |s(t)| \left[ \eta_2 dt - K_p \int \Theta dt \right]
\]
\[
< \eta_1 |s(t)||s(t)|^{1/2} \leq 0.
\]
(41)

Accordingly, we can easily conclude that the sliding surface $s(t)$ will be bounded as $|s(t)| \leq \ell$.

Step 2: we use the similar procedure from the step 2 of the proof of the FOSMC. It can be obtained that
\[
\dot{V} = s(t) \cdot \dot{s}(t)
\]
\[
= s(t) \left[ K_p \dot{e}(t) + K_i \int_{t_0}^t \dot{e}(r) dr + K_d \dot{e}(t) \right]
\]
\[
= s(t) \left[ K_p \left( \omega_c(t) + \frac{B}{J} \omega(t) - \frac{3P_n \phi_f}{2J} \right) + K_i \int_{t_0}^t \dot{e}(r) dr + K_d \dot{e}(t) \right]
\]
\[
= s(t) \left[ -\eta_1 |s(t)| \left( \int \eta_2 |s(t)| dt - K_p \psi \tilde{u}(t) + K_p \Delta d(t) \right) \right]
\]
\[
\leq -\eta_1 |s(t)||s(t)|^{1/2} - |s(t)| \left[ \eta_2 dt + \int |s(t)||K_p \psi \tilde{u}(t)| dt + K_p |s(t)||\Delta d(t)| \right]
\]
\[
= -\eta_1 |s(t)||s(t)|^{1/2} - |s(t)| \left[ \eta_2 dt + K_p \psi |s(t)| \right] \left[ \int \tilde{u}(t) dt + K_p |s(t)| \right] \left| \Delta d(t) dt \right|
\]
(44)

where $\psi = 3P_n \phi_f / 2J$.

Assumption 3. According to the continuous and derivable function (20), the derivative of the value of $f_{new}(x, a, \delta)$ has a boundary layer, which satisfies the following condition:
\[
\left| f_{new}(x, a, \delta) \right| < \Lambda, \quad (45)
\]

According to Assumption 3 and if $K_p \psi \Lambda (k_\psi + k_\delta) + K_p \Theta < \eta_2$ is satisfied, one can obtain
\[
\dot{V} < -\eta_1 |s(t)||s(t)|^{1/2} - |s(t)| \left[ \eta_2 dt + K_p \psi |s(t)| \right] \left[ (k_\psi + k_\delta) \Delta dt + K_p |s(t)| \right] \Theta dt
\]
\[
= -\eta_1 |s(t)||s(t)|^{1/2} - |s(t)| \left[ \eta_2 dt - K_p \psi k_\psi \right] \left[ \Delta dt - K_p |s(t)| \right] \left| \Theta dt \right|
\]
\[
< -\eta_1 |s(t)||s(t)|^{1/2} \leq 0.
\]
(46)

Proof. Step 1: the Lyapunov function for the compound NLSEF-STFOSMC is defined as
\[
V = \frac{1}{2} s(t)^2 > 0. \quad (43)
\]
Taking derivative of (43), it can be obtained that
\[
\dot{V} = \frac{1}{2} s(t)^2 > 0. \quad (46)
\]

where $\Lambda$ is the upper limit of the derivative of the value of $f_{new}(x, a, \delta)$, and it is a constant.

According to Assumption 3 and if $K_p \psi \Lambda (k_\psi + k_\delta) + K_p \Theta < \eta_2$ is satisfied, one can obtain
Accordingly, the sliding surface $s(t)$ will be bounded in finite time as $|s(t)| \leq \ell$.

Step 2: the similar procedure from the step 2 of the proof of the FOSMC can be used.

Sequentially, it can be obtained that

$$
|e(t)| = \left| \frac{1}{K_p} s(t) - \frac{K_i}{K_p} D^\nu \Phi(e(t), \tau) - \frac{K_d}{K_p} D^\xi \Phi(e(t), \tau) \right|
\leq \left| \frac{1}{K_p} s(t) \right| + \frac{K_i}{K_p} D^\nu \Phi(e(t), \tau) + \frac{K_d}{K_p} D^\xi \Phi(e(t), \tau) \leq 3\ell \frac{K_p}{K_p}
$$

(47)

According to (47), the control error of the proposed compound NLSEF-STFOSMC can converge to a small neighborhood of zero, and the stable condition has been achieved.

4. Comparative Simulations

To verify the effectiveness of the proposed FOSMC, STFOSMC, and NLSEF-STFOSMC, we implement the comparative simulations based on the Lenovo Ideapad 320s, Intel(R) Core i7 7500U, CPU @2.70 GHz, RAM 8 GB, 1T BHDD, NVIDIA GeForce 920MX + IntelGMAHD620, 64 bit operating system, Matlab (R2014a)/Simulink.

Tables 1–6 show the parameters of the PMSM, traditional SMC, FOSMC, STFOSMC, novel ESO, and traditional ESO, respectively.

Figure 7 demonstrates the time responses of the sliding surfaces under the traditional SMC, the proposed FOSMC, the proposed STFOSMC, and the proposed compound NLSEF-STFOSMC. Figure 7(a) shows that the time response of the novel FOPID sliding surface is closer to $s = 0$ than the time response of the traditional FOPID sliding surface. Furthermore, it is clear that the reaching time under the proposed STFOSMC and the proposed compound NLSEF-STFOSMC is shorter than the reaching time under the
Figure 7: (a) Time response curves of the sliding functions under the traditional SMC, the proposed FOSMC, the proposed STFOSMC, and the proposed compound NLSEF-STFOSMC. (b) Partial enlarged views of the image on the left.

Figure 8: (a) $Z_{22}$ curves under the novel ESO and the traditional ESO. (b) Partial enlarged views of the image on the left.

Figure 9: (a) Speed response curves under the SMCs with $\omega_r = 100$. (b) Partial enlarged views of the image on the left.
FOSMC and the traditional SMC. Figure 7(b) reveals that the time response curve of the sliding surface under the proposed compound NLSEF-STFOSMC is the smoothest of the four sliding mode surfaces.

The reference speed is set to 100 rad/s at 0s, and the motor starts with 3 N·m load torque. Figure 8 shows the estimated disturbance results of the novel ESO and the traditional ESO. Due to the application of the nonlinear saturation function, the high-frequency flutter phenomenon is weakened significantly and the error of observer estimation is also reduced.

Figures 9 and 10 show the speed response curves and $i_q(t)$ current curves, respectively. Table 7 shows the speed adjustment time with load start-up of the motor under the four control strategies. From Figure 9(a) and Table 7, the results show that the traditional SMC has longer adjustment time than those of the proposed STFOSMC, the proposed FOSMC, and the proposed compound NLSEF-STFOSMC. The proposed compound NLSEF-STFOSMC has shorter adjustment time than the other control strategies. Figure 9(a) shows that the proposed compound NLSEF-STFOSMC exhibits better dynamic performance than the other control strategies. From Figure 9(b), it is obvious that the proposed three strategies have better tracking performances and that the tracking errors in steady state are smaller than those of the traditional SMC. Figure 10 shows that the chattering phenomenon of the traditional SMC, the FOSMC, and the STFOSMC is significantly larger than that of the compound NLSEF-STFOSMC. Noticeably, the comparison results obviously show that the proposed compound NLSEF-STFOSMC has excellent dynamic and static performance.

The simulation results compare the robustness of the proposed STFOSMC, the traditional SMC, the proposed FOSMC, and the proposed compound NLSEF-STFOSMC under the external disturbance. In the first simulation, when the external load becomes 0 N·m from 6 N·m at 1 s, the corresponding curves of the speed of the different SMC strategies are shown in Figure 11. In the second simulation, when the external load becomes 6 N·m from 0 N·m at 1 s, the corresponding curves of the speed of the different SMC strategies are shown in Figure 12. In order to better compare the robustness of the four control strategies, speed perturbation and speed recovery time are compared in Table 8. It can be seen from Figures 11–12 and Table 8 that the four SMC strategies have experienced different levels of impact caused by the external load disturbances. The simulation results obviously demonstrate that the proposed compound NLSEF-STFOSMC exhibits the shortest speed recovery time and the smallest speed perturbation. Therefore, the simulation results demonstrate that the proposed compound NLSEF-STFOSMC has stronger robustness than other control strategies.

### 5. Conclusions and Future Work

This study successfully presents a novel compound NLSEF-STFOSMC for the speed control of PMSM servo drive based on a novel ESO. Firstly, a novel FOPID switching manifold with a nonlinear integral term and a nonlinear differential term is designed. An STFOSMC involving the novel FOPID switching manifold and an ST reaching law is successfully suggested. Secondly, a novel nonlinear ESO is
designed, which can accurately estimate the external disturbance without the high-frequency flutter phenomenon. Thirdly, a compound NLSEF-STFOSMC is proposed combining the advantages of the modified SMC and the NLSEF. Lyapunov approach and Fractional calculus are used to show the stability for the system. Finally, the comparison results show that the proposed hybrid control strategy has excellent dynamic and static performance and strong robustness.

In the future, the proposed compound NLSEF-STFOSMC strategy will be further verified by the cSPACE semiphysical control system based on TI TMS320F28335 DSP and Matlab/Simulink. Figure 13 shows the PMSM verification platform.
Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest.

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