

Research Article

Detection of Weak Signal Based on Parameter Identification of Delay Differential System with Noise Disturbance

QiuBao Wang , Yuejuan Yang, and Xing Zhang

Department of Mathematical and Physics Shijiazhuang Tiedao University, Shijiazhuang 050043, Hebei, China

Correspondence should be addressed to QiuBao Wang; wangqiubao12@sina.com

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Extraction of the weak signals is crucial for fault prognostics in which case features are often very weak and masked by noise. In the time domain, the detection of weak signals depends on the identification of nonlinear parameters. A new signal detection and estimation method based on the incremental harmonic balance (IHB) is developed from the stochastic Van der Pol–Duffing equation with delayed feedback under parametric excitation. This is the first time that the IHB has been applied for the identification of parameters in stochastic delay differential equations (SDDEs). Compared to the method of intermittency transition between order and chaos to detect weak signals, this new method is more direct and the calculation result is what we want to obtain. This new method is suitable for the generalization and application of SDDEs.

1. Introduction

It is well known that a self-excited system can exhibit a remarkable complex dynamical behaviour. The earliest mathematical model has been proposed by Van der Pol to describe the vacuum tube circuit [1], and subsequently, many researchers have studied the complex phenomena observed from experiments and computer simulations in the fields of mechanical, electric, ecological, and chemical systems and so on. For rotating machinery, undesirable bifurcations, high-amplitude vibrations caused by resonance, quasiperiodic motion, and chaotic behaviour may occur and cause degradation or catastrophic failure of the system.

Bearing failure is one of the foremost causes of breakdowns in rotating machinery, and such failure can be catastrophic, resulting in huge economic losses. To prevent these kinds of failures from happening, various bearing condition monitoring techniques have been developed. Among them, vibration analysis has been used extensively due to its intrinsic advantage of revealing bearing failure [2–4]. Detecting the dangerous vibrations is very crucial for engineering science, and various papers have been dedicated for the control of resonantly forced systems in various applicative fields; Atay [5] studied the effect of delayed

position feedback on the response of a Van der Pol oscillator, showing that the delay can change the amplitude of limit cycle oscillations or suppress them altogether. Lai and Leng [6] considered a two-dimensional Duffing oscillator and proposed the weak-signal detection approach based on the GPASR model. A filtering technique through chaotic regime-switching is proposed by Denis et al. [7]. The inherent deficiency of the measuring mechanism leads to a mass of various noises to the effective signal inevitably. The signal of a defective bearing is spread across a wide frequency band and hence can become masked by noise and low-frequency signals with ease [8]. Prognostics is achieved by detecting the failure and defect at its initial stage and alerting maintenance personnel before it develops into a sharp loss. Nevertheless, in the early failure stage, it is difficult to detect the weak rolling bearing failure signal.

To solve the problem of detecting effective weak failure signals under strong noise, numerous scholars have proposed various methods: (1) apply the transform to achieve a goal of signal demodulation. Pang et al. [9] present an improved Hilbert time transform (IHTT), and the wavelet transform has been used for signal demodulation [10]. (2) Construct filters based on noise type and application to conduct the denoising [11, 12]. (3) Utilize the characteristics

of chaos system which is immune to noise and sensitive to weak signals at the same time to detect the weak effective failure signals. Zhao [13] detects the weak signal submerged in strong noise successfully by the transition from the chaotic state to a large-scale periodic state. (4) Use noise to enhance the amplitude of weak failure signal. Yang and Cao [14] uses the principle of primary resonance and stochastic resonance to study time delay-controlled SD oscillator with stiffness nonlinearities, and it shows that the weak harmonic signal can be enhanced with appropriate noise and time delay. (5) In recent years, detecting weak signals based on the deep network has become a new way in the background of machine learning artificial intelligence that can automatically extract bearing signals and identify faults [15, 16]. Xia et al. [17] propose an ensemble framework based on convolutional bidirectional long short-term memory with multiple time windows (MTW CNN-BLSTM ensemble) for accurately predicting remaining useful life.

It is worth noting that stochastic resonance is a nonlinear system involving noise and weak periodic signals; under the right conditions, the noise enhances the periodic signal of the weak signal where Kramers rate [18] is the key. And, the chaotic threshold is of great importance for detecting harmonic weak signal in the way of transition from the chaos to large-scale period, where the method of Melnikov function [19] can solve this problem very well. Moreover, a mass of data is needed to train the network layer with deep network detecting weak signals. In addition, Dong et al. [20] integrate a Bayesian updating prognostic model using sensor-based degradation information for computing each machine's time-to-failure (TTFs), with an opportunistic maintenance policy handling flexible system structures for optimizing the maintenance scheduling. In this paper, a new method for detecting weak signals is proposed based on SDDEs.

In the time domain, the basis of detecting weak signals is parameter identification, starting from the 1970s, showing a variety of system parameter recognition methods. Nayfeh [21] proposed a nonlinear system parameter recognition method, which is based on the response of nonlinear systems and the comparison between the system hypothesis model.

A method of recognition parameters based on the wavelet transform principle is proposed by Staszewski [22], which can identify parameters in multi-DOF nonlinear systems, but this method can only deal with free vibration system. Chen et al. [23] carried out parameter identification of nonlinear systems based on neural networks. In recent years, parameter identification is applied in various fields, such as automatic identification and medical research [24–26].

The incremental harmonic balance (IHB) method was originally developed by Lau et al. [27] for treating periodic structural vibrations and which is a semianalytical method; moreover, it is remarkably effective in computer implementation for obtaining both the stable and unstable solutions. Then, through the efforts of many scholars, the IHB method has been successfully applied to many fields of nonlinear vibration, and Shen [28] successfully applied the IHB method to study the bifurcation and the route-to-chaos analyses. Dou and Ye [29, 30] raised the method of IHB to

rectify the harmonic coefficients in frequency response calculating, which may help the identification algorithm to correct the noise-induced distortion in the experiment, and compared with the harmonic balance (HB) method, the accuracy of parameter identification has obvious advantages. Moreover, these methods are applied in ordinary differential equations (ODEs), and it is necessary to study identifying parameters with the delay differential equations (DDEs). The study of time-delay system is more consistent with the real vibration because a state in the past will affect the vibration state now [31]. Recently, Zhang [32] applies the IHB method to solve the superharmonic resonance solution of nonlinear time-delay differential equations. The research methods of parameter estimation in time-delay chaotic systems are endless, for example, neural network method [33] and self-synchronization method [34].

In this paper, for the first time, the IHB method is applied to SDDEs for the identification of weak signals, and from the result of parameter identification, it is more direct than the chaos detection method. We take stochastic delay Van der Pol–Duffing equation, for example, to identify parameters. The delay τ is reduced by scaling, and the results show that it does not affect the identification. Calculating the explicit expressions K, P , and R is the important point of this paper, and then the explicit expressions of K, P , and R are substituted into equation (9) and iterated until the $\|R\|$ is small enough; then, the result is the value of the parameter to be identified, which is clearly expressed. Finally, the ability of stochastic delay Van der Pol–Duffing system to detect weak signals is evaluated through the signal to noise ratio (SNR).

2. Stochastic Van der Pol–Duffing Oscillator under Time-Delayed Feedback

The Van der Pol–Duffing oscillator is a representative kind of system in the nonlinear system, and the system can show rich nonlinear dynamic behaviour with the change in strength in the period force [19, 35, 36]. As a classic chaotic system, Van der Pol–Duffing oscillator is often used in the modeling of dynamical systems; nowadays, there are many nonlinear problems in the fields of material resources, biology, engineering, psychiatry, and economics, which can be discussed and analyzed based on the modeling of nonlinear systems [37]. With the further study of vibration equation and the process of modeling, the stochastic Van der Pol–Duffing equation under the time-delayed feedback is more consistent with the process of actual measurement and signal acquisition, which has rich dynamic behaviour. The characteristic of being sensitive to a weak signal with strong noise perturbation makes it possible to be used in the field of weak signal detection. For stochastic Van der Pol oscillator with time-delayed feedback, some scholars have studied its response and stability analysis [38].

The stochastic Van der Pol–Duffing oscillator under time-delayed feedback is in the form of the following:

$$\ddot{x}(t) + \alpha(x^2(t - \tau) - 1)\dot{x}(t) + \beta x(t) = f \cos(\Omega t) + \sigma \xi(t), \quad (1)$$

where α and β are the damping coefficient and stiffness coefficient, respectively; $f \cos(\Omega t)$ is the external excitation; f and Ω are the amplitude and angular frequency of external excitation, respectively; τ is the time delay; $x^2(t - \tau)\dot{x}(t)$ indicates that the square of displacement at $t - \tau$ will affect the velocity at t , namely, displacement feedback; and $\xi(t)$ is the Gaussian white noise with a constant power spectral density \bar{K} and autocorrelation function $R(v) = E[\xi(t)\xi(t + v)] = \delta(v)$, where v is autocorrelation time and δ is Dirac function; moreover, $E[\cdot]$ denotes the expectation operator and $\sigma = \sqrt{2\pi\bar{K}}$ denotes noise intensity.

For various parameters f, τ , and Ω , the system has rich nonlinear dynamic behaviours, seen from Figures 1–3.

3. The Process of Parameter Identification

The influence of noise on the result of parameter identification can be fully considered in the process of numerical simulation. We can write equation (1) without noise as the equation of state, as follows:

$$\begin{cases} \dot{y}_1(t) = y_2(t), \\ \dot{y}_2(t) = f \cos(\Omega t) - \alpha y_1^2(t - \tau)y_2(t) + \alpha y_2(t) - \beta y_1(t). \end{cases} \quad (2)$$

Then, take scale transformation $t \rightarrow (t/\tau)$ and write it as follows for convenience:

$$\ddot{y}(t) - \alpha\tau^2 \dot{y}(t) + \beta\tau^2 y(t) + \sum_{i=1}^M b_i g_i(y(t), \dot{y}(t), y(t-1), \tau, t) = 0, \quad (3)$$

where $M = 2$, $b_1 = \tau^2 \alpha$, $b_2 = -\tau^2 f$, $g_1 = y^2(t-1) \dot{y}(t)$, $g_2 = \cos(\Omega\tau t)$, and b_1, b_2 are nonlinear parameters to be identified.

4. Construct Increment Process

y , $y(t-1)$, and $\{b_i | i = 1, 2\}$ are as follows:

$$\begin{aligned} y(t) &= y_0(t) + \Delta y(t), \\ y(t-1) &= y_0(t-1) + \Delta y(t-1), \\ b_i &= b_{i0} + \Delta b_i, \quad i = 1, 2. \end{aligned} \quad (4)$$

Working with initial values $\{b_{i0} = 0 | i = 1, 2\}$. Determine the initial values of the displacement $y_0(t)$ and $y_0(t-1)$ based on the experimental data, that is, depending on the displacement response on the $[t_0, t_0 + T]$ when the system enters the steady-state period. Let $\omega = \Omega/\mu$ (μ is an integer), period $T = 2\pi/\omega$. The initial values of displacement are expanded into the Fourier series in plural form, and they are written in the following matrix form, where N is the degree of expansion of harmonic term:

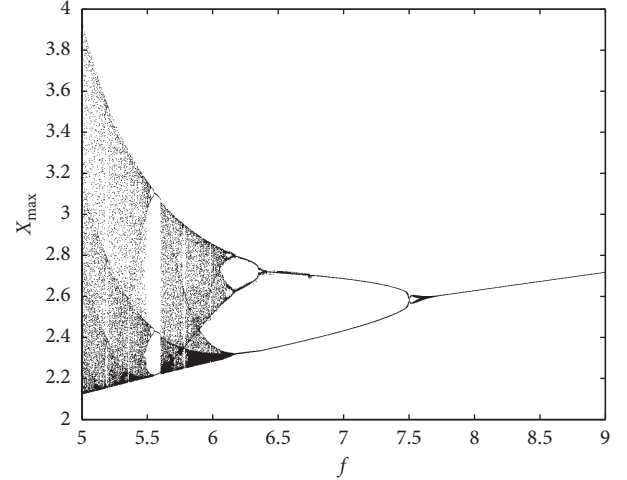


FIGURE 1: The bifurcation diagram of f with certain fixed parameters $\tau = 2$, $\alpha = 5$, $\beta = 1$, and $\Omega = 1$ shows that chaotic state \rightarrow quasiperiodic motion \rightarrow period-doubling bifurcation \rightarrow periodic motion with the increasing f .

$$y_0(t) = \sum_{k=-N}^N e^{jk\omega t} \cdot a_k = Y_1 \cdot A_0, \quad (5)$$

$$y_0(t-1) = \sum_{k=-N}^N e^{jk\omega(t-1)} \cdot a_k = Y_2 \cdot A_0, \quad (6)$$

where

$$\begin{aligned} Y_1 &= [e^{j(-N)\omega t} \dots 1 \dots e^{jN\omega t}], \\ Y_2 &= [e^{j(-N)\omega(t-1)} \dots 1 \dots e^{jN\omega(t-1)}], \\ A_0 &= [a_{-N} \dots a_0 \dots a_N]^T. \end{aligned} \quad (7)$$

Each element in A_0 is the Fourier coefficient of each order, where the k -order Fourier coefficient is obtained by numerical integral transformation of the data series of the displacement response:

$$\begin{aligned} a_{k1} &= \frac{1}{T} \int_0^T \tilde{y}_0(t) \cdot e^{-jk\omega t} dt, \\ a_{k2} &= \frac{1}{T} \int_0^T \tilde{y}_0(t-1) \cdot e^{-jk\omega(t-1)} dt, \end{aligned} \quad (8)$$

where $\tilde{y}_0(t)$ and $\tilde{y}_0(t-1)$ are a sequence of displacement response measured and collected in experiments. To the extent that the error allows $a_{k1} \approx a_{k2}$, $a_k = (1/T) \int_0^T \tilde{y}_0(t) \cdot e^{-jk\omega t} dt$. Working with the incremental processes. Write $\Delta y(t)$ and $\Delta y(t-1)$ in the form of Fourier series in the plural form:

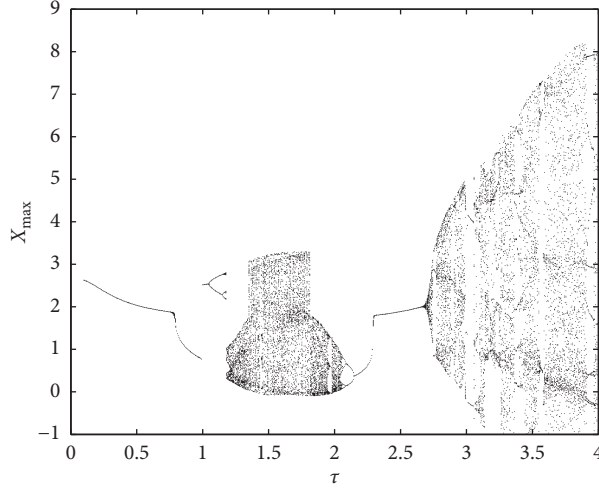


FIGURE 2: The bifurcation diagram of τ with certain fixed parameters $\alpha = 0.5$, $\beta = 1$, $\Omega = 1$, and $f = 1$.

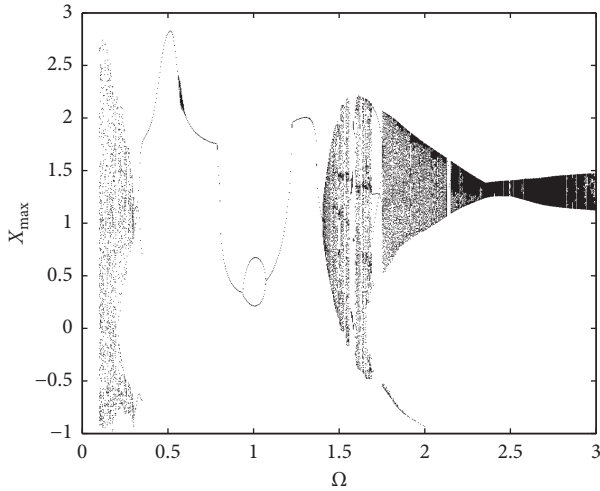


FIGURE 3: The bifurcation diagram of Ω with certain fixed parameters $\alpha = 0.45$, $\beta = 1$, $\tau = 2$, and $f = 1$.

$$\begin{aligned}\Delta y(t) &= Y_1 \cdot \Delta A, \\ \Delta y(t-1) &= Y_2 \cdot \Delta A, \\ \Delta A &= [\Delta a_{-N} \dots \Delta a_0 \dots a_N]^T.\end{aligned}\quad (9)$$

The increments of the parameters are $\Delta b = [\Delta b_1 \ \Delta b_2]^T$.

Formulas in (4) are substituted into equation (3), by the means of Taylor unfold, and omitting the incremental two times and more than two high-order items, we can obtain

$$\begin{aligned}\Delta \ddot{y}(t) - \alpha \tau^2 \Delta \dot{y}(t) + \beta \tau^2 \Delta y(t) + 2b_{10}y_0(t-1)\dot{y}_0(t)\Delta y(t-1) \\ + b_{10}y_0^2(t-1)\Delta \dot{y}(t) + y_0^2(t-1)\dot{y}(t)\Delta b_1 + \cos(\Omega \tau t)\Delta b_2 \\ = -\ddot{y}_0(t) + \alpha \tau \dot{y}_0(t) - \beta \tau^2 y_0(t) - b_{10}y_0^2(t-1)\dot{y}_0(t) - b_{20} \cos(\Omega \tau t).\end{aligned}\quad (10)$$

5. Construct Increment Equation

With the method of Galerkin and the regular $\Delta y(t) = Y_1 \Delta A = \Delta A^T Y_1^T = (\Delta A Y_1)^T = (\Delta y(t))^T$, we can obtain

$$\begin{aligned}(\delta \Delta A)^T \int_{t_0}^{t_0+T} Y_1^T [\ddot{Y}_1 - \alpha \tau \dot{Y}_1 + \beta \tau^2 Y_1 + 2b_{10}y_0(t-1)\dot{y}_0(t)Y_2 \\ + b_{10}y_0^2(t-1)\dot{Y}_1] dt \cdot \Delta A \\ + (\delta \Delta A)^T \int_{t_0}^{t_0+T} Y_1^T [y_0^2(t-1)\dot{y}(t)] dt \cdot \Delta b_1 \\ + (\delta \Delta A)^T \int_{t_0}^{t_0+T} Y_1^T [\cos(\Omega \tau t)] dt \cdot \Delta b_2 \\ = (\delta \Delta A)^T \int_{t_0}^{t_0+T} Y_1^T [-\ddot{y}_0(t) + \alpha \tau \dot{y}_0(t) - \beta \tau^2 y_0(t) \\ - b_{10}y_0^2(t-1)\dot{y}_0(t) - b_{20} \cos(\Omega \tau t)] dt.\end{aligned}\quad (11)$$

With arbitrary of $\delta \Delta A$, we can obtain the incremental equation as follows:

$$K \Delta A + P \Delta b = R. \quad (12)$$

The elements of K , P , and R are

$$\begin{aligned}K &= \frac{1}{T} \int_{t_0}^{t_0+T} Y_1^T [\ddot{Y}_1 - \alpha \tau \dot{Y}_1 + \beta \tau^2 Y_1 + 2b_{10}y_0(t-1)\dot{y}_0(t)Y_2 \\ &\quad + b_{10}y_0^2(t-1)\dot{Y}_1] dt, \\ P_1 &= \frac{1}{T} \int_{t_0}^{t_0+T} Y_1^T [y_0^2(t-1)\dot{y}(t)] dt, \quad P_2 = \frac{1}{T} Y_1^T [\cos(\Omega \tau t)] dt, \\ R &= \frac{1}{T} \int_{t_0}^{t_0+T} Y_1^T [-\ddot{y}_0(t) + \alpha \tau \dot{y}_0(t) - \beta \tau^2 y_0(t) \\ &\quad - b_{10}y_0^2(t-1)\dot{y}_0(t) - b_{20} \cos(\Omega \tau t)] dt.\end{aligned}\quad (13)$$

TABLE 1: Parameter recognition results of incremental harmonic equilibrium method in different states of time-delay system, where $b_1^* = \alpha\tau$ and $b_2^* = -\tau^2 f$ are exact values.

Parameters	σ	0%			1%			5%		
		$f = 6$	$f = 7$	$f = 8$	$f = 6$	$f = 7$	$f = 8$	$f = 6$	$f = 7$	$f = 8$
b_1	10	10.00968	10.00962	10.00515	10.00962	10.00962	10.00512	10.00975	10.00965	10.00515
	15	10.00956	10.00957	10.00497	10.00961	10.00951	10.00511	10.00957	10.00950	10.00503
	20	10.00959	10.00954	10.00503	10.00961	10.00951	10.00506	10.00972	10.00952	10.00503
	25	10.00961	10.00951	10.00501	10.00957	10.00950	10.00510	10.00963	10.00954	10.00502
	30	10.00959	10.00949	10.00503	10.00973	10.00950	10.00500	10.00951	10.00963	10.00501
b_2	10	24.00924	28.00882	32.00723	24.00920	28.00860	32.00723	24.00903	28.00873	32.00723
	15	24.00941	28.00886	32.00769	24.00950	28.00871	32.00710	24.00933	28.00871	32.00810
	20	24.00937	28.00889	32.00769	24.00934	28.00882	32.00820	24.00932	28.00872	32.00841
	25	24.00940	28.00882	32.00774	24.00931	28.00881	32.00720	24.00891	28.00881	32.00790
	30	24.00932	28.00886	32.00769	24.00943	28.00881	32.00770	24.00937	28.00877	32.00757

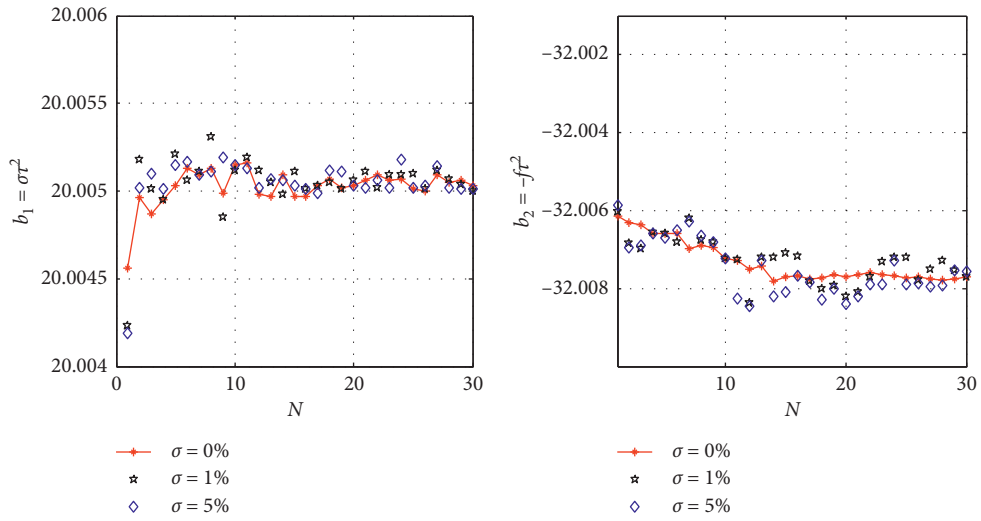


FIGURE 4: The influence of noise on parameter recognition in a periodic state with $f = 8$ and $\tau = 2$.

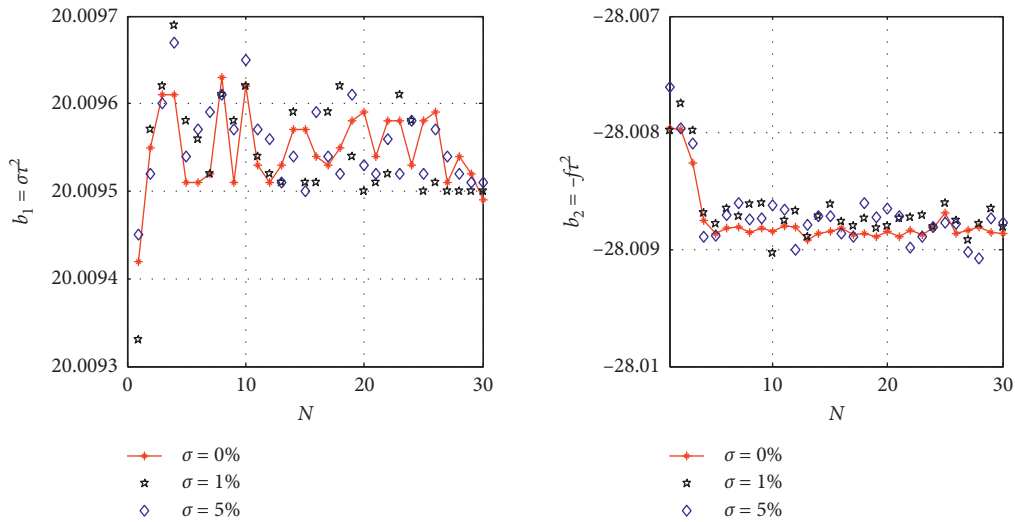


FIGURE 5: The influence of noise on parameter recognition in period-2 state with $f = 7$ and $\tau = 2$.

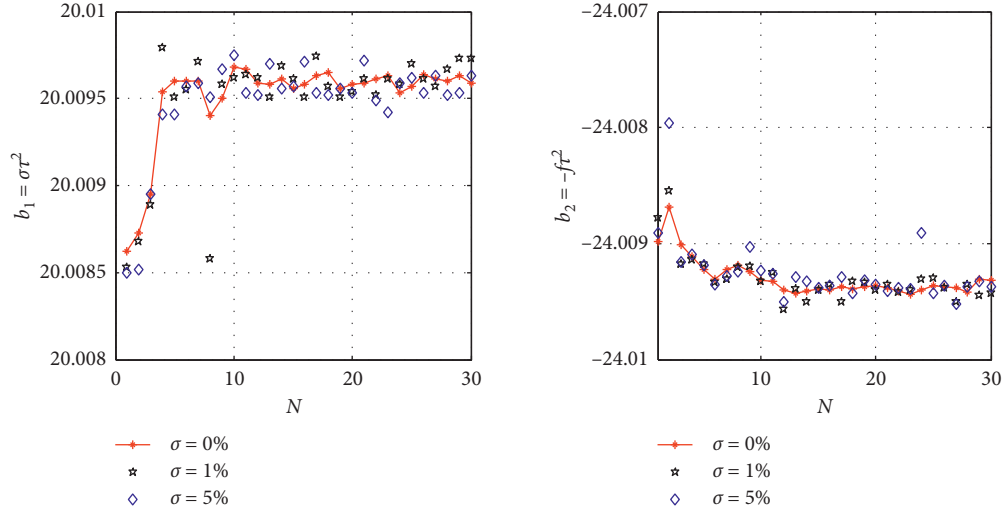


FIGURE 6: The influence of noise on parameter recognition in a chaotic state with $f = 6$ and $\tau = 2$.

To know the explicit expression of elements in K , P , and R , according to the plural form of Fourier series of the initial displacement expansion, the coefficients of the Fourier series in the plural form corresponding to $\dot{y}_0(t)$, $y_0^2(t-1)$, $y_0^2(t-1)\dot{y}_0(t)$, $\ddot{y}_0(t)$, $\cos(\Omega t)$ are obtained:

$$\begin{aligned}
 \dot{y}_0(t) &= \sum_{k=-N}^N a_{k3} \cdot e^{jk\omega t}, & a_{k3} &= a_m \cdot jm\omega \delta_m^k, \\
 y_0^2(t-1) &= \sum_{k=-N}^N a_{k4} \cdot e^{jk\omega t}, & a_{k4} &= a_m a_1 \cdot e^{-j\omega(m+1)} \delta_{m+1}^k, \\
 y_0(t-1)\dot{y}_0(t) &= \sum_{k=-N}^N a_{k5} \cdot e^{jk\omega t}, & a_{k5} &= a_m a_{n3} e^{-jm\omega} \delta_{m+n}^k, \\
 y_0^2(t-1)\dot{y}_0(t) &= \sum_{k=-N}^N a_{k6} \cdot e^{jk\omega t}, & a_{k6} &= a_{m4} a_{13} \delta_{m+1}^k, \\
 \ddot{y}_0(t) &= \sum_{k=-N}^N a_{k7} \cdot e^{jk\omega t}, & a_{k7} &= -m^2 \omega^2 a_m \delta_m^k.
 \end{aligned} \tag{14}$$

Calculate the Fourier coefficient q_k of the harmonic excitation item $\cos(\Omega t)$ based on the experimental data:

$$\cos(\Omega t) = \sum_{k=-N}^N q_k \cdot e^{jk\omega t}, \quad q_k = \frac{1}{T} \int_0^T \frac{\tilde{F}(t)}{\rho} e^{-jk\omega t} dt, \tag{15}$$

where $\tilde{F}(t)$ is the excitation force signal collected experimentally and ρ is shaking force amplitude. Let $\bar{m} = m + N + 1$, $\bar{n} = n + N + 1$, then we can write the explicit expression of elements in K , P , and R :

$$\begin{aligned}
 K(\bar{m}, \bar{n}) &= (-n^2 \omega^2 - \alpha \tau j n \omega + \beta \tau^2) \delta_{m+n}^0 \\
 &\quad + \sum_{k=-N}^N (2b_{10} a_{k2} + b_{10} a_{k3} j n \omega) e^{-jn\omega} \delta_{m+n+k}^0, \\
 P_1(\bar{m}, 1) &= a_{k4} \delta_{m+k}^0, \quad P_2(\bar{m}, 2) = q_k \delta_{m+k}^0, \\
 R(\bar{m}, 1) &= (-a_{k5} + \alpha \tau a_{k1} - \beta \tau^2 a_k - b_{10} a_{k4} - b_{20} q_k) \delta_{m+k}^0.
 \end{aligned} \tag{16}$$

Equation (16) is substituted into equation (12) iteratively. It is clear that equation (12) has $2N + 1 + 2$ unknowns $\{\Delta a_k, k = -N, \dots, N; \Delta b_1, \Delta b_2\}$, but there are $2N + 1$ equations. So the increment of the harmonic item coefficients ΔA and Δb as the tuning amount is obtained by equation (12) on condition that no less than 2 principal harmonic coefficients increment Δa_k zero. Δy , y , and b_1, b_2 can be obtained from equations (4) and (9), respectively, and then replace the initial values of y_0 and b_{10}, b_{20} to (16) and enter the next iteration, and so on. We do not stop iterating until $\|R\|$ is small enough. When the iteration stops, the results of b_1 and b_2 are the value of the parameter to be identified.

6. The Results of Identification

Consider the parameters $b_1 = \alpha \tau^2$ and $b_2 = -f \tau^2$ to be identified, and given parameter initial value $b_{10} = b_{20} = 0$; constructing incremental equations, select $a_0, a_{\pm 1}, a_{\pm 2}$ as the main harmonic coefficients, that is, $\Delta a_0 = \Delta a_{\pm 1} = \Delta a_{\pm 2} = 0$; solve the remaining harmonic item increments and parameter increments. and iterate over and over until the algorithm converges steadily. The identification results are shown below (Table 1).

TABLE 2: Results of parameters identification with the method of IHB.

Parameter	σ	0%	1%	5%
	N	Results of parameter identification		
b ($b^* = -\tau^2(f+r)$)	10	-32.020632	-32.020633	-32.020623
	15	-32.020632	-32.020637	-32.020638
	20	-32.020633	-32.020643	-32.020641
	25	-32.020641	-32.020631	-32.020630
	30	-32.020619	-32.020620	-32.020625

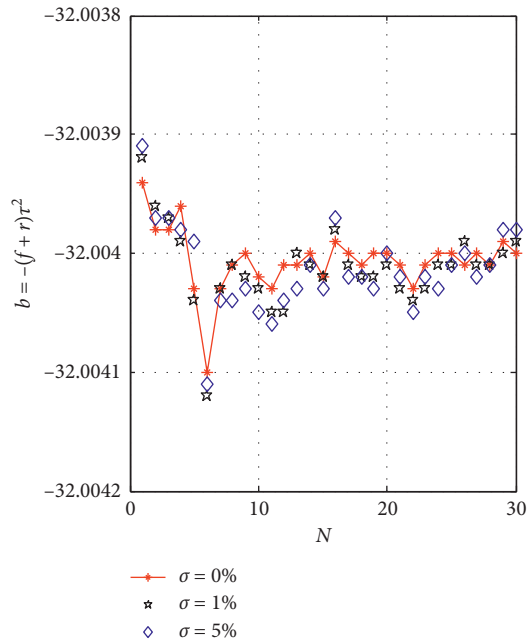


FIGURE 7: The influence of noise on parameter recognition with fixed parameters $f = 8, r = 0.001, \alpha = 5, \beta = 1, \Omega = 1,$ and $\tau = 2.$

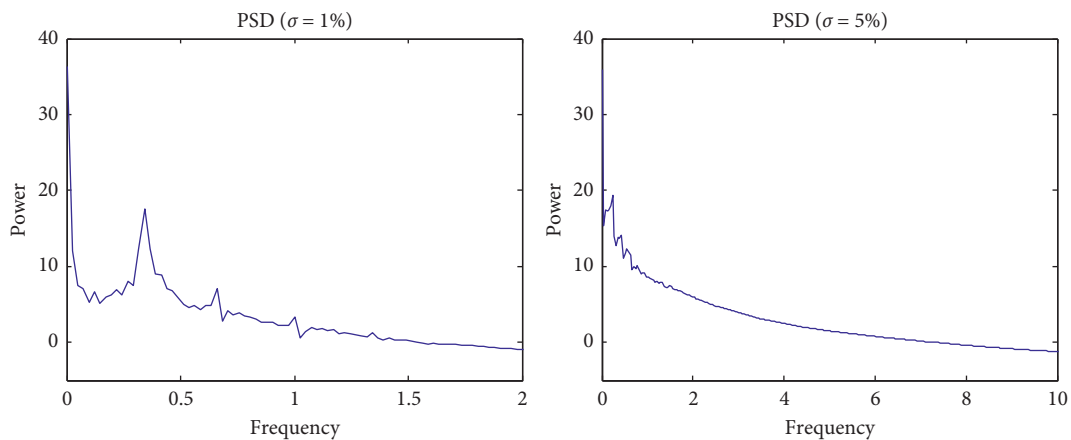


FIGURE 8: PSD under different noise intensity $\sigma = 1%$ and $\sigma = 5%.$

Figures 4–6 show that the recognition results of parameters b_1 and b_2 when the system is in different states, such as period, period-2, and chaos. We can get from the figures that the results of parameter recognition are stable when the

system is in different states. It is worth noting that the difficulty of parameter recognition using the IHB method is finding the appropriate period initial displacement value from the steady-state response to recognize the parameters.

7. Identify the Amplitude of Weak Signals

Add the weak signal under strong noise $r\tau^2 \cos(\Omega\tau t) + \sigma\tau^2\xi(t)$ to equation (3) with $r = 0.001$ and $\tau = 2$, and we can obtain

$$\begin{aligned} \ddot{y} - \alpha\tau^2 \dot{y}(t) + \beta\tau^2 y(t) + \alpha\tau^2(y^2(t-1))\dot{y}(t) \\ = (f+r)\tau^2 \cos(\Omega\tau t) + \sigma\tau^2\xi(t), \end{aligned} \quad (17)$$

where the parameter to be identified is $b = -(f+r)\tau^2$. From the results of identifying Table 2 and Figure 7 with certain parameters, we can obtain that $b \approx -32.004$ and then the amplitude of the weak harmonic signal is $r = 0.001$, which is consistent with the added signal amplitude so that the weak signal can be identified well.

Consider the SNR to judge how well the system detects weak signal under the noise intensity $\sigma = 0.05$:

$$\text{SNR} = 10 \lg\left(0.5 \frac{r^2}{\sigma^2}\right) = 10 \lg\left(0.5 \frac{0.001^2}{0.05^2}\right) = -36.99 \text{dB}. \quad (18)$$

The amplitude of the weak signal can be detected under the condition of low SNR, and it is proved that this method is worthy of application and popularization in the field of DDEs.

According to the system response power spectral density function under different noises, we can obtain the power spectral density (PSD) images, as shown in Figure 8; the frequency at the highest energy is $fre = (1/T) = 0.3180$ obviously. The angular frequency of a weak signal is $\Omega\tau = (2\pi/T)$, so $T = \pi$, and then, the frequency of a weak signal is $(1/T) = 0.3182$. We can detect weak signals from strong noise.

8. Conclusions

In this paper, there is a rich dynamic behaviour of the stochastic Van der Pol–Duffing oscillator with noise interference, and the parameters can be identified by the method of IHB in different states. For a very low SNR, the detection results are very good and this method can be extended to detect weak signals in nonlinear DDEs containing the certain noise.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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