Hindawi
Mathematical Problems in Engineering
Volume 2020, Article ID 2648508, 12 pages
https://doi.org/10.1155/2020/2648508

Research Article

Research on Velocity Fluctuation of High Pressure and High Flow Double Booster Cylinder Hydraulic System

Fuqiang Chen, Rendong Wu, Chaolong Yuan, Wei Wei, and Wei Jiao

Department of Mechanical Engineering,
Key Laboratory under Ministry of Education for Advanced Material Processing Technology, Tsinghua University, Beijing 100084, China

Correspondence should be addressed to Chaolong Yuan; yuanchl@tsinghua.edu.cn

Received 1 March 2020; Accepted 27 April 2020; Published 30 May 2020

Academic Editor: Krzysztof Puszynski

In the extrusion machine with double booster cylinder hydraulic system, the stability of the extrusion velocity is affected because of the switching process of the two booster cylinders in the hydraulic system. In this paper, the analytical solution of the extrusion velocity fluctuation caused by the double booster cylinder hydraulic system is derived. Firstly, the model of the double booster cylinder hydraulic system was established, and the switching process of the two booster cylinders was studied under the condition of high pressure and high flow. Secondly, the traditional control method cannot obtain stable extrusion velocity under the condition of high pressure and high flow rate, and the new control method was proposed to significantly reduce the fluctuation of extrusion velocity. Thirdly, the fluctuation of extrusion velocity caused by the booster system was derived and the influence of the parameters of the hydraulic components such as the hydraulic cylinder and the hydraulic valve and the working conditions such as the flow rate and the pressure on the fluctuation of extrusion velocity was analyzed. Finally, based on the AMESIM software, the double booster cylinder hydraulic system of the 35 MN extrusion machine was simulated and analyzed. The numerical simulation results were used to verify the analytical solution of extrusion velocity fluctuation. The analytical solution can be applied to the engineering design of the hydraulic system and more in-depth optimization analysis.

1. Introduction

The hydraulic system of extrusion machine needs to achieve high pressure and high flow to meet the requirements of extrusion pressure and extrusion velocity. Stable extrusion velocity is the key to produce smooth and crack free products [1], and research on constant velocity extrusion is the focus at present [2, 3]. The booster system of existing extrusion machine basically uses a booster cylinder to generate high pressure. If a single-acting booster cylinder is used, the operation of the booster cylinder is limited by the piston stroke, so the system cannot achieve continuous boosting [4]. Compared with single-acting booster cylinder, the system which uses a double-acting booster cylinder can achieve continuous boosting [5]. However, when the system is working, there must be a process of forward movement, deceleration, stop, reverse acceleration, and reverse movement of the piston in the booster cylinder. This process results in unstable oil output from the booster system [6–8].

The use of two booster cylinders for continuous boosting can ensure that the deceleration and reset phase of one booster cylinder piston is just the acceleration and uniform motion phase of the other booster cylinder piston, which eliminates the flow fluctuations of the booster system [9, 10]. Based on numerical calculation, Yu et al. and Zhuo et al. compared the high-pressure water jet hydraulic system using one booster cylinder and two booster cylinders. The results show that the output pressure and flow of the hydraulic system with two booster cylinders are more stable [11, 12]. In the switching process of two booster cylinders, Zeng and Chen gave slope signals to two flow valves for system control, and the influence of different hydraulic system parameters on the flow fluctuation was studied [13]. Xu and You and Sun et al. applied the booster system to a low flow

water jet cutter and conducted experiments. The pressure fluctuation of this system was only 2% [14, 15]. In the water jet equipment, the outlet of the booster system is connected with the nozzle [16], and the outlet of the booster cylinder can be equipped with an accumulator or a pressurizer to reduce the fluctuation of pressure and flow [17, 18]. Different from the water jet equipment, the outlet of the booster system of the extrusion machine is to be connected with the working cylinder, so it is unrealistic to set the accumulator or pressurizer. Although the double booster cylinder hydraulic system is widely used in water jet cutting equipment, its research is limited to the working condition of high pressure and low flow. Under the conditions of high pressure and high flow required by the extrusion machine, there is less research on this system. The research methods are mostly numerical simulations and physical experiments, and they only target hydraulic systems with specific parameters. Therefore, the current research cannot quickly obtain the magnitude of the velocity fluctuation of the hydraulic system with different parameters under different pressure and flow conditions, and it is difficult to guide the hydraulic system engineering design and in-depth optimization analysis.

In this paper, a mathematical model of the double booster cylinder hydraulic system is established, and the switching process of the booster cylinders under high pressure and high flow condition is studied in detail. The expression of the extrusion velocity fluctuation caused by the booster system under the condition was derived, and the analytical solution of the extrusion velocity fluctuation was verified by numerical analysis results.

2. Working Principle of System

The principle of the double booster cylinder hydraulic system studied in this paper is shown in Figure 1. As shown in Figure 1(a), the booster cylinder B performs the boosting action while the booster cylinder A performs the reset action. The charge system can provide low-pressure oil to the rod chamber of the booster cylinder when the booster cylinder is reset. As shown in Figure 1(b), when the piston of the booster cylinder B is about to complete stroke, the booster cylinder A starts to enter the boosting action, and its output flow gradually increases to the rated flow. The booster cylinder B starts to exit the boosting action, and its output flow gradually reduces from the rated value to 0. As shown in Figure 1(c), after booster cylinder B completely exits the boosting action, the booster cylinder A performs the boosting action while the boosting cylinder B performs the reset action. The two booster cylinders alternately cycle in this way, and there is a switching stage in which the two booster cylinders boost at the same time. After alternating several times, the pressurization of the entire extrusion process is completed.

3. Comparative of Low Flow Conditions and High Flow Conditions

Under the condition of high pressure and low flow, the switch process of the booster cylinder is controlled by giving the slope signals corresponding to two flow valves. This control method makes the deceleration stage of one booster piston just the acceleration stage of the other, and the sum of the velocity of the two pistons is constant. The output flow of the booster cylinders can be superposed so as to reduce the output flow fluctuation during the switching process of the two booster cylinders and make the extrusion velocity more stable.

Taking the double booster cylinder hydraulic system of a 35 MN metal extrusion machine as an example, the AMISIM software is used to analyze the hydraulic system under high pressure and high flow conditions. Firstly, the control method under the condition of high pressure and low flow is used to control the switching process, that is, when a certain booster cylinder works normally, the signal r_0 is given to the corresponding flow valve; when the two booster cylinders are switched, the corresponding slope signals are given to the two flow valves for control. The corresponding control signal of the booster cylinder B which exits the boosting action is $r_2 = r_0(1 - (t/t_g))$, and the corresponding control signal of the booster cylinder A which enters the boosting action is $r_1 = r_0(t/t_g)$, where t_g is the time of the switching process of the booster cylinder. If t_g is too small, it will cause great hydraulic shock, and if t_g is too large, it will affect the working rhythm of the booster cylinders. For the system studied in this paper, 0.6 s is preferred. Apply corresponding load to make the output pressure of the booster cylinder reach 50 MPa. Other hydraulic system parameters of the extrusion machine are shown in Table 1.

The model shown in Figures 2 and 3 is established in AMESIM software. The master system is cylinder, pipeline, and various on-off valves and relief valves. The slave systems are the flow valves and the pump station, which, respectively, control the input flow of two booster cylinders.

Through numerical simulation, booster cylinder B is about to complete a stroke at 29.45 s and starts to exit the boosting action; booster cylinder A starts to enter the boosting action at the same time. After 0.6 s, the switching process of the two booster cylinders is completed. During the switching process, the extrusion velocity is as shown in Figure 4. The ramp control signals r_1 and r_2 given to the flow valve A and flow valve B are shown in Figure 5. The extrusion velocity fluctuates greatly during the switching process. It quickly drops to 0 at the beginning of the switching and rises to the normal velocity with a large overshoot after a period of time. Figure 6 shows the output flow of the booster cylinder. Figure 7 shows the pressure in the rod chamber of the booster cylinder and the rodless chamber of the main cylinder.

At the beginning of the switching process, the control signal r_1 of the flow valve A increases in the form of a ramp, the velocity of piston in the booster cylinder A also increases in the form of a ramp. The oil in the rod chamber of the booster cylinder A is compressed. Although the pressure of this chamber gradually increases from a low pressure close to 0, it does not reach the pressure in the rodless chamber of the main cylinder. Therefore, the check valve at the outlet of the booster cylinder A is not opened, and only the booster cylinder B outputs oil to the main cylinder, thereby reducing the extrusion velocity.

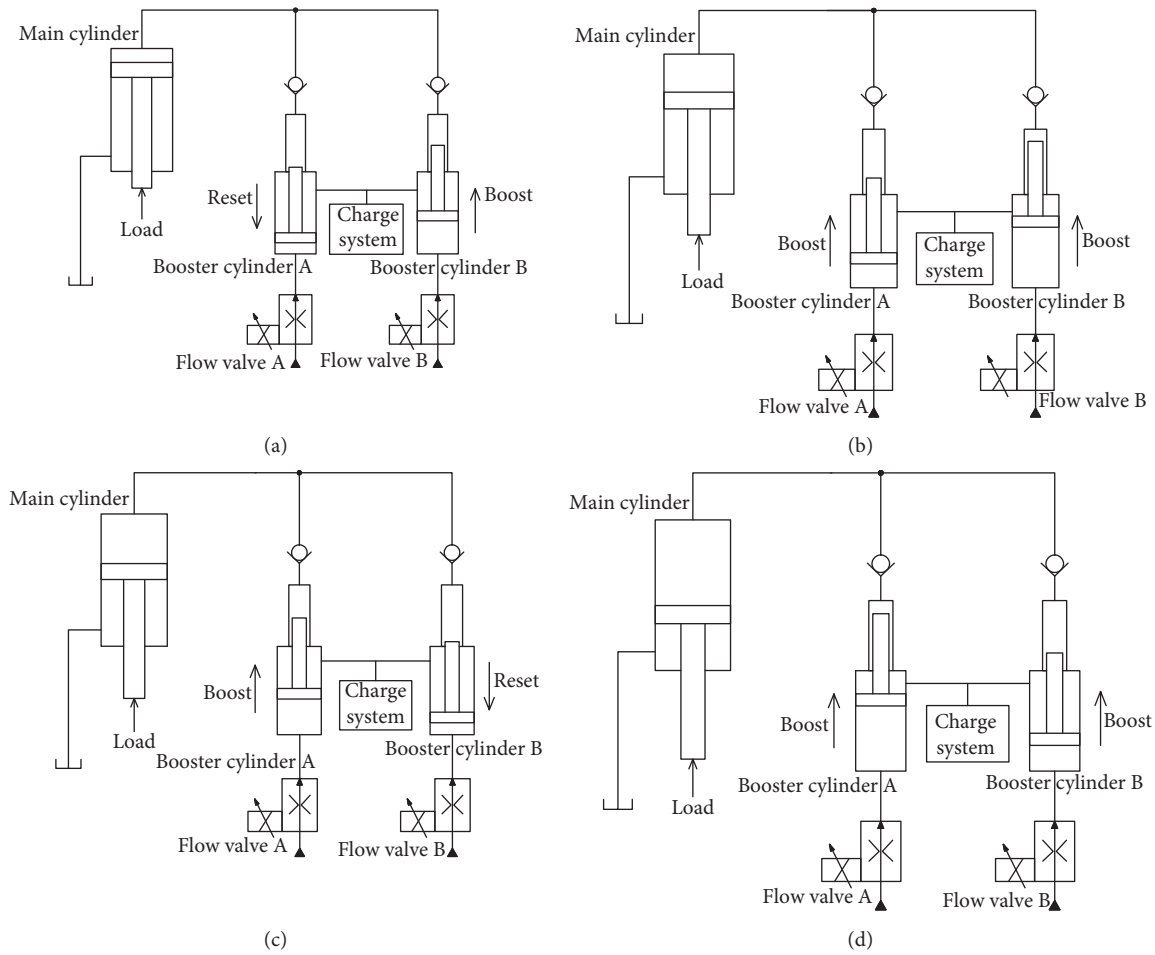

Figure 1: Principle of the double booster cylinder hydraulic system. (a) Operation phase of single booster cylinder A. (b) Switching process of the two booster cylinders. (c) Operation phase of single booster cylinder B. (d) Switching process of the two booster cylinders.

Table 1: Main parameters of the hydraulic system.

Parameters	Value
Bulk modulus of hydraulic oil (Pa)	8×10^8
Density of hydraulic oil (kg/m^3)	850
Absolute viscosity of hydraulic oil (cP)	51
Mass of piston in booster cylinder (kg)	350
Piston area of rodless chamber of booster cylinder (m^2)	0.20
Piston area of rod chamber of booster cylinder (m^2)	0.066
Stroke of booster cylinder (m)	0.75
Mass of piston in main cylinder and its attachment parts (kg)	5.0×10^4
Piston area of rodless chamber of main cylinder (m^2)	0.50
Piston area of rod chamber of main cylinder (m^2)	0.16
Stroke of main cylinder (m)	1.2

After a period of time, the pressure in the rod chamber of booster cylinder A is greater than that in the rodless chamber of the main cylinder, and the check valve at its outlet is opened. At this time, the control signal r_2 has been reduced to 0, and the booster cylinder B has also been out of work. Therefore, the extrusion velocity is increased from 0 to a constant value with a large overshoot.

From the above analysis, since the volume of the booster cylinder suitable for high flow conditions is huge. When switching starts, the booster cylinder A needs to spend more time compressing a large amount of oil in order to output the flow to the main cylinder. Therefore, when the booster cylinder is switched, the velocity of the main cylinder fluctuates sharply or even decreases to zero.

In view of the long time of oil compression in the rod chamber of the booster cylinder, the traditional control method of given the slope signal cannot meet the demand of high pressure and high flow system. Therefore, a control method for firstly compressing oil and then switching the booster cylinder A is considered. When the piston of booster cylinder B has not completed a stroke and the piston of booster cylinder A has been reset, the pressure compensation signal of flow valve A is given as $r_1 = r_p$, and the normal control signal of flow valve B is given as $r_2 = r_0$. r_p can be determined by

$$r_p = \begin{cases} r_{max}, & P_2 - P_A \geq \Delta p, \\ 0, & P_2 - P_A < \Delta p, \end{cases} \tag{1}$$

where r_p is determined by the pressure P_2 in the rodless chamber of the main cylinder and the pressure P_A in the rod

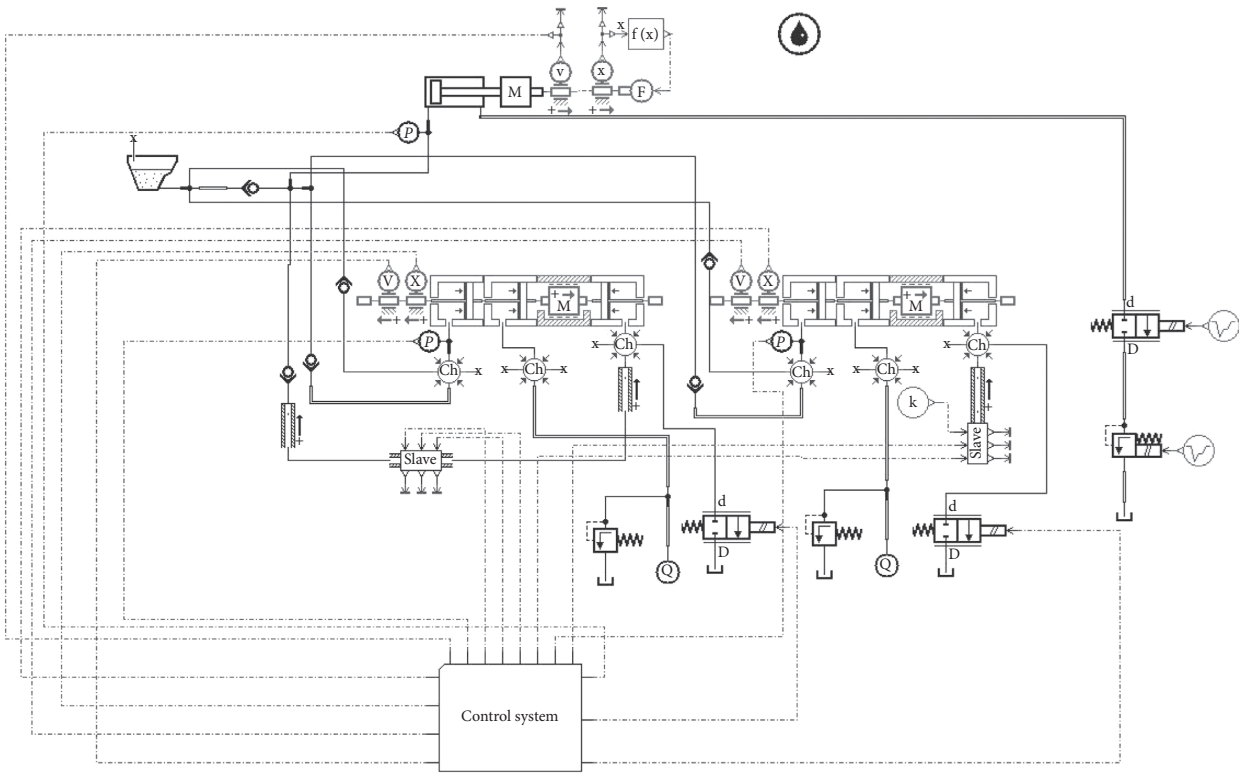

Figure 2: Master system model.

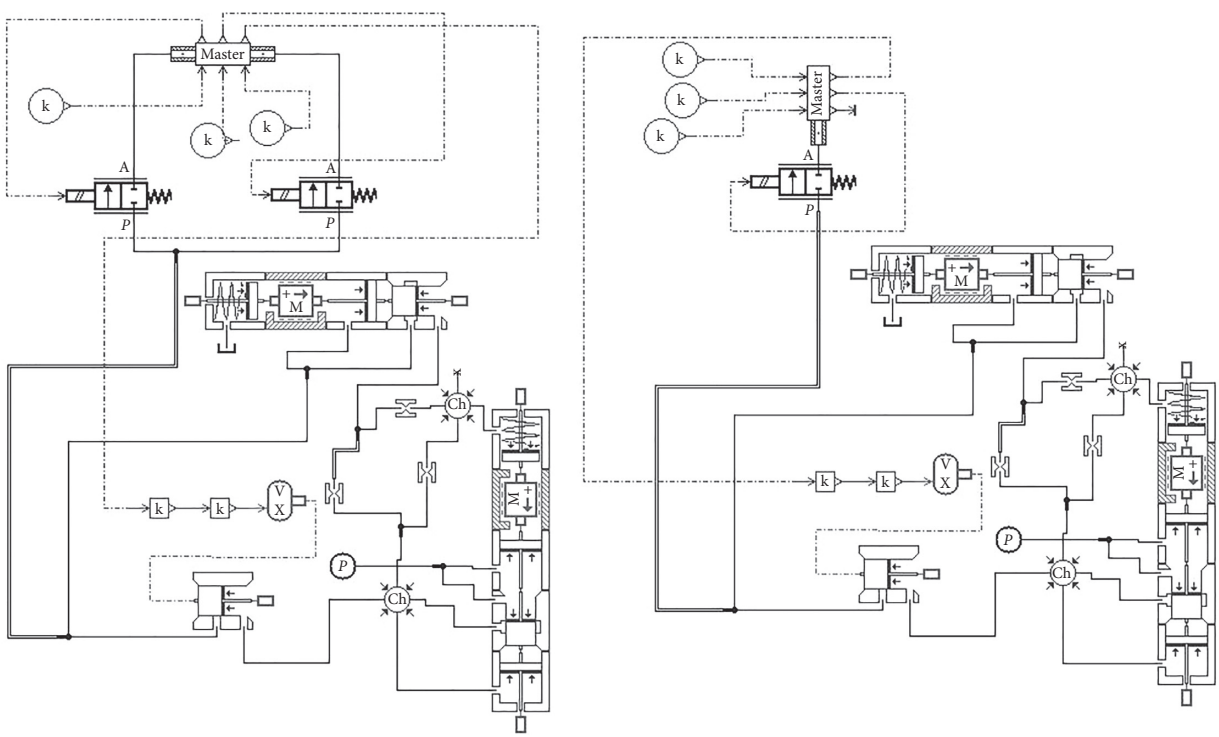

Figure 3: Slave systems model.

chamber of the booster cylinder A. When P_A is far less than P_2, it means that the pressure in the rod chamber of booster cylinder A fails to meet the requirements, and the oil in this chamber needs to be compressed at the fastest velocity. When P_A is close to P_2, it means that the oil compression of the rod chamber of booster cylinder A is completed and it is

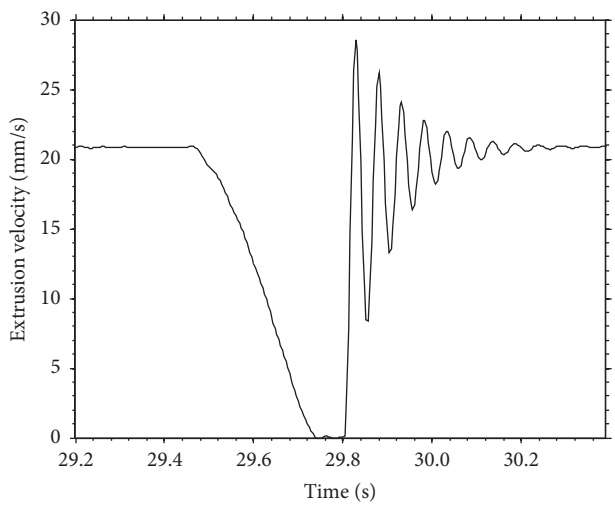

Figure 4: Extrusion velocity.

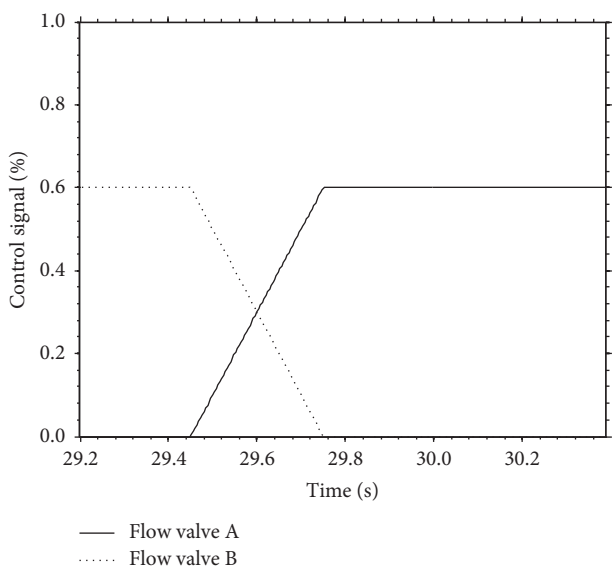

Figure 5: Control signal of the flow valve.

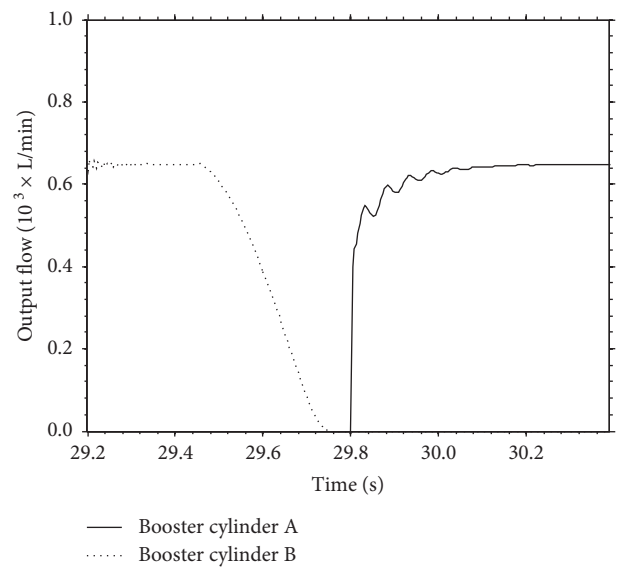

Figure 6: Output flow of the booster cylinder.

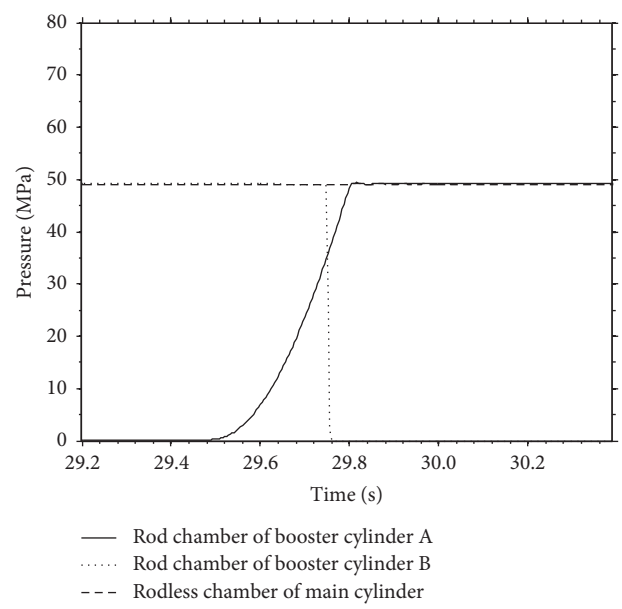

Figure 7: Pressure in the rod chamber of the booster cylinder and the rodless chamber of the main cylinder.

necessary to stop compressing the oil in the chamber. r_{max} is the maximum control signal to saturate the output flow of the flow valve. The compensation signal r_p is reduced to 0 after the completion of oil compression. However, there is a certain overshoot in the pressure P_A. In order to make the check valve at the outlet of the rod chamber closed at this stage, Δp can be taken as 0.15 MPa temporarily. For the convenience of description, this stage is referred to as the pressure compensation phase.

When the control mode of firstly oil compression and then switching is adopted, the control signal is as shown in Figure 8. The extrusion velocity obtained by numerical simulation is shown in Figure 9. Figure 10 shows the pressure of the main cylinder and the booster cylinders, and Figure 11 shows the output flow of the booster cylinders and the input flow of the main cylinder. As shown in Figure 10, the pressure P_A in the rod chamber of booster cylinder A is close to the pressure P_2 in the rodless chamber of the main cylinder during the pressure compensation phase. As shown in Figure 11, in the subsequent switching process, the pressure P_A can reach the pressure P_2 more quickly and booster cylinder A rapidly outputs the flow to the main cylinder. Therefore, by using the control method of first oil compression and then switching, a smaller velocity fluctuation can be obtained under high pressure and high flow conditions.

4. Analytical Solution of Velocity Fluctuation

The parameters that affect the extrusion velocity in the switching process are divided into three categories: the first

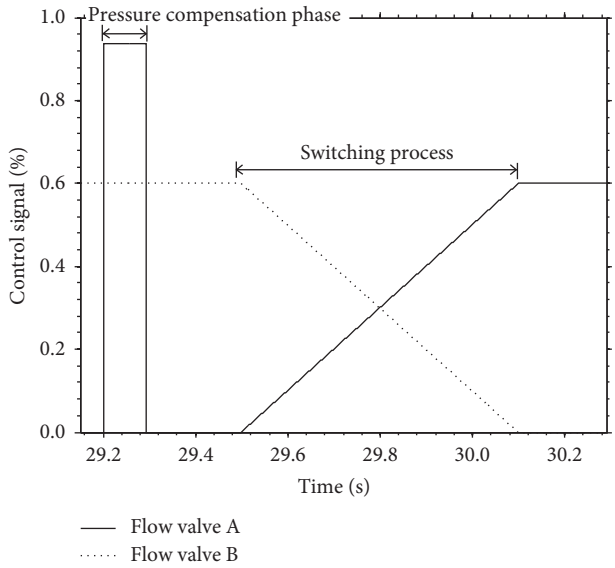

Figure 8: Control signal of the flow valve.

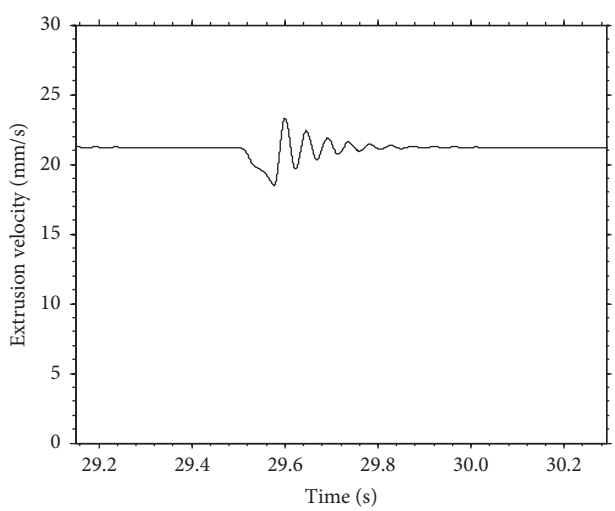

Figure 9: Extrusion velocity.

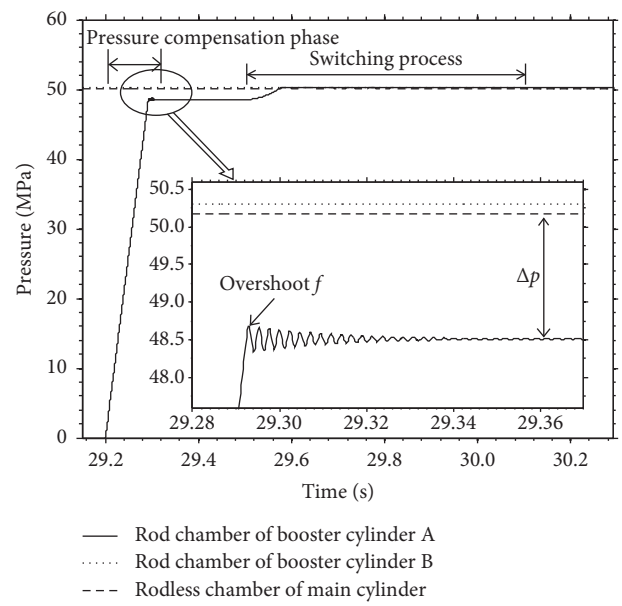

Figure 10: Pressure in the rod chamber of the booster cylinder and the rodless chamber of the main cylinder.

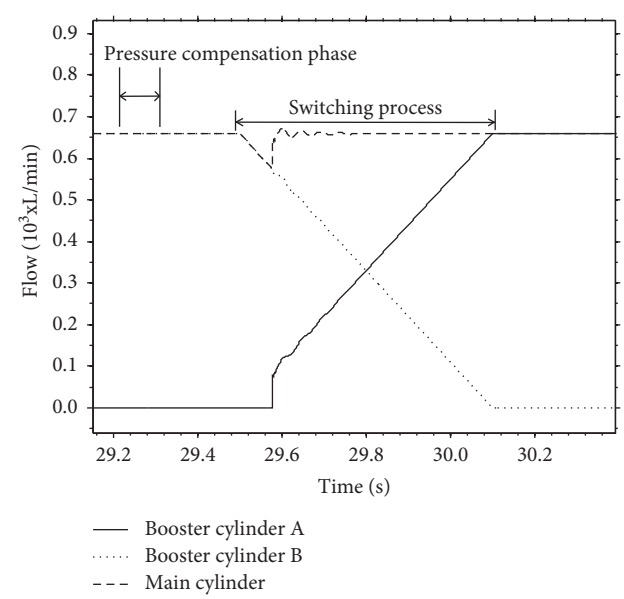

Figure 11: Output flow of the booster cylinder and input flow of the main cylinder.

category is the parameters of the hydraulic system, including the parameters of the hydraulic components such as valves and cylinders; the second category is the parameters of working condition, which are expressed by the pressure and flow; the third category is the control parameters, such as the parameters such as t_g and Δp already described above.

In order to make the velocity fluctuation as small as possible, at the beginning of the switching process, it is necessary to make the time of oil compression in booster cylinder A as short as possible, that is, to minimize Δp. In the pressure compensation phase, in order to make the check valve at the outlet of the booster cylinder A closed, Δp must be greater than a certain value. As shown in Figure 10, when the Δp is equal to the pressure overshoot $f(G_0, p_2)$ of the rod chamber in booster cylinder A, the minimum value of extrusion velocity fluctuation can be obtained. Among them, G_0 represents the mathematical model of the booster cylinder, including various parameters such as the mass of the piston, the volume of the chamber, and the area of the piston; p_2 can be simplified to the pressure of the rodless chamber of the main cylinder under high pressure. As shown in Figure 12, according to the continuous equation of oil in the rodless chamber and the rod chamber of the booster cylinder and the motion equation of the piston of the booster cylinder, the mathematical models (2)~(4) of the booster cylinder A during the pressure compensation phase can be obtained:

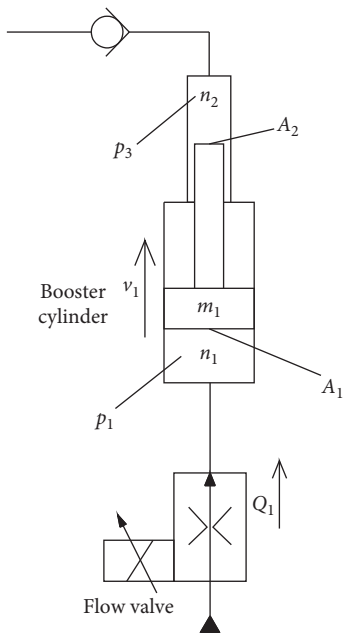

Figure 12: Model of the booster cylinder.

$$\dot{p}_3 = \frac{EA_2 v_1}{n_2}, \tag{2}$$

$$\dot{p}_1 = \frac{E}{n_1}\left(Q_1 - A_1 v_1\right), \tag{3}$$

$$p_1 A_1 = m_1 \dot{v}_1 + B_1 v_1 + p_3 A_2. \tag{4}$$

A_1 and A_2 are the area of piston in the rodless chamber and rod chamber in the booster cylinder, n_1 and n_2 are the volume of the rodless chamber and rod chamber of the booster cylinder, m_1 is the mass of the piston of the booster cylinder, E is the bulk modulus of hydraulic oil, B_1 is the equivalent viscous friction coefficient of the booster cylinder, Q_1 is the output flow from the flow valve to the rodless chamber of the booster cylinder, p_1 and p_3 are the pressure of the rodless chamber and the rod chamber of the booster cylinder, and v_1 is the movement velocity of piston.

Combining equations (2)~(4) and through Laplace transformation, the relationship between the output flow of the flow valve and the pressure p_3 in the rod chamber of the booster cylinder in the frequency domain is obtained:

$$p_3 = \frac{\left(E^2 A_1 A_2 / n_1 n_2 m_1\right) Q_1}{s\left[s^2 + \left(B_1/m_1\right)s + \left(E/m_1\right)\left(\left(A_1^2/n_1\right) + \left(A_2^2/n_2\right)\right)\right]}. \tag{5}$$

If the control mode described in the second section is adopted, the input flow Q_1 from the pressure compensation phase to the switching stage can be expressed as follows:

$$Q_1 = Q_{\mathrm{m}} \cdot 1(t) - Q_{\mathrm{m}} \cdot 1\left(t - \Delta t_1\right), \tag{6}$$

where Δt_1 is the total time of pressure compensation phase and Q_{m} is the output flow of flow valve when the control signal is r_{m}. Combining equations (5) and (6), the expression of the pressure p_3 in the rod chamber of booster cylinder A from the pressure compensation phase to the switching stage can be obtained. t represents the time elapsed from the beginning of the pressure compensation phase:

$$p_3 = \begin{cases} \left\lfloor t - \dfrac{2\alpha}{\alpha^2 + \omega^2} + \dfrac{1}{\omega} e^{-at} \sin(\omega t + \theta) \right\rfloor \dfrac{KQ_{\mathrm{m}}}{\alpha^2 + \omega^2} \cdot 1(t), \\ \left\{ \Delta t_1 + \dfrac{1}{\omega}\left[e^{-at} \sin(\omega t + \theta) - e^{-a\left(t - \Delta t_1\right)} \sin\left(\omega\left(t - \Delta t_1\right) + \theta\right) \right] \right\} \dfrac{KQ_{\mathrm{m}}}{\alpha^2 + \omega^2} \cdot 1\left(t - \Delta t_1\right), \end{cases} \tag{7}$$

where $K = E^2 A_1 A_2 / n_1 n_2 m_1$, $\alpha = B_1 / 2m_1$, $\alpha^2 + \omega^2 = E/m_1\left(\left(A_1^2/n_1\right) + \left(A_2^2/n_2\right)\right)$, and $\theta = 2\tan^{-1}(\omega/\alpha)$.

Since the bulk modulus E of hydraulic oil is large at high pressure and the equivalent viscous friction coefficient B_1 is small, the damping angular frequency ω is very large. From equation (7), it can be seen that, during the pressure compensation phase, the pressure in the rod chamber of the booster cylinder can be regarded as increasing uniformly, that is,

$$\Delta t_1 = \frac{\alpha^2 + \omega^2}{KQ_{\mathrm{m}}} p_2. \tag{8}$$

Let $p_{3\max}$ be the maximum value of pressure p_3 during the pressure compensation phase to the switching stage. According to the definition of overshoot,

$$\Delta p = f\left(G_0, p_2\right) = \frac{p_{3\max} - \lim_{t \longrightarrow \infty} p_3}{\lim_{t \longrightarrow \infty} p_3} p_2 = \frac{e^{-\alpha\pi/2\omega}}{\Delta t_1\, \omega} p_2. \tag{9}$$

During the switching process, the piston of booster cylinder A moves at approximately a ramp velocity. When it has not output oil to the main cylinder, the pressure in its rod chamber changes according to the following expression:

$$\begin{aligned} \dot{p}_3 &= \frac{EA_2}{n_2} \frac{v_0}{t_{\mathrm{g}}} t, \\ p_3 &= \frac{EA_2}{n_2} \frac{v_0}{t_{\mathrm{g}}} \frac{t^2}{2} + p_{30}, \end{aligned} \tag{10}$$

where v_0 is the velocity of the piston in the booster cylinder A during the operation phase of single booster cylinder and t represents the time elapsed from the beginning of the

switching process. In the above analysis, p_{30} is the difference between P_2 and Δp, so the oil compression time of booster cylinder A in the switching process is

$$\Delta t = \sqrt{\frac{2\Delta p n_2 t_g}{E A_2 v_0}}. \tag{11}$$

At the beginning of the switching process, the input flow of the main cylinder and the corresponding main cylinder control signal decreases in the form of a ramp, after time Δt, and it returns to the normal value in the form of a step. It can be expressed as follows:

$$\begin{aligned} u = u_0 \cdot 1(t) - [kt \cdot 1(t) - k(t-\Delta t) \\ \cdot 1(t-\Delta t) - k\Delta t \cdot 1(t-\Delta t)], \end{aligned} \tag{12}$$

where u_0 is the extrusion velocity during the operation phase of the single booster cylinder and Q_0 is the input flow of the main cylinder during the operation phase of the single booster cylinder. $u_0 = Q_0/A$, $k = u_0/t_g$, and $Q_0 = A_2 v_0$. A is the piston area of the rodless chamber in the main cylinder.

As shown in Figure 13, the main cylinder mathematical models (13) and (14) are obtained from the continuous equation of the oil in the rodless chamber and the motion equation of the piston. Equation (15) of the extrusion velocity v and u in frequency domain is derived:

$$\dot{p} = \frac{E}{n}(Q - Av), \tag{13}$$

$$pA = m\dot{v} + Bv + F_L, \tag{14}$$

$$v = \frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2} u, \tag{15}$$

where n is the volume of the rodless chamber of the main cylinder, F_L is the load force of the main cylinder, B is the equivalent viscous friction coefficient of the main cylinder, p is the pressure of the rodless chamber of the main cylinder, m is the total mass of the piston in main cylinder and its attachment parts, $\omega_n^2 = EA^2/nm$, and $2\zeta\omega_n = B/m$.

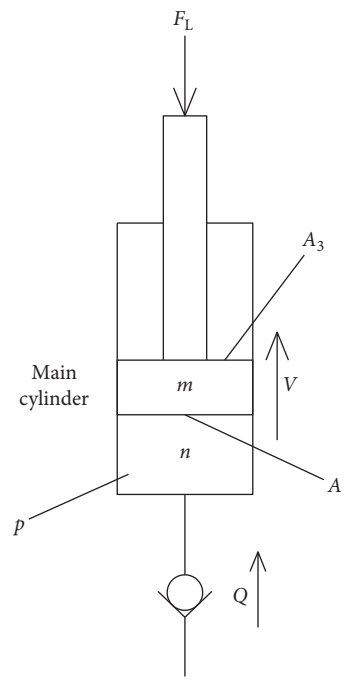

Figure 13: Model of the main cylinder.

By linearizing the model of the valve on the oil circuit near the operating point, a calculation method of the equivalent viscous friction coefficient of the main cylinder is obtained:

$$B = \sum_{i=1}^{l} \frac{A_3^2}{k_l m}, \tag{16}$$

where A_3 is the area of the rodless chamber of the main cylinder, l is the number of valves connected to the return oil circuit of the main cylinder, and k_l is the slope of the pressure-flow curve of the lth valve at the operating point. Equations (12) and (16) are introduced into equation (15) to obtain the analytical solution of the extrusion velocity v in the switching stage:

$$\begin{aligned} v &= u_0 - k\left[t - \frac{2\zeta}{\omega_n} + \frac{e^{-\zeta\omega_n t}}{\omega_n\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t + \varphi\right)\right] (t < \Delta t), \\ v &= u_0 - k\left[\frac{e^{-\zeta\omega_n t}}{\omega_n\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2}\,t + \varphi\right) - \frac{e^{-\zeta\omega_n(t-\Delta t)}}{\omega_n\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2}\,(t-\Delta t) + \varphi\right) + \frac{\Delta t e^{-\zeta\omega_n(t-\Delta t)}}{\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2}\,(t-\Delta t) + \phi\right)\right] (t \geq \Delta t), \end{aligned} \tag{17}$$

where $\varphi = 2\tan^{-1}((\sqrt{1-\zeta^2})/\zeta)$ and $\phi = \tan^{-1}((\sqrt{1-\zeta^2})/\zeta)$. $\delta_{\min}$ and $\delta_{\max}$ represent the decrease and increase of extrusion velocity in the switching process, then the fluctuation of extrusion velocity is $\delta = \delta_{\max} - \delta_{\min}$. When t is equal to Δt, v gets the minimum value $v_{\min}$. In the actual hydraulic system, Δt is relatively large, which makes $e^{-\zeta\omega_n \Delta t}$ tend to 0, that is, the minimum velocity in the switching process is simplified into the following expression:

$$v_{\min} = u_0 - k\left(\Delta t - \frac{2\zeta}{\omega_n}\right). \tag{18}$$

Then, the expression of $\delta_{\min}$ is as follows:

$$\delta_{\min} = \frac{u_0 - v_{\min}}{u_0} = \frac{1}{t_g}\left(\Delta t - \frac{2\zeta}{\omega_n}\right). \tag{19}$$

In theory, the analytical expression of $\delta_{\max}$ cannot be obtained by equation (17). Considering that $e^{-\zeta\omega_n\Delta t}$ tends to 0, the increment y of v relative to u_0 is obtained from equation (17):

$$\begin{aligned} y = {} & k\frac{e^{-\zeta\omega_n(t-\Delta t)}}{\omega_n\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2}\,(t-\Delta t)+\varphi\right) \\ & - k\frac{\Delta t e^{-\zeta\omega_n(t-\Delta t)}}{\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2}\,(t-\Delta t)+\phi\right). \end{aligned} \tag{20}$$

That is, $\delta_{\max}$ can be converted into

$$\delta_{\max} = \frac{y_{\max}}{u_0}. \tag{21}$$

As shown in equations (22)~(25), let $y = y_1 + y_2$, and obtain the extreme values of y_1 and y_2, respectively:

$$y_1 = k\frac{e^{-\zeta\omega_n(t-\Delta t)}}{\omega_n\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2}\,(t-\Delta t)+\varphi\right), \tag{22}$$

$$y_2 = k\frac{\Delta t e^{-\zeta\omega_n(t-\Delta t)}}{\sqrt{1-\zeta^2}}\sin\left(\omega_n\sqrt{1-\zeta^2}\,(t-\Delta t)+\phi\right), \tag{23}$$

$$y_{1\max} = \frac{2\zeta k}{\omega_n}, \tag{24}$$

$$y_{2\min} = -k\Delta t e^{-\zeta\pi/\left(\sqrt{1-\zeta^2}\right)}. \tag{25}$$

Since y is divided into two parts, extreme values are calculated separately. Therefore, the extrusion velocity increment during the switching process will not exceed $y_{1\max} - y_{2\min}$, that is,

$$\delta_{\max} > \frac{y_{1\max} - y_{2\min}}{u_0} = \frac{k}{u_0}\left(\frac{2\zeta}{\omega_n} + \Delta t e^{-\zeta\pi/\left(\sqrt{1-\zeta^2}\right)}\right). \tag{26}$$

The fluctuation of extrusion velocity can be obtained by combining equations (11), (19), and (26):

$$\delta = \sqrt{\frac{2\Delta p n_2}{E Q_0 t_g}}\left(e^{-\zeta\pi/\left(\sqrt{1-\zeta^2}\right)} + 1\right). \tag{27}$$

5. Verification and Analysis of Analytical Solution of Velocity Fluctuation

Take the parameters of 35 MN extrusion machine mentioned above as reference parameters. It is assumed that, during the switching process, pressure of the rodless chamber in the main cylinder is about $p_2 = 50$ MPa due to the load force such as billet deformation resistance and the switching time is $t_g = 0.6$ s. In the working stage of a single booster cylinder, in order to meet the extrusion velocity, the flow $Q_1 = 1800$ L/min to be provided by the flow valve, the flow input to the main cylinder after boosting is $Q_0 = 604$ L/min. From equation (9), we can get the overshoot during the pressure compensation phase:

$$\Delta p = \frac{e^{-\alpha\pi/2\omega}}{\Delta t_1 \omega} p_2 = 0.50\,\text{MPa}. \tag{28}$$

By introducing it into equation (27), we can get the velocity fluctuation under the pressure and flow conditions:

$$\delta = \sqrt{\frac{2\Delta p n_2}{E Q_0 t_g}}\left(e^{-\zeta\pi/\left(\sqrt{1-\zeta^2}\right)} + 1\right) = 12.05\%. \tag{29}$$

Through the simulation of AMESIM software, the extrusion velocity fluctuation is 12.50% under the pressure and flow conditions, and the analytical solution is basically consistent with the simulation results.

On the basis of reference parameters, the condition parameters, hydraulic system parameters, and control parameters of the extrusion machine are modified correspondingly. Through AMESIM software, the model after modification of parameters is simulated and calculated, and the results of velocity fluctuation are compared with the analytical solutions.

Figures 14–16 show the relationship between fluctuation of extrusion velocity and extrusion velocity v, switching time t_g, and pressure overshoot Δp. The coordinate point is the result of numerical simulation and the curve is the analytical solution. As shown in Figure 14, in low-velocity extrusion, the velocity fluctuation caused by the switching process is larger than that in high-velocity extrusion. As shown in Figure 15, the velocity fluctuation decreases with the increase of t_g. As shown in Figure 16, the velocity fluctuation increases with the increase of Δp. The analytical solution is not much different from the simulation result.

Figure 17 shows the relationship between fluctuation of extrusion velocity and total mass m of the main cylinder piston and its attachment parts. Figure 18 shows the relationship between fluctuation of extrusion velocity and volume n of the rodless chamber of the main cylinder during the switching process. The coordinate point is the numerical simulation result and the curve is the analytical solution. As shown in Figures 17 and 18, parameters m and n have little effect on the fluctuation of extrusion velocity. The velocity fluctuation increases slightly with the increase of m and decreases slightly with the increase of n. The analytical solution is not much different from the simulation result, but it is slightly smaller than the simulation result. The maximum error between the analytical solution and the simulation result are 6% and 7%.

Figure 19 shows the relationship between fluctuation of extrusion velocity and the piston area A of the main cylinder. The coordinate point is the numerical simulation result and the curve is the analytical solution. The velocity fluctuation decreases with the increase of A. The analytical solution is slightly smaller than the simulation result, and the maximum error is 18%.

The equivalent viscous friction coefficient B of the main cylinder is a comprehensive reflection of the leakage, the

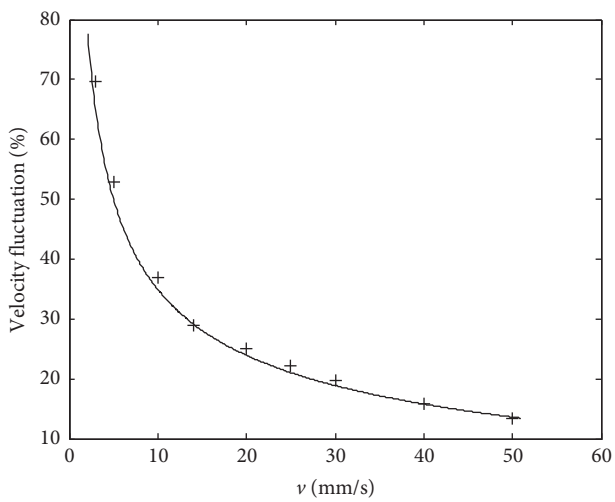

Figure 14: Relationship between extrusion velocity fluctuation and extrusion velocity v.

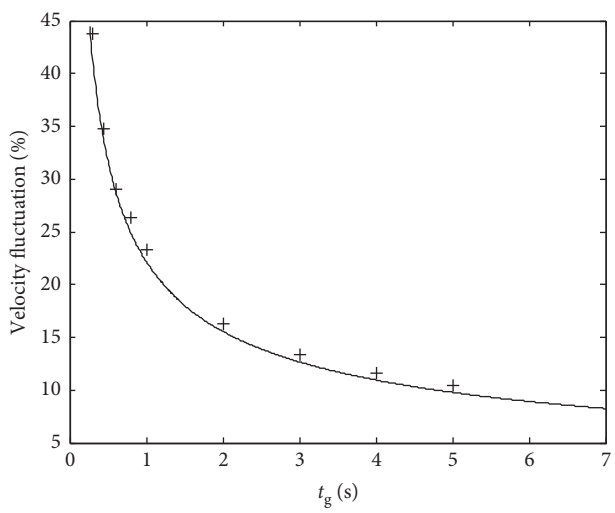

Figure 15: Relationship between velocity fluctuation and switching time t_g.

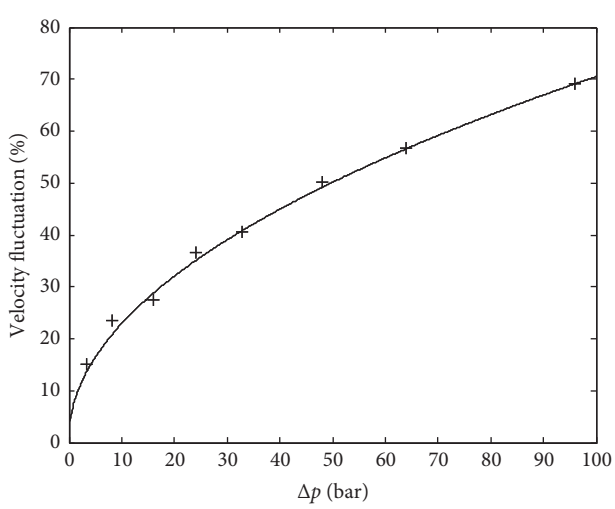

Figure 16: Relationship between velocity fluctuation and Δp.

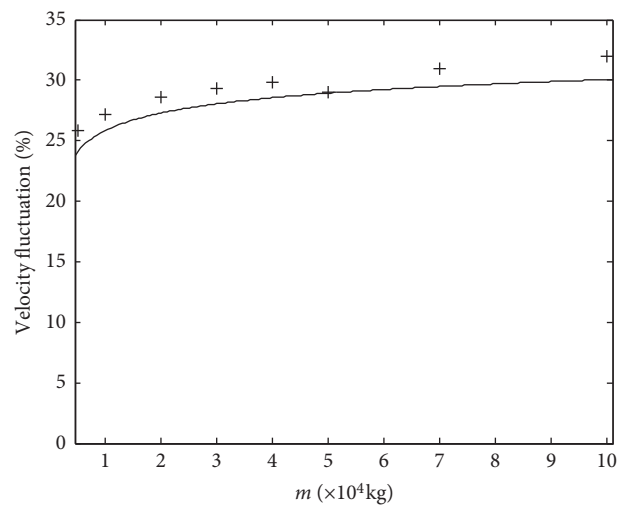

Figure 17: Relationship between velocity fluctuation and m.

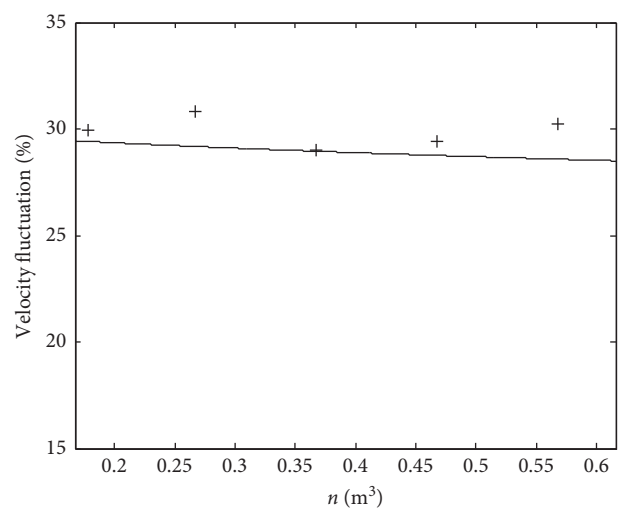

Figure 18: Relationship between velocity fluctuation and n.

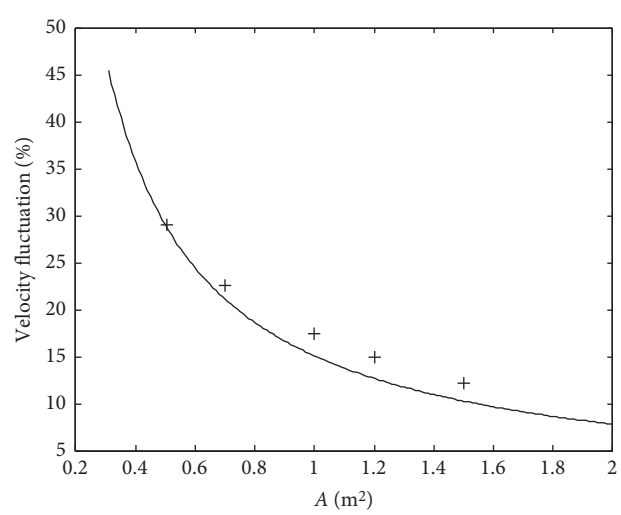

Figure 19: Relationship between velocity fluctuation and A.

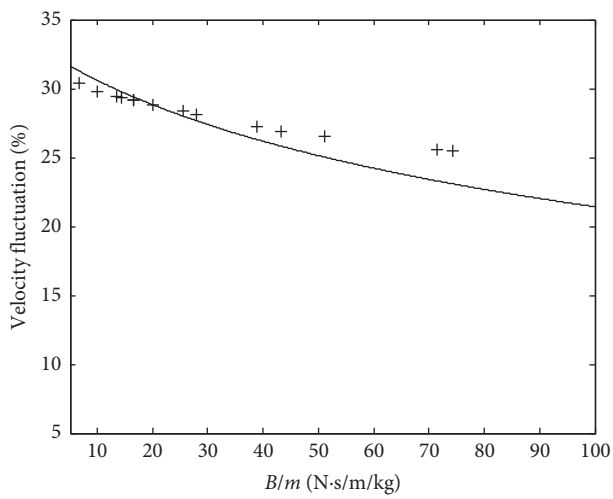

Figure 20: Relationship between velocity fluctuation and B.

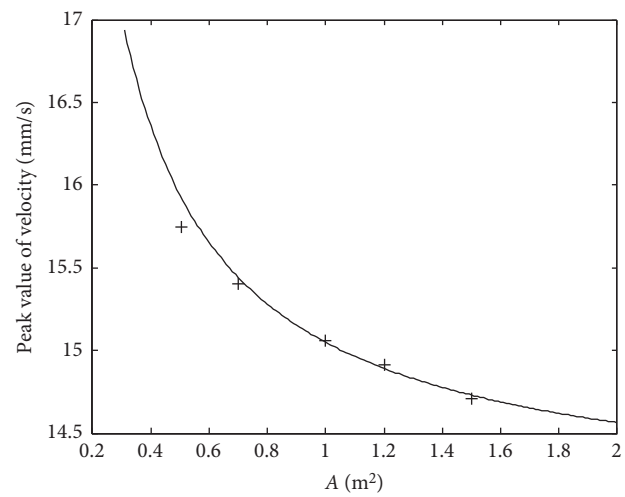

Figure 21: Relationship between peak values of the velocity and A.

flow-pressure curve of the valve on the return line, the area of the rodless chamber of the main cylinder, and the total mass m of the main cylinder piston and its attachment parts. Figure 20 shows the relationship between the fluctuation of extrusion velocity and the equivalent viscous friction coefficient B of the main cylinder. The coordinate point is the numerical simulation result and the curve is the analytical solution. The velocity fluctuation decreases with the increase of B. There is not much difference between the analytical solution and the numerical solution. When B is small, the analytical solution is small. With the increase of B, the analytical solution gradually exceeds the simulation result. The maximum error between the analytical solution and the simulation result is 8%.

Through the above analysis, the velocity fluctuation is most sensitive to the changes of the control parameters t_g and Δp and condition parameter v. The hydraulic system parameters A and B also have a greater impact on the velocity fluctuation, while the hydraulic system parameters n and m have a smaller effect on the velocity fluctuation. Compared with the numerical solution, the analytical solution is smaller when the conditions parameters, hydraulic system parameters, and control parameters change. The relationship between the peak and valley values of the velocity and the piston area A of the rodless chamber in the main cylinder can be obtained through equations (19) and (26). As shown in Figures 21 and 22, the coordinate point is the simulation result and the curves are the analytical solution. The peak velocity obtained from equation (26) is the upper limit result, which leads to the larger analytical solution than the simulation result. Since the approximate treatment is used to calculate the valley value of velocity, i.e., $e^{-\zeta\omega_n\Delta t}$ tends to 0, the analytical solution obtained from equation (19) is not the result of the lower limit. The analytic solution of valley velocity is larger than the numerical solution. The deviation between the analytical solution of the valley velocity and the simulation result is much greater than that between the analytical solution of the peak velocity and the simulation result. Therefore, the analytical solution of the

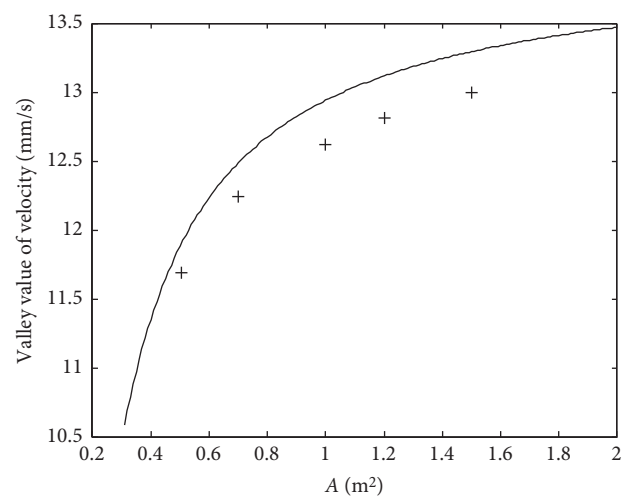

Figure 22: Relationship between valley values of the velocity and A.

fluctuation amplitude is smaller than the simulation result. For other parameters, except A, the deviation between the analytical solution and the numerical simulation result can also be explained by the same reason.

6. Conclusion

In this paper, the mathematical model of the double booster cylinder hydraulic system is established, and the switching process of the booster cylinders is studied by two methods of theoretical derivation and numerical simulation. The conclusion is as follows.

In the switching process of booster cylinders, the traditional control method is not applicable in high-pressure and high-flow hydraulic systems. In the high-flow hydraulic system, the time of oil compression in the rod chamber of the booster cylinder is long, so the control method that the first compresses the oil and then the switching should be adopted to reduce the velocity fluctuation in the switching process of the booster cylinder.

The expression of the fluctuation value of the extrusion velocity caused by the hydraulic system during the switching process is derived. Through the expression, the influence of the parameters of hydraulic components, such as hydraulic cylinder and hydraulic valve, condition parameters, such as flow and pressure, and control parameters, such as switching time on extrusion velocity fluctuation, are analyzed. The velocity fluctuation is most sensitive to the changes of the control parameters t_g and Δp and condition parameter v. The hydraulic system parameters A and B also have a greater impact on the velocity fluctuation, while the hydraulic system parameters n and m have a smaller effect on the velocity fluctuation.

Taking the hydraulic system of a 35 MN extrusion machine as an example, based on AMESIM software, the extrusion velocity fluctuation in the switching process is numerically solved. The simulation results are used to verify the analytical solution, and the reason for the error between the analytical solution and the simulation results is analyzed. The analytical solution can be applied to the engineering design of specific hydraulic system and more in-depth optimization analysis.

This study is contributing to reduce the fluctuation of extrusion velocity caused by the booster system, which is conducive to the production of high-quality extrusion products. It also provides a reference for the engineering design of the booster system.

The future research focuses on the introduction of the closed-loop control method in the switching process of booster cylinders in order to further improve the control accuracy of extrusion velocity. For example, the common PID algorithm, fuzzy control, sliding mode control, adaptive control, or other methods will be applied.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

References

[1] W. Z. Li, T. Sun, Y. C. Hu et al., "Research on constant velocity extruding process control for 36,000-ton vertical extrusion press," *Journal of Dynamic Systems Measurement & Control*, vol. 135, no. 4, Article ID 041009, 2013.

[2] C. Wang, W. Z. Li, Q. C. Zhao et al., "Consensus control design for 360 MN extrusion machine producing process," *Mathematical Problems in Engineering*, vol. 2014, Article ID 623921, 16 pages, 2014.

[3] Z.-Q. Zhu, X.-J. Lu, J. Lei, and M.-H. Huang, "Nonlinear-dynamic-analysis based fuzzy PID control approach for complex hydraulic driving process," *International Journal of Precision Engineering and Manufacturing*, vol. 19, no. 7, pp. 947–958, 2018.

[4] A. C. Günaydın, M. Halkacı, F. Ateş et al., "Experimental research of the usability on double acting intensifiers in hydroforming," in *Proceedings of the 2018 the 2nd International Conference on Mechanical, System and Control Engineering*, Moscow, Russia, June 2018.

[5] Y. Q. Shao, J. S. Liu, and S. J. Zhou, "Application of double-acting reciprocating hydraulic intensifier in cubic apparatus," *Superhard Material Engineering*, vol. 25, no. 3, pp. 31–33, 2013.

[6] J. Xu, B. You, and X. Kong, "Design and experiment research on abrasive water-jet cutting machine based on phased intensifier," *IFAC Proceedings Volumes*, vol. 41, no. 2, pp. 14846–14851, 2008.

[7] B. C. Fabien, M. Ramulu, and M. Tremblay, "Dynamic modelling and identification of a waterjet cutting system," *Mathematical and Computer Modelling of Dynamical Systems*, vol. 9, no. 1, pp. 45–63, 2003.

[8] J. Baralić, P. Janković, and B. Nedić, "The waviness of an abrasive water jet generated surface," in *Proceedings of the 13th international conference on tribology*, pp. 217–221, Kragujevac, Serbia, May 2013.

[9] P. J. Singh and D. Benson, "Development of phased intensifier for waterjet cutting," in *Proceedings of the 11th International Conference on Jet Cutting Technology*, pp. 305–318, Scotland, UK, September 1992.

[10] L. S. Cao, "Design of water-jet cutting system based on phased intensifier," *Applied Science and Technology*, vol. 34, no. 8, pp. 48–50, 2007.

[11] F. Yu, M. G. Yang, J. H. Cai et al., "Analysis of fluctuation of pressure on high-pressure system of two intensifiers," *Fluid Machinery*, vol. 5, no. 37, pp. 22–25, 2009.

[12] W. G. Zhuo, Y. Hu, K. X. Li et al., "Simulation research of high pressure water jet hydraulic system based on AMESim," *Fluid Power Transmission and Control*, vol. 14, no. 2, pp. 19–23, 2016.

[13] Y. L. Zeng and K. S. Chen, "Pressure and flow stability for hydraulic driven reciprocating booster waterjet system," *Chinese Hydraulics & Pneumatics*, vol. 39, no. 5, pp. 134–138, 2015.

[14] J. Z. Xu and B. You, "Design of ultrahigh pressure water-jet cutting machine based on phased intensifier," in *Proceedings of the 7th World Congress on Intelligent Control and Automation*, pp. 9368–9373, Chongqing, China, June 2008.

[15] X. B. Sun, H. Y. Wang, X. B. Kong et al., "Water-jet cutting system based on phased intensifier," in *Proceedings of the 2007 2nd IEEE Conference on Industrial Electronics and Applications*, pp. 1053–1057, Harbin, China, May 2007.

[16] H. Liu, J. Wang, N. Kelson, and R. J. Brown, "A study of abrasive waterjet characteristics by CFD simulation," *Journal of Materials Processing Technology*, vol. 153-154, no. 1, pp. 488–493, 2004.

[17] Y. L. Zeng, K. S. Chen, F. L. Zheng, Q. H. Xi, and C. Yu, "Pressure and flow stability for ultra-high pressure water jet system," *Advanced Materials Research*, vol. 926–930, no. 2, pp. 810–813, 2014.

[18] Q. G. Tang, Z. Bai, X. D. Sun et al., "Stability analysis of pressure and flowrate for a high pressure waterjet cutting system based on AMESim," *Chinese Hydraulics & Pneumatics*, vol. 38, no. 4, pp. 31–33, 2014.