Hindawi
Mathematical Problems in Engineering
Volume 2020, Article ID 3672439, 9 pages
https://doi.org/10.1155/2020/3672439

Research Article

Distributed Optimal Control for a Class of Switched Nonlinear Systems with the State Time Delay

Yuxing Duan and Baili Su

College of Engineering, Qufu Normal University, Rizhao, Shandong, China

Correspondence should be addressed to Baili Su; subaili111@126.com

Received 13 January 2020; Revised 5 April 2020; Accepted 15 April 2020; Published 11 May 2020

Academic Editor: Jürgen Pannek

This paper is focused on a kind of distributed optimal control design for a class of switched nonlinear systems with the state time delay which have a prescribed switching sequence. Firstly, we design a bounded controller to make the system stable for each mode of the nominal system. Then, a distributed optimal controller which can satisfy input constraint is designed based on the bounded stabilization controller. A sufficient condition to guarantee ultimate boundedness of the system is given based on appropriate assumption. The significance of this paper is that distributed optimal control method is applied to switched nonlinear systems with the state time delay. Finally, a simulation example is given to verify the effectiveness of the proposed method.

1. Introduction

Switched time-delay systems are an important kind of hybrid systems, which have attracted extensive attention in recent years. There are some results for switched time-delay systems [1–4]. However, the input of switched time-delay systems is often constrained by objective conditions, and some switched systems need to operate according to a prescribed switching sequence in many practical control systems. So, there are still many challenging control problems need to be solved for this class of nonlinear systems.

Regression optimal control is an optimal control method that can deal with system constraints. It obtains the current control action by solving a finite time open loop optimal control problem at every sampling moment. It has well dynamic control effect and is helpful to improve the stability of the system. Therefore, its study has received considerable attention [5–8]. Moreover, the method of combining regression optimal control with the control Lyapunov function-based bounded control has made further development in the fields of single-objective control, multiobjective control, and so on. For example, the objective function in [9] adopts the nonquadratic cost function and studies a regression optimal control based on the Lyapunov economic model, which can directly solve economic problems. In [10], a fast feedback controller that stabilizes the fast dynamics and a regression optimal controller that stabilizes the slow dynamics and enforces desired performance objectives in the slow subsystem are designed for a kind of nonlinear singularly perturbed systems. He et al. [11] study an alternative utopia-tracking multiobjective economic regression optimal control scheme of constrained nonlinear systems with guaranteed asymptotic stability and convergence of average performance.

In most existing results, centralized optimal control methods are adopted. When this kind of control method is applied to the system, the computational complexity reduces the performance of the system with the increase of variables and the expansion of model size. Distributed optimal control can reduce computational burden and fault tolerance of the system by using communication and cooperation among multiple controllers. Therefore, more and more scholars study distributed optimal control. The method of combining distributed optimal control with the control Lyapunov function-based bounded control also has made further development. Heidarinejad et al. [12] study a distributed optimal control of the switched nonlinear system. It redesigns the local control system (LCS) and the new control system (NCS) by using an optimal control method based on Lyapunov function to coordinate their respective operations. The distributed structure adopted in [12] enables LCS to stabilize the closed-loop system, and NCS can communicate and cooperate with LCS, which can further improve the closed-loop performance. A distributed optimal control scheme with such distributed structure is considered for switched nonlinear time-delay systems with delayed measurements and communication disruption in [13]. Liu et al. [14] propose a distributed optimal control strategy for nonlinear systems with asynchronous and time-delay measurements. Heidarinejad et al. [15] study a distributed optimal control scheme of nonlinear systems subject to communication disruptions—communication channel noise and data losses—between distributed controllers. It is noted that the distributed optimal control method is seldom applied to nonlinear time-delay systems. Hence, it is necessary to study the problem of distributed optimal control for switched nonlinear systems with the state time delay.

In this paper, a distributed optimal control method is applied to switched nonlinear systems with the state time delay. The main ideas are as follows. A bounded controller to make system stable for each mode of the nominal system is designed. Then, a kind of distributed optimal control which can satisfy input constraint is designed based on the bounded stabilization controller. A sufficient condition to guarantee ultimate boundedness of the switched system is given. Finally, a simulation example is given to verify the effectiveness of the proposed distributed optimal control strategy.

2. Problem Statement and Preliminaries

Consider a class of switched nonlinear systems with the state time delay as follows:

$$\begin{aligned}\dot{x}(t) = {} & f_{\sigma(t)}(x(t)) + g_{1_{\sigma(t)}}(x(t))u_{1_{\sigma(t)}}(t) + g_{2_{\sigma(t)}}(x(t))u_{2_{\sigma(t)}}(t) \\ & + q_{\sigma(t)}\left(x\left(t-\tau_{\sigma(t)}\right)\right) + k_{\sigma(t)}(x(t))\omega_{\sigma(t)}(t),\end{aligned} \tag{1}$$

where $x(t) \in R^{n_x}$ is the state, $u_{j_{\sigma(t)}}(t) \in R^{n_{u_j}}$ $(j = 1, 2)$ is the input, $\omega_{\sigma(t)}(t) \in R^{n_\omega}$ is external disturbance, respectively, $f_{\sigma(t)}(\cdot)$, $g_{1_{\sigma(t)}}(\cdot)$, $g_{2_{\sigma(t)}}(\cdot)$, $q_{\sigma(t)}(\cdot)$, and $k_{\sigma(t)}(\cdot)$ are local Lipschitz functions satisfying $f_{\sigma(t)}(0) = 0$, $g_{1_{\sigma(t)}}(0) = 0$, $g_{2_{\sigma(t)}}(0) = 0$, $q_{\sigma(t)}(0) = 0$, and $k_{\sigma(t)}(0) = 0$, and $\tau_{\sigma(t)}$ denotes time delay. The initial value is $x(\xi) = \varphi_{\sigma(t)}(\xi)$, $\forall \xi \in [-\tau_{\sigma(t)}, 0)$ with $\varphi_{\sigma(t)}(\xi)$ being a continuous function and $\|\varphi_{\sigma(t)}\| = \max_{-\tau_{\sigma(t)} \leq \xi \leq 0} \|\varphi_{\sigma(t)}(\xi)\|$, $\|\cdot\|$ denotes the Euclidean norm. $u_{j_{\sigma(t)}}(t)$ is restricted on a nonempty set $U_{j_{\sigma(t)}}(t) := \left\{u_{j_{\sigma(t)}}(t) \in R^{n_{u_j}}: \|u_{j_{\sigma(t)}}(t)\| \leq u^{\max}_{j_{\sigma(t)}}\right\}$ with $u^{\max}_{j_{\sigma(t)}}$ being positive constant $(j = 1, 2)$. $\omega_{\sigma(t)}(t) \in W_{\sigma(t)} := \left\{\omega_{\sigma(t)}(t) \in R^{n_\omega}: \|\omega_{\sigma(t)}(t)\| \leq \theta_{\sigma(t)}\right\}$ with $\theta_{\sigma(t)}$ being a positive constant. $\sigma(t)$: $[0, \infty) \longrightarrow S = \{1, 2, \ldots, s\}$ is the right continuous switching symbol with s which denotes the number of switched modes. In other words, $\sigma(t_i) \triangleq \lim_{t \longrightarrow t_i^+} \sigma(t)$ for all i, which means that a finite number of switches can only be switched in a finite time interval.

In this paper, $t_{i_r^{in}}$ and $t_{i_r^{out}}$ denote the time of the rth switch in and out for the ith mode respectively, i.e., $\sigma(t^+_{i_r^{\text{in}}}) = \sigma(t^-_{i_r^{\text{out}}}) = i$. So, if $t_{i_r^{\text{in}}} \leq t < t_{i_r^{\text{out}}}$, then system (1) can be described as

$$\begin{aligned}\dot{x}(t) = {} & f_i(x(t)) + g_{1_i}(x(t))u_{1_i}(t) + g_{2_i}(x(t))u_{2_i}(t) \\ & + q_i(x(t-\tau_i)) + k_i(x(t))\omega_i(t).\end{aligned} \tag{2}$$

$T_{i^{\text{in}}} = \left\{t_{i_1^{\text{in}}}, t_{i_2^{\text{in}}}, \ldots\right\}$ and $T_{i^{\text{out}}} = \left\{t_{i_1^{\text{out}}}, t_{i_2^{\text{out}}}, \ldots\right\}$ denote the time sets of switching in and out of the ith mode, respectively.

The objective of this paper is to propose a distributed optimal control scheme for a class of switched nonlinear systems with the state time delay. System (1) adopts the distributed structure (see Figure 1), where u_1 and u_2 are designed by controller 1 and controller 2, respectively.

The following assumption is satisfied at mode i $(i = 1, 2, \ldots, s)$.

Assumption 1. For some $a_i \in (0, 1)$, there holds

$$\left\|\frac{\partial V_i}{\partial x} q_i(x(t-\tau_i))\right\| \leq \Psi_i(V_i(x(t))) + a_i \Psi_i(V_i(x(t-\tau_i))), \tag{3}$$

where $\Psi_i(\cdot)$ is a continuous positive definite nondecreasing function.

Remark 1. Assumption 1 illustrates that the decoupling between the delay state and the current state can be realized by the derivative of the Lyapunov function. This assumption is also used to realize the separation of time-delay state and nontime-delay state in [16].

Remark 2. The distributed optimal control can set up multiple part controllers to make the system stable and improve its performance. For simplicity, only two parts are considered in this paper. Distributed optimal controller designed in this paper is composed of controller 1 and controller 2. Controller 1 can stabilize the closed-loop system. Controller 2 can improve the performance of the closed-loop system by communicating with controller 1. If the distributed optimal controller is composed by multiple controllers, then controller 1 makes the closed-loop system stable, remaining controllers improve the performance of the closed-loop system through cooperation and communication with controller 1.

3. Main Results

3.1. Design of Bounded Controller. For each mode i $(i = 1, 2, \ldots, s)$, we can design bounded controllers acting on the following proposition.

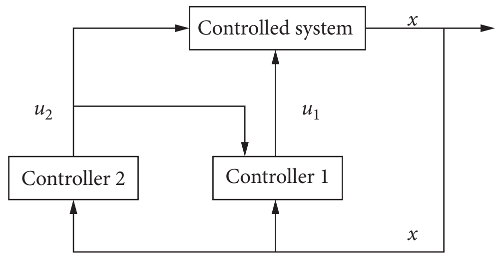

Figure 1: The system structure.

Proposition 1. *If the nominal system of system (2)*

$$\begin{aligned}\dot{x}(t) = {} & f_i(x(t)) + g_{1_i}(x(t))u_{1_i}(t) + g_{2_i}(x(t))u_{2_i}(t) \\ & + q_i(x(t-\tau_i)),\end{aligned} \tag{4}$$

satisfies Assumption 1, we can choose bounded controllers $u_{1_i}(t) = h_i(x(t)) = -c_i(x)((\partial V_i/\partial x)g_{1_i}(x))$ *and* $u_{2_i}(t) = 0$ *such that*

$$\frac{\partial V_i}{\partial x}\left(f_i(x(t)) + g_{1_i}(x(t))u_{1_i}(t)\right) \le -2\Psi_i(V_i(x(t))), \tag{5}$$

where

$$c_i(x) = \begin{cases} \dfrac{\alpha_i(x) + \sqrt{\alpha_i^2(x) + \left(u_{1_i}^{\max}\beta_i(x)\right)^4}}{\beta_i^2(x)\left(1 + \sqrt{1 + \left(u_{1_i}^{\max}\beta_i(x)\right)^2}\right)}, & \beta_i(x) \ne 0, \\ 0, & \beta_i(x) = 0, \end{cases} \tag{6}$$

$$\begin{aligned}\alpha_i(x) &= \frac{\partial V_i}{\partial x} f_i(x(t)) + 2\Psi_i(V_i(x(t))), \\ \beta_i(x) &= \left\|\frac{\partial V_i}{\partial x} g_{1_i}(x(t))\right\|.\end{aligned} \tag{7}$$

Then, system (4) is asymptotically stable under the control of $u_{1_i}(t) = h_i(x(t))$ *and* $u_{2_i}(t) = 0$.

Proof. Based on controller (6), from [17,18], the following stable set is given:

$$\Phi\left(u_{1_i}^{\max}\right) = \left\{x(t) \in R^{n_x}\colon \alpha_i(x) \le u_{1_i}^{\max}\beta_i(x)\right\}. \tag{8}$$

The maximum estimation of $\Phi(u_{1_i}^{\max})$ can be described by

$$\Omega_{\rho_i} = \left\{x(t) \in R^{n_x}\colon V_i(x) \le \rho_i\right\}, \tag{9}$$

where $\rho_i > 0$ is the largest number for $\Omega_{\rho_i} \setminus \{0\} \subseteq \Phi(u_{1_i}^{\max})$ and Ω_{ρ_i} is invariant set of system (4).

Next, for all initial conditions in Ω_ρ, we can prove that the controllers

$$\begin{aligned}u_{1_i}(t) &= h_i(x(t)) = -c_i(x)\left(\frac{\partial V_i}{\partial x} g_{1_i}(x(t))\right), \\ u_{2_i}(t) &= 0,\end{aligned} \tag{10}$$

are bounded. From (6), we have

$$\begin{aligned}\left\|u_{1_i}(x)\right\| &\le \left\|\frac{\alpha_i(x) + \sqrt{\alpha_i^2(x) + \left(u_{1_i}^{\max}\beta_i(x)\right)^4}}{\beta_i(x)\left(1 + \sqrt{1 + \left(u_{1_i}^{\max}\beta_i(x)\right)^2}\right)}\right\| \\ &\le \left\|\frac{u_{1_i}^{\max}\beta_i(x) + \sqrt{\left(u_{1_i}^{\max}\beta_i(x)\right)^2 + \left(u_{1_i}^{\max}\beta_i(x)\right)^4}}{\beta_i(x)\left(1 + \sqrt{1 + \left(u_{1_i}^{\max}\beta_i(x)\right)^2}\right)}\right\| \\ &= \left\|u_{1_i}^{\max}\right\|.\end{aligned} \tag{11}$$

For system (4), Razumikhin's original idea is that if $V_i(x(t)) \ge V_i(x(t-\tau_i))$ and $dV_i/dt \le 0$ are satisfied, then the system is stable. The derivative of function $V_i(x(t))$ is

$$\begin{aligned}\dot{V}_i(x(t)) &\le \frac{\partial V_i}{\partial x}\left(f_i(x(t)) + g_{1_i}(x(t))h_{1_i}(t)\right) + \left\|\frac{\partial V_i}{\partial x} q_i(x(t-\tau_i))\right\| \\ &\le \frac{\partial V_i}{\partial x} f_i(x(t)) - \frac{\alpha_i(x) + \sqrt{\alpha_i^2(x) + \alpha_i^2(x)\left(u_{1_i}^{\max}\beta(x)\right)^2}}{1 + \sqrt{1 + \left(u_{1_i}^{\max}\beta(x)\right)^2}} + \left\|\frac{\partial V_i}{\partial x} q_i(x(t-\tau_i))\right\| \\ &\le -2\Psi_i(V(x(t))) + \left\|\frac{\partial V_i}{\partial x} q_i(x(t-\tau_i))\right\|.\end{aligned} \tag{12}$$

Because system (4) satisfies Assumption 1, we can obtain

$$\left\|\frac{\partial V_i}{\partial x} q_i\left(x\left(t-\tau_i\right)\right)\right\| \leq -\Psi_i\left(V_i\left(x(t)\right)\right)+a_i \Psi_i\left(V_i\left(x\left(t-\tau_i\right)\right)\right). \tag{13}$$

If $V_i(x(t-\tau_i)) \leq V_i(x(t))$, then $\Psi_i(V_i(x(t-\tau_i))) \leq \Psi_i(V_i(x(t)))$. We have

$$\dot{V}_i(x(t)) \leq -\left(1-a_i\right)\Psi_i\left(V_i(x(t))\right). \tag{14}$$

Because $a_i \in (0,1)$, we have $\dot{V}_i(x(t)) \leq 0$. System (4) is asymptotically stable under control of $u_{1_i}(t)=h_i(x(t))$ and $u_{2_i}(t)=0$. □

3.2. Distributed Optimal Control. For each mode i ($i=1,2,\ldots,s$), a distributed optimal control scheme is designed.

Firstly, we design controller 2 based on the bounded controller and the received measurement $x(t_k)$ at time t_k. The input u_{2_i} can be calculated by the following optimal problem:

$$\min_{u_{2_i}\in p(\Delta)} \int_{t_k}^{t_k+N\Delta} \left(\tilde{x}(t)^T Q_i \tilde{x}(t)+u_{1_i}^T(t) R_{1_i} u_{1_i}(t)+u_{2_i}^T(t) R_{2_i} u_{2_i}(t)\right) dt, \tag{15}$$

s.t.

$$\dot{\tilde{x}}(t)=f_i(\tilde{x}(t))+g_{1_i}(\tilde{x}(t)) u_{1_i}(t)+g_{2_i}(\tilde{x}(t)) u_{2_i}(t)+q\left(\tilde{x}\left(t-\tau_i\right)\right), \quad \forall t \in \left[t_k, t_k+N\Delta\right), \tag{16}$$

$$u_{1_i}(t)=h_i\left(\tilde{x}\left(t_k+m\Delta\right), \quad \forall t \in \left[t_k+m\Delta, t_k+(m+1)\Delta\right), \quad m=0,1,\ldots,N-1, \tag{17}$$

$$u_{2_i}(t) \in U_{2_i}, \quad \forall t \in \left[t_k, t_k+N\Delta\right), \tag{18}$$

$$\dot{\hat{x}}(t)=f_i(\hat{x}(t))+g_{1_i}(\hat{x}(t)) h_i\left(x\left(t_k+m\Delta\right)\right)+q_i\left(\hat{x}\left(t-\tau_i\right)\right), \quad \forall t \in \left[t_k, t_k+N\Delta\right), \tag{19}$$

$$\hat{x}\left(t_k\right)=\tilde{x}\left(t_k\right), \tag{20}$$

$$\frac{\partial V_i}{\partial x} g_{2_i}\left(x\left(t_k\right)\right) u_{2_i}(t) \leq 0, \tag{21}$$

where $p(\Delta)$ is a family of piecewise constants function with a sampling time Δ, $N\Delta$ is the predicted range, Q_i, R_{1_i}, and R_{2_i} are the right of positive definite matrix, $\tilde{x}(t)$ is the trajectory of system (4) with $u_{1_i}=h_i(x(t))$, and u_2 is the input trajectory computed by (15)–(21).

The optimal solution is denoted by $u_{2_i}^*(t \mid t_k)$ ($t \in [t_k, t_k+N\Delta)$).

Once $u_{2_i}^*(t \mid t_k)$ is calculated, it is sent to controller 1 and the corresponding control actuator. Then, we design controller 1 based on the measurement $x(t_k)$ and $u_{2_i}^*(t \mid t_k)$. The input u_{1_i} is calculated by the following optimal problem:

$$\min_{u_1\in p(\Delta)} \int_{t_k}^{t_k+N\Delta} \left(\tilde{x}(t)^T Q_i \tilde{x}(t)+u_{1_i}^T(t) R_{1_i} u_{1_i}(t)+u_{2_i}^T(t) R_{2_i} u_{2_i}(t)\right) dt, \tag{22}$$

s.t.

$$\tilde{x}(t)=f_i(\tilde{x}(t))+g_{1_i}(\tilde{x}(t)) u_{1_i}(t)+g_{2_i}(\tilde{x}(t)) u_{2_i}(t)+q_i\left(\tilde{x}\left(t-\tau_i\right)\right), \quad \forall t \in \left[t_k, t_k+N\Delta\right), \tag{23}$$

$$\dot{\tilde{x}}(t)=f_i(\tilde{x}(t))+g_{1_i}(\tilde{x}(t)) h_i\left(\tilde{x}\left(t_k+j\Delta\right)\right)+g_{2_i}(\tilde{x}(t)) u_{2_i}(t) + q_i\left(\tilde{x}\left(t-\tau_i\right)\right), \quad \forall t \in \left[t_k+m\Delta, t_k+(m+1)\Delta\right), \quad m=0,1,\ldots,N-1, \tag{24}$$

$$u_{2_i}(t)=u_{2_i}^*\left(t \mid t_k\right), \quad \forall t \in \left[t_k, t_k+N\Delta\right), \tag{25}$$

$$u_{1_i}(t) \in U_{1_i}, \quad \forall t \in \left[t_k, t_k+N\Delta\right), \tag{26}$$

$$\tilde{x}\left(t_k\right)=\tilde{x}\left(t_k\right), \tag{27}$$

$$\frac{\partial V}{\partial x} g_{1_i}\left(x\left(t_k\right)\right) u_{1_i}(t) \leq \frac{\partial V}{\partial x} g_{1_i}\left(x\left(t_k\right)\right) h_i\left(x\left(t_k\right)\right). \tag{28}$$

The optimal solution is expressed as $u_{1_i}^*(t \mid t_k)$ ($t \in [t_k, t_k+N\Delta)$).

Remark 3. We emphasize two points: (i) (18) and (26) ensure that system input satisfies the constraint. (ii) (21) and (28) guarantee the stability of mode i.

3.3. Stability Analysis. In this section, the stability of distributed optimal control scheme is demonstrated.

Theorem 1. *Considering that switched nonlinear system (1) satisfies Assumption 1, and under distributed optimal control of (15)–(21) and (23)–(28),*

(i) When there is no switch, let Δ_i, $\varepsilon_i>0$, $0<\rho_{s_i}<\rho_i$ satisfy the following inequality:

$$-(1-a_i)\Psi_i\left(\rho_{s_i}\right)+L_x^{f_i} M_{1_i}\Delta+L_x^{g_{1_i}} M_{2_i} u_{1_i}^{\max}\Delta+L_x^{g_{2i}} M_{3_i} u_{2_i}^{\max}\Delta + L_x^k \theta_i+L_x^{q_i}\left(M_{4_i}\Delta+2\left\|\varphi_i\right\|\right) \leq -\varepsilon_i, \tag{29}$$

if initial state starts from the set Ω_{ρ_i}, and $\rho_{m_i}<\rho_i$, where $\rho_{m_i}=\max_{\rho_{s_i}}\left\{V_i(x(t+\Delta)): V_i(x(t)) \leq \Omega_{\rho_{s_i}}\right\}$, then $x(t)$ is ultimately bounded in $\Omega_{\rho_{m_i}}$.

(ii) When there is switch, the following constraints need to be added to distributed optimal control of (15)–(21) and (23)–(28):

$$V_j\left(x\left(t_{j_w^{in}}\right)\right) \le \begin{cases} V_j\left(x\left(t_{j_{w-1}^{in}}\right)\right) - \varepsilon^*, & w > 1,\ V_i\left(x\left(t_{j_{w-1}^{in}}\right)\right) - \rho_{m_j} > 0, \\ \rho_{m_j}, & w > 1,\ V_i\left(x\left(t_{j_{w-1}^{in}}\right)\right) - \rho_{m_j} \le 0, \\ \rho_j, & w = 1, \end{cases} \tag{30}$$

where $V_j(x(t_{j_w^{in}}))$ denotes the value of Lyapunov functions of mode j for wth time and $V_j(x(t_{j_{w-1}^{in}}))$ is the value of Lyapunov functions of mode j for $(w-1)$th time. If (29) is satisfied, then $x(t)$ is ultimately bounded in $\Omega_{\rho_{m_i}}$.

Proof. The proof is divided into two parts.

(i) When there is no switch, if $x(t_k) \in \Omega_{\rho_i}$, from (21) and (28), we have

$$\begin{aligned} \dot{V}_i(x(t_k)) &= \frac{\partial V_i}{\partial x}\left(f_i(x(t_k)) + g_{1_i}(x(t_k))u_{1_i}^*\left(t \mid t_k\right)\right) + g_{2_i}(x(t_k))u_{2_i}^*\left(t \mid t_k\right) + q_i(x(t_k - \tau))) \\ &\le \frac{\partial V_i}{\partial x}\left(f_i(x(t_k)) + g_{1_i}(x(t_k))h_i(x(t))\right) + q_i(x(t_k - \tau_i))). \end{aligned} \tag{31}$$

From (14), it can be deduced that

$$\dot{V}_i(x(t_k)) \le -(1 - a_i)\Psi_i(V_i(x(t_k))). \tag{32}$$

The derivative of $V_i(x(t))$ along the actual state trajectory in $t \in [t_k, t_{k+1})$ is

$$\dot{V}_i(x(t)) = \frac{\partial V_i}{\partial x}\left(f_i(x(t)) + g_{1_i}(x(t))u_{1_i}^*\left(t \mid t_k\right)\right) + g_{2_i}(x(t))u_{2_i}^*\left(t \mid t_k\right) + q_i(x(t - \tau_i)) + k_i(x(t))\omega_i(t)\Big). \tag{33}$$

It follows from (31) and (33) that

$$\begin{aligned} \dot{V}_i(x(t)) \le{}& -(1 - a_i)\Psi_i(V_i(x(t_k))) + \frac{\partial V_i}{\partial x}\left(f_i(x(t)) + g_{1_i}(x(t))u_{1_i}^*\left(t \mid t_k\right) + g_{2_i}(x(t))u_{2_i}^*\left(t \mid t_k\right)\right. \\ &+ q_i(x(t - \tau)) + k_i(x(t))\omega_i(t)) - \frac{\partial V_i}{\partial x}\left(f_i(x(t_k)) + g_{1_i}(x(t_k))u_{1_i}^*\left(t \mid t_k\right)\right. \\ &\left. + g_{2_i}(x(t))u_{2_i}^*\left(t \mid t_k\right) + q_i(x(t_k - \tau_i))\right). \end{aligned} \tag{34}$$

$V_i(x(t))$ is acontinuous differentiable and $f_i(\cdot)$, $g_{1_i}(\cdot)$, $g_{2_i}(\cdot)$, $q_i(\cdot)$, and $k_i(\cdot)$ are local Lipschitz. So, there are positive numbers $L_x^{f_i}$, $L_x^{g_{1_i}}$, $L_x^{g_{2_i}}$, $L_x^{q_i}$, and $L_x^{k_i}$ which make the following equations true for $\xi_1, \xi_2 \in \Omega_{\rho_i}$:

$$\left\|\frac{\partial V_i}{\partial x}f_i(\xi_1(t)) - \frac{\partial V_i}{\partial x}f_i(\xi_2(t))\right\| \le L_x^{f_i}\left\|\xi_1(t) - \xi_2(t)\right\|, \tag{35}$$

$$\left\|\frac{\partial V_i}{\partial x}g_{1_i}(\xi_1(t)) - \frac{\partial V_i}{\partial x}g_{1_i}(\xi_2(t))\right\| \le L_x^{g_{1_i}}\left\|\xi_1(t) - \xi_2(t)\right\|, \tag{36}$$

$$\left\|\frac{\partial V_i}{\partial x}g_{2_i}(\xi_1(t)) - \frac{\partial V_i}{\partial x}g_{2_i}(\xi_2(t))\right\| \le L_x^{g_{2_i}}\left\|\xi_1(t) - \xi_2(t)\right\|, \tag{37}$$

$$\begin{aligned} &\left\|\frac{\partial V_i}{\partial x}q_i(\xi_1(t - \tau_i)) - \frac{\partial V_i}{\partial x}q_i(\xi_2(t - \tau_i))\right\| \\ &\quad \le L_x^{q_i}\left\|\xi_1(t - \tau_i) - \xi_2(t - \tau_i)\right\|, \end{aligned} \tag{38}$$

$$\left\|\frac{\partial V_i}{\partial x}k_i(\xi_1(t - \tau_i))\right\| \le L_x^{k_i}. \tag{39}$$

Using (35)–(39), we can rewrite (34) as follows:

$$\begin{aligned}\dot{V}_i(x(t)) \leq & -(1-a_i)\Psi_i(V_i(x(t_k))) + L_x^{f_i}\|x(t)-x(t_k)\| \\ & + L_x^{g_{1_i}}\|x(t)-x(t_k)\|u_{1_i}^{\max} + L_x^{g_{2_i}}\|x(t)-x(t_k)\|u_{2_i}^{\max} \\ & + L_x^{q_i}\|x(t-\tau_i)-x(t_k-\tau_i)\| + L_x^{k_i}\theta_i.\end{aligned} \tag{40}$$

For all $x(t_k) \in \Omega_{\rho_i}/\Omega_{\rho_{s_i}}$, it follows from (32) that

$$-(1-a_i)\Psi_i(V_i(x(t_k))) \leq -(1-a_i)\Psi_i\left(\Omega_{\rho_{s_i}}\right). \tag{41}$$

According to the smoothness of functions $f_i(\cdot)$, $g_{1_i}(\cdot)$, $g_{2_i}(\cdot)$, and $q_i(\cdot)$, there exists positive constants M_{j_i} ($j = 1, 2, 3, 4$) such that

$$\begin{aligned}\|f_i(x(t))\| &\leq M_{1_i}, \\ \|g_{1_i}(x(t))\| &\leq M_{2_i}, \\ \|g_{2_i}(x(t))\| &\leq M_{3_i}, \\ \|q_i(x(t))\| &\leq M_{4_i}.\end{aligned} \tag{42}$$

Because of the continuity of $x(t)$, for $t \in [t_k, t_{k+1})$, there are

$$\begin{aligned}& L_x^{f_i}\|x(t)-x(t_k)\| + L_x^{g_{1_i}}\|x(t)-x(t_k)\|u_{1_i}^{\max} + L_x^{g_{2_i}}\|x(t)-x(t_k)\|u_{2_i}^{\max} \\ & \quad \leq L_x^{f_i}M_{1_i}\Delta + L_x^{g_{1_i}}u_{1_i}^{\max}M_{2_i}\Delta + L_x^{g_{2_i}}u_{2_i}^{\max}M_{3_i}\Delta, \\ & L_x^{q_i}\|x(t-\tau_i)-x(t_k-\tau_i)\| \leq L_x^{q_i}\max\left\{M_{4_i}\Delta, 2\|\varphi_i\|, M_{4_i}\Delta + 2\|\varphi_i\|\right\} = L_x^{q_i}\left(M_{4_i}\Delta + 2\|\varphi_i\|\right).\end{aligned} \tag{43}$$

By (40) and all initial states $x(t_k) \in \Omega_i/\Omega_{\rho_{s_i}}$, we have

$$\begin{aligned}\dot{V}_i(x(t_k)) \leq & -(1-a_i)\Psi_i\left(\rho_{s_i}\right) + L_x^{f_i}M_{1_i}\Delta + L_x^{g_{1_i}}M_{2_i}u_{1_i}^{\max}\Delta \\ & + L_x^{g_{2_i}}M_{3_i}u_{2_i}^{\max}\Delta + L_x^{k_i}\theta_i + L_x^{q_i}\left(M_{4_i}\Delta + 2\|\varphi_i\|\right).\end{aligned} \tag{44}$$

For $x(t_k) \in \Omega_i/\Omega_{\rho_{s_i}}$, if (29) is true, there exists $\varepsilon > 0$ such that

$$\dot{V}_i(x(t)) \leq -\varepsilon_i. \tag{45}$$

Integrating this bound on $t \in [t_k, t_{k+1})$, we have

$$\begin{gathered}V_i(x(t_{k+1})) \leq V_i(x(t_k)) - \varepsilon_i\Delta, \\ V_i(x(t)) \leq V_i(x(t_k)), \\ \forall t \in [t_k, t_{k+1}).\end{gathered} \tag{46}$$

By recursively applying (46), we can obtain that if the initial state starts from $\Omega_i/\Omega_{\rho_{s_i}}$, the state converges to $\Omega_{\rho_{s_i}}$. If the state converges to $\Omega_{\rho_{s_i}} \subset \Omega_{\rho_{m_i}}$ (or starts at $\Omega_{\rho_{s_i}}$), the state remains in $\Omega_{\rho_{m_i}}$, determined by the definition of ρ_{m_i}. So, $x(t)$ is ultimately bounded in $\Omega_{\rho_{m_i}}$.

(ii) When there is switch, there are two cases.

In case of $t_{i_r^{\text{in}}} \leq t < t_{i_r^{\text{out}}}$ and $t_{j_w^{\text{in}}} = t_{i_r^{\text{out}}} < \infty$, we consider that the system has an infinite number of switches. At this point, ith mode is activated. If $V_i(x) > \rho_{m_i}$, then the constraints of (21) and (28) guarantee that $V_i(x(t_{i_r^{\text{out}}})) < V_i(x(t_{i_r^{\text{in}}}))$, and the constraints of (30) ensure that $V_i(x(t_{i_r^{\text{in}}})) < V_i(x(t_{i_{r-1}^{\text{in}}})) < \cdots < \rho_{m_i}$. So, $V_i(x)$ is always decrease. Even if ith mode is not activated, there is lth mode ($l \in S$) which is activated and guarantees that $V_l(x)$ is decrease and $V_l(x) < \rho_{m_l}$. So, $x(t)$ is ultimately bounded stable in $\Omega_{\rho_{m_i}}$.

In case of $t_{i_r^{\text{in}}} \leq t < t_{i_r^{\text{out}}}$ and $t_{j_w^{\text{in}}} = t_{i_r^{\text{out}}} = \infty$, we consider that the system has an finite number of switches. Similarly, $V_i(x(t_{i_r^{\text{in}}})) < V_i(x(t_{i_{r-1}^{\text{in}}})) < \cdots < \rho_{m_i}$. When switching to the ith mode, there is $x(t_{i_r^{\text{in}}}) \in \rho_{m_i}$. From this point on, distributed optimal control strategy is not subject to any switching constraints. So, $x(t)$ is ultimately bounded stable.

In conclusion, Theorem 1 is proved.

The manipulation inputs of the distributed optimal control scheme are defined:

$$\begin{gathered}u_{1_i}(t) = u_{1_i}^*\left(t \mid t_k\right), \\ u_{2_i}(t) = u_{2_i}^*\left(t \mid t_k\right), \\ \forall t \in [t_k, t_{k+1}).\end{gathered} \tag{47}$$

The executive strategy of optimal control in this paper is given as follows:

Step 1: controller 2 receives the measurement $x(t_k)$ at time t_k, and then controller 2 calculates the optimal trajectory of u_{2_i}.

Step 2: the optimal trajectory of u_{2_i} is transmitted to the actuator and controller 1.

Step 3: once the optimal trajectory of u_{2_i} is received by controller 1, the optimal trajectory of u_{1_i} is calculated.

Step 4: controller 1 sends the optimal trajectory of u_{1_i} to the actuator.

Step 5: go to the next moment. □

4. Simulation Example

We consider the following switched system to verify the effectiveness of the proposed distributed optimal control scheme:

$$\begin{aligned}\dot{x}(t) = {} & f_i(x(t)) + g_{1_i}(x(t))u_{1_i}(t) + g_{2_i}(x(t))u_{2_i}(t) \\ & + q_i(x(t-\tau_i)) + k_i(x(t))\omega_i(t),\end{aligned} \tag{48}$$

where $f_1(x) = (3x_2^3(t), 0)^T$ and $f_2(x) = (x_2^3, 0)^T$; $g_{1_1}(x) = (0, 1)^T$ and $g_{1_2}(x) = (0, 1)^T$; $g_{2_1}(x) = (1, 0)^T$ and $g_{2_2}(x) = (1, 0)^T$; $q_1((x(t-1))) = (2x_2^2(t-1), 2x_1^2(t-1))^T$ and $q_2((x(t-1))) = (x_2^2(t-1), x_1^2(t-1))^T$; $k_1(x(t)) = (0.2, 0)^T$ and $k_2(x(t)) = (0.23, 0)^T$; and $\omega_1(t) = (\sin(x_2(t)), 0)^T$ and $\omega_2(t) = (\sin(x_2(t)), 0)^T$. Our goal is to adjust the initial state x_0 to the stable state $x_s^T = [0, 0]$ for the system. The control constraints are $u_{1_i} \in \{u_{1_i} \in R^1 \mid \|u_{1_i}\| \le 20\}$ and $u_{2_i} \in \{u_{2_i} \in R^1 \mid \|u_{2_i}\| \le 15\}$, $x(t) = \varphi_i(t) = [1.2, -0.9]$, $t \in [-1, 0]$.

We choose the Lyapunov function $V_1(x_1, x_2) = 0.25x_1^2 + x_2^2$, $\Psi_1(V) = V_1 + 22V_1^2$, and $a_1 = 0.85$, Lyapunov function $V_2(x_1, x_2) = 0.25x_1^2 + x_2^2$, $\Psi_2(V) = V_2 + 20V_2^2$, and $a_2 = 0.79$ to satisfy Assumption 1. We use Proposition 1 to construct the bounded controller $u_{1_i} = h_i(x)$ and $u_{2_i} = 0$. We apply the distributed optimal control and the centralized optimal control, respectively. Both control methods use the same parameters. The prediction horizon is $N = 6$, the sampling interval is $\Delta = 0.01$, and the weighted matrices are chosen as $Q_i = \mathrm{diag}[10, 10]$, $R_{1_i} = 0.50$, and $R_{2_i} = 0.60 (i = 1, 2)$. Select the initial state as $x_0^T = [-2.6, 1.9]$.

Set the system switching mode 2 from mode 1 at time $t = 3.50$ s (the system state enters the stable region of mode 2). The simulation results are shown in Figures 2–5:

The solid lines and the dotted lines represent the state trajectories under distributed optimal control and centralized optimal control, respectively. Simulation results indicate that optimal control via distributed optimal control method can ensure that the time-delay system state is ultimately bounded and stable. It also shows that both control designs give similar closed-loop performance and drive the state and the state close to the stable state in about 7.5 s and 8.5 s, respectively. To further illustrate that the two control designs have similar closed-loop performance, we illustrate this point from the perspective of the performance index. A series of simulation comparisons are made between the distributed optimal control and the centralized optimal control with the same parameters by using ten sets of different initial values. We calculated the total performance cost along the closed-loop system trajectories in 10s at different initial values. The comparison results are shown in Table 1.

It can be seen from Table 1 that the cost of distributed optimal control method is lower than that of the centralized optimal control method in 6 out of 10 simulations. This shows that the closed-loop performance of the distributed optimal control method is comparable to that of the centralized optimal control method. Among the 10 simulation results, we also compared the computation

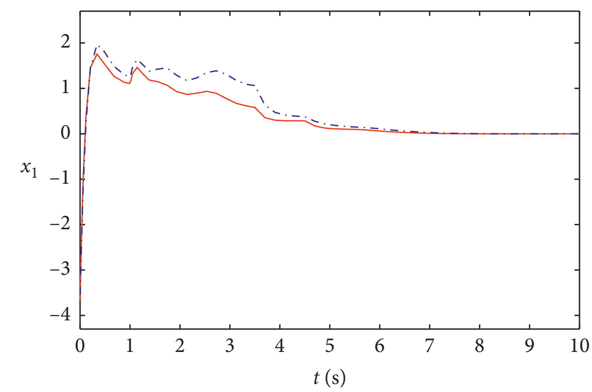

Figure 2: The trajectories of x_1.

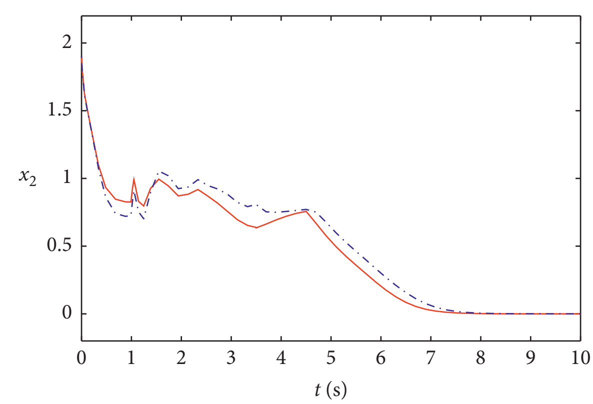

Figure 3: The trajectories of x_2.

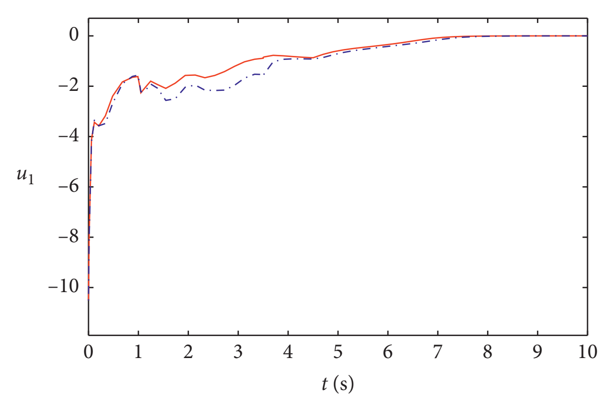

Figure 4: The trajectories of u_1.

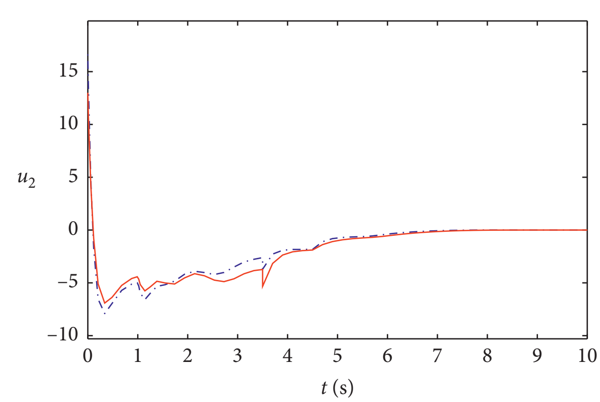

Figure 5: The trajectories of u_2.

Table 1: Total performance cost along the closed-loop system trajectories.

Number	1	2	3	4	5	6	7	8	9	10
Initial value	[−1.1, 0.9]	[1.2, −0.9]	[−0.7, 1.0]	[0.8, 1.05]	[−0.97, −1.3]	[0.86, −1]	[−0.5, 1.4]	[1.5, −1.7]	[−1.4, 0.97]	[1.6, −1.8]
Distributed optimal control	1778	1693	1860	1792	2120	1673	1732	3170	1973	2970
Centralized optimal control	1687	1801	1778	1933	1946	1854	1973	3457	1669	2532

time of the distributed optimal control method and centralized optimal control method. The average time to calculate the centralized optimal control is 2.64 s. The average time to calculate controller 1 and controller 2 is 0.9 s and 1.05 s, respectively. Obviously, the computation time of the distributed optimal control method is less than that of centralized optimal control. This is because centralized optimization control must optimize two inputs simultaneously in an optimization problem, while distributed optimization control is to solve two optimization problems which only have one decision variable.

5. Conclusions

This paper solves the problem of distributed optimal control for switched nonlinear systems with the state time delay. For each mode, a bounded controller is designed to stabilize the system. Then, a distributed optimal control scheme which can satisfy the input constraint is designed based on the bounded stabilization controller. Finally, it is proved that the system state is ultimately bounded stable.

Data Availability

No data were used to support this study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

This research was supported by the Natural Science Foundation of China (Grant nos. 61374004, 61773237, and 61473170) and Key Research and Development Programs of Shandong Province (Grant no. 2017GSF18116).

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