

## Research Article

# An Adaptive First-Order Reliability Analysis Method for Nonlinear Problems

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The HL-RF algorithm of the first-order reliability method (FORM) is a widely useful tool in structural reliability analysis. However, the iteration results of HL-RF algorithm may not converge due to periodic cycles for some highly nonlinear reliability problems. In this paper, an adaptive first-order reliability method (AFORM) is proposed to improve solution efficiency for some highly nonlinear reliability problems by introducing an adaptive factor. In AFORM, based on the two-parameter approximate first-order reliability method, the new iteration point and the previous iteration point are used to obtain the corresponding angle, and the result of convergence is judged by angle condition. According to the convergence degree of the results, two iteration parameters of the approximate reliability method are adjusted continuously by adaptive factor. Moreover, iteration step size is adjusted by changing the parameters to improve the efficiency and robustness of FORM. Finally, four numerical examples and one mechanical reliability analysis example are used to verify the proposed method. Compared with the different algorithms, the results show that AFORM has better efficiency and robustness for some highly nonlinear reliability problems.

## 1. Introduction

In engineering practice, with the improvement of product performance requirements and the complexity of service environment, its safety has become a growing concern. In general, the safety factor method is usually used in engineering design where design variables are regarded as deterministic variables according to experience. However, because of the influence of machining errors, internal dispersion of materials, and accidental factors, the dimensional parameters, material characteristics, and external loads of the structure all have certain uncertainties [1–5]. Therefore, when there are many uncertain factors in structure design, it is difficult to ensure the reliability of the structures when using the safety factor method without considering these uncertain factors. Unlike the safety factor method, the reliability method is an effective tool to ensure the safety of structures by considering the impact of structural uncertainties [6–9].

The reliability methods including the simulation method and approximation method are mainly used in reliability

analysis [10, 11]. Among them, the simulation method is a traditional method. The common simulation methods include direct Monte Carlo simulation (MCS) [12, 13] and important sampling (IS) [14, 15]. The sampling points of IS are concentrated on the important area, so IS has a smaller number of sampling and a higher calculation efficiency when the accuracy is the same. However, in engineering practice, since the failure probability of engineering structure is very small, large amount of simulations are needed to solve the reliability problems of a small failure probability when using the simulation method. Therefore, the simulation method has certain limitations in the reliability engineering analysis. Unlike the simulation method, the approximation method is widely used in reliability engineering because of its simplicity. The common approximation methods include the response surface method [16–18] and moment method [19–21], such as the FORM and second-order reliability method (SORM). The response surface method is mainly aimed at the reliability evaluation problem in the case of implicit function, while the moment method is aimed to avoid the iteration procedure and difficulty to obtain the

design point both for explicit and implicit performance functions.

Among moment approximate reliability methods, FORM is often used in structural reliability analysis [22, 23]. The Hasofer-Lind and Rackwitz-Fiessler (HL-RF) method is widely used in engineering practice because of its simplicity and high efficiency [24, 25]. However, in reliability analysis, the HL-RF method may produce periodic and chaotic solutions for highly nonlinear performance functions. To improve the convergence of HL-RF in solving nonlinear problems, Elegbede [26] applied the particle swarm optimization method to calculate the failure probability of the structure, which can obtain relatively higher accurate results. Santosh et al. [27] proposed a modified HL-RF method by using step size selection criteria. The chaos control method was introduced by Yang [28] in FORM, which can make the solution results of nonlinear reliability problems converge well, but the convergence speed is slow. To control iterative convergence speed, an improved HL-RF method with finite step size robust iterative algorithm was proposed by Gong and Yi [24]. The conjugate gradient optimization technique was applied by Keshtegar and Miri [29] in HL-RF, and they proposed a modified HL-RF in order to overcome the problem that the results of HL-RF do not converge in the reliability analysis of complex structures. Recently, there are some new FORM algorithms in the literature. Keshtegar [30] developed a new FORM which controls instability solutions using chaotic conjugate map. Meng et al. [31] proposed a new directional stability transformation method of chaos control for first-order reliability analysis. Besides, Keshtegar and Chakraborty [32] improved FORM by introducing a conjugate search direction approach. Moreover, Roudak et al. [33] proposed an approximate first-order reliability method with two parameters for nonlinear reliability problems, which has good robustness and efficiency. However, for different nonlinear problems, different parameter values have great influence on the solution results. Therefore, how to get the proper parameters becomes a problem to be solved.

Based on the two-parameter approximate first-order reliability method, an AFORM method for nonlinear problems is proposed in this paper. To measure the convergence degree of the result, AFORM judges the convergence degree of iteration by the angle condition. To improve the efficiency and robustness of convergence, according to the convergence degree of iteration results, the iteration parameters are constantly updated by introducing adaptive factors. Then, the iteration step size is adaptively changed. Finally, the method is verified by highly nonlinear numerical examples and mechanical engineering examples. The results show that the proposed AFORM is efficient and stable compared with other methods.

## 2. Approximation of Failure Probability and First-Order Reliability Methods

**2.1. Approximation of Failure Probability.** To ensure the safety of structures with various uncertainties, it is usually necessary to carry out reliability analysis. In the reliability

analysis, the failure probability  $P_f$  of the structure can be used to measure the reliability of the structure by

$$P_f = \int_{g(\mathbf{X}) \leq 0} f_{\mathbf{X}}(\mathbf{X}) d\mathbf{X}, \quad (1)$$

where  $g(\mathbf{X})$  is performance function and  $f_{\mathbf{X}}(\mathbf{X})$  is probability density function of random variable  $\mathbf{X}$ .

However, in the actual engineering reliability analysis, because the structural performance function contains multiple random variables, the multidimensional integral calculation is very complicated, especially for the performance function of complex structures. On the contrary, the approximate reliability method is simple and efficient. The approximation for the failure probability can be expressed as

$$P_f \approx \Phi(-\beta) = \Phi(-\|\mathbf{U}^*\|), \quad (2)$$

where  $\Phi(\cdot)$  represents the cumulative distribution function of the standard normal distribution,  $\beta$  is reliability index, and  $\mathbf{U}^*$  is the most probable point (MPP).

**2.2. HL-RF Method.** The HL-RF method is widely used in engineering practice, which mainly includes two parts of standard normal space transformation and linear approximation [34, 35]. Standard normal space transformation is mainly to transform nonstandard normal space into standard normal space. The conversion principle is

$$\mu_i = \frac{x_i - \mu'_{X_i}}{\sigma'_{X_i}}, \quad (3)$$

where  $x_i$  is the  $i$ th random variable in the nonstandard normal space ( $X$ -space),  $\mu_i$  is the  $i$ th random variable in the standard normal space ( $U$ -space) corresponding to  $X_i$ , and  $\sigma'_{X_i}$  and  $\mu'_{X_i}$  are the equivalent standard deviation and mean of  $i$ th normal random variables, which can be respectively expressed as [36, 37]

$$\sigma'_{X_i} = \frac{\varphi\{\Phi^{-1}[F_{X_i}(x_i^*)]\}}{f_{X_i}(x_i^*)}, \quad (4)$$

$$\mu'_{X_i} = x_i^* - \sigma'_{X_i} \Phi^{-1}[F_{X_i}(x_i^*)], \quad (5)$$

where  $\varphi(\cdot)$  represents the probability density function of the standard normal distribution.  $x_i^*$  is the design point, which represents the nonstandard random variable. Also,  $\Phi^{-1}(\cdot)$  is the inverse function of the standard normal distribution.  $F_{X_i}(x_i^*)$  and  $f_{X_i}(x_i^*)$  are the cumulative distribution function and probability density function of  $X_i$ , respectively.

The linear approximation of HL-RF mainly uses Taylor expansion at the most probable point (MPP) of the performance function, which can be expressed as

$$g(\mathbf{U}) \approx \bar{g}(\mathbf{U}) = g(\mathbf{U}^*) + \nabla^T g(\mathbf{U}^*)(\mathbf{U} - \mathbf{U}^*), \quad (6)$$

where  $\bar{g}(\cdot)$  stands for linear approximation expressions of the performance function.  $\nabla g(\mathbf{U}^*)$  is the gradient vector and  $\mathbf{U}^*$  represents MPP.

According to (6), let  $\bar{g}(\cdot) = 0$ , and then the new design point is obtained by

$$\mathbf{U}_{k+1} = \frac{\nabla^T g(\mathbf{U}_k) \mathbf{U}_k - g(\mathbf{U}_k)}{\|\nabla g(\mathbf{U}_k)\|^2} \nabla g(\mathbf{U}_k). \quad (7)$$

Equation (7) is an algorithm for searching MPP by HL-RF. When the iteration point  $\mathbf{U}_{k+1}$  converges, the MPP can be obtained.

**2.3. CC Method.** To overcome the problem of non-convergence when HL-RF is used to solve highly nonlinear structural performance function, Yang [28] applied the theory of chaotic dynamics in FORM and proposed a FORM based on chaos control (CC). Although it can converge to the results of nonlinear reliability problems, it has a relatively low efficiency. This method gives a new iteration point which can be computed by

$$\mathbf{U}_{k+1} = \mathbf{U}_k + \alpha \mathbf{C}[\mathbf{f}(\mathbf{U}_k) - \mathbf{U}_k], \quad (8)$$

$$\mathbf{f}(\mathbf{U}_k) = \frac{\nabla^T g(\mathbf{U}_k) \mathbf{U}_k - g(\mathbf{U}_k)}{\|\nabla g(\mathbf{U}_k)\|^2} \nabla g(\mathbf{U}_k), \quad (9)$$

where  $\alpha$  is the control factor ranging from 0 to 1,  $\mathbf{C}$  represents the  $m \times m$  dimensional involution matrix, usually taken as the identity matrix  $\mathbf{I}$ , and  $\mathbf{f}(\cdot)$  stands for the iterative function vector.

**2.4. Approximate FORM with Two Parameters.** To overcome the instability of HL-RF with highly nonlinear reliability problems, Roudak et al. [33] proposed an efficient and robust algorithm by introducing two adjustable parameters based on HL-RF. We call this method proposed by Roudak as the Roudak method in the following paper. Based on (6), the parameter  $\lambda$  is introduced in this method, and then equation (6) is reformulated as

$$\bar{g}(\mathbf{U}_{k+1}) = g(\mathbf{U}_k) + \nabla^T g(\mathbf{U}_k)(\mathbf{U}_{k+1} - \mathbf{U}_k) = \lambda g(\mathbf{U}_k). \quad (10)$$

The Roudak method also introduces a parameter  $\xi$  and defines a auxiliary point by

$$\mathbf{A}_{k+1} = \mathbf{U}_k - \xi \frac{\nabla g(\mathbf{U}_k)}{\|\nabla g(\mathbf{U}_k)\|}, \quad (11)$$

where  $\xi$  is the step size in the opposite direction of the gradient vector.

To solve the new iteration point, let the direction of  $\mathbf{U}_k$  cosine vector be

$$\mathbf{V}_{k+1} = \frac{\mathbf{A}_{k+1}}{\|\mathbf{A}_{k+1}\|}. \quad (12)$$

According to (12), the new iteration point is computed by

$$\mathbf{U}_{k+1} = \frac{\nabla^T g(\mathbf{U}_k) \mathbf{U}_k - (1 - \lambda)g(\mathbf{U}_k)}{\nabla^T g(\mathbf{U}_k) \mathbf{V}_{k+1}} \mathbf{V}_{k+1}. \quad (13)$$

**2.5. Adaptive First-Order Reliability Method (AFORM).** Although the approximate first-order reliability method with two parameters has the characteristics of fast convergence and high accuracy for nonlinear reliability problems, the choice of parameters for different nonlinear problems greatly affects the results and efficiency. Therefore, how to choose appropriate parameters is still a problem needed to be solved. To overcome this problem, this paper updates the iteration parameters by using the angle  $\theta_{k+1}$  between the iteration points  $\mathbf{U}_k$  and  $\mathbf{U}_{k+1}$  and by introducing an adaptive factor  $\delta$ . Obviously, only the initial values of the parameters  $\xi^S$  and  $\lambda^S$  are given, and the iteration value of each parameter can be automatically updated according to the following formula. The algorithm is described as

$$\lambda^S = \begin{cases} \delta \lambda^S, & \delta \theta_{k+1} > \theta_k, \\ \lambda^S \frac{\theta_k}{\theta_{k+1}}, & \delta \theta_{k+1} \leq \theta_k < \theta_{k+1}, \\ \max(\lambda^S, 0.2), & \theta_{k+1} \leq \theta_k, \end{cases} \quad (14)$$

$$\xi^S = \begin{cases} \delta \xi^S, & \delta \theta_{k+1} > \theta_k, \\ \xi^S \frac{\theta_k}{\theta_{k+1}}, & \delta \theta_{k+1} \leq \theta_k < \theta_{k+1}, \\ \max(\xi^S, 0.2), & \theta_{k+1} \leq \theta_k. \end{cases} \quad (15)$$

To avoid nonconvergence of the iteration result as the parameters  $\xi^S$  and  $\lambda^S$  are too small, let  $\xi^S$  and  $\lambda^S$  be not less than 0.2. Moreover, to measure the degree of convergence of the iteration result,  $\theta_{k+1}$  is used as a condition for updating parameters. From the algorithm, it can be found that  $\theta_{k+1}$  reflects the convergence of  $\mathbf{U}_k$ . When the error between  $\mathbf{U}_k$  and  $\mathbf{U}_{k+1}$  approaches 0, the corresponding value of  $\theta_{k+1}$  also tends to 0.  $\theta_{k+1}$  can be expressed as

$$\theta_{k+1} = \cos^{-1} \left[ \frac{\mathbf{U}_{k+1} \cdot \mathbf{U}_k}{\|\mathbf{U}_{k+1}\| \cdot \|\mathbf{U}_k\|} \right]. \quad (16)$$

By continuously adjusting the parameters  $\xi^S$  and  $\lambda^S$  in each iteration, (11) and (12) are rewritten as

$$\mathbf{A}_{k+1}^S = \mathbf{U}_k - \xi^S \frac{\nabla g(\mathbf{U}_k)}{\|\nabla g(\mathbf{U}_k)\|}, \quad (17)$$

$$\mathbf{V}_{k+1}^S = \frac{\mathbf{A}_{k+1}^S}{\|\mathbf{A}_{k+1}^S\|}. \quad (18)$$

Equation (13) is reformulated as

$$\mathbf{U}_{k+1} = \frac{\nabla^T g(\mathbf{U}_k) \mathbf{U}_k - (1 - \lambda^S)g(\mathbf{U}_k)}{\nabla^T g(\mathbf{U}_k) \mathbf{U}_{k+1}} \mathbf{V}_{k+1}^S. \quad (19)$$

In this paper, the initial values of the parameters in the proposed AFORM algorithm are  $\xi^S = 0.6$  and  $\lambda^S = 0.8$ . In order to improve the efficiency of the algorithm, the initial

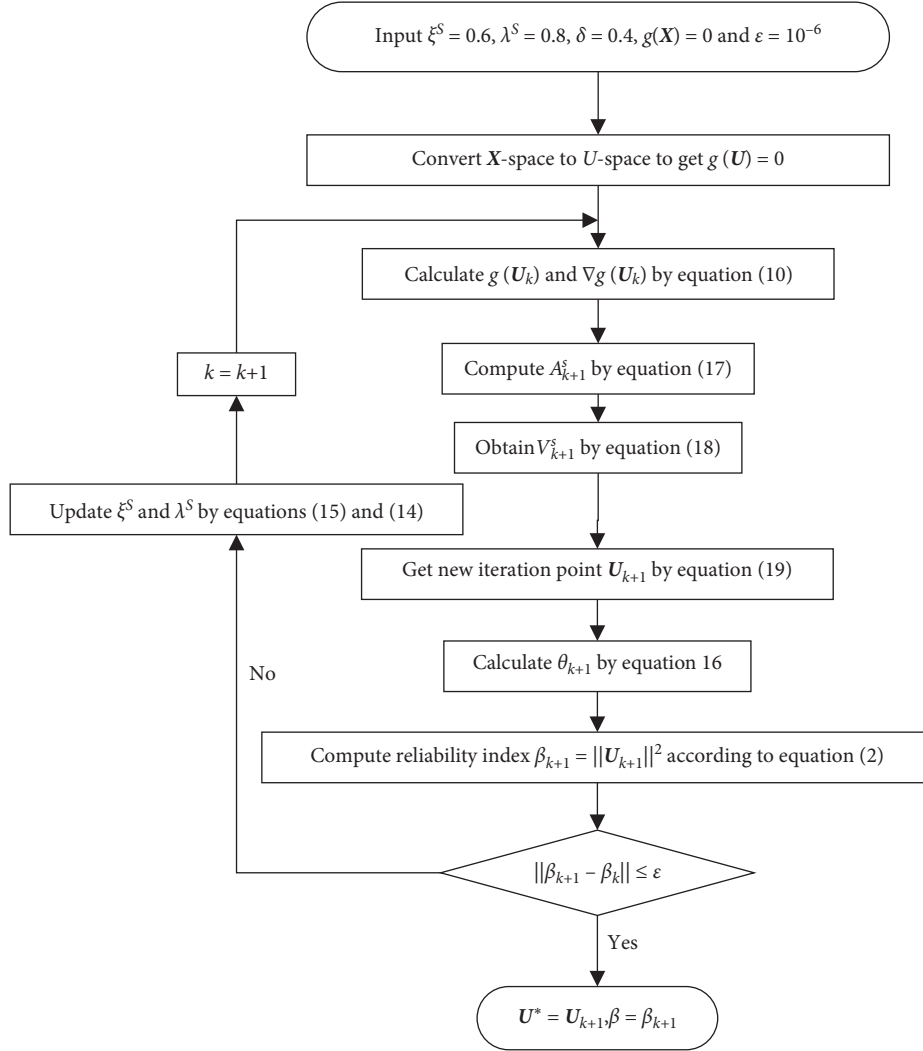


FIGURE 1: Flowchart of AFORM algorithm.

adaptive factor  $\delta = 0.4$  is taken. The specific flowchart of the algorithm is shown in Figure 1.

### 3. Example Analyses

**3.1. Nonlinear Numerical Example 1.** To test the performance of the reliability algorithm, a common nonlinear function is used to verify it. This example uses a cubic polynomial function, which can be expressed as [30]

$$g = x_1^3 + x_1^2 x_2 + x_2^3 - 18, \quad (20)$$

where  $x_1$  and  $x_2$  follow the normal distribution with means  $\mu_1 = 10.0$ ,  $\mu_2 = 9.9$  and standard deviations  $\sigma_1 = \sigma_2 = 5.0$ .

Reliability index calculated by Wang and Grandhi [36] and Gong et al. [6] is 2.2983. Meng et al. [31] and Keshtegar and Miri [29] obtain that the result of the reliability index is 2.2983. In the reliability analysis of example 1, the reliability index obtained by Monte Carlo simulation using  $10^6$  samples is 2.5265. The computation results and iterative process of reliability index using the different methods in example 1 are shown in Table 1 and

TABLE 1: Calculation results of different methods in example 1.

	HL-RF	CC	Roudak	AFORM
$\beta$	1.5355	2.2982	2.2983	2.2983
Iteration number	40	144	35	17

Figure 2, respectively. We can see that it can quickly converge to 1.5355 using the HL-RF method to solve the reliability index. However, it has a larger error compared with the other methods. The reliability indexes calculated by the CC method and Roudak method are 2.2982 and 2.2983, respectively. But the efficiency of these two methods is relatively low (the number of iterations is 144 and 35, respectively). Unlike HL-RF, CC, and Roudak, the reliability index calculated by the AFORM not only has a small error but also has a high efficiency. The number of iterations is about 8 times and 4 times less than the CC method and Roudak method, respectively. Therefore, for nonlinear reliability problems, the proposed AFORM method has the characteristics of high efficiency and small error.

**3.2. Nonlinear Numerical Example 2.** This example uses a nonlinear performance function with a fourth-order polynomial, which is described as [27]

$$g = x_1^4 + 2x_2^4 - 20, \quad (21)$$

where  $x_1$  and  $x_2$  follow a normal distribution with means  $\mu_1 = \mu_2 = 10.0$  and standard deviations  $\sigma_1 = \sigma_2 = 5.0$ .

The reliability index calculated in [36] is 2.3633. The calculated result in [6] is 2.3628. Moreover, the reliability index given in reference [38] is 2.3654. The reliability indexes obtained in [33, 39] are 2.3655. The reliability index obtained after  $10^6$  calculations is 2.8404 by Monte Carlo simulation. We also give the computation results and iterative process of reliability index by different methods for example 2 in Table 2 and Figure 3, respectively. As shown in Figure 3, the results show a second-order periodic oscillation by HL–RF. This is because that it is difficult for HL–RF to solve the reliability model with high nonlinearity. Unlike HL–RF, all reliability indexes of CC, Roudak, and AFORM are converged, and their convergence results are 2.3654, 2.3655, and 2.3655, respectively. However, their efficiency is different. CC has the lowest efficiency with 163 iterations. Also, Roudak is more efficient than CC. AFORM has the highest efficiency. Therefore, the AFORM has higher efficiency for nonlinear reliability problems.

**3.3. Nonlinear Numerical Example 3.** A highly nonlinear performance function of a pipeline is obtained by response surface fitting, which can be formulated by [6]

$$\begin{aligned} g = & 1.1 - 0.00115x_1x_2 + 0.00157x_2^2 + 0.00117x_1^2 + 0.0135x_2x_3 \\ & - 0.0705x_2 - 0.00534x_1 - 0.0149x_1x_3 - 0.0611x_2x_4 \\ & + 0.0717x_1x_4 - 0.226x_3 + 0.0333x_3^2 - 0.558x_3x_4 \\ & + 0.998x_4 - 1.339x_4^2, \end{aligned} \quad (22)$$

where  $x_1$  follows the extreme-II distribution with means  $\mu_1 = 10$  and standard deviations  $\sigma_1 = 5$ ,  $x_2$  and  $x_3$  are normal random variables with means  $\mu_2 = 10$ ,  $\mu_3 = 0.8$  and standard deviations  $\sigma_2 = 5$ ,  $\sigma_3 = 0.2$ , and  $x_4$  is a lognormal distribution random variable with means  $\mu_4 = 0.0625$  and standard deviations  $\sigma_4 = 0.0625$ . According to [33], the reliability index is 1.3961 computed by MCS using  $10^6$  samples. The calculating results and iterative process for different methods are shown in Table 3 and Figure 4, respectively. Although reliability indexes computed by CC, Roudak, and AFORM are almost convergent to the same value, the convergence rate is not the same. From Table 3, it can be seen that the efficiency of the CC and Roudak method is low, but the AFORM method has the highest efficiency which is about 2 times faster than CC and Roudak. Therefore, compared with other methods, the proposed AFORM method has better efficiency and robustness for example 3 with highly nonlinear performance function.

**3.4. Nonlinear Numerical Example 4.** A nonlinear performance function with nonnormal random variables is used in this example, which is expressed as [30]

$$g = x_1^4 + x_2^2 - 50, \quad (23)$$

where  $x_1$  follows the lognormal distribution with mean  $\mu_1 = 5$  and standard deviation  $\sigma_1 = 1$ , respectively; moreover,  $x_2$  follows the Gumbel distribution with mean  $\mu_2 = 10$  and standard deviation  $\sigma_2 = 10$ , respectively.

Table 4 and Figure 5 show the computation results and iterative process of reliability index with different methods for example 4, respectively. As shown in Figure 5, the reliability index of the traditional HL–RF method has a periodic cycle, which produces unstable results as chaotic solutions in example 4. Although the results of CC and Roudak methods can reach convergence, their efficiency is relatively low. For the nonlinear reliability problems of nonnormal distribution, the results of AFORM can both converge and have high efficiency. The number of iterations of AFORM is only 17. Therefore, for this example, the AFORM method is more efficient than other methods.

**3.5. Example 5 about Reliability Analysis of the Two-Degree-of-Freedom Primary-Secondary Dynamic System.** This example uses a highly nonlinear performance function to validate the proposed method. The performance function represents the performance of the two-degree-of-freedom primary and secondary power system based on the force capacity of the second spring. The dynamic system is shown in Figure 6. Moreover, the performance function corresponding to Figure 6 is expressed as [33]

$$g = F_s - k_s P [E(x_s^2)]^{0.5}, \quad (24)$$

where the subscripts  $p$  and  $s$  represent the primary and secondary springs, respectively.  $F_s$  and  $k_s$ , which represent the force and stiffness of the secondary spring, are random variables,  $P$  is a deterministic peak factor and equals to 3, and  $E(x_s^2)$  is the mean square response of the relative displacement of secondary spring, which can be computed by

$$E(x_s^2) = \frac{\pi S_0}{4\zeta_s \omega_s^3} \left[ \frac{\zeta_s \zeta_a}{\zeta_p \zeta_s (4\zeta_a^2 + \eta^2) + \nu \zeta_a^2} \frac{(\zeta_p \omega_p^3 + \zeta_s \omega_s^3) \omega_p}{4\zeta_a \omega_a^4} \right], \quad (25)$$

where  $S_0$  is the intensity of a white noise base excitation of the system and  $\zeta_p$  and  $\zeta_s$  represent the damping ratios of the primary and secondary springs, respectively.  $\omega_p = (k_p/m_p)^{0.5}$  and  $\omega_s = (k_s/m_s)^{0.5}$  represent the natural frequency of the primary and secondary oscillators, respectively.  $k_p$  is the stiffness of the secondary spring, and  $m_s$  and  $m_p$  are the mass.  $\zeta_a = (\zeta_p + \zeta_s)/2$  and  $\omega_a = (\omega_p + \omega_s)/2$  represent the average frequency and damping coefficient, respectively.  $\eta = (\omega_p - \omega_s)/\omega_a$  and  $\nu = m_s/m_p$  are the mass ratio and the tuning parameter, respectively, where,  $S_0$ ,  $k_p$ ,  $m_p$ ,  $m_s$ ,  $\zeta_p$ , and  $\zeta_s$  are random variables. Also, distribution characteristics of each random variable are shown in Table 5.

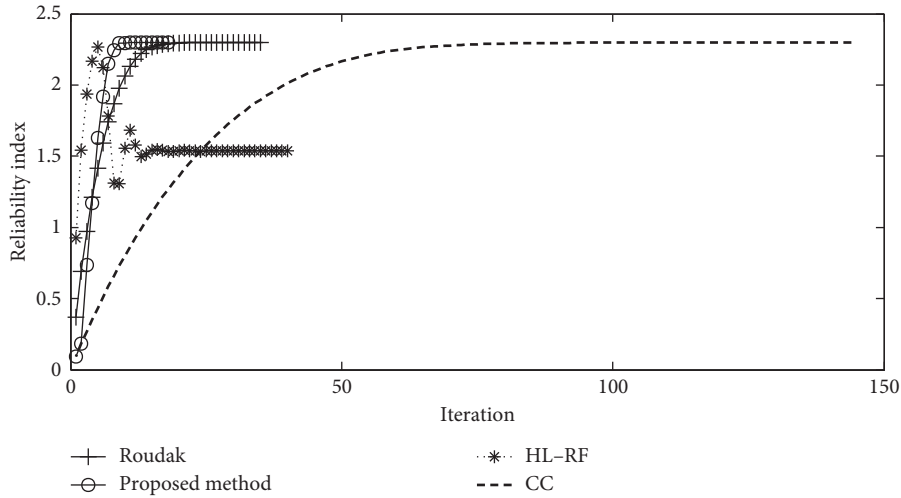


FIGURE 2: Iterative process of reliability index of example 1.

TABLE 2: Calculation results of different methods in example 2.

	HL-RF	CC	Roudak	AFORM
$\beta$	—	2.3654	2.3655	2.3655
Iteration number	—	163	40	18

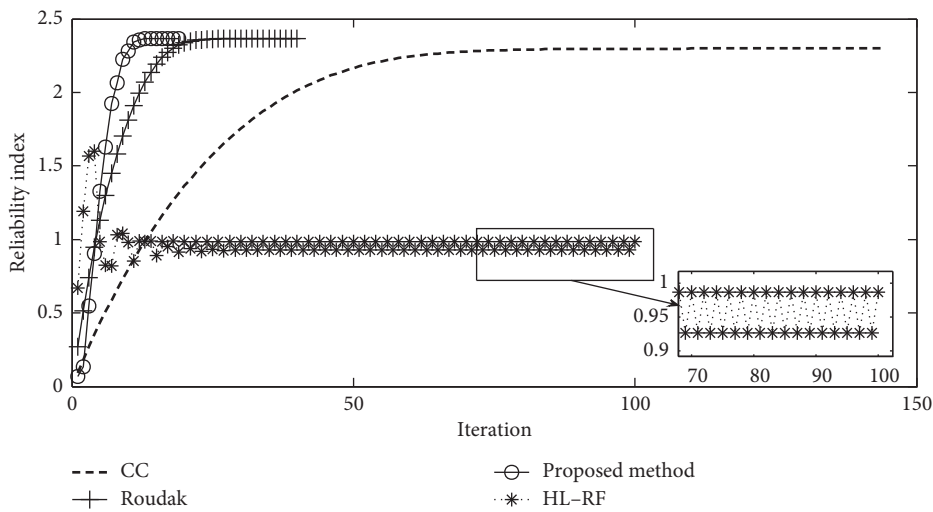


FIGURE 3: Iterative process of reliability index of example 2.

TABLE 3: Calculation results of different methods in example 3.

	HL-RF	CC	Roudak	AFORM
$\beta$	—	1.3303	1.3303	1.3304
Iteration number	—	110	109	44

For this dynamic system, we give the computation results and iterative process of reliability index using the different methods in Table 6 and Figure 7, respectively. As shown in Figure 7, the results obtained by HL-RF method

show a periodic oscillation. It shows that the HL-RF method has certain limitations in reliability evaluation of the complex highly nonlinear performance function. Although CC, Roudak, and AFORM all obtained convergence results,

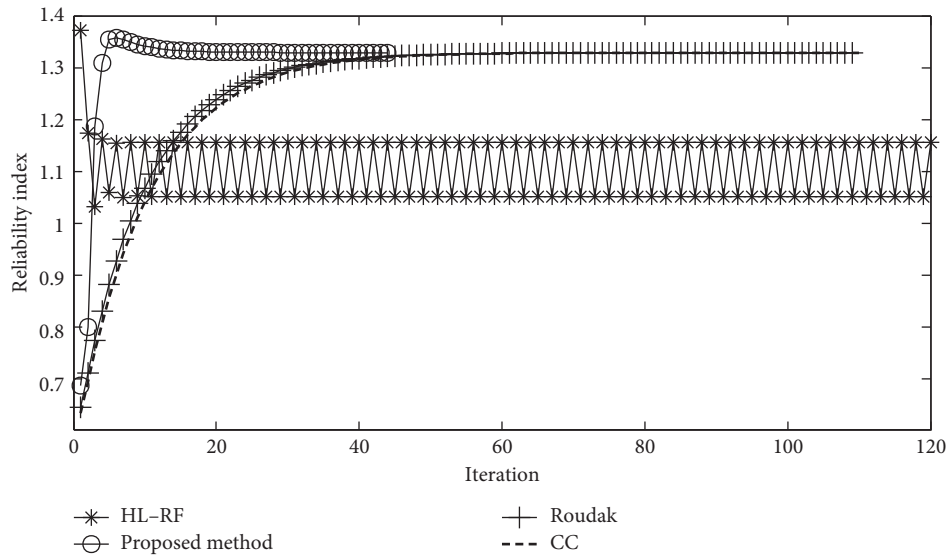


FIGURE 4: Iterative process of reliability index of example 3.

TABLE 4: Calculation results of different methods in example 4.

	HL-RF	CC	Roudak	AFORM
$\beta$	—	3.2593	3.2593	3.2593
Iteration number	—	138	33	17

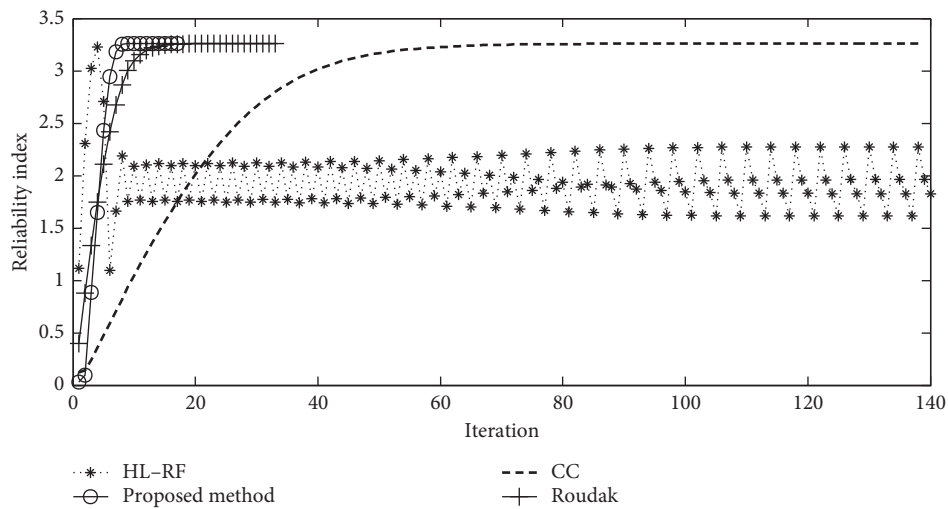


FIGURE 5: Iterative process of reliability index of example 4.

the CC and Roudak methods converge slowly. This shows that CC and Roudak methods are inefficient for nonlinear reliability problems. According to Table 6, the reliability indexes obtained by CC, Roudak, and AFORM methods all converge to 2.1231. However, the number of iterations of the CC and Roudak methods is 118 and 116, respectively, which is about twice of the AFORM method. Therefore, the AFORM method is significantly better than the other methods.

In order to select the proper initial values of the parameters in the AFORM method, the effects of different initial values of  $\lambda^S$ ,  $\xi^S$ , and  $\delta$  on the calculation results of different examples are analyzed.

The influence of the initial value  $\lambda^S$  on the iteration results of each example is shown in Figure 8 when the initial values  $\xi^S=0.6$  and  $\delta=0.4$ . Figure 8 shows that when the initial values  $\xi^S=0.6$  and  $\delta=0.4$ , the changes of the initial value  $\lambda^S$  have little effect on the iteration results of example 2,

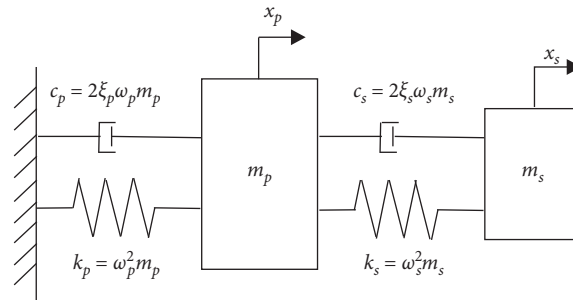


FIGURE 6: Diagram of the primary and secondary dynamic system.

TABLE 5: Distribution characteristics of each random variable.

Random variable	Distribution	Mean	Standard deviation
$k_p$	Lognormal	1	0.2
$k_s$	Lognormal	0.01	0.002
$m_p$	Lognormal	1	0.1
$m_s$	Lognormal	0.01	0.001
$\zeta_p$	Lognormal	0.05	0.02
$\zeta_s$	Lognormal	0.02	0.01
$F_s$	Lognormal	15	1.5
$S_0$	Lognormal	100	10

TABLE 6: Calculation results of different methods for the dynamic system.

	HL-RF	CC	Roudak	AFORM
$\beta$	—	2.1231	2.1231	2.1231
Iteration number	—	118	116	43

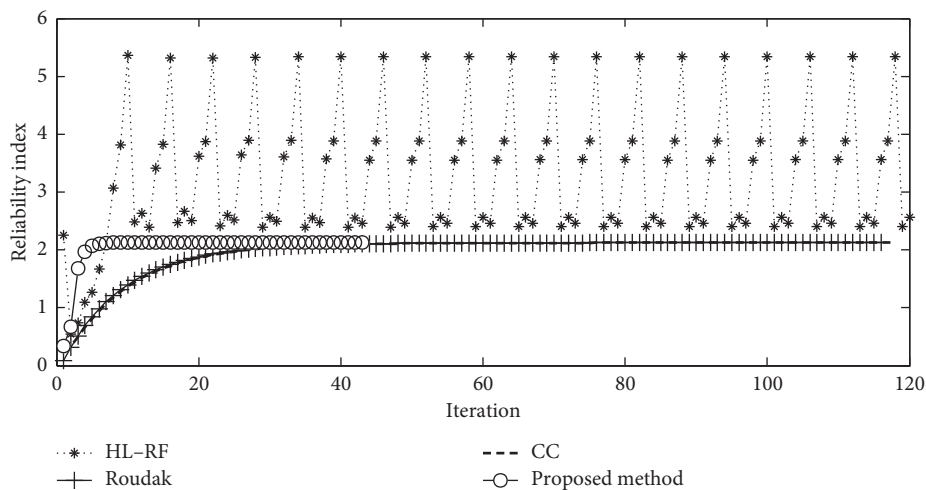


FIGURE 7: Iterative process of reliability index of example 5.

but have an obvious impact on examples 1, 3, 4, and 5. When the initial value  $\lambda^S = 0.8$ , the iteration result of each example is relatively low. Therefore, the value of  $\lambda^S$  is recommended to be 0.8.

The effect of the initial value  $\xi^S$  on the iteration results of each example is given in Figure 9 when the initial values

$\lambda^S = 0.8$  and  $\delta = 0.4$ . When  $\lambda^S = 0.8$ ,  $\delta = 0.4$ , and  $\xi^S = 0.7$ , the solution does not converge for example 5, so the number of iterations is defined as 0. Figure 9 suggests that when the initial values  $\lambda^S = 0.8$  and  $\delta = 0.4$ , the changes of initial values  $\xi^S$  have little effect on the iteration results of examples 1 and 2, but have an apparent effect on examples 3, 4, and 5. Also,



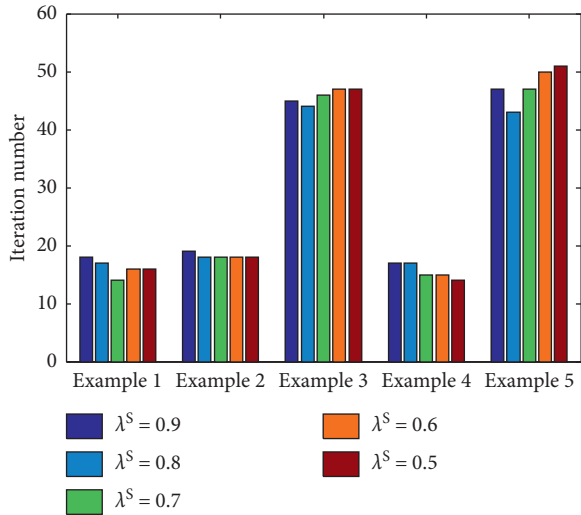


FIGURE 8: Influence of different initial values of  $\lambda^S$  on the iteration results of each example.

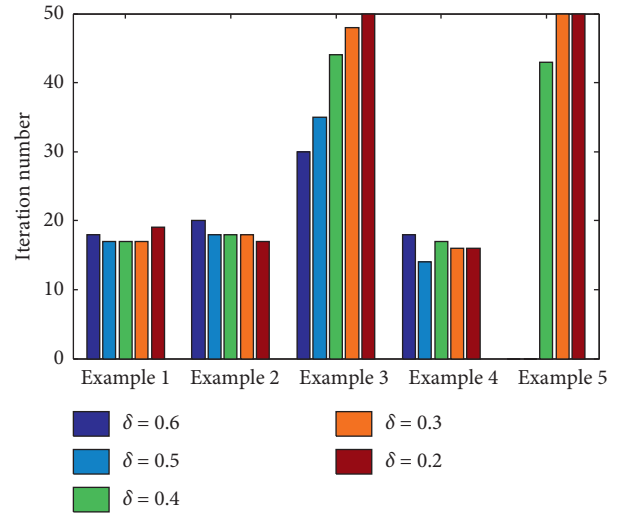


FIGURE 10: Influence of different initial values of  $\delta$  on the iteration results of each example.

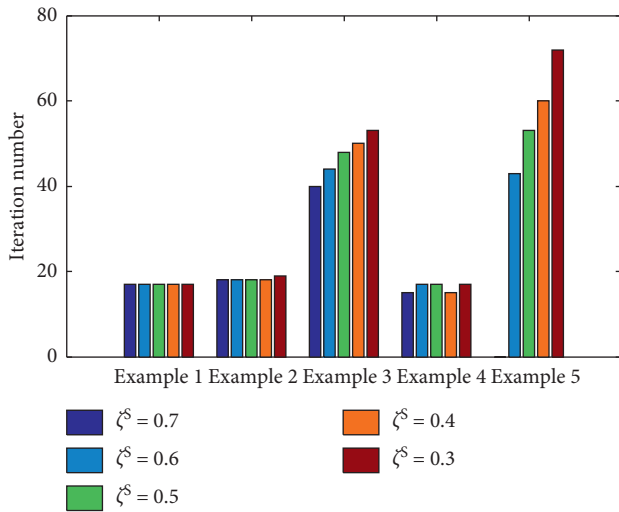


FIGURE 9: Influence of different initial values of  $\xi^S$  on the iteration results of each example.

when the initial value  $\xi^S = 0.6$ , the number of iterations for solving each example is low, so it can be used as the initial parameter value for the AFORM method.

In addition, Figure 10 shows the effect of the initial value  $\delta$  on the iteration results of each example when the initial values  $\lambda^S = 0.8$  and  $\xi^S = 0.6$ . Among them, when  $\delta = 0.6$  and  $\delta = 0.5$ , the calculation results of the example 5 do not converge, so the number of iterations of example 5 is defined as 0. Figure 10 shows that there is a big difference in iteration number for each example when using the AFORM method with the different  $\delta$ . When  $\delta = 0.4$ , the number of iterations of the AFORM method to solve each example is low, so  $\delta = 0.4$  can be used as the parameter value of the AFORM method.

#### 4. Conclusions

Traditional HL-RF algorithm in FORM is convergent or has inefficient solution for some highly nonlinear reliability evaluation problems. In this paper, an adaptive first-order reliability method is developed to improve the efficiency and stability of FORM for highly nonlinear reliability evaluation problems. In AFORM, iteration parameters are continuously updated by introducing adaptive factors and angular conditions. Besides, to select the proper initial values of the parameters in the AFORM method, the effects of different initial values of parameters on the calculation results of different examples are analyzed. The rationality of the method is verified by four numerical examples and one engineering example. Compared with the other methods (HL-RF, CC, and Roudak methods), the results show that the proposed AFORM method has better efficiency and robustness for some highly nonlinear reliability evaluation problems.

#### Data Availability

The data used to support the findings of this study are included within the article.

#### Conflicts of Interest

The authors declare no conflicts of interest regarding the publication of this paper.

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