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*Research Article*

# Method for Inferring the Design Value of the Resistance Based on Probabilistic Model

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Methods for inferring the design value of the resistance based on test have long been studied extensively, but the existing methods have several limitations on unified guarantee rate ensurance and reliability control. Firstly, the rationales and deficiencies of the present methods in ISO 2394 : 2015 and EN 1990 : 2002 were generalized. Secondly, in view of the disadvantages, a new inferring method combining the probability model of resistance with statistical approach was put forward. The proposed method established a relationship among design resistance, probability characteristics of known factors, and statistical results of unknown factors and possessed a rigorous and sound theoretical basis on both conditions that the coefficient of variation of model uncertainty was unknown and full known. Lastly, a contrast work was carried out between the Eurocode method and the proposed method; the results showed that the latter method had a higher inferring value, which means a better inferring result.

## 1. Introduction

The method for structure design based on test, aiming at ultimate limit state, was proposed in ISO 2394 : 1998 [1] and EN 1990 : 2002 [2] consecutively. Owing to its practicability and applicability to large batch of similar components, major or new structures, this method is adopted in GB 50153-2008 [3] and ISO 2394 : 2015 [4] and has become one of the major trends of structural design.

The experiment-aided design method combines experimental data and statistical approach to infer the design value of resistance (i.e., design resistance). The inference is usually based on resistance variable, which relates to the test results only, or based on resistance analysis model, which adopts probability characteristics of influence factors. The latter method reduces the uncertainty to a large extent and obtains a better inference result [1, 4], thus it has higher design value. For instance, the ISO 2394 : 2015 method has been introduced in Refs. [5–7] and EN 1990 : 2002 in Refs. [8, 9], while the detailed deducing processes are different. They are approximate and empirical separately, having certain limitations on the control of structural reliability.

Due to the limitations of the abovementioned two methods, an inferring method on the basis of the probability model of resistance and statistical theory is put forward which provides the ensurance of structural reliability control.

## 2. International Inferring Methods Based on the Analysis Model

*2.1. Method in ISO 2394 : 2015.* The design resistance of a structural element obtained from test, whose direct basis is the analysis model, can be calculated as follows [4]:

$$R_d = \frac{1}{\gamma_d}\eta_d g(x_d, w),$$
$$\gamma_d = \frac{1}{\theta_d}, \tag{1}$$

where $x_d$ and $w$ are the design value of random variable and the set of measurable deterministic variables, respectively; they are preknown; $g(\cdot)$ is the resistance model; $\eta_d$ is the design value of the conversion factor with the consideration

of differences between reality and test conditions and can be determined by experience directly; $\theta_d$ is the design value of the model uncertainty $\theta$ and should be deduced by test results.

In this international standard, $\theta$ is assumed as a lognormal distribution and its design value $\theta_d$ can be inferred from the Bayesian estimation method based on Jeffreys noninformative prior distribution [4, 10, 11]:

$$\begin{aligned} \theta_d &= \exp\left(m_{\ln\theta}\right)\exp\left(\pm t_{vd} s_{\ln\theta}\sqrt{1+\frac{1}{n}}\right), \\ m_{\ln\theta} &= \frac{1}{n}\sum_{i=1}^{n}\ln\theta_i, \\ s_{\ln\theta} &= \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}\left(\ln\theta_i - m_{\ln\theta}\right)^2}, \\ \theta_i &= \frac{R_i}{g\left(x_i, w_i\right)}, \quad i = 1, 2, \ldots, n, \end{aligned} \tag{2}$$

where $x_i$ and $R_i$ are the experimental values of $X$ and $R$, respectively, which have to be measured in every experiment; $w_i$ is the value of $W$ used in test; $\theta_i$ is the value of $\theta$ decided by test, $i = 1, 2, \ldots, n$, and $n$ is the number of specimen; $t_{vd}$ is the $\Phi(\beta_R)$ quantile of the student distribution with a freedom degree of $n-1$, where $\Phi(\cdot)$ is the standard normal distribution function, $\beta_R = \alpha_d\beta$, where $\beta$ is the target reliability index and $\alpha_d$ is the design value of sensitivity factor according to first-order reliability method (FORM). If the uncertainty of resistance is dominating during reliability analysis, $\alpha_d = 0.8$; if not, $\alpha_d = 0.3$.

The abovementioned method can be used in both the design value method and partial factor method. In the former, the guaranteed rate of the design resistance is $\Phi(\beta_R)$[4], while in the latter, this rate should be determined by probability analysis with $\beta_R$ replaced by $\Phi^{-1}(p_d)$.

To ensure the accuracy of reliability control, design resistance should adopt a uniform guarantee rate (i.e., $\Phi(\beta_R)$ or $p_d$) in the abovementioned two methods. While the implied rates are not all $\Phi(\beta_R)$ or $p_d$ in fact for the reason that the model reflecting the relationship between resistance and influence variables is only approximate. Of particular note is that $\Phi(\beta_R)$ is obtained just from the design value of model uncertainty in the partial factor method, resulting in a lower rate since many other influence variables exist, causing a higher referring value.

*2.2. Method in EN 1990 : 2002.* A probability model is used to calculate the design resistance in EN 1990 : 2002; furthermore, the weighted average method of the design resistance under two ideal conditions (i.e., no prior knowledge and full knowledge of the coefficient of variation) is applied.

Assuming that the resistance follows a lognormal distribution $\mathrm{LN}(\mu_{\ln R}, \sigma^2_{\ln R})$, the design resistance $R_d$ can be expressed as the following equation:

$$R_d = \exp\{\mu_{\ln R} - k\sigma_{\ln R}\}, \tag{3}$$

$$\mu_{\ln R} = \ln\frac{\mu_R}{\sqrt{1+V_R^2}}, \tag{4}$$

$$\sigma_{\ln R} = \sqrt{\ln\left(1+V_R^2\right)}, \tag{5}$$

$$k = \Phi^{-1}(p), \tag{6}$$

where $\mu_R$ and $V_R$ are the mean and coefficient of variation of the resistance; $p$ is the guarantee rate of $R_d$.

Supposing that $R_1, R_2, \ldots, R_n$ are the samples of resistance, the inference of $R_d$ can be expressed as equation (7) when the coefficient of variation $V_R$ (which means the distribution parameter $\sigma_{\ln R}$) is unknown and equation (8) when $V_R$ is known according to the Bayesian estimation method [2]:

$$R_d = \exp\left(m_{\ln R} - k_n s_{\ln R}\right), \tag{7}$$

$$R_d = \exp\left(m_{\ln R} - k_\infty \sigma_{\ln R}\right), \tag{8}$$

where $m_{\ln R}$ and $s_{\ln R}$ are the realized values of the mean and coefficient of variation of samples $\ln R_1, \ln R_2, \ldots, \ln R_n$ separately; $k_n = t_{(n-1,p)}\sqrt{1+1/n}$, where $t_{(n-1,p)}$ is the $p$ quantile of the student distribution with a freedom degree of $n-1$; $k_\infty = n_p\sqrt{1+1/n}$, where $n_p$ is the $p$ quantile of the standardized normal distribution.

The design resistance using the probability model can be expressed as the following equation [2]:

$$R = \theta b g_{rt}(X), \tag{9}$$

where $g_{rt}(\cdot)$ is the theoretical model of resistance; $b$ is the ratio between the experimental value and the theoretical value obtained by least squares regression; the mean value of model uncertainty $\theta$ is about 1.

Assuming that $\theta$ and $g_{rt}(X)$ obey lognormal distributions $LN(\mu_{\ln\theta}, \sigma^2_{\ln\theta})$ and $LN(\mu_{\ln rt}, \sigma^2_{\ln rt})$ separately, the following simplification can be made:

$$Q^2 = Q_\theta^2 + Q_{rt}^2, \tag{10}$$

with

$$\begin{aligned} Q &= \sigma_{\ln R} = \sqrt{\ln\left(1+V_R^2\right)}, \\ Q_\theta &= \sigma_{\ln\theta} = \sqrt{\ln\left(1+V_\theta^2\right)}, \\ Q_{rt} &= \sigma_{\ln rt} = \sqrt{\ln\left(1+V_{rt}^2\right)}, \end{aligned} \tag{11}$$

where $V_\theta$ and $V_{rt}$ are the coefficient of variation of $\theta$ and $g_{rt}(X)$ separately. In most cases, $Q$ is known partly for $Q_\theta$ being unknown, while $Q_{rt}$ is full known. Therefore, the inferring value of design resistance ranges from the value obtained by no prior knowledge about $Q$ to full knowledge of $Q$. Moreover, with the rising of the ratio of $Q_\theta^2$ to $Q^2$, the inferring result will get closer to the case where $Q$ is unknown, from the value obtained by no prior knowledge

about $Q$ to full knowledge of $Q$. Thus, the weighting factors can take the following expressions:

$$\alpha_\theta^2 = \frac{Q_\theta^2}{Q^2} = \frac{\ln\left(1+V_\theta^2\right)}{\ln\left(1+V_R^2\right)},$$
$$\alpha_{rt}^2 = \frac{Q_{rt}^2}{Q^2} = \frac{\ln\left(1+V_{rt}^2\right)}{\ln\left(1+V_R^2\right)}. \tag{12}$$

After weighted average of the inferring results was obtained by equations (7) and (8), the following equation can be obtained:

$$\begin{aligned}\ln R_d &= \alpha_{rt}^2\left(m_{\ln R} - k_\infty s_{\ln R}\right) + \alpha_\theta^2\left(m_{\ln R} - k_n Q\right)\\ &= m_{\ln R} - k_\infty \alpha_{rt}^2 s_{\ln R} - k_n \alpha_\theta^2 Q.\end{aligned} \tag{13}$$

Assuming the following approximate equations are available,

$$Q_\theta \approx s_{\ln\theta}, \tag{14}$$

$$Q = \sqrt{Q_\theta^2 + Q_{rt}^2} \approx \sqrt{s_{\ln\theta}^2 + Q_{rt}^2}, \tag{15}$$

$$s_{\ln R} \approx \sqrt{s_{\ln\theta}^2 + Q_{rt}^2} \approx Q, \tag{16}$$

$$\begin{aligned}m_{\ln R} &\approx \ln\frac{m_R}{\sqrt{1+V_R^2}}\\ &= \ln\left[m_\theta b g_{rt}\left(\mu_X\right)\right] - 0.5Q^2,\end{aligned} \tag{17}$$

the inference design resistance can be expressed as follows [2]:

$$R_d = m_\theta b g_{rt}\left(\mu_X\right)\exp\left(-k_\infty \alpha_{rt} Q_{rt} - k_n \alpha_\theta Q_\theta - 0.5Q^2\right), \tag{18}$$

where $m_R$ is the realized value of mean of samples $R_1, R_2, \ldots, R_n$; $\mu_X$ is the vector of mean of influence factors of resistance; $m_\theta$ is the realized value of mean of samples $\theta_1, \theta_2, \ldots, \theta_n$, when fitting the test results by the least squares method; the value is 1.

If the coefficient of variation of $\theta$ is known, the inference design resistance can be expressed as the following equation [2]:

$$R_d = m_\theta b g_{rt}\left(\mu_X\right)\exp\left(-k_\infty Q - 0.5Q^2\right). \tag{19}$$

It is experienced to adopt the weighted average method to infer the design resistance in EN 1990 : 2002. More seriously, there are four drawbacks in the abovementioned method:

Firstly, $k_\infty$ should be $n_p\sqrt{1+1/n}$, not $n_p$ in equations (18) and (19), for large errors will be produced when the number of specimen is not large;

Secondly, the relationship between $m_{\ln R}$ and $m_R$ in equation (17) is not valid when the number of experiments is small, while the available relationship should be expressed as follows:

$$\mu_{\ln R} = \ln\frac{\mu_R}{\sqrt{1+\delta_R^2}}, \tag{20}$$

where $\mu_{\ln R}$ and $\mu_R$ are the true values of $m_{\ln R}$ and $m_R$, respectively;

Thirdly, equations (7) and (8) respectively represent the conditions that the coefficient of variation of resistance are unknown and full known, where $s_{\ln R}$ is a statistical value and $\sigma_{\ln R}$ is a given distribution parameter. However, in the process of establishing equation (18), distribution parameter $Q$ is determined by $s_{\ln\theta}$ in accordance with equation (15), which does not represent a full known condition.

Finally, equation (16) which calculates the statistical value $s_{\ln R}$ does not conform to the calculation of standard deviation in statistics. The correct equation should be expressed as follows [12]:

$$s_{\ln R} = \sqrt{s_{\ln\theta}^2 + 2s_{\ln\theta,\ln rt} + s_{\ln rt}^2}, \tag{21}$$

where $s_{\ln\theta,\ln rt}$ is the covariance of $\ln\theta$and $\ln g_{rt}(X)$; $s_{\ln rt}$ is the standard deviation of $\ln g_{rt}(X)$.

These four drawbacks are necessary approximations to calculate the design resistance, but not rigorous in theory.

## 3. An Inferring Method Based on the Probability Model of Resistance

The inferring method based on the probability model builds the relationship among design resistance, probability characteristics of known factors, and statistical results of unknown factors by statistical theory. This kind of model has also been used in EN 1990 : 2002, while just in analyzing the probability characteristics of resistance. Thus, the inferring method of Eurocode is based on the resistance variable in essence.

The logarithm of design resistance can be expressed as follows in the probability model inferring method:

$$\begin{aligned}\ln R_d &= \mu_{\ln\theta} + \ln b + \mu_{\ln rt} - k\sqrt{\sigma_{\ln\theta}^2 + \sigma_{\ln rt}^2}\\ &= \mu_{\ln\theta} + \ln b + \ln g_{rt}\left(\mu_X\right) - k\sqrt{Q_\theta^2 + Q_{rt}^2}.\end{aligned} \tag{22}$$

The approximate equation is available within a practical scope for $0.05 \le V_\theta \le 0.25$ and $0.05 \le V_{rt} \le 0.25$:

$$\begin{aligned}\sqrt{Q_\theta^2 + Q_{rt}^2} &\approx 0.7\left(Q_\theta + Q_{rt}\right) + 0.2\left|Q_\theta - Q_{rt}\right|\\ &= \max\left\{0.9Q_\theta + 0.5Q_{rt}, 0.5Q_\theta + 0.9Q_{rt}\right\}.\end{aligned} \tag{23}$$

Equation (23) can be regarded as an accurate formula with a range of absolute errors from −0.005 to 0.008 and relative errors from −0.019 to 0.029 for the errors are small. By assigning this to equation (22), we obtain

$$\ln R_d = \min\begin{cases}\left(\mu_{\ln\theta} - 0.9kQ_\theta\right) + \left[\ln b + \ln g_{rt}\left(X_m\right) - 0.5kQ_{rt}\right],\\ \left(\mu_{\ln\theta} - 0.5kQ_\theta\right) + \left[\ln b + \ln g_{rt}\left(X_m\right) - 0.9kQ_{rt}\right].\end{cases} \tag{24}$$

Taking

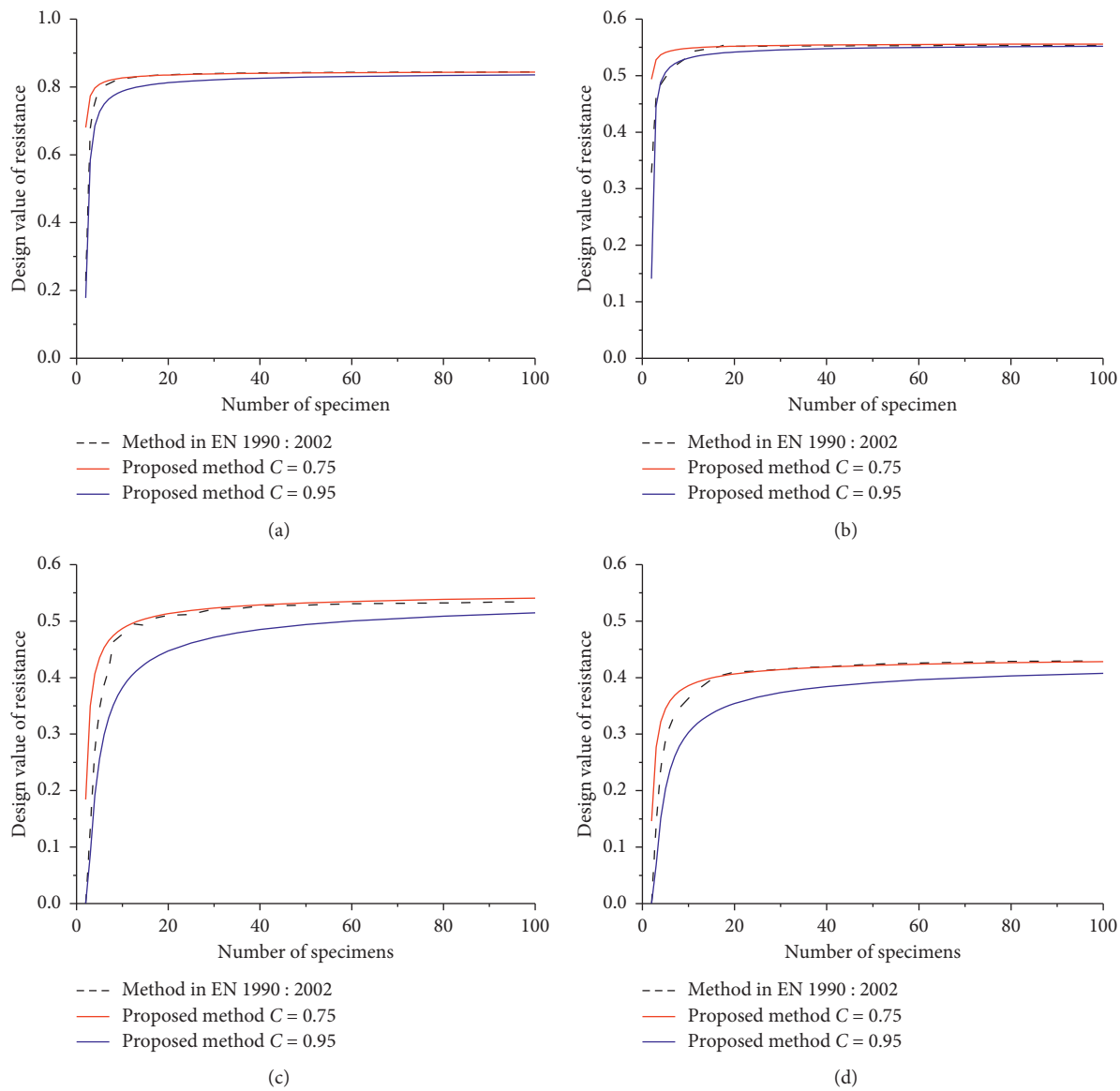


Figure 1: Inferring results when the coefficient of variation of $\theta$ is known. (a) $S_{\ln\theta} = 0.05$ , $Q_{rt} = 0.05$. (b) $S_{\ln\theta} = 0.05$ , $Q_{rt} = 0.25$. (c) $S_{\ln\theta} = 0.25$ , $Q_{rt} = 0.05$. (d) $S_{\ln\theta} = 0.25$ , $Q_{rt} = 0.25$.

$$\begin{aligned} T_1 &= \frac{\overline{\ln\theta} - (\mu_{\ln\theta} - 0.9kQ_\theta)}{S_{\ln\theta}/\sqrt{n}} \\ &= \frac{\overline{\ln\theta} - \ln R_d + [\ln b + \ln g_{rt}(\mu_X) - 0.5kQ_{rt}]}{S_{\ln\theta}/\sqrt{n}}, \\ T_2 &= \frac{\overline{\ln\theta} - (\mu_{\ln\theta} - 0.5kQ_\theta)}{S_{\ln\theta}/\sqrt{n}} \\ &= \frac{\overline{\ln\theta} - \ln R_d + [\ln b + \ln g_{rt}(\mu_X) - 0.9kQ_{rt}]}{S_{\ln\theta}/\sqrt{n}}, \end{aligned} \tag{25}$$

where $\overline{\ln\theta}$ and $S_{\ln\theta}$ are the mean and coefficient of variation of samples $\ln\theta_1, \ln\theta_2, \ldots, \ln\theta_n$ separately, $T_1$ and $T_2$ follow the noncenter student distribution with a freedom degree of both $n-1$ and noncentrality parameters of $0.9k\sqrt{n}$ and $0.5k\sqrt{n}$, respectively [13]. The design resistance can be expressed as equation (26) when $(s_{\ln\theta}/Q_{rt}) \geq (0.4k/k_{n,0.9} - k_{n,0.5})$ or equation (26) when $(s_{\ln\theta}/Q_{rt}) < (0.4k/k_{n,0.9}k_{n,0.5})$by the method of interval estimation.

$$\begin{aligned} R_d = \{ & m'_\theta b g_{rt}(\mu_X) \exp\left[-\left(k_{n,0.9}s_{\ln\theta} + 0.5kQ_{rt}\right)\right], m'_\theta b g_{rt}(\mu_X) \\ & \cdot \exp\left[-\left(k_{n,0.5}s_{\ln\theta} + 0.9kQ_{rt}\right)\right]. \} \end{aligned} \tag{26}$$

The realized value of the mean of samples $\theta_1, \theta_2, \ldots, \theta_n$, $k_{n,0.9}$, and $k_{n,0.5}$ can be calculated in the following expressions:

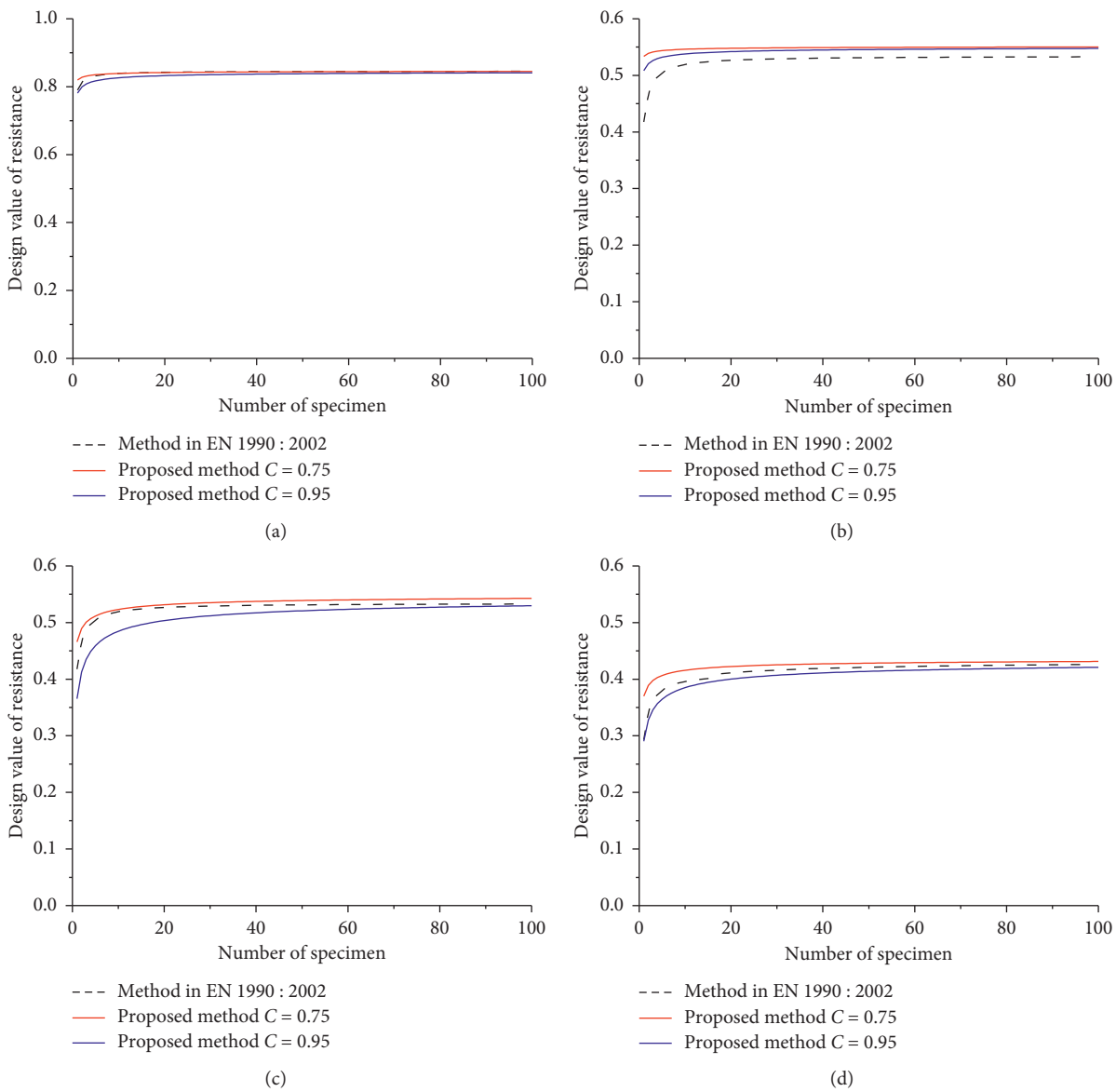


FIGURE 2: Inferring results when the coefficient of variation of $\theta$ is unknown. (a) $Q_\theta = 0.05$ , $Q_{rt} = 0.05$. (b) $Q_\theta = 0.05$ , $Q_{rt} = 0.25$. (c) $Q_\theta = 0.25$ , $Q_{rt} = 0.05$. (d) $Q_\theta = 0.25$ , $Q_{rt} = 0.25$.

$$\begin{aligned} m'_\theta &= \exp\left(\frac{1}{n}\sum_{i=1}^{n}\ln\theta_i\right) = \left(\prod_{i=1}^{n}\theta_i\right)^{1/n}, \\ k_{n,0.9} &= \frac{t_{(n-1,0.9k\sqrt{n},C)}}{\sqrt{n}}, \\ k_{n,0.5} &= \frac{t_{(n-1,0.5k\sqrt{n},C)}}{\sqrt{n}}, \end{aligned} \tag{27}$$

where $t_{(n-1,0.9k\sqrt{n},C)}$ and $t_{(n-1,0.5k\sqrt{n},C)}$ are the $C$ quantiles of the noncenter student distribution with a freedom degree of both $n-1$ and noncentrality parameters of $0.9k\sqrt{n}$ and $0.5k\sqrt{n}$, respectively, whose values can be supplied in the form of a table. $C$ is the confidence level with a desirable value for 0.75 [4, 14–17].

$T'$, subjected to the standardized normal distribution, can be taken as the following expression when the coefficient of variation of $\theta$ is known:

$$T' = \frac{\overline{\ln\theta} - \mu_{\ln\theta}}{\sigma_{\ln\theta}/\sqrt{n}} = \frac{\overline{\ln\theta} - \ln R_d + [\ln b + \ln g_{rt}(\mu_X) - kQ]}{Q_\theta/\sqrt{n}}. \tag{28}$$

The design resistance can be expressed as equation (29) by the method of interval estimation:

$$R_d = m'_\theta b g_{rt}(\mu_X)\exp[-(k_\infty{}'Q_\theta + kQ)], \tag{29}$$

$$k_\infty{}' = \frac{n_C}{\sqrt{n}}, \tag{30}$$

where $n_C$ is the $C$ quantile of the standardized normal distribution. In normal case, the coefficient of variation of $\theta$ is unknown. If a larger value of $V_\theta$ (or $Q_\theta$) is adopted by experience, a better inferring result will be acquired from equation (29) than equation (26), which means a higher referring design value is obtained.

The abovementioned method, which is based on the probability model of resistance and builds a relationship among design resistance, probability characteristics of known factors, and statistical results of unknown factors by statistical theory directly, has a rigorous and sound theoretical basis for both conditions where the coefficient of variation of $\theta$ is unknown and full known.

## 4. Comparison and Analysis

The inferring results obtained by methods in ISO 2394 : 2015, EN 1990 : 2002, and this paper are different. To make a comparison, Figure 1 shows the differences between the EN 1990 : 2002 method and the proposed method when the coefficient of variation of $\theta$ is known, and Figure 2 shows the differences when the coefficient of variation of $\theta$ is unknown. In order to facilitate the comparison, set $m'_\theta bg_{rt}(\mu_X) = m_\theta bg_{rt}(\mu_X) = 1$, $Q_\theta = s_{\ln\theta}$, $p = 0.99$.

From Figures 1 and 2, it can be seen that

(1) The inferring results are better when the coefficient of variation of $\theta$ is known rather than unknown, especially when the number of specimen is small.

(2) The inferring results are same obtained by the EN 1990 : 2002 method and the proposed method when the coefficient of variation of $\theta$ is known, while the results are different when the coefficient of variation of $\theta$ is unknown.

(3) It shows that the calculated results using the EN 1990 : 2002 method are almost between those using the proposed method when $C = 0.75$ and $C = 0.95$; the EN 1990 : 2002 method is too conservative since the inferring results are lower, even reducing to 0 when the number is small; a better inferring result (that is, a higher design resistance) will be obtained when $C = 0.75$ in the proposed method.

(4) The greater the variability of resistance influence factors ($Q_{rt}$, $Q_\theta$, or $s_{\ln\theta}$), the smaller the inferring values of design resistance.

The differences between the proposed method in this study and EN 1990 : 2002 result from the following reasons:

(1) The method in EN 1990 : 2002 adopts the probability model in analyzing the probability characteristics of resistance. Thus, the method is based on the resistance variable in essence, while the proposed method based on the probability model builds the relationship among design resistance, probability characteristics of known factors, and statistical results of unknown factors by statistical theory.

(2) The method in EN 1990 : 2002 adopts the weighted average method of the design resistance under two ideal conditions (i.e., no prior knowledge and full knowledge of the coefficient of variation), which is a simplified and experienced method merely, while the proposed method is established on the basis of statistics strictly, thus it has a more accurate outcome.

## 5. Conclusion

This study provides a new inferring method of design resistance based on the probability model of resistance, and the proposed method is superior to the method in ISO 2394 : 2015 and EN 1990 : 2002. Based on the analysis and comparison, several clear conclusions can be drawn as follows:

(1) The inferring method presented in ISO 2394 : 2015 which is based on the analysis model reflects an approximate relationship between resistance and the influence variables from the viewpoint of probability. By this method, a unified guarantee rate cannot be ensured, which will affect the reliability control to some extent. It is also worth noting that the method takes the guarantee rate of model uncertainty as the implied rate of design resistance, causing it to be lower than the specified one.

(2) The inferring method put forward in EN 1990 : 2002, which is based on the probability model in the process of analyzing the probability characteristic of resistance, is a method on the basis of resistance variable. In addition, it is empirical to use the weighted average method and approximate calculations.

(3) The proposed method, which is based on the probability model of resistance, establishes a relationship among design resistance, probability characteristics of known factors, and statistical results of unknown factors by statistical theory directly and has a rigorous and sound theoretical basis.

(4) The case comparison shows that the discrepancy between the values acquired from EN 1990 : 2002 and the proposed method is small when the coefficient of variation of $\theta$ is known, while it becomes wider when the coefficient of variation of $\theta$ is unknown. Furthermore, when the specimen number is small, the inferring results are lower, even reducing to 0 by the EN 1990 : 2002 method; while it is higher by the proposed method, it means better inferences.

## Data Availability

No data were used to support this study.

## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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