

## Research Article

# Tristage Bargaining Dynamic Game-Based Preventive Maintenance for Electric Multiple Unit

Lü Xiong,<sup>1,2</sup> Hong Wang ,<sup>1</sup> and Zuhua Jiang<sup>3</sup>

<sup>1</sup>School of Mechatronic Engineering, Lanzhou Jiaotong University, Lanzhou 730070, China

<sup>2</sup>School of Rail Traffic, Guangdong Communication Polytechnic, Guangzhou 510650, China

<sup>3</sup>School of Mechanical Engineering, Shanghai Jiao Tong University, Shanghai 200240, China

Correspondence should be addressed to Hong Wang; [wh@mail.lzjtu.cn](mailto:wh@mail.lzjtu.cn)

Received 29 December 2019; Accepted 11 June 2020; Published 4 July 2020

Academic Editor: Neale R. Smith

Copyright © 2020 Lü Xiong et al. This is an open access article distributed under the Creative Commons Attribution License, which permits unrestricted use, distribution, and reproduction in any medium, provided the original work is properly cited.

The operation and maintenance sectors of electric multiple unit (EMU) are considered as game participants to optimize the preventive maintenance (PM) schedule of EMU components. The total cost of a component over a life cycle includes failure risk and maintenance cost. The failure risk of EMU components is assessed quantitatively by using an analytic hierarchy process and scoring and weighting the factors that affect the failure risk of such components. The operation sector expects failure risk to be minimized, whereas the maintenance sector expects maintenance costs to be low, and their interests interfere with each other to some extent. This study establishes a tristage bargaining dynamic game model of the operation sector priority bid and maintenance sector priority bid considering the PM reliability threshold  $R$  as the bargaining object. A numerical example demonstrates that the result is more beneficial to the sector that bids first, and the operation priority bid allows the component to maintain a higher reliability level over a life cycle, especially in the latter half of the component life cycle.

## 1. Introduction

Cost and interest allocation problems arise in many real-life situations [1, 2], where individuals, who may have different purposes, decide to work together. In such situations, the problem occurs when the times come to divide the joint costs and interests resulting from cooperation among participants. Eltoukhy et al. [3] developed a coordinated configuration of a scenario-based stochastic operational aircraft maintenance routing problem and maintenance staffing problem as a Stackelberg game to reduce costs for airlines and maintenance companies. The operation of production and utility systems is commonly optimized separately without feedback between them. In general, this separated planning increases total costs. Conceptually, a joint optimization of production and utility systems is desirable, but it is often not possible in practice. Leenders et al. [4] overcame such practical limitations by proposing a repeated Stackelberg game. Xiao et al. [5] considered the effects of differences between plan products and actual products. The two

heterogeneous players always adopt suitable strategies that can improve their benefits most, and a nonlinear duopoly Stackelberg competition model on output between heterogeneous players is developed.

As for PM issues [6–11], many models have been developed and implemented to improve system reliability, prevent system failures, and reduce maintenance costs [12–16]. In fact, the PM issues of a system are ultimately economic issues. How to reduce the PM cost or increase the availability of system under the premise of ensuring the necessary system reliability is a contended topic among scholars. In recent years, scholars and maintenance engineers have introduced game theory into the field of PM [17, 18] and have achieved numerous significant advancements. Hu et al. [19] adopted a game theory model to handle the interactive effect between the production plan and PM schedule. An extended imperfect PM model is proposed for a system running with a time-varying production rate. Tayeb et al. [20] proposed a novel game theory approach to the problem of integrating a periodic and flexible PM schedule

and production scheduling for permutation flow shops. Pourahmadi et al. [21] considered that electric power system operators would have to be able to manage the system operation expenses more effectively, as waves of maintenance costs and equipment investments would be anticipated within a few years. A cost-effective reliability-centered maintenance approach based on game theory is proposed to assess the component criticality for overall system reliability and further maintenance focus and for reducing the maintenance cost of the electric power system.

With the development of high-speed railway technology, high-speed EMU is becoming increasingly used around the world. The PM strategy of high-speed EMU that followed has also received more attention. With regard to the operation and maintenance sectors of EMU, owing to their respective interests, they present a potential competitive relationship. Both the operation and maintenance sectors want to achieve higher benefits and maximize their own interests. The operation sector expects the EMU to transport passengers to their destination safely and comfortably at the lowest risk, whereas the maintenance sector expects to reduce maintenance costs as much as possible while maintaining the reliability of the EMU at a reasonable level. However, it is not difficult to understand that the higher the PM reliability threshold  $R$  is, the more PM measures will be adopted, the higher the PM cost will be, and the lower the failure times and risks will be. On the contrary, the lower the PM reliability threshold  $R$  is, the fewer PM measures will be carried out, the lower the maintenance cost will be, and the more failure times and risks will be. However, both maintenance sector and operation sector want the EMU to operate safely and smoothly, and they have some common interests to some extent. Hence, they present a cooperative relationship as well. To balance the competitive and cooperative nature of the relationship between the operation and maintenance sectors, a tristage bargaining dynamic game model is established to study the game behavior of these sectors.

In this study, a key EMU component is considered as the study object. Based on the tristage bargaining dynamic game model under symmetric information, considering the EMU operation and maintenance sectors as game participants, we develop a tristage bargaining dynamic game model to obtain the PM schedule of the EMU component. The following assumptions are made for the model:

- (1) The initial component reliability is "1."
- (2) The failure rate function of the component follows a Weibull distribution.
- (3) The imperfect PM measure of the component in a life cycle is only junior maintenance and senior maintenance.
- (4) During each PM interval, minimal repair is carried out when failure occurs. Minimal repair only restores the machine to working conditions, as bad as it was before, which has been applied in numerous studies.
- (5) Different imperfect PM measures consistently restore the performance of nonfailure components.

## 2. Model Establishment

**2.1. Failure Rate Evolution Rule.** Malik [22] introduced the concept of age reduction factor to describe the system function. Age after the  $i$ th PM is reduced to  $a_i\lambda(t)$  when it is  $\lambda(t)$  in period  $i$  of PM, the failure rate function after the  $i$ th PM becomes  $\lambda_i(t + a_iT_i)$  for  $t \in (0, T_i + 1)$ , where  $0 < a_i < 1$  is the age reduction factor due to the imperfect PM action. This implies that each imperfect PM changes the initial failure rate value immediately after the PM to  $\lambda_i(a_iT_i)$ , but not to zero. Nakagawa [23] proposed another model based on the failure rate increase factor. According to Nakagawa, the failure rate function becomes  $b_i\lambda_i(t)$  for  $t \in (0, T_i + 1)$  after the  $i$ th PM, where  $b_i > 1$  is the failure rate increase factor. This indicates that each PM makes the failure rate increase at a higher rate. Combining the advantages of the age reduction factor and failure rate increase factor, Lin et al. [15] established a hybrid failure rate evolution model. The age after the  $i$ th PM is reduced to  $a_i\lambda(t)$  when it is  $\lambda(t)$  in period  $i$  of PM, and the failure rate becomes  $b_i\lambda_i(t)$  after the  $i$ th PM, where  $0 < a_i < 1$  and  $b_i > 1$ . The hybrid failure rate evolution function can be defined as

$$\lambda_{i+1}(t) = b_i\lambda_i(t + a_iT_i), \quad 0 < t < T_{i+1}. \quad (1)$$

Scheduled PM is performed whenever the system reaches the reliability threshold  $R$ . Based on this policy, a reliability equation can be constructed as

$$\begin{aligned} R &= \exp\left[-\int_0^{t_1} \lambda_1(t)dt\right] = \exp\left[-\int_0^{t_2} \lambda_2(t)dt\right] \\ &= \dots = \exp\left[-\int_0^{t_i} \lambda_i(t)dt\right]. \end{aligned} \quad (2)$$

According to the bilevel imperfect PM strategy, there are two maintenance measures to choose at the scheduled PM time.

- (1) Junior maintenance: the failure rate of the system after the  $(i-1)$ th PM (junior maintenance) becomes  $\lambda_{j,i-1}^+$  when it was  $\lambda_{s,i-2}^+$  after  $(i-2)$ th PM (senior maintenance). The failure rate of the system after the  $i$ th PM (junior maintenance) becomes  $\lambda_{j,i}^+$  when it was  $\lambda_{j,i-1}^+$  after  $(i-1)$ th PM, and  $\lambda_{s,i-2}^+ < \lambda_{j,i-1}^+ < \lambda_{j,i}^+$ . This indicates that regardless of the last scheduled imperfect PM, and whether junior maintenance or senior maintenance is adopted, the failure rate of the system after junior maintenance is higher than that of the last scheduled imperfect PM, as depicted in Figure 1(a).
- (2) Senior maintenance: the failure rate of the system after the  $(i-1)$ th PM (junior maintenance) becomes  $\lambda_{j,i-1}^+$  when it was  $\lambda_{s,i-2}^+$  after  $(i-2)$ th PM (senior maintenance); the failure rate of the system after the  $i$ th PM (senior maintenance) becomes  $\lambda_{s,i}^+$  when it was  $\lambda_{j,i-1}^+$  after the  $(i-1)$ th PM, and  $\lambda_{s,i-2}^+ < \lambda_{s,i}^+ < \lambda_{j,i-1}^+$ . This implies that the failure rate after senior maintenance is  $(\lambda_{s,i}^+)$  only higher than that of the last senior maintenance and  $(\lambda_{s,i-2}^+)$  may be lower than that of the last junior maintenance

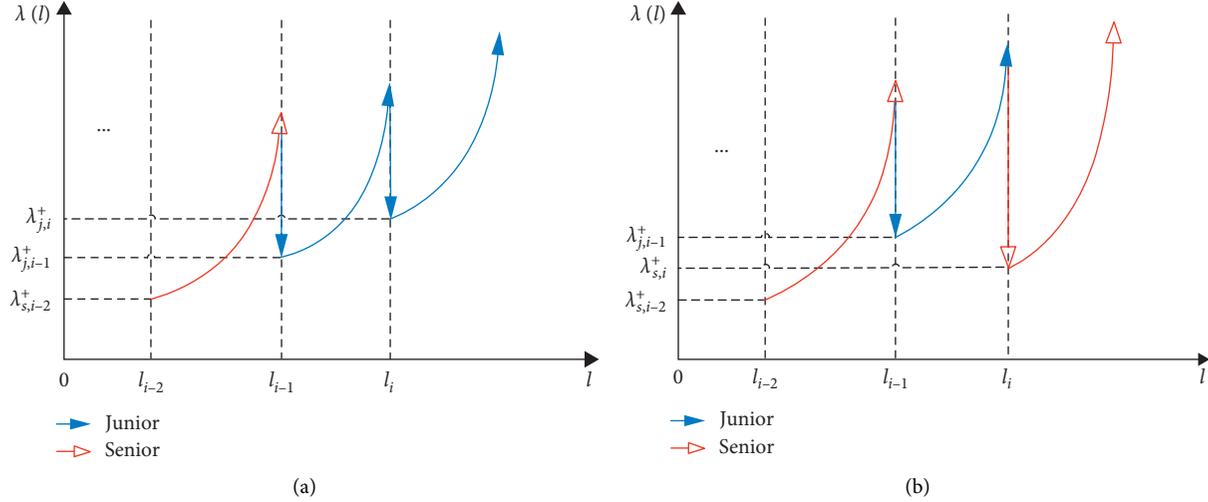


FIGURE 1: Bilevel imperfect PM failure rate evolution rule: (a) adopt junior maintenance at  $l_i$  and (b) adopt senior maintenance at  $l_i$ .

( $\lambda_{j,i-1}^+$ ). Hence, after senior maintenance, the failure rate of the system is only higher than that of the last senior maintenance, and it is not related to the last junior maintenance, as depicted in Figure 1(b).

**2.2. Maintenance Method Selection.** Whenever the reliability of the system reaches the PM threshold  $R$ , a scheduled imperfect PM action will be performed. According to the bilevel imperfect PM strategy, there are two imperfect PM measures to adopt when the scheduled PM time arrives. However, which specific imperfect PM measure to adopt, junior maintenance or senior maintenance, still needs to be determined. Here, we adopt a cost-effectiveness analysis to address this problem. The maintenance measure selection factor  $\omega_i$  is introduced:

$$\omega_i = \begin{cases} 0, & \text{junior maintenance,} \\ 1, & \text{senior maintenance.} \end{cases} \quad (3)$$

The failure rate increase rate factor  $a_i$  and age reduction rate factor  $b_i$  will be different as different imperfect PM measures are applied. The functions of  $a_i$  and  $b_i$  under different PM measures can be defined as

$$a_i = (1 - \omega_i)a_i^j + \omega_i a_i^s. \quad (4)$$

The failure rate of system before the  $i$ th PM is the same as the end of the  $(i-1)$ th PM interval, and the failure rate of system after the  $i$ th PM is equal to that at the beginning of the  $(i+1)$ th PM interval. The functions of  $\lambda_i^-$  and  $\lambda_i^+$  can be defined as

$$\begin{cases} \lambda_i^- = \lambda_i(T_i) = b_{i-1}\lambda_{i-1}(T_i + a_{i-1}T_{i-1}), \\ \lambda_i^+ = \lambda_{i+1}(0) = b_i\lambda_i(a_iT_i). \end{cases} \quad (5)$$

The improvement of the failure rate after PM can be constructed as  $\Delta\lambda_i = \lambda_i^- - \lambda_i^+$ . Herein, we consider the cost-effectiveness ratio  $\phi$  as an economic analysis indicator.  $\phi_i^j(t)$  and  $\phi_i^s(t)$  represent the junior maintenance and senior maintenance cost-effectiveness ratio, respectively, and are expressed as

$$\phi_i^j(t) = \frac{c_j}{\Delta\lambda_i} = \frac{c_j}{b_{i-1}\lambda_{i-1}(T_i + a_{i-1}T_{i-1}) - \delta_i^j\lambda_i(\theta_i^jT_i)}, \quad (6)$$

$$\phi_i^s(t) = \frac{c_s}{\Delta\lambda_i} = \frac{c_s}{b_{i-1}\lambda_{i-1}(T_i + a_{i-1}T_{i-1}) - \delta_i^s\lambda_i(\theta_i^sT_i)}. \quad (7)$$

As shown by (6) and (7), junior maintenance is a more cost-effective maintenance measure if  $\phi_i^j(t) \leq \phi_i^s(t)$ , and senior maintenance is a more cost-effective maintenance measure if  $\phi_i^j(t) > \phi_i^s(t)$ . In addition,  $\omega_i$  can be further expressed as

$$\omega_i = \begin{cases} 0, & \phi_i^s(t) \geq \phi_i^j(t), \\ 1, & \phi_i^s(t) < \phi_i^j(t). \end{cases} \quad (8)$$

**2.3. Maintenance Cost Modeling.** The maintenance cost of a component in a life cycle includes PM, failure maintenance, and replacement costs.

- (1) PM costs include junior maintenance, senior maintenance, and site occupation costs:

$$\begin{aligned} C_p &= \sum_{i=1}^{N-1} (c_i^p(t_i) + c_i^o(t_i)), \\ c_i^p(t_i) &= (1 - \omega_i)c_j + \omega_i c_s, \\ c_i^o(t_i) &= \tau[(1 - \omega_i)t_j + \omega_i t_s]. \end{aligned} \quad (9)$$

- (2) Failure maintenance costs include minimal repair and site occupation costs:

$$C_f = (c_m + \tau t_m) \sum_{i=1}^N \int_{t_{i-1}}^{t_i} \lambda_i(t) dt. \quad (10)$$

(3) Replacement costs for the system include labor, waste, and site occupation costs:

$$C_r = c_l + c_w + \tau t_r. \quad (11)$$

In summary, the maintenance cost of a system in a life cycle can be expressed as

$$C_x = C_p + C_f + C_r. \quad (12)$$

**2.4. Failure Risk Modeling.** Risk is defined as the likelihood of a particular event occurring and the severity of outcome [24]. For high-speed EMUs, the damage of unpredicted failure may not only be downtime losses but also cause a delay of EMU, affecting the normal operation of other trains in the operating range. Moreover, it affects safe operation of EMU, threatening the safety of passengers and property. For factors that influence the failure risk of EMU component, we mainly consider the following three aspects: the effect of failure on EMU safety during operation, the effect of failure on EMU-delaying, and the complexity of failure maintenance. We could then evaluate the failure risk of the EMU component by weighting and scoring these three influencing factors.

In order to establish the failure risk assessment function, the failure risk factor is introduced and expressed as

$$\psi = \sum_{i=1}^n s_i \tau_i, \quad (13)$$

where  $s_i$  is the score of the  $i$ th influence factor,  $\tau_i$  represents the weight of the  $i$ th influence factor, and  $n$  indicates the number of influence factors.

Referring to the evaluation method of component importance in [25–28], the scoring and weighting of each influence factor is performed. Considering that the failure risk evaluation of the EMU component is neither too cumbersome nor in line with the actual situation, the scoring criteria of each influence factor are divided into 4 to 5 levels. The scoring criteria for each influence factor are listed in Tables 1–3.

The weight of each influence factor is obtained using an analytic hierarchy process. First, a judgment matrix of relative risk between the influence factors is constructed as follows:

$$M = \begin{bmatrix} u_{11} & u_{12} & \dots & u_{1n} \\ u_{21} & u_{22} & \dots & u_{2n} \\ \dots & \dots & \dots & \dots \\ u_{n1} & u_{n2} & \dots & u_{nn} \end{bmatrix}, \quad (14)$$

where  $u_{ab}$  is the relative risk of  $a$  to  $b$ . The relative risk values are shown in Table 4.

The value of relative risk is shown in Table 4.

Second, the largest eigenvalue of the judgment matrix  $\zeta_{\max}$  is calculated and substituted into the homogeneous linear equations:

TABLE 1: Scoring criteria for failure effect on operation safety.

Serial number	Severity	Score
1	General failure	1–2
2	Dangerous failure	3–4
3	Big failure	5–6
4	Major failure	7–8
5	Particularly significant failure	9–10

TABLE 2: Scoring criteria for failure effect on EMU-delaying.

Serial number	Delay time (h)	Score
1	<2	1–2
2	2–10	3–6
3	10–30	7–9
4	>30	10

TABLE 3: Scoring criteria for failure maintenance complexity.

Serial number	Degree of difficulty	Score
1	General	1–2
2	Medium	3–4
3	High	5–7
4	Very high	8–9
5	Unable to maintenance	10

TABLE 4: Relative risk scales and implications.

Relative risk scales	Implications
1	The same
3	Slightly larger
5	Quite large
7	Very large
9	Great large
2, 4, 6, 8	Between the above

$$\begin{cases} (u_{11} - \zeta)\alpha_1 + u_{12}\alpha_2 + \dots + u_{1n}\alpha_n = 0, \\ u_{21}\alpha_1 + (u_{22} - \zeta)\alpha_2 + \dots + u_{2n}\alpha_n = 0, \\ \dots \\ u_{n1}\alpha_1 + u_{n2}\alpha_2 + \dots + (u_{nn} - \zeta)\alpha_n = 0. \end{cases} \quad (15)$$

The feature vector corresponding to the maximum eigenvalue  $\zeta_{\max}$  is the weight value of each determinant.

Finally, it is necessary to test the consistency of the judgment matrix. The test formula is as follows:

$$U_P = \frac{U_C}{U_M}, \quad (16)$$

where  $U_P$  is the random consistency ratio of the judgment matrix;  $U_C$  is the general consistency indicator of the judgment matrix, and  $U_C = (\lambda_{\max} - n)/(n - 1)$ ; and  $U_M$  is the average random consistency indicator of the judgment matrix. Table 5 shows the value of order 1–7 in the judgment matrix  $U_M$ .

If  $U_P < 0.1$ , it can be judged that  $U_M$  has satisfactory consistency and its weight distribution is reasonable. Otherwise, the judgment matrix needs to be adjusted until it achieves a satisfactory consistency.

TABLE 5: Values of order 1~7 in the judgment matrix.

$n$	1	2	3	4	5	6	7
$U_M$	0.00	0.00	0.58	0.90	1.12	1.24	1.32

The failure risk of component during the  $i$ th PM interval can be expressed as

$$r = \psi c_d \int_{l_{i-1}}^{l_i} \lambda_i(l) dl. \quad (17)$$

**2.5. Total Cost Modeling.** The total cost of a component in a life cycle can be divided into two parts: maintenance cost and failure risk. The total cost can be expressed as

$$C = C_x + r. \quad (18)$$

The weighting factors  $w_1$  and  $w_2$  ( $w_1 + w_2 = 1$ ) are introduced to determine the propensity of PM strategies:

$$V = w_1 C_x + w_2 r_x, \quad (19)$$

where  $w_1$  and  $w_2$  are the maintenance cost and the failure risk weighting factors, respectively.

### 3. Tristage Bargaining Dynamic Game Model

Bargaining is a game process in which participants with common interests try to reach an agreement when confronted with conflicts. It is a typical negotiation activity. In the negotiation process, when sector A puts forward an offer to sector B, together with the main contract terms, sector B analyzes all its contents, judges its intention through sector A's offer, and gives a reoffer and other responses to make the transaction develop in a direction that is beneficial to itself and satisfies certain requirements of the other sector, so as to facilitate the exchange of interests in the negotiation. Because of negotiation costs and time loss, the cost function of both sectors should add an additional loss  $\delta$  ( $0 < \delta < 1$ ) to the original basis. Assuming that the game process only involves three rounds, it is referred to as a tristage bargaining dynamic game model.

First, sector A proposes a PM plan, and sector B chooses to accept or reject it. If sector B rejects, it proposes another PM plan for sector A to choose to accept or reject, and so it goes on like this. Once a condition is rejected, it is no longer binding and it is no longer relevant to the latter game. In this cycle, the game ends when either sector accepts the other sector's PM plan. Each time a sector proposes a PM plan and the other sector chooses whether to accept it can be considered as a round. According to which sector A or sector B has the priority to bid, the tristage bargaining dynamic game can be divided into Models 1 and 2.

**3.1. Model 1: Operation Sector Priority Bids.** The game process is as follows:

- (1) The operation sector takes  $\min(r)$  as the decision-making goal and obtains PM reliability threshold  $R_1$  by solving the function of  $\min(r)$ .  $R_1$  is the solution that minimizes  $r$ .
- (2) The maintenance sector chooses to accept or reject the conditions put forward by the operation sector. If the maintenance sector accepts, the game is over. The maintenance cost of the EMU component is then  $C_x(R_1)$ , the risk cost is  $r(R_1)$ , and the total cost is  $C(R_1)$ . If the maintenance sector rejects, another PM reliability threshold  $R_2$  will be provided by the maintenance sector.
- (3) The operation sector chooses to accept or reject the conditions put forward by the maintenance sector. If the operation sector accepts, the game is over. The maintenance cost of the EMU component is then  $(1 + \delta)C_x(R_2)$ , the risk cost is  $(1 + \delta)r(R_2)$ , and the total cost is  $C(R_2)$ . If the operation sector rejects, another PM reliability threshold  $\bar{R}$  will be provided by the operation sector. The maintenance sector must then accept the conditions put forward by the operation sector, and the game is over. The maintenance cost of the EMU component is  $(1 + \delta)^2 C_x(\bar{R})$ , the risk cost is  $(1 + \delta)^2 r(\bar{R})$ , and the total cost is  $C(\bar{R})$ . The game process is shown in Figure 2.

**3.2. Model 2: Maintenance Sector Priority Bids.** The game between the two sectors is as follows:

- (1) The maintenance sector takes  $\min(C_x)$  as the decision-making goal and obtains the PM reliability threshold  $R'_1$  by solving the function of  $\min(C_x)$ .  $R'_1$  is the solution that minimizes  $C_x$ .
- (2) The operation sector chooses to accept or reject the conditions put forward by the maintenance sector. If the operation sector accepts, the game is over. The maintenance cost of the EMU component is then  $C_x(R'_1)$ , the failure risk is  $r(R'_1)$ , and the total cost is  $C(R'_1)$ . If the operation sector rejects, another PM reliability threshold  $R'_2$  will be provided by the operation sector.
- (3) The maintenance sector chooses to accept or reject the conditions proposed by the operation sector. If the maintenance sector accepts, the game is over. The maintenance cost of the EMU component is then  $(1 + \delta)C_x(R'_2)$ , the failure risk is  $(1 + \delta)r(R'_2)$ , and the total cost is  $C(R'_2)$ . If the maintenance sector rejects, another PM reliability threshold  $\bar{R}$  will be provided by the maintenance sector. The operation sector must then accept the conditions put forward

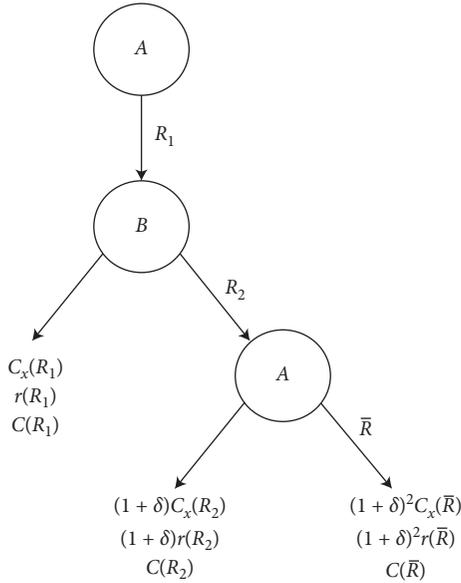


FIGURE 2: Game process of model 1.

by the maintenance sector, and the game is over. The maintenance cost of the EMU component is then  $(1 + \delta)^2 C_x(\bar{R})$ , the failure risk is  $(1 + \delta)^2 r(\bar{R})$ , and the total cost is  $C(\bar{R})$ . The game process is shown in Figure 3.

### 3.3. Model Solving Method

#### 3.3.1. Solution Method of Model 1

(1) *First Round.* Operation sector obtains the PM reliability threshold  $R_1$  by optimizing the objective function  $\min(r)$ . Maintenance sector chooses to accept or reject the conditions proposed by the operation sector. If maintenance sector accepts, the game is over. Otherwise, the maintenance sector will put forward another PM reliability threshold,  $R_2$ .

(2) *Second Round.* The condition for game to enter the second round is that the maintenance cost of the second round should be lower than that of the first round, and the total cost of the second round should be lower than that of the first round. Thus, the game enters the second round, where  $R_2$  should meet the following condition:

$$(1 + \delta)C_x(R_2) < C_x(R_1) \text{ and } (1 + \delta)C(R_2) < C(R_1). \quad (20)$$

(3) *Third Round.* The condition for game to enter the third round is that the failure risk of the third round must be less than that of the second round, and the total cost of the third round should be lower than that of the second round. Therefore, game enters the third round, where  $R^*$  should meet the following condition:

$$(1 + \delta)^2 r(R^*) < (1 + \delta)r(R_2) \text{ and } (1 + \delta)^2 C(R^*) < (1 + \delta)C(R_2). \quad (21)$$

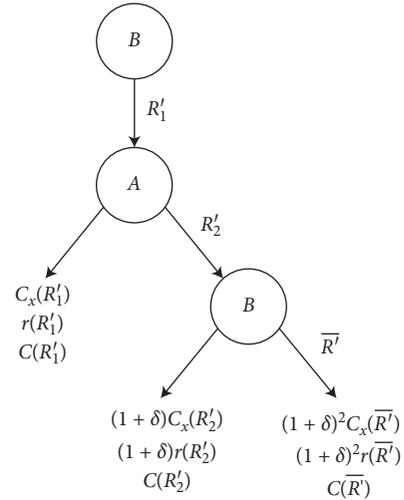


FIGURE 3: Game process of model 2.

#### 3.3.2. Solution Method of Model 2

(1) *First Round.* The maintenance sector obtains the PM reliability threshold  $R'_1$  by optimizing the objective function  $\min(C_x)$ . The operation sector chooses to accept or reject the conditions proposed by the maintenance sector through calculation. If the operation sector accepts, the game is over. Otherwise, the operation sector will propose another PM reliability threshold,  $R'_2$ .

(2) *Second Round.* The condition for the game to enter the second round is that the failure risk of the second round must be less than that of the first round, and the total cost of the second round is lower than that of the first round. Thus, the game enters the second round, where  $R'_2$  should meet the following condition:

$$(1 + \delta)r(R'_2) < r(R'_1) \text{ and } (1 + \delta)C(R'_2) < C(R'_1). \quad (22)$$

(3) *Third Round.* The condition for the game to enter the third round is that the maintenance cost of the third round must be less than that of the second round, and the total cost of the third round is lower than that of the second round. Therefore, the game enters the third round, where  $R^*$  should meet the following condition:

$$(1 + \delta)^2 C_x(R^{*'}) < (1 + \delta)C_x(R'_2) \text{ and } (1 + \delta)^2 C(R^{*'}) < (1 + \delta)C(R'_2). \quad (23)$$

## 4. Numerical Example

The failure rate distribution function of component is described by Weibull distribution. The Weibull distribution is expressed as

$$\lambda_1(l) = \frac{m}{\eta} \left( \frac{l}{\eta} \right)^{m-1}, \quad (24)$$

where  $m$  is the shape parameter and  $\eta$  is the characteristic life parameter. In order to obtain the values of  $m$  and  $\eta$ , the actual maintenance records of a key component of the CRH3C EMU are investigated and collated, and a total of 216 life data points are obtained as follows (unit:  $10^3$  km):

$$D = \begin{bmatrix} 36.56 & 37.26 & 37.26 & \dots & 41.33 & 42.83 & 43.72 \\ 43.72 & 43.76 & 44.02 & \dots & 47.80 & 48.15 & 49.31 \\ 49.32 & 52.31 & 52.51 & \dots & 54.79 & 56.03 & 56.29 \\ \dots & \dots & \dots & \dots & \dots & \dots & \dots \\ 118.34 & 118.96 & 119.36 & \dots & 120.91 & 123.17 & 125.26 \\ 126.25 & 126.41 & 128.92 & \dots & 132.27 & 134.15 & 134.17 \\ 136.23 & 138.15 & 141.87 & \dots & 145.97 & 147.13 & 148.14 \end{bmatrix}. \quad (25)$$

Using MATLAB software (MathWorks Inc., Natick, MA, USA) and the maximum likelihood estimation method, the above 216 life data points were analyzed. Finally, the shape parameter  $m = 3$  and characteristic life parameter  $\eta = 100$  were obtained. To compute the optimal reliability threshold  $R$ , the adjustment factors ( $a_k, b_k$ ), cost parameters ( $c_j, c_s, c_m, c_r, c_w, \psi, \delta$ ), and time parameters ( $t_j, t_s, t_m, t_r$ ) need to be known. Usually, maintenance engineers are responsible for the determination of these parameters, which are listed in Table 6.

The judgment matrix of relative risk between various influence factors is as follows:

$$M = \begin{bmatrix} 1 & \frac{1}{3} & \frac{1}{4} \\ 3 & 1 & 2 \\ 4 & \frac{1}{2} & 1 \end{bmatrix}. \quad (26)$$

According to Tables 1–3, the influence factors of failure risk are scored, and the failure risk factor of component can be obtained.

Table 7 presents the results of PM optimization of component with different weight coefficients.

As observed in Table 7

- (1) The PM reliability threshold  $R$  decreases while the maintenance cost weight  $w_1$  increases, and it increases as the failure risk weight  $w_2$  increases as well.
- (2) The maintenance cost  $C_x$  decreases while  $w_1$  increases, and the failure risk  $r$  decreases as  $w_2$  increases. This reveals the impact of  $w_1$  and  $w_2$  on maintenance cost and failure risk.

Figures 4 and 5 reveal the relationship between  $C$  and  $R$  under conditions of operation sector priority bid and maintenance sector priority bid, respectively. These figures indicate that the total cost of the system over a life cycle steadily declines at first and rises rapidly as  $R$  increases. Additionally, the total cost is lower at a certain value of  $R$ , which represents a best-case scenario.

TABLE 6: Maintenance parameters.

Parameter	Value
$c_j$ (yuan)	300
$c_s$ (yuan)	600
$c_m$ (yuan)	600
$c_r$ (yuan)	100
$c_w$ (yuan)	1000
$\Delta$	0.04
$A_j$	$i/3i + 1$
$A_s$	$i/5i + 1$
$t_j$ (h)	8
$t_s$ (h)	10
$t_m$ (h)	10
$t_r$ (h)	5
$c_d$ (yuan)	2000
$l_{max}/10^4$ km	240
$b_j$	$12i + 1/8i + 1$
$b_s$	$12i + 1/11i + 1$

TABLE 7: PM optimization results with different weight coefficients.

$w_1$	$w_2$	$R$	$C_x$ (yuan)	$R$ (yuan)	$C$ (yuan)
1	0	0.54	4799	14613	19412
0.9	0.1	0.63	4941	13167	18108
0.8	0.2	0.63	4941	13167	18108
0.7	0.3	0.76	5603	10775	16378
0.6	0.4	0.82	6405	9407	15812
0.5	0.5	0.84	6696	9052	15748
0.4	0.6	0.87	7590	8349	15939
0.3	0.7	0.89	8199	8047	16246
0.2	0.8	0.89	8199	8047	16246
0.1	0.9	0.92	10125	7659	17784
0	1	0.92	10125	7659	17784

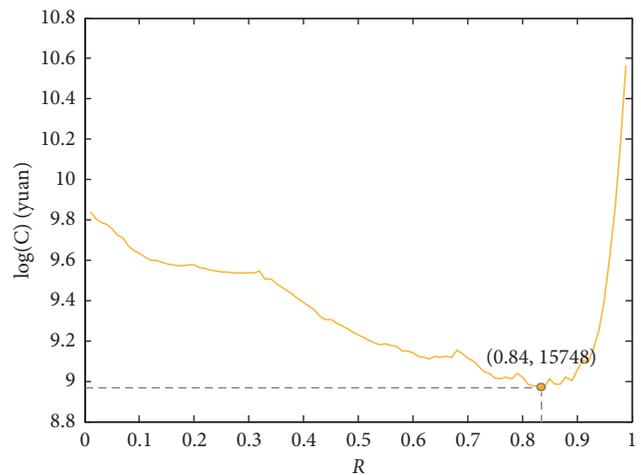


FIGURE 4: C-R curve of operation sector priority bid.

Tables 8 and 9 present the optimization results and PM schedules of the two game models, respectively. Figure 6 shows the reliability evolution comparison between the two models.

As shown in Tables 8 and 9 and Figure 6,

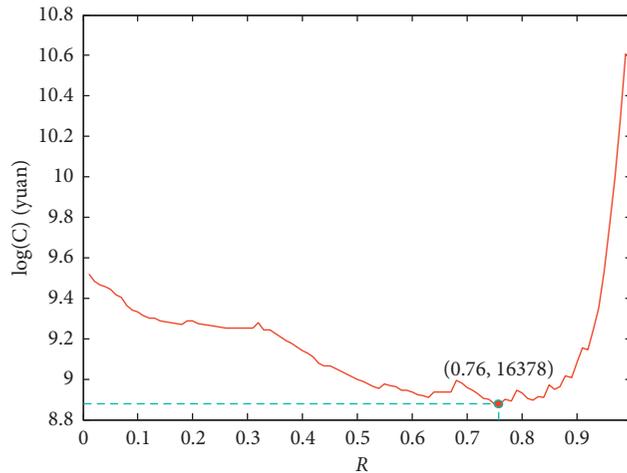


FIGURE 5: C-R curve of maintenance sector priority bid.

TABLE 8: Optimization results of the two game models.

Model	$R$	$w_1$	$w_2$	$C_x$ (yuan)	$R$ (yuan)	$C$ (yuan)
1	0.84	0.5	0.5	6696	9052	15748
2	0.76	0.7	0.3	5603	10775	16378

TABLE 9: PM schedules under the two game models (0-junior and 1-senior).

Model	$R$	PM mileage interval ( $10^4$ km)	Maintenance measure	Failure maintenance times
1	0.84	52-92-114-128-160-176-186-208-218-236	0-0-0-1-0-0-1-0-1-0	1.91
2	0.76	66-108-134-150-186-204-216	0-0-0-1-0-0-1	2.27

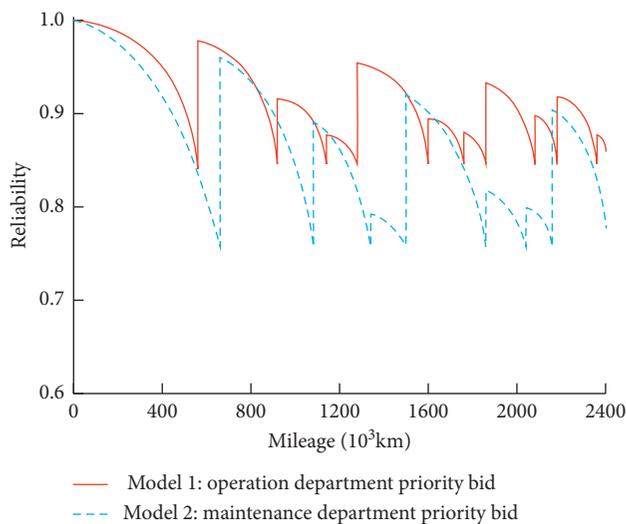


FIGURE 6: Reliability evolution comparison between the two models.

- (1) Under the tristage bargaining dynamic game model, the PM reliability threshold  $R$  of Model 1 (operation sector priority bid) is higher than that of Model 2 (maintenance sector priority bid), and the failure risk of Model 1 is significantly lower than that of Model 2.

The maintenance cost of Model 1 is significantly higher than that of Model 2, and the total cost of the two models is similar.

- (2) Model 1 allows components to maintain a higher reliability level than that of Model 2 during a life cycle, especially in the latter half of the component life cycle (1.2 million km to 2.4 million km).
- (3) Model 1 adopted two more junior maintenance measures and one more senior maintenance measure than Model 2, which results in the number of failure maintenance time being reduced by 15.9%.
- (4) Maintenance sector is stronger than operation sector. Irrespective of the operation sector priority bid or maintenance sector priority bid, game results show that maintenance cost weight  $w_1$  is always equal to or greater than that of failure risk weight  $w_2$ .

### 5. Conclusion

The PM schedule of the EMU component is jointly developed by the maintenance and operation sectors. This study established a tristage bargaining dynamic game model between these EMU sectors to balance the allocation of resources between them. The numerical example shows that operation sector priority bid allows

components to maintain a higher level of reliability during a life cycle, especially in the latter half of the component life cycle, and its failure probability is reduced by nearly 1/6 than that under maintenance sector priority bid. In the tristrage bargaining dynamic game model, the game outcome is more beneficial to the sector that priority bids. Therefore, if the status of the two game participants is equal, the participants should attempt to obtain the right to bid first and take the initiative in the bargaining process.

The imperfect PM method applied in this paper is a bilevel strategy where there are only two types of imperfect PM measures available for EMU components, namely, senior maintenance and junior maintenance. However, in the field of EMU PM, engineers may adopt a multi-level imperfect PM measure strategy for components according to its actual situation, i.e., there are multiple types of imperfect PM measures for EMU components at scheduled PM time. Further studies will attempt to develop such multi-level imperfect PM strategies for the EMU component to adopt according to their actual PM situation.

## Notation

$c_j$ :	Junior maintenance cost
$\lambda_i(t)$ :	Failure rate function within the $i$ th PM
$c_s$ :	Senior maintenance cost
$\lambda_i^-$ :	Failure rate before the $i$ th PM
$\tau$ :	Site occupation rate
$\lambda_i^+$ :	Failure rate after the $i$ th PM
$t_j$ :	Junior maintenance time
$a_j$ :	Age reduction factor
$t_s$ :	Senior maintenance time
$b_j$ :	Failure rate increase factor
$R$ :	Reliability threshold for scheduled PM
$T_i$ :	Time interval between $t_i$ and $t_{i-1}$
$C_f$ :	Failure maintenance cost
$\lambda_i^j$ :	Failure rate after junior maintenance
$c_m$ :	Minimal repair cost
$\lambda_i^s$ :	Failure rate after senior maintenance
$t_m$ :	Minimal repair time
$\lambda_{last}^s$ :	Failure rate after last senior maintenance
$C_r$ :	Replacement cost
$\omega_i$ :	Maintenance method selection factor
$c_l$ :	Labor cost
$a_i^j$ :	Age reduction factor after junior maintenance
$c_w$ :	Waste cost
$a_i^s$ :	Age reduction factor after senior maintenance
$t_r$ :	Replacement time
$b_i^j$ :	Failure rate increase factor after junior maintenance
$C_x$ :	Maintenance cost
$b_i^s$ :	Failure rate increase factor after senior maintenance
$\psi$ :	Failure risk factor
$C_p$ :	PM cost
$c_d$ :	Unit failure risk cost
$c_i^p$ :	Maintenance (junior and senior) cost
$C$ :	Total cost
$c_o$ :	Site occupation cost
$\delta$ :	Negotiation loss factor.

## Data Availability

The maintenance record data used to support the findings of this study are included within the supplementary information file.

## Conflicts of Interest

The authors declare that there is no conflicts of interest regarding the publication of this paper.

## Acknowledgments

This work is supported by the National Nature Science Foundation of China (no. 71361019) and Youth Innovative Talents Project of Universities in Guangdong Province (no. 2018GkQNCX075).

## Supplementary Materials

Maintenance record data. The records the actual maintenance data of CRH3C Electric Multiple Unit (EMU) brake pads, a total of 216 items. And the shape parameter ( $m$ ) and scale parameter ( $\eta$ ) in numerical illustration are obtained by analyzing these data. (*Supplementary Materials*)

## References

- [1] Z. Zhang, J. R. Ren, K. C. Xiao et al., "Cost allocation mechanism design for urban utility tunnel construction based on cooperative game and resource dependence theory," *Energies*, vol. 12, no. 17, pp. 1–16, 2019.
- [2] G. Bonamini, E. Colombo, N. Llorca, and J. Sanchez-Soriano, "Cost allocation for rural electrification using game theory: a case of distributed generation in rural India," *Energy for Sustainable Development*, vol. 50, no. 6, pp. 139–152, 2019.
- [3] A. E. E. Eltoukhy, Z. X. Wang, F. T. S. Chan, and S. H. Chung, "Joint optimization using a leader-follower Stackelberg game for coordinated configuration of stochastic operational aircraft maintenance routing and maintenance staffing," *Computers & Industrial Engineering*, vol. 125, no. 11, pp. 46–68, 2018.
- [4] L. Leenders, B. Bahl, M. Hennen, and A. Bardow, "Coordinating scheduling of production and utility system using a Stackelberg game," *Energy*, vol. 175, no. 10, pp. 1283–1295, 2019.
- [5] Y. Xiao, Y. Peng, Q. Lu, and X. Wu, "Chaotic dynamics in nonlinear duopoly Stackelberg game with heterogeneous players," *Physica A: Statistical Mechanics and Its Applications*, vol. 492, no. 2, pp. 1980–1987, 2018.
- [6] M. Finkelstein and I. Gertsbakh, "On preventive maintenance of systems subject to shocks," *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, vol. 230, no. 2, pp. 220–227, 2016.
- [7] K. T. Huynh, A. Barros, and C. Bérenguer, "Adaptive condition-based maintenance decision framework for deteriorating systems operating under variable environment and uncertain condition monitoring," *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, vol. 226, no. 6, pp. 602–623, 2012.
- [8] X. J. Zhou, C. J. Wu, Y. T. Li, and L. F. Xi, "A preventive maintenance model for leased equipment subject to internal

- degradation and external shock damage,” *Reliability Engineering & System Safety*, vol. 154, pp. 1–7, 2016.
- [9] Y. Liu, Y. Li, H. Z. Huang, and Y. Kuang, “An optimal sequential preventive maintenance policy under stochastic maintenance quality,” *Structure and Infrastructure Engineering*, vol. 7, no. 4, pp. 315–322, 2011.
- [10] J. W. Hu, Z. H. Jiang, and H. Wang, “Preventive maintenance for a single-machine system under variable operational conditions,” *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, vol. 230, no. 4, pp. 391–404, 2016.
- [11] T. T. Le, F. Chatelain, and C. Bérenguer, “Multi-branch hidden Markov models for remaining useful life estimation of systems under multiple deterioration modes,” *Proceedings of the Institution of Mechanical Engineers, Part O: Journal of Risk and Reliability*, vol. 230, no. 5, pp. 473–484, 2016.
- [12] M. Park, K. M. Jung, and D. H. Park, “Optimization of periodic preventive maintenance policy following the expiration of two-dimensional warranty,” *Reliability Engineering & System Safety*, vol. 170, pp. 1–9, 2018.
- [13] J. H. Cha, M. Finkelstein, and G. Levitin, “On preventive maintenance of systems with lifetimes dependent on a random shock process,” *Reliability Engineering & System Safety*, vol. 168, pp. 90–97, 2017.
- [14] Y. S. Huang, W. Y. Gau, and J. W. Ho, “Cost analysis of two-dimensional warranty for products with periodic preventive maintenance,” *Reliability Engineering & System Safety*, vol. 134, pp. 51–58, 2015.
- [15] D. Lin, M. J. Zuo, and R. C. M. Yam, “Sequential imperfect preventive maintenance models with two categories of failure modes,” *Naval Research Logistics*, vol. 48, no. 2, pp. 172–183, 2001.
- [16] J. Schutz, N. Rezg, and J. B. Léger, “Periodic and sequential preventive maintenance policies over a finite planning horizon with a dynamic failure law,” *Journal of Intelligent Manufacturing*, vol. 22, no. 4, pp. 523–532, 2011.
- [17] F. T. Chang, G. H. Zhou, W. Cheng, C. Zhang, and C. Tian, “A service-oriented multi-player maintenance grouping strategy for complex multi-component system based on game theory,” *Advanced Engineering Informatics*, vol. 42, Article ID 100970, 2019.
- [18] C. Wang, Z. Wang, Y. Hou, and K. Ma, “Dynamic game-based maintenance scheduling of integrated electric and natural gas grids with a bilevel approach,” *IEEE Transactions on Power Systems*, vol. 33, no. 5, pp. 4958–4971, 2018.
- [19] J. W. Hu, Z. H. Jiang, and H. Wang, “Joint optimization of production plan and preventive maintenance schedule by stackelberg game,” *Asia-Pacific Journal of Operational Research*, vol. 34, no. 4, pp. 1–28, 2017.
- [20] F. B. S. Tayeb, K. Benatchba, and A. E. Messiaid, “Game theory-based integration of scheduling with flexible and periodic maintenance planning in the permutation flowshop sequencing problem,” *Operational Research*, vol. 18, no. 1, pp. 211–255, 2018.
- [21] F. Pourahmadi, M. Fotuhi-Firuzabad, and P. Dehghanian, “Application of game theory in reliability-centered maintenance of electric power systems,” *IEEE Transactions on Industry Applications*, vol. 53, no. 2, pp. 936–946, 2017.
- [22] M. A. K. Malik, “Reliable preventive maintenance scheduling,” *A I I E Transactions*, vol. 11, no. 3, pp. 221–228, 1979.
- [23] T. Nakagawa, “Sequential imperfect preventive maintenance policies,” *IEEE Transactions on Reliability*, vol. 37, no. 3, pp. 295–298, 1988.
- [24] W.-K. Lee, “Risk assessment modeling in aviation safety management,” *Journal of Air Transport Management*, vol. 12, no. 5, pp. 267–273, 2006.
- [25] N. Pancholi and M. Bhatt, “FMECA-based maintenance planning through COPRAS-G and PSI,” *Journal of Quality in Maintenance Engineering*, vol. 24, no. 2, pp. 224–243, 2018.
- [26] M. Bevilacqua, M. Braglia, and R. Gabbrielli, “Monte Carlo simulation approach for a modified FMECA in a power plant,” *Quality and Reliability Engineering International*, vol. 16, no. 4, pp. 313–324, 2000.
- [27] M. Shafiee, A. Labib, J. Maiti, and A. Starr, “Maintenance strategy selection for multi-component systems using a combined analytic network process and cost-risk criticality model,” *Proceedings of the Institution of Mechanical Engineers Part O-Journal of Risk and Reliability*, vol. 233, no. 2, pp. 89–104, 2019.
- [28] F. B. Yeni and G. Özçelik, “Interval-valued Atanassov intuitionistic Fuzzy CODAS method for multi criteria group decision making problems,” *Group Decision and Negotiation*, vol. 28, no. 2, pp. 433–452, 2019.