Hindawi
Mathematical Problems in Engineering
Volume 2020, Article ID 3645858, 10 pages
https://doi.org/10.1155/2020/3645858

Research Article

Developing a Reliability Model of CNC System under Limited Sample Data Based on Multisource Information Fusion

Chong Peng,[1] **Yuzhen Cai**,[1] **Guangpeng Liu**,[1] **and T. W. Liao**[2]

[1]*Department of Mechanical Engineering and Automation, Beihang University, Beijing 100191, China*
[2]*Department of Mechanical and Industrial Engineering, Louisiana State University, Baton Rouge, LA 70803, USA*

Correspondence should be addressed to Chong Peng; pch@buaa.edu.cn

Received 6 June 2019; Revised 4 September 2019; Accepted 19 December 2019; Published 16 January 2020

Academic Editor: Guangming Xie

The reliability of the computer numerical control (CNC) system affects its processing performance and is a major concern in the manufacturing industry today. However, existing reliability models to assess the reliability of the CNC system often exhibit relatively large errors due to inadequate treatment of small samples. In order to get around the constraint of limited lifetime failure data and take full advantage of existing reliability parameters in traditional reliability models, a multisource information fusion-based reliability model grounded on Bayesian inference is proposed to deal with the small sample size. The prior distributions are derived by using the probability encoding method and conjugate distribution based on the idea of multisource information fusion. Then, using the Jensen–Shannon divergence (JSD) to measure the similarity between prior information and field observation data, a constrained optimization problem is established to determine the respective weight of prior information and field observation data. Finally, by conducting the reliability analysis of repairable CNC systems, the validity of the proposed model and its prior distribution derivation method are verified.

1. Introduction

Enterprises and end users are increasingly demanding CNC systems with high reliability due to the rapid advancement in technology. The CNC system, as the core part of the CNC machine tool, plays a significant role in the manufacturing industry today, whose reliability directly affects production costs and the performance of the machine tool. A reliability model is the premise and basis of reliability research of product life cycle. How to build a more accurate reliability model based on available reliability data is the key to improve the reliability of electromechanical equipment, especially the CNC system. Traditional reliability modeling [1], based on the law of large numbers and the central limit theorem, combined with statistical principles such as maximum likelihood estimation (MLE) and the least square method, has achieved good results in the research of reliability modeling and assessment with small lifetime failure data. Yalcinkaya and Birgoren [2] proposed using the Bayesian–Weibull method to solve a problem for which confidence interval was difficult to estimate under the condition of small sample. Yang et al. [3] put forward the Weibull neural network model and validated the model using field failure data obtained from 23 sets of CNC machine tools. Peng et al. [4] proposed a bathtub-shaped failure rate model based on the piecewise intensity function and verified the validity of the proposed model as well.

However, as the result of increased reliability of the CNC system in modern days, only limited lifetime failure data are available. Applying traditional method to modeling system reliability based on small sample without any modification will inevitably lead to decreasing model accuracy in the obtained model parameters and hence lose the confidence level of the assessment outcome.

Information fusion can integrate multiple sources of information to interpret data better and provide a more accurate assessment of the system. Many scholars carried out research studies along this line based on the multisource information fusion method. Mathon et al. [5] used the Dempster–Shafer evidence theory (DST) to fuse collected

field data and expert judgement information and obtained a better model applied to uncertainty surrounding permeability. Xu et al. [6] proposed a multisource fuzzy incomplete information fusion method based on the information entropy theory and verified its effectiveness.

The methods mentioned above are not appropriate for building the reliability models of the CNC system because of highly complex calculations involved. With the development of the Markov Chain Monte Carlo (MCMC) methods [7–10], the limitations of computational complexity have been overcome. Accordingly, another fusion method, Bayesian inference, has been further studied. Yang et al. [11] obtained field failure time data of machine tools and carried out reliability assessment and modeling based on the Bayesian theory. Wilson and Fronczyk [12] held the belief that developing a prior distribution based on the Bayes theory using similar systems and expert information is useful for future work in those complex systems with limited reliability information. Liu et al. [13] merged degradation data into failure data of a measurement system based on the Bayes theory and combined it with the degenerate trajectory model to obtain a distribution of pseudofailure life to assess the reliability of metering equipment accurately.

This paper proposes a new information fusion-based method (as shown in Figure 1) for building a reliability model for the CNC system grounded on Bayesian inference, which can solve the problem of inadequate evaluation accuracy caused by lack of reliability data under limited conditions such as time and cost. By obtaining data that can reflect reliability information, combined with expert-provided information, the accuracy of reliability assessment can be greatly improved. Through the information fusion technology, reliability index can be quickly obtained to shorten test time and improve test efficiency, leading to a new approach for reliability modeling of the CNC system.

The remainder of the paper is organized as follows. Section 2 describes the derivation of prior distribution and the optimization of respective weights of different prior distributions. Section 3 studies the parameter estimation of the model. Section 4 gives an exemplary application for verifying the proposed method. Finally, the paper is concluded.

2. Multisource Information Fusion Method Based on Bayesian Inference

The method of multisource information fusion comes from the theory of multisensor fusion, which collects and fuses data from different sources. The related issues in developing a reliability model mainly include the choice of model, quantification of multisource information, and fusion of different prior distributions [14], as shown in Figure 1. Choosing a model depends on the nature of highly reliable and long-life equipment. Weibull distribution, as a traditional reliability modeling method, can be used to analyse the reliability of the CNC system [15]. Accordingly, the failure rate function is

$$\lambda(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1}, \quad \theta > 0,\ \beta > 0,\ t \geq 0, \tag{1}$$

where β is the shape parameter of Weibull. When taking different β values, the shape of Weibull distribution is greatly affected. The scale parameter θ determines the mean value of a Weibull distribution.

The quantization of information mainly uses the method of probability encoding and conjugate distribution. The fusion of different prior distributions uses the JSD to measure the similarity between prior information and field data, and a constrained optimization problem is established to determine the respective weights of prior information and field data. Finally, the reliability model for limited sample data is developed, by combining the probability encoding and the JSD method with Bayesian inference.

2.1. Bayesian Model. The Bayesian model is used to integrate different prior distributions [16]. The posterior distribution is

$$\pi(\theta \mid x) = \frac{h(x,\theta)}{m(x)} = \frac{p(x \mid \theta)\pi(\theta)}{\int_{\Theta} p(x \mid \theta)\pi(\theta)\mathrm{d}\theta}, \tag{2}$$

where $\pi(\theta)$ is the prior distribution for the model parameter. Let $p(\theta \mid x)$ be the likelihood function that is obtained based on the probability density function of the chosen reliability model.

For the reliability analysis of the CNC system, the commonly used reliability model is Weibull distribution. The corresponding Bayesian model [17] is

$$\begin{aligned}\pi(\beta,\theta \mid t) &= \frac{L(t \mid \beta,\theta)\pi(\beta,\theta)}{\iint_{\substack{\beta>0\\ \theta>0}} L(t \mid \beta,\theta)\pi(\beta,\theta)\mathrm{d}\beta\,\mathrm{d}\theta} \\ &= \frac{L(t \mid \beta,\theta)\sum_i^n w_i\pi_i(\beta,\theta)}{\iint_{\substack{\beta>0\\ \theta>0}} L(t \mid \beta,\theta)\sum_i^n w_i\pi_i(\beta,\theta)\mathrm{d}\beta\,\mathrm{d}\theta},\end{aligned} \tag{3}$$

where $L(t \mid \beta,\theta)$ is the likelihood function for the collected lifetime failure data. $\sum_i^n w_i\pi_i(\beta,\theta)$ is the fusion prior distribution that is generated by information fusion using data gathered from multiple sources.

2.2. Derivation of Prior Distribution Based on Failure Time Data. Deriving the prior distribution following Weibull distribution is one of the key issues in Bayesian analysis of lifetime data, which requires a great deal of integration and calculation. Therefore, an alternative approach is taken here [18], which involves transforming Weibull distribution into exponential distribution whose conjugate prior distribution determines the prior distribution of Weibull distribution.

Based on the failure rate function given in equation (1), the reliability function is

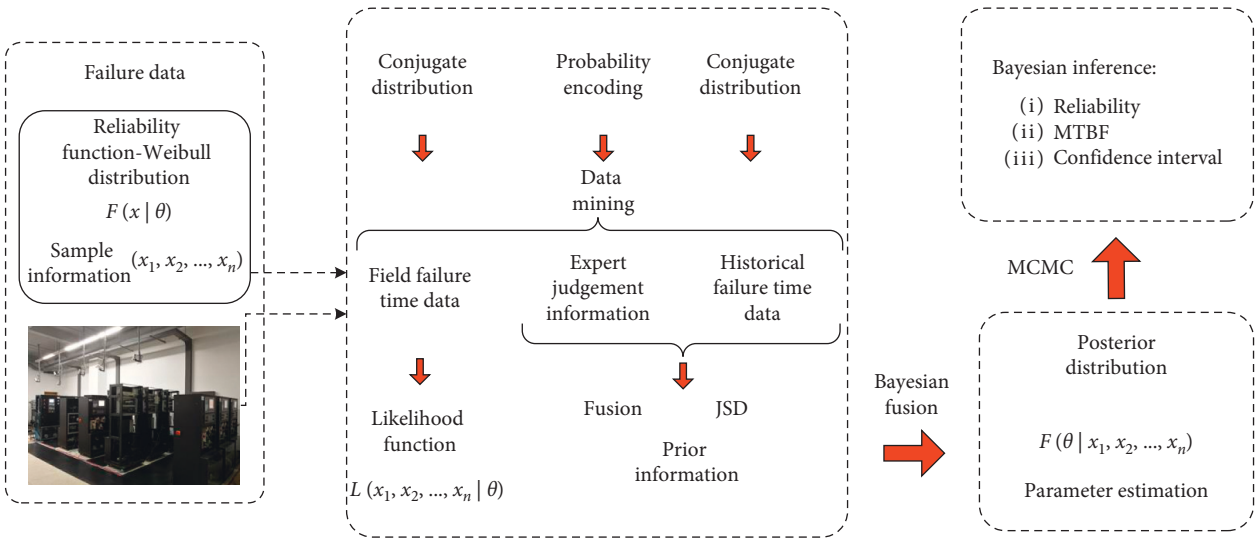

Figure 1: Structure of the information fusion method.

$$R(t) = \exp\left[\left(-\frac{t}{\theta}\right)^{\beta}\right]. \tag{4}$$

The probability density function is

$$f(t) = \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right]. \tag{5}$$

The cumulative probability distribution function is

$$F(t) = \int_0^{t_1} \frac{\beta}{\theta}\left(\frac{t}{\theta}\right)^{\beta-1} \exp\left[-\left(\frac{t}{\theta}\right)^{\beta}\right] \mathrm{d}t. \tag{6}$$

Assuming $h = t$, $\alpha = \theta$, and $X = t^{\beta}$, equation (6) turns into

$$F(X) = \int_0^{X} \frac{1}{\alpha} \exp\left[-\frac{h}{\alpha}\right] \mathrm{d}h. \tag{7}$$

The probability density function after transformation is

$$f(x) = \frac{1}{\alpha} \exp\left[-\left(\frac{x}{\alpha}\right)\right], \tag{8}$$

where $f(x)$ is exponential distribution and $1/\alpha$ is the parameter of exponential distribution. If β is known, then $1/\alpha$ is only associated with θ; subsequently, the prior distribution of θ can be introduced by the conjugate distribution of exponential distribution. Under the same working environment and the same type of system, the shape parameters β of a particular CNC system are theoretically identical. If the assumed shape parameter deviates from the actual, there will be some errors. Therefore, it is possible to reduce the error to some extent by deriving the shape parameter through historical failure time data. Subsequently, the prior distribution of Weibull distribution [19] can be obtained according to the conjugate prior distribution of exponential distribution, that is,

$$\pi(\alpha) = \frac{b^a}{\Gamma(a)}\left(\frac{1}{\alpha}\right)^{a+1} \exp\left[-\left(\frac{b}{\alpha}\right)\right]. \tag{9}$$

2.3. Derivation of Prior Distribution Based on Expert Information. Information provided by experts can be used to increase sample size and to improve the accuracy of reliability assessment for small samples. Therefore, the probability encoding method as shown in the data mining section of Figure 1 [20], specifically the probabilities and values method is applied to obtain statistical information of expert information data in the form of probability density functions obtained by setting parameter type and quantifying expert information data. The information obtained by experts includes values of scale parameters and shape parameters and corresponding probability values, respectively [21]. Since the scale parameter and the shape parameter correspond to failure type and life expectancy of the CNC system, respectively, experts can give parameter values easily based on their experience. Using equation (10), expert information can be transformed into parameter information and the prior distribution of Weibull distribution can be generated from the expert information.

The inverse cumulative distribution function (CDF) is

$$P_{\theta} = \int_{-\infty}^{V_{\theta}} f(\theta; \mu_{\theta}, \sigma_{\theta}) \mathrm{d}\theta, \tag{10}$$

where $f(\theta; \mu_{\theta}, \sigma_{\theta})$ is a parametric CDF.

The probabilistic encoding method has the following steps:

Step 1: determine variables to be collected and collect expert information

Step 2: select an appropriate Bayesian update method

Step 3: convert the expert information into model parameters using the inverse CDF function

Step 4: update model parameters using the Bayesian method

2.4. JSD Method for Determination of Weights. Multisource information fusion aims at integrating information from different dimensions or levels and obtaining more accurate reliability assessment result than using the single information source alone. The ultimate goal of information fusion is to obtain a joint probability distribution. In detail, prior distributions are reasonably assigned proper fusion weights and then integrated into the joint probability distribution. Information entropy measures the total uncertainty in a probability distribution; thus, the similarity between prior information and field test information can be quantified through their respective information entropy. Accordingly, the respective weights of different prior distributions can be determined prior to being used in the integrated prior distribution.

The probability density information entropy is

$$H(m(x)) = -E\left(\log\frac{m(x)}{n(x)}\right) = -\int_{\Omega}\pi(x)\log\frac{m(x)}{n(x)}\mathrm{d}x, \tag{11}$$

where $E(\cdot)$ is mathematical expectation of $m(\cdot)$. In equation (11), $n(x)$ denotes the noninformation prior density function, and the noninformation means that information of the sample is completely unknown. The noninformation density function of the Weibull position parameter can be taken as $n(x) = 1$ [22].

The Kullback–Leibler divergence (KLD), a.k.a. relative entropy, can be used to calculate the similarity between two probability distributions. For probability distributions $A(t)$ and $B(t)$, the KLD is defined as

$$\mathrm{KLD}(A \parallel B) = \int_{\Theta} a(t)\log\frac{a(t)}{b(t)}\mathrm{d}x, \tag{12}$$

where $a(t)$ and $b(t)$ are the probability densities of $A(t)$ and $B(t)$, respectively. Note that $\mathrm{KLD}(A \parallel B) \neq \mathrm{KLD}(B \parallel A)$. Thus, the KLD is an asymmetric and noncommutative measure.

The Jensen–Shannon Divergence [23] is another measure that can be used to measure the similarity between two probability distributions based on the KLD:

$$\mathrm{JSD}(A \parallel B) = \frac{1}{2}(\mathrm{KLD}(A \parallel M) + \mathrm{KLD}(B \parallel M)), \tag{13}$$

where M is the mean probability distribution function of distributions A and B.

$$M = \frac{1}{2}(\pi_i(\theta) + \pi_0(\theta)). \tag{14}$$

The JSD has the following properties:

$$\begin{aligned} &\mathrm{JSD}(A \parallel B) \geq 0, \\ &\mathrm{JSD}(A \parallel B) = \mathrm{JSD}(B \parallel A), \\ &\mathrm{JSD}(A \parallel B) = 0. \end{aligned} \tag{15}$$

when $A = B$.

Compared with the KLD, the JSD can better measure the similarity between the probability density function of prior distribution and the probability density function of field test data. The importance of each prior distribution is determined by the JSD, then weights w_i of each distribution are assigned, and the multisource information is fused using the respective weights w_i [24].

The fusion prior distribution is defined as

$$\pi(\theta) = \sum_{i}^{n} w_i\pi_i(\theta). \tag{16}$$

According to the properties of the JSD, the smaller the JSD value obtained by combining the fusion prior distribution, π_θ and the prior distribution of field test data $\pi_0(\theta)$, the greater the weight is

$$\begin{aligned} \mathrm{JSD}\left(\sum_{i}^{n} w_i\pi_i(\theta) \parallel \pi_0(\theta)\right) &= \frac{1}{2}\mathrm{KLD}\left(\sum_{i}^{n} w_i\pi_i(\theta) \parallel M\right) \\ &\quad + \mathrm{KLD}\left(M \parallel \pi_0(\theta)\right) \\ &= H(M) - \frac{1}{2}\left(H\left(\sum_{i}^{n} w_i\pi_i(\theta)\right)\right. \\ &\quad \left. + H(\pi_0(\theta))\right). \end{aligned} \tag{17}$$

The weights are bounded between zero and one and summed to one. Hence, a constrained optimization problem can be formulated as follows:

$$\begin{cases} \min\left\{H(M) - \frac{1}{2}\left(H\left(\sum_{i}^{n} w_i\pi_i(\theta)\right) + H(\pi_0(\theta))\right)\right\}, \\ 0 \leq w_i \leq 1, \\ \sum_{i}^{n} w_i = 1. \end{cases} \tag{18}$$

3. Parameter Estimation Based on MCMC

Solving the integral problem of the posterior distribution of the Bayesian method is difficult and obtaining the result through mathematical integral is also complex, so the Bayesian model after the fusion is used to perform analysis of lifetime failure data by the MCMC method, as shown in the Bayesian inference of Figure 1. MCMC [25] is a Markov chain establishing a posteriori stationary distribution. MCMC establishes the Markov chain of posterior smooth distribution, and it can then be used to generate samples to obtain posterior distribution samples for statistical inference. By doing so, it can eliminate the need for numerical

calculation of integrand to ensure the effective implementation and stability of the solution process.

The posterior distribution $\pi(\beta, \theta \mid t)$ as target distribution, according to the experience of selecting a uniform distribution as proposed distribution, generates a Markov chain $\Theta_i = \{(\beta_i, \theta_i), i = 1, 2, \ldots, n\}$ with the following specific steps:

(1) Generate an initial value Θ_0 from the proposed distribution

(2) Iterate on $i = 1, 2, \ldots$

  (a) Produce values of Θ' from the proposed distribution

  (b) Generate a random value u from uniform distribution

  (c) Calculate the acceptance probability and if

$$u \leq \frac{\pi(\Theta' \mid t) g(\Theta_i \mid \Theta')}{\pi(\Theta_i \mid t) g(\Theta' \mid \Theta_i)}, \tag{19}$$

then $\Theta_{i+1} = \Theta'$; Otherwise, $\Theta_{i+1} = \Theta_i$, where $g(\Theta_i \mid \Theta)$ is the proposed distribution.

4. Case Study

4.1. Data Source and Determination of Prior Distribution. The development of the CNC machine tool is strategically important to the equipment manufacturing industry in the globally competitive market. The CNC system is the core of CNC machine tools, and its importance is hence self-evident. Because of its long life and high reliability, it is difficult to collect lifetime failure data, and data size is often too small for reliability modeling. Therefore, in order to test the reliability of the CNC system, a long-term multisample test under the same environment was carried out in the test laboratory as shown in Figure 2.

Figures 2(a) and 2(b) show the laboratory environment specially built for collecting lifetime failure data, which correspond to the laboratory environment of field data and historical data, respectively. Among them, field data refer to data collected from March 2016 to June 2018 of 30 sets of new CNC systems of the same type running in laboratory, as shown in Figure 2(a). On the other hand, historical data refer to data collected between March 2013 and June 2016, as shown in Figure 2(b).

This test has been taken to record the failures of CNC systems, and a small sample of lifetime failure data is obtained, as shown in Tables 1 and 2. Table 1 is field failure time data and Table 2 is historical failure time data. Expert judgment information is obtained from two experts, as given in Table 3.

Then, the proposed model is applied to perform reliability analysis of a repairable CNC system.

Based on the inverse CDF given in equation (10) and expert provided information in Table 3, combining the Bayesian updating theory with the probability encoding method, the distribution function of the expert information is obtained as follows:

$$\begin{aligned}\pi_1(\beta, \theta) &= \frac{1}{2\pi\sqrt{\sigma_\theta^2 \sigma_\beta^2}} \exp\left(-\frac{(\theta - \overline{\theta})^2}{2\sigma_\theta^2} - \frac{(\beta - \overline{\beta})^2}{2\sigma_\beta^2}\right) \\ &= \frac{1}{2\pi\sqrt{20 \times 0.1}} \exp\left(-\frac{(\theta - 214.79)^2}{2 \times 20} - \frac{(\beta - 0.78)^2}{2 \times 0.1}\right).\end{aligned} \tag{20}$$

In the last section, the process of solving the prior distribution using failure time data was described. According to the collected historical and field failure time data, the mean and variance of the sample data are computed as 85.188 and 5168.521, respectively. On this basis, according to equations (7) and (8), the hyperparameters in equation (9) are calculated as $a_2 = 2.576$ and $b_2 = 243.181$, and the prior distribution of historical failure time data is obtained as

$$\pi_2(\beta, \theta) = \pi(\alpha) = \frac{204.792^{3.404}}{\Gamma(3.404)} \left(\frac{1}{\alpha}\right)^{4.404} \exp\left(-\left(\frac{204.792}{\alpha}\right)\right). \tag{21}$$

Similarly, $a_0 = 4.03$ and $b_0 = 1100.25$. The prior distribution of failure time data can be obtained as

$$\pi_0(\beta, \theta) = \pi(\alpha) = \frac{394.02^{2.8}}{\Gamma(2.8)} \left(\frac{1}{\alpha}\right)^{3.8} \exp\left(-\left(\frac{394.02}{\alpha}\right)\right). \tag{22}$$

The probability density curve of each prior distribution is plotted in Figure 3. As shown in Figure 3, the weight ratio of each prior distribution to fuse prior distribution can be predicted by its shape. For example, the expert judgement information is subjective and differs greatly from the field failure data source, so its weight ratio should be relatively small. Recall that the specific weights need to be obtained by calculating the JSD optimization model given in equation (18), to be further described next.

4.2. Prior Distribution Fusion Based on JSD. In the previous section, we obtained prior distribution functions based on expert information and historical fault data. Because of the different importance of expert information and historical information, we use the JSD to obtain the optimal weights for quantitatively analysing their respective importance. Subsequently, the fusion prior distribution is computed as

$$\pi(\beta, \theta) = w_1 \times \pi_1(\beta, \theta) + w_2 \times \pi_2(\beta, \theta). \tag{23}$$

Using the numerical solution method provided in the MATLAB optimization toolbox, based on the constrained optimization problem given in equation (18), the weights are obtained as $w_1 = 0.175$ and $w_2 = 0.825$, respectively.

4.3. Bayesian Reliability Assessment Based on Information Fusion. The probability distribution function of posterior probability can be obtained by using the likelihood function to combine the field failure data, the acquired prior distribution, and the weights of different prior distributions.

The likelihood function of Weibull distribution is

(a) (b)

Figure 2: The environment of CNC system reliability test laboratory.

Table 1: Field failure time data.

Failure instance	Failure time (h)
1	1162.69
2	156.94
3	200.56
4	49.56
5	395.54
6	231.95
7	318.76
8	125.11
9	231.78
10	126.35
11	174.50
12	91.50

Table 2: Historical failure time data.

Failure instance	Failure time (h)
1	829.92
2	366.55
3	94.08
4	147.99
5	450.25
6	73.15
7	152.51
8	23.50
9	280.28
10	17.02
11	24.47
12	9.31
13	342.61
14	203.93
15	269.73
16	113.85

$$L(t \mid \beta, \theta) = \prod_{i=1}^{n}\left(\frac{\beta}{\theta}\left(\frac{t_i}{\theta}\right)^{\beta-1} \exp\left(-\left(\frac{t_i}{\theta}\right)^{\beta}\right)\right) = \frac{\beta^n}{\theta^{n\beta}} \prod_{i=1}^{n}\left(t_i^{\beta-1}\right) \exp\left(-\sum_{i=1}^{n}\left(\frac{t_i}{\theta}\right)^{\beta}\right). \tag{24}$$

Table 3: Expert judgment information and PV analysis result.

Judgment information	V_θ	P_θ	μ_θ	V_β	P_β	μ_β
Expert 1	207	0.17	226	0.62	0.29	0.6753
	215	0.42	219	0.71	0.57	0.6924
	221	0.86	199	0.84	0.65	0.7015
	227	0.89	202	0.97	0.83	0.8764
	235	0.96	200	0.99	0.92	0.8495
	258	0.97	220	1.02	0.98	0.8164
	274	0.99	227	1.13	0.99	0.8974
Expert 2	200	0.10	226	0.64	0.15	0.7436
	212	0.32	221	0.69	0.29	0.7453
	219	0.65	211	0.73	0.32	0.7768
	224	0.85	203	0.84	0.65	0.8015
	231	0.91	204	0.87	0.83	0.7764
	246	0.93	216	0.95	0.95	0.7855
	259	0.98	218	0.99	0.98	0.7864

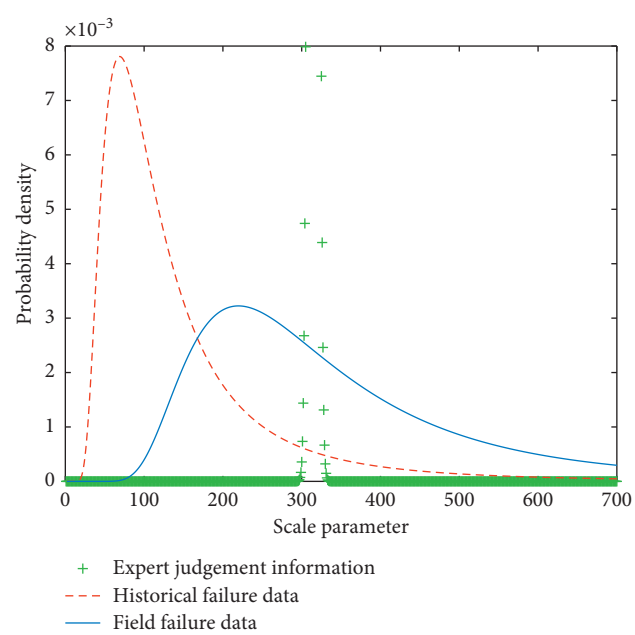

Figure 3: Prior distribution probability density curve.

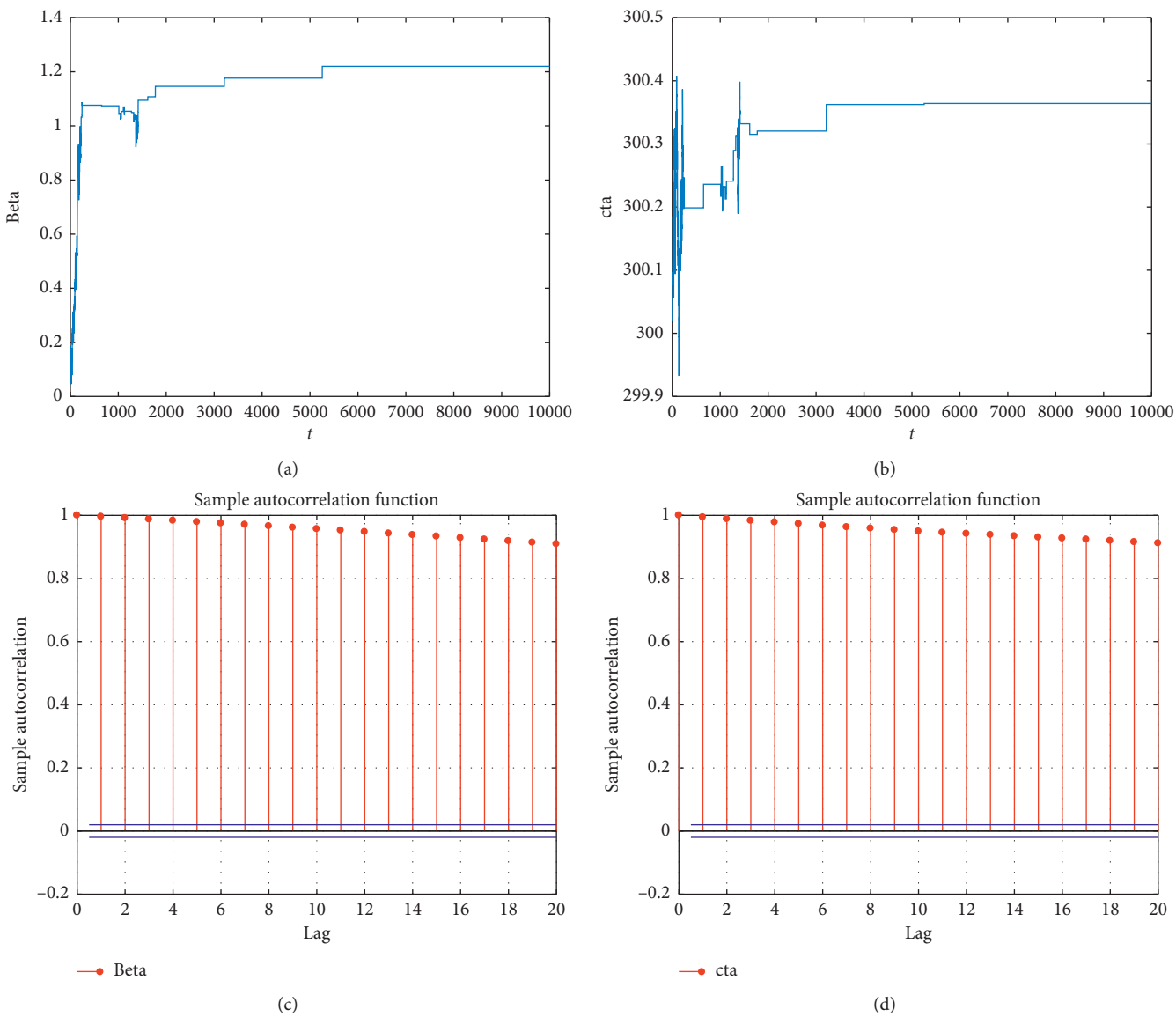

Figure 4: Parameter iteration analysis process and autocorrelation function diagram for multisource information fusion. (a) The relationship between the number of iterations t and hyperparameter β. (b) The relationship between the number of iterations t and hyperparameter θ. (c) Sample autocorrelation function graph for β. (d) Sample autocorrelation function graph for θ.

The Bayesian posterior distribution is

$$\pi(\beta,\theta\mid t)=\frac{L(t\mid\beta,\theta)\pi(\beta,\theta)}{\iint_{\substack{\beta>0\\ \theta>0}}L(t\mid\beta,\theta)\pi(\beta,\theta)\mathrm{d}\beta\,\mathrm{d}\theta}=\frac{L(t\mid\beta,\theta)\sum_i^n w_i\pi_i(\beta,\theta)}{\iint_{\substack{\beta>0\\ \theta>0}}L(t\mid\beta,\theta)\sum_i^n w_i\pi_i(\beta,\theta)\mathrm{d}\beta\,\mathrm{d}\theta}$$
$$=\frac{\left(\beta^n/\theta^{n\beta}\right)\prod_{i=1}^n\left(t_i^{\beta-1}\right)\exp\left(-\sum_{i=1}^n\left(t_i/\theta\right)^\beta\right)\pi(\beta,\theta)}{\iint_{\substack{\beta>0\\ \theta>0}}\left(\beta^n/\theta^{n\beta}\right)\prod_{i=1}^n\left(t_i^{\beta-1}\right)\exp\left(-\sum_{i=1}^n\left(t_i/\theta\right)^\beta\right)\pi(\beta,\theta)\mathrm{d}\beta\,\mathrm{d}\theta}. \tag{25}$$

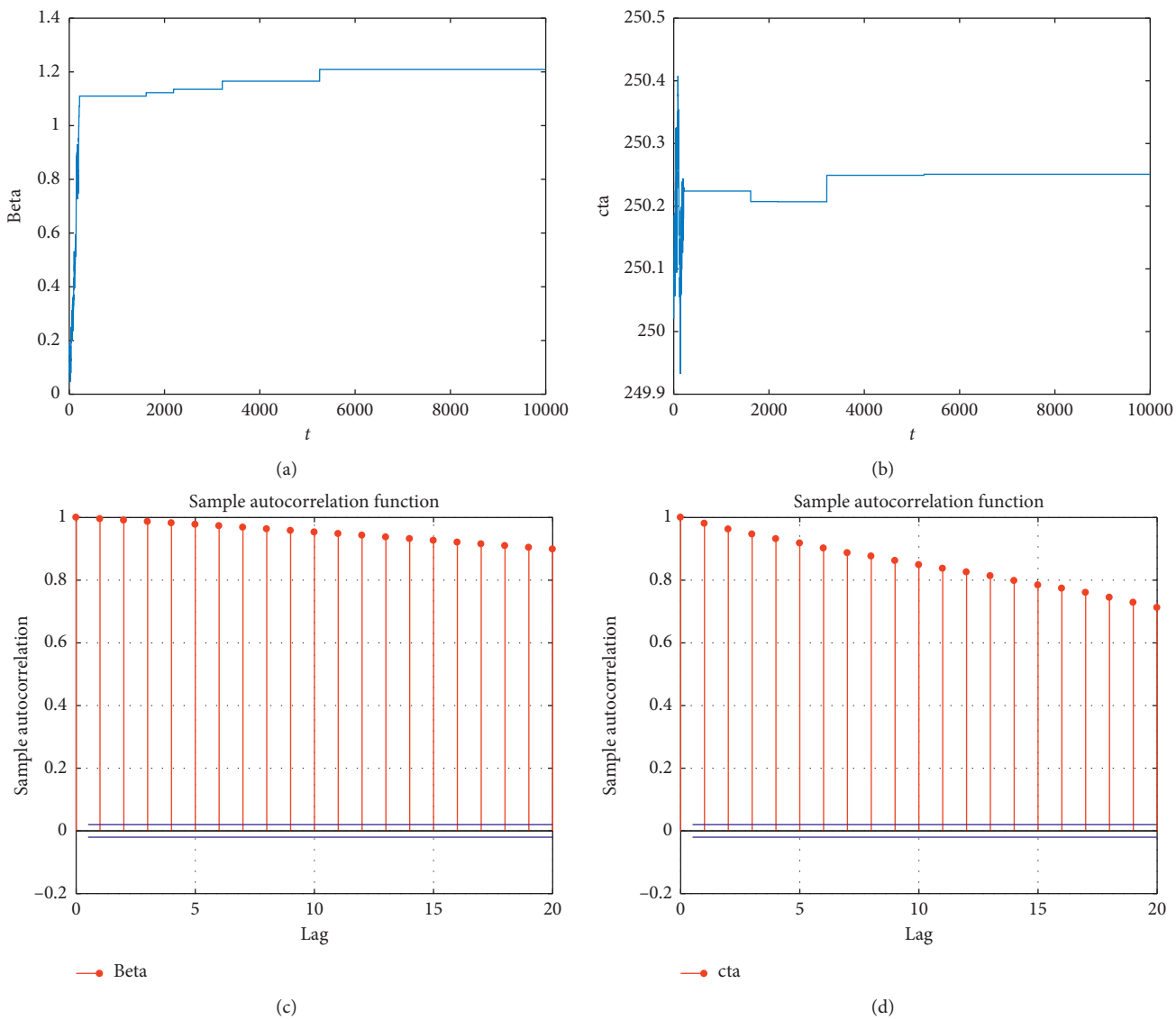

Figure 5: Parameter iteration analysis process and autocorrelation function diagram for noninformation prior distribution. (a) The relationship between the number of iterations t and hyperparameter β. (b) The relationship between the number of iterations t and hyperparameter θ. (c) Sample autocorrelation function graph for β. (d) Sample autocorrelation function graph for θ.

In order to get around the numerical calculation difficulty brought by high calculus, the MCMC method is used to estimate the parameters. Figures 4(a) and 4(b) show the convergence of MCMC in estimating prior distribution parameters in the multisource information fusion process. The corresponding autocorrelation functions shown in Figures 4(c) and 4(d) can be used to diagnose the convergence of the Markov chain.

Likely, Figures 5(a) and 5(b) show the convergence of MCMC in estimating noninformation prior distribution parameters. The corresponding autocorrelation functions shown in Figures 5(c) and 5(d) can be used to diagnose the convergence of the Markov chain. The change of the value indicates the convergence of the Markov chain. Figures 4(c) and 5(d) have higher correlation values and slower convergence indicated by a slight downward trend. Figures 4(d) and 5(c) have an obvious downward trend, indicating a good convergence of MCMC.

Using the above-described parameter estimation by MCMC and MLE, the results of final estimated parameters are obtained as shown in Table 4, and then, the reliability index, i.e., mean time between failure (MTBF) is calculated.

For comparison of different parameter estimation processes, the MCMC method based on the fusion prior distribution, the MCMC method without prior information, and the MLE method are, respectively, used to estimate parameters. As can be seen from the table, the results obtained based on the MCMC method of fusion prior distribution and confidence intervals of MCMC without prior information are both less than those of the traditional MLE method ($0.005 < 0.012 < 0.72$), indicating that the MCMC method has better accuracy. The confidence interval of the

Table 4: Comparison of parameter estimation results.

Parameter	Multisource fusion prior information			MLE			Noninformation priori distribution		
	Mean	Confidence interval		Mean	Confidence interval		Mean	Confidence interval	
β	1.162	1.11591	1.1639	1.0	0.64	1.36	1.158	1.152	1.164
Confidence interval		0.005			0.72			0.012	
MTBF		287.487			250			256.672	

MCMC with prior distribution is smaller than that of MCMC without prior information. Based on the above test results, the validity of the proposed model is verified.

5. Conclusions

This paper has presented a new method for developing a reliability model based on multisource information fusion to deal with small sample size. The following achievements are made:

(1) In order to solve the problems of small lifetime failure data, low accuracy of reliability assessment, and incomplete use of reliability parameters, a new method for developing a reliability assessment model based on multisource information fusion is proposed. The prior distribution of combining information from different sources is obtained by using the conjugate distribution and probability encoding method.

(2) A JSD-based method for optimizing weights of multisource information is developed, which is designed to obtain more reasonable weight information through optimization. Moreover, with respect to the difficulty of parameter estimation methods in solving integral equations, MCMC is used to estimate relevant model parameters and guarantee the feasibility of the model in a selected application.

(3) The practical use of the proposed model is demonstrated by using a set of lifetime failure data of repairable CNC systems. For comparison, the MCMC method based on fusion prior distribution, the MCMC method without prior information, and the method of MLE are, respectively, used to estimate parameters. The corresponding confidence intervals are obtained, and the validity of the proposed model and its parameter estimation method are verified. The study is expected to be useful for reliability analysis of repairable systems, especially for small samples, and provides important insights for guiding the development of the preventive maintenance strategy.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Acknowledgments

This work was supported by the Natural Science Foundation of China (Grant no. 51875029), the National Science and Technology Major Project "High-Grade CNC Machine Tools and Basic Manufacturing Equipment" (Grant no. 2016ZX04004006), and the Project of Fault information Dictionary Standards for CNC equipment and its verification.

References

[1] H. Liu and V. Makis, "Cutting-tool reliability assessment in variable machining conditions," *IEEE Transactions Reliability*, vol. 45, no. 4, pp. 573–581, 1996.

[2] M. Yalcinkaya and B. Birgoren, "Confidence interval estimation of Weibull lower percentiles in small samples via Bayesian inference," *Journal of the European Ceramic Society*, vol. 37, no. 8, pp. 2983–2990, 2017.

[3] Z. Yang, C. Chen, J. Wang, and G. Li, "Reliability assessment of CNC machining center based on Weibull neural network," *Mathematical Problems in Engineering*, vol. 2015, Article ID 292197, 8 pages, 2015.

[4] C. Peng, G. Liu, and L. Wang, "Piecewise modelling and parameter estimation of repairable system failure rate," *SpringerPlus*, vol. 5, no. 1, p. 1477, 2016.

[5] B. R. Mathon, M. M. Ozbek, and G. F. Pinder, "Dempster-shafer theory applied to uncertainty surrounding permeability," *Mathematical Geosciences*, vol. 42, no. 3, pp. 293–307, 2010.

[6] W. Xu, M. Li, and X. Wang, "Information fusion based on information entropy in fuzzy multi-source incomplete information system," *International Journal of Fuzzy Systems*, vol. 19, no. 4, pp. 1200–1216, 2017.

[7] M.-Y. Liao, "Efficient technique for assessing actual non-normal quality loss: Markov chain Monte Carlo," *Quality and Reliability Engineering International*, vol. 33, no. 5, pp. 945–957, 2017.

[8] Y. W. Hu, H. C. Zhang, S. J. Liu, and H. T. Lu, "Sequential Monte Carlo method toward online RUL assessment with applications," *Chinese Journal of Mechanical Engineering*, vol. 31, no. 1, p. 5, 2018.

[9] E. A. Ahmed, "Bayesian estimation based on progressive type-II censoring from two-parameter bathtub-shaped lifetime model: an Markov chain Monte Carlo approach," *Journal of Applied Statistics*, vol. 41, no. 4, pp. 752–768, 2014.

[10] R. Medjoudj, D. Aissani, A. Boubakeur, and K. D. Haim, "Interruption modelling in electrical power distribution systems using the Weibull-Markov model," *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, vol. 223, no. 2, pp. 145–157, 2009.

[11] Z. Yang, Y. Kan, F. Chen, B. Xu, C. Chen, and C. Yang, "Bayesian reliability modeling and assessment solution for NC machine tools under small-sample data," *Chinese Journal of Mechanical Engineering*, vol. 28, no. 6, pp. 1229–1239, 2015.

[12] A. G. Wilson and K. M. Fronczyk, "Bayesian reliability: combining information," *Quality Engineering*, vol. 29, no. 1, p. 11, 2016.

[13] X. Liu, Y. Liu, and X. Liu, "Fusion method of metering equipment reliability assessment based on degradation information and failure data," in *Proceedings of the First International Conference on Reliability Systems Engineering*, pp. 1–6, Beijing, China, October 2015.

[14] H. Z. Huang, P. L. Cui, W. Peng, and H. Gao, "Fatigue lifetime assessment of aircraft engine disc via multi-source information fusion," *International Journal of Turbo & Jet-Engines*, vol. 31, no. 2, pp. 167–174, 2014.

[15] F. Azizi, F. Haghighi, E. Ghadiri, and L. Torabi, "Statistical inference for masked interval data with Weibull distribution under simple step-stress test and tampered failure rate model," *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, vol. 231, no. 6, pp. 654–665, 2017.

[16] W. Peng, H.-Z. Huang, M. Xie, Y. Yang, and Y. Liu, "A bayesian approach for system reliability analysis with multilevel pass-fail, lifetime and degradation data sets," *IEEE Transactions on Reliability*, vol. 62, no. 3, pp. 689–699, 2013.

[17] S. K. Upadhyay and A. Gupta, "A Bayes analysis of modified Weibull distribution via Markov chain Monte Carlo simulation," *Journal of Statistical Computation and Simulation*, vol. 80, no. 3, pp. 241–254, 2010.

[18] P. O'Connor and A. Kleyner, *Practical Reliability Engineering*, Wiley Press, Hoboken, NJ, USA, 2012.

[19] L. C. Méndez-González, L. A. Rodríguez-Picón, D. J. Valles-Rosales, R. Romero-López, and A. E. Quezada-Carreón, "Reliability analysis for electronic devices using beta-Weibull distribution," *Quality and Reliability Engineering International*, vol. 33, no. 8, pp. 2521–2530, 2017.

[20] C. S. Spetzler and C.-A. S. Staël Von Holstein, "Exceptional paper-probability encoding in decision analysis," *Management Science*, vol. 22, no. 3, pp. 340–358, 1975.

[21] Y. Liu, J. Fan, and Y. Li, "One system reliability assessment method for CNC grinder," *Eksploatacja I Niezawodnosc-Maintenance and Reliability*, vol. 16, no. 1, pp. 97–104, 2014.

[22] H. Li, R. Yuan, W. Peng, Y. Liu, and H. Huang, "Bayesian inference of Weibull distribution based on probability encoding method," in *Proceedings of International Conference on Quality, Reliability, Risk, Maintenance and Safety Engineering*, pp. 365–369, Xi'an, China, 2011.

[23] H. Jeffreys, *Theory of Probability*, Oxford University Press, New York, NY, USA, 1961.

[24] Q. Liu, G. Jin, J. Feng, Z. Zhou, J. Zhou, and M. Xi, "GeMiBi: a general multiple sources information Bayesian fusion for performance evaluation and an application to HPC cluster," *Engineering Failure Analysis*, vol. 18, no. 6, pp. 1465–1476, 2011.

[25] S. Singh, S. Abdullah, N. A. Nik Mohamed, and M. S. Mohd Noorani, "Markov chain modelling of reliability analysis and prediction under mixed mode loading," *Chinese Journal of Mechanical Engineering*, vol. 28, no. 2, pp. 307–314, 2015.