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Research Article

Pricing of Power Exchange Option with Jumps under the Double Risk of Exchange and Default

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Power exchange option is an exotic option which combines power option and exchange option. In this paper, we consider the pricing of the power exchange option under exchange rate volatility risk and issuing company bankruptcy risk. Meanwhile, considering the major events between the two countries, we add the Poisson jump process to the option model in order to reflect the impact of sudden factors on the price of transnational derivatives in the international market. According to the no-arbitrage principle, a mathematical model for pricing such problems is established, and explicit solutions are obtained. The numerical examples show that the model established in this paper is effective.

1. Introduction

As the influence of fierce market competition and homogenization of financial products, the traditional European option or American option can no longer meet the needs of investors. Various types of overseas derivatives and risk management financial products have been designed by many financial institutions. Some of them are customized products which can cater to the special risk hedging needs of institutional investors. Power exchange option is an exotic option which is derived from exchange option and power option; both of them are powerful financial instruments for hedging nonlinear risks or incentive design of the executive stock option [1, 2], which are mainly traded in the OTC market. Compared with the traditional options, the application scenarios of power exchange options are more diversified, with better flexibility, innovation, and practicability. As the option is trade in the OTC market, the power exchange option is typically nonstandardized contracts. The elements are designed and adjusted on the basis of the specific needs of both parties, and there is no unified and standardized standard. On the one hand, the OTC option meets the needs of both buyers and sellers, so the volume of trading and the volume of trading far exceed the OTC option. On the other hand, different security firms or future companies have different pricing standards for royalties, and only the contracting parties can understand the content of the contract, let alone obtain market data, which leads to the almost opaque phenomenon of the OTC option. Because the exchange could not monitor whether the parties to the contract have fulfilled their obligations, when the issuing company is in bankruptcy, liquidation, etc., the counterparty would not be able to complete the payment according to the contract when the option expires; thus, a breach of contract occurred.

Many scholars have conducted in-depth research on the power exchange option and jump risk.

Margrabe [3] introduced the definition of exchange option firstly, which was a new option whose underlying assets are two risky assets. It allowed option holders to exchange the two risky assets at the maturity date. At the same time, Margrabe proposed that the value of American exchange options with short-term maturity was consistent with that of European exchange options with standard conditions. That is to say, American exchange options with early exercise rights were not superior to the European

exchange options. Fischer [4] studied the pricing of an exchange option and explained that the exercise price was the price of untranslated assets.

Blenman and Clark [5] summarized the exchange option of Fischer-Margrabe [3, 4] and Tompkins [1] as the power exchange option, whose payoffs were $(\lambda_1 S_1^{\alpha_1}(T) - \lambda_2 S_2^{\alpha_2}(T))^+$. Under the risk-neutral probability, assumed that the values of underlying assets were controlled by geometric Brownian motion, they found the pricing formula of the European power exchange option:

$$\mathrm{PE}(t, S_1, S_2, \xi; T) = \gamma_1(t, S_1, \xi; T) N(d_1) - \gamma_2(t, S_2, \xi; T) N(d_2), \tag{1}$$

where

$$\begin{aligned}
d_1 &= \frac{\ln(\lambda_1 S_1^{\alpha_1}/\lambda_2 S_2^{\alpha_2}) + [\alpha_1(r-\delta_1) - \alpha_2(r-\delta_2) - \alpha_1(1-\alpha_1)(\sigma_1^2/2)] + \alpha_2(1-\alpha_2)(\sigma_2^2/2) + (1/2)v^2(T-t)}{v\sqrt{(T-t)}},\\
d_2 &= \frac{\ln(\lambda_1 S_1^{\alpha_1}/\lambda_2 S_2^{\alpha_2}) + [\alpha_1(r-\delta_1) - \alpha_2(r-\delta_2) - \alpha_1(1-\alpha_1)(\sigma_1^2/2) + \alpha_2(1-\alpha_2)(\sigma_2^2/2) - (1/2)v^2](T-t)}{v\sqrt{(T-t)}},\\
\Upsilon_1(t, S_1, \xi; T) &= \lambda_1 S_1^{\alpha_1} \exp\left\{\left[(\alpha_1 - 1)r - \alpha_1\delta_1 - \alpha_1(1-\alpha_1)\frac{\sigma_1^2}{2}\right](T-t)\right\},\\
\Upsilon_1(t, S_1, \xi; T) &= \lambda_1 S_1^{\alpha_1} \exp\left\{\left[(\alpha_1 - 1)r - \alpha_1\delta_1 - \alpha_1(1-\alpha_1)\frac{\sigma_1^2}{2}\right](T-t)\right\},
\end{aligned} \tag{2}$$

where $N(\cdot)$ denotes the cumulative normal density function.

With this formula, they proved that perfect hedging could be achieved by holding multiple positions of asset S_1 value $\alpha_1 N(d_1)\Upsilon_1$, short positions of asset S_2 value $\alpha_2 N(d_2)\Upsilon_2$, and buying or selling portfolios of riskless assets.

Blenman and Clark also proved the equivalence of American power exchange option and European power exchange option under certain conditions. For the American power exchange option with parameters $\xi = (r, \alpha_1, \alpha_2, \lambda_1, \lambda_2, \delta_1, \delta_2)$, if $\alpha_2 \le 1, \alpha_1(r-\delta_1) \ge r$ and $\alpha_2(r-\delta_2) \le r$, it is not optimal to exercise option rights early, and its value is the same as that of European options.

The appearance of new information in option pricing is likely to bring about discontinuous changes in asset prices. In order to capture discontinuous asset prices, Merton [6] based on Black–Scholes model by using underlying assets included a compound Poisson process to simulate discontinuous changes in stock prices. The pricing formulas of several kinds of exotic options in the jump process were given, which laid a foundation for further research.

By using the jump-diffusion model, Tian et al. [7] assumed that the dynamics of asset prices are controlled by jump-diffusion and that the two assets were interrelated, and a closed valuation formula for the fragile European option was calculated. It was proved that the price of the fragile call option was fragile. If there was no consideration of jump risk, the value of fragile options and stocks were calculated. Price may run counter to reality.

Wang [8] incorporated the discontinuous change of risky asset prices into the power-exchange option model. The jump-diffusion process was used to describe the dynamic change of asset prices. Not only the common jump time but also the jump strength and the difference of the impact of common jump components on asset prices were considered. The total jump risk was divided into special components and general jump risk, and the differences between special jump risk and general jump risk are considered. In addition, the correlation between two kinds of risky assets was included in both the continuous part and the discontinuous part, while the correlation between the discontinuous part was linked through a common jump process, which improved the pricing framework of the power exchange option of Blenman and Clark [5] and obtained a clear pricing formula of the power exchange option.

Owing to the fact that the exchange does not assume the responsibility of the over-the-counter (OTC) parties to fulfill their obligations, the holders of over-the-counter (OTC) contracts are vulnerable to the risk of counterparties, that is, the risk that one counterparty in a financial contract fails to meet the agreed terms. Hull and White [9] calculated the pricing formulas of defaultable European option and compared the pricing analytic formulas of defaultable European option, American option, and ordinary European option by numerical methods. Klein [10] put forward a more realistic assumption that the final payment of default depended on the final market value of assets and other companies with the same level of liability. Brigo and Mercurio [11] showed how to consider the event that counterparties may default in a risk-neutral valuation of financial returns.

Wang et al. [12] proposed a power-exchange option pricing model including default risk and jump risk. In their model, they not only included jump components in all asset price processes but also made these processes interrelated. In the calculation process, the pricing formulas of the power exchange option under counterparty risk and jump risk are obtained by using measure transformation technology.

Xu et al. [13] studied the pricing of power exchange options with default risk. They considered the possibility of bankruptcy and liquidation of the company at any time before the expiration of the option. That is to say, the time

when the counterparty actually defaults was uncertain. Furthermore, the pricing formula of the power exchange option with uncertain default risk was given.

With the integration of the international financial environment, cross-trading of various types of asset securities is frequent, and various pricing problems between the two kinds of currency transactions have been considered. Reiner [14] linked foreign stock and currency risk on the premise that both underlying asset value and exchange rate fluctuations met the B-S equation. For the first time, the pricing model of dual-currency stock options in four cases was proposed, which progressively progressed according to the degree of complexity.

At present, the research on foreign exchange options has formed a relatively complete theoretical structure, while the deeper level of dual-currency options is still under continuous research. Kwok and Wong [15] gave the pricing formula of the singular option with dual-currency assets. Li and Zhou [16] studied the pricing of the power exchange option under the influence of exchange rate fluctuations. Gao and Wang [17], under the dual-currency model, used the method of measure transformation to give the European option pricing formula of asset price with jump process. Huang and He [18] studied the pricing theory of dual-currency European option and reset option under jump risk related to floating exchange rate. Furthermore, underlying asset can be considered drived by Lévy process. Yu et al. [19] modeled the price of the found by an exponential Lévy process. Zhang et al. [20] linked to the performance of the underlying asset, which is modeled by an exponential Lévy process.

According to the research path of the above literature studies, this paper comprehensively considers the power exchange option with the dual risk of exchange rate and default. Also, the Poisson jump process is added into the option to analyze the pricing of the power exchange options with dual currency with jump process.

2. Model Building

In recent years, financial market emergencies and important policies occur frequently, and the release of such major financial events and important policies has a seismic impact on the entire financial market. In order to reflect the unexpected and unpredictable major events in the market, we add the Poisson jump process to the basic assets, company value, and exchange rate, based on which we analyze the dual-currency power-exchange option model with default risk.

Assume that the probability space (Ω, F, P) represents an economic environment with uncertainty, where P is the risk-neutral probability measure. We define the market environment that includes two risky assets. At the expiration date T, the payoffs of European power exchange options are as follows:

$$V\left(S_1(T), S_2(T), T\right) = \left(\lambda_1 S_1^{\alpha_1}(T) - \lambda_2 S_2^{\alpha_2}(T)\right)^+, \tag{3}$$

where $\lambda_i (i = 1, 2)$ and $\alpha_i (i = 1, 2)$ are positive constants. Especially, for $\lambda_1 = \lambda_2 = 1$ and $\alpha_1 = \alpha_2 = 1$, it simplifies to the standard exchange option.

According to Merton's [6] assumption, the jump risks common to the underlying assets are diversified in the market, and the risk premium is 0. We use the method of partial differential equation to research the pricing of dual-currency power exchange options with jump process.

With the risk-neutral probability measure P, according to the different combinations of stocks that investors hold, we divide the dual-currency power exchange options with jump process into two types for discussion.

Type 1. A domestic investor invests in a foreign stock and a domestic stock at the same time. Over the life of the investment, he will sell foreign stocks and buy domestic ones if they outperform the foreign ones. So, to reduce the transaction cost of switching the underlying asset, he bought a power exchange option that would exchange the two assets at maturity time.

Assume the underlying assets $S_i(t), i = 1, 2$ and the exchange rate $X(t)$ follow the following equations; each of these equations contains a jump representing a major event $N(t)$.

$$\begin{aligned}
\mathrm{d}S_1(t) &= \left(r_f - \rho_{1X}\sigma_1\sigma_X - k_1\lambda\right)S_1(t)\mathrm{d}t + \sigma_1 S_1(t)\mathrm{d}W_1(t) + \left(e^{Z_1(t-)} - 1\right)S_1(t)\mathrm{d}N(t),\\
\mathrm{d}S_2(t) &= \left(r_d - k_2\lambda\right)S_2(t)\mathrm{d}t + \sigma_2 S_2(t)\mathrm{d}W_2(t) + \left(e^{Z_2(t-)} - 1\right)S_2(t)\mathrm{d}N(t),\\
\mathrm{d}X(t) &= \left(r_d - r_f\right)X(t)\mathrm{d}t + \sigma_X X(t)\mathrm{d}W_X(t) + \left(e^{Z_X(t-)} - 1\right)X(t)\mathrm{d}N(t),
\end{aligned} \tag{4}$$

where r_f is the foreign risk-free interest rate; r_d is the risk-free interest rate of the country; σ_1 and σ_2 are the instantaneous standard deviations of foreign stock return volatility and domestic stock return, respectively; σ_X is the instantaneous standard deviation of the exchange rate; $\rho_{1X} = \mathrm{Cov}\left(\mathrm{d}W_1(t), \mathrm{d}W_X(t)\right)$ represents the instantaneous covariance of foreign stocks and exchange rates; and $W_1(t)$, $W_2(t)$, and $W_X(t)$ are standard Brownian motion under the measure, which meet $\mathrm{d}W_1(t) \cdot \mathrm{d}W_2(t) = \rho_{12}\mathrm{d}t$, $\mathrm{d}W_1(t) \cdot \mathrm{d}W_X(t) = \rho_{1X}\mathrm{d}t$, and $\mathrm{d}W_2(t) \cdot \mathrm{d}W_X(t) = \rho_{2X}\mathrm{d}t$. In addition, as described by Merton [6], $N(t)$ is a Poisson process with an intensity of λ, and it is independent of other Brownian motions. It is used to simulate the discontinuous changes of asset prices and affect the general changes of asset prices. If the general jump occurs at time t, then the jump of the asset is $S_i(t), i = 1, 2$ controlled by $Z_i(t)$, where $Z_i(t)$ is the normal distribution with mean μ_i and standard deviation $\gamma_i > 0$.

Type 2. A domestic investor is interested in two foreign stocks, so he buys foreign power exchange options which use

the two stocks as the underlying asset. Since he buys in his own currency, he should exchange the option at the exchange rate. In this case, assume that the price of the underlying asset $S_i(t), i = 1, 2$ and the exchange rate $X(t)$ follow the following equations, where the parameters have the same meaning as those of Type 1.

$$\begin{aligned}
\mathrm{d}S_1(t) &= \left(r_f - \rho_{1X}\sigma_1\sigma_X - k_1\lambda\right)S_1(t)\mathrm{d}t + \sigma_1 S_1(t)\mathrm{d}W_1(t) + \left(e^{Z_1(t-)} - 1\right)S_1(t)\mathrm{d}N(t),\\
\mathrm{d}S_2(t) &= \left(r_f - \rho_{2X}\sigma_2\sigma_X - k_2\lambda\right)S_2(t)\mathrm{d}t + \sigma_2 S_2(t)\mathrm{d}W_2(t) + \left(e^{Z_2(t-)} - 1\right)S_2(t)\mathrm{d}N(t),\\
\mathrm{d}X(t) &= \left(r_d - r_f - k_X\lambda\right)X(t)\mathrm{d}t + \sigma_X X(t)\mathrm{d}W_X(t) + \left(e^{Z_X(t-)} - 1\right)X(t)\mathrm{d}N(t).
\end{aligned} \tag{5}$$

3. Explicit Solution of the Model

3.1. Solution of the Pricing Model of Type 1. According to the principle of no arbitrage and using ITO's lemma, we get the following conclusions for the first kind of the dual-currency power exchange option model.

Theorem 1. *Pricing formula of dual-currency power exchange options under the first kind of jump risk is*

$$\begin{aligned}
C^* &= e^{-r_d T} E\left[\lambda_2 S_2^{\alpha_2}(\mathrm{T})\right] \cdot \sum_{n=0}^{\infty} \frac{\left(\lambda^Q T\right)^n e^{-\lambda^Q T}}{n!}\left(K_1 - K_2 + K_3 - K_4\right)\\
&= \lambda_2 e^{-r_d T} \cdot S_2^{\alpha_2}(0) \cdot \exp\left[\left(\alpha_2\left(r_d - \frac{\sigma_2^2}{2} - k_2\lambda\right) + \frac{\alpha_2^2\sigma_2^2}{2}\right)T\right] \cdot \exp\left[\lambda\left(e^{\alpha_2\mu_2 + (1/2)\alpha_2^2\gamma_2^2} - 1\right)T\right] \cdot \sum_{n=0}^{\infty} \frac{\left(\lambda^Q T\right)^n e^{-\lambda^Q T}}{n!}\left(K_1 - K_2 + K_3 - K_4\right),
\end{aligned} \tag{6}$$

where

$$\begin{aligned}
K_1 &= \frac{\lambda_1}{\lambda_2} e^{M_1 + (1/2)H_1} N_2\left(\frac{M_1 - \ln(\lambda_2/\lambda_1) + H_1}{\sqrt{H_1}}, \frac{M_2 - \ln L^* + \rho\sqrt{H_1}\sqrt{H_2}}{\sqrt{H_2}}, \rho\right),\\
K_2 &= N_2\left(\frac{M_1 - \ln(\lambda_2/\lambda_1)}{\sqrt{H_1}}, \frac{M_2 - \ln L^*}{\sqrt{H_2}}, \rho\right),\\
K_3 &= \frac{\lambda_1}{\lambda_2} \cdot \frac{(1-\alpha)}{L} \cdot e^{M_1 + M_2 + (1/2)H_1 + (1/2)H_2 + \rho\sqrt{H_1}\sqrt{H_2}} \cdot N_2\\
&\quad \cdot \left(\frac{M_1 - \ln(\lambda_2/\lambda_1) + H_1 + \rho\sqrt{H_1}\sqrt{H_2}}{\sqrt{H_1}}, -\frac{\ln L^* - M_2 - \rho\sqrt{H_1}\sqrt{H_2} - H_2}{\sqrt{H_2}}, -\rho\right),\\
K_4 &= \frac{(1-\alpha)}{L} e^{M_2 + (1/2)H_2} N_2\left(\frac{M_1 - \ln(\lambda_2/\lambda_1) + \rho\sqrt{H_1}\sqrt{H_2}}{\sqrt{H_1}}, -\frac{M_2 - \ln L^* + H_2}{\sqrt{H_2}}, -\rho\right).
\end{aligned} \tag{7}$$

Proof. Let $G(t) = X(t) \cdot S_1(t)$, so $G(t)$ satisfy the following random process:

$$\begin{aligned}
\mathrm{d}G(t) &= \mathrm{d}X(t)S_1(t) = X(t)\mathrm{d}S_1(t) + S_1(t)\mathrm{d}X(t) + \mathrm{d}S_1(t)\mathrm{d}X(t) + \left(X(t)S_1(t) - X(t-)S_1(t-)\right)\mathrm{d}N(t)\\
&= X(t)S_1(t)\left[\left(r_f - \rho_{1X}\sigma_1\sigma_X - k_1\lambda\right)\mathrm{d}t + \sigma_1\mathrm{d}W_1(t)\right] + S_1(t)X(t)\left[\left(r_d - r_f - k_X\lambda\right)\mathrm{d}t + \sigma_X\mathrm{d}W_X(t)\right] + \sigma_1 S_1(t)\sigma_X X(t)\mathrm{d}W_1\\
&\quad \cdot (t)\mathrm{d}W_X(t) + \left[\left(\left(e^{Z_X(t-)} - 1\right)X(t-) + X(t-)\right) \cdot \left(\left(e^{Z_1(t-)} - 1\right)S_1(t-) + S_1(t-)\right) - X(t-)S_1(t-)\right]\mathrm{d}N(t)\\
&= G(t)\left(r_d - k_1\lambda - k_X\lambda\right)\mathrm{d}t + G(t)\left[\sigma_1\mathrm{d}W_1(t) + \sigma_X\mathrm{d}W_X(t)\right] + \left(e^{Z_X(t-) + Z_1(t-)} - 1\right)G(t-)\mathrm{d}N(t)\\
&= \left(r_d - k_1\lambda - k_X\lambda\right)G(t)\mathrm{d}t + \sigma_G G(t)\mathrm{d}W_G(t) + \left(e^{Z_G(t-)} - 1\right)G(t-)\mathrm{d}N(t),
\end{aligned} \tag{8}$$

where $W_G(t)$ is the Brownian motion under risk-neutral measure and $\sigma_G^2 = \sigma_1^2 + \sigma_X^2 - 2\sigma_1\sigma_X\rho_{1X}$.

According to $\sigma_G\rho_{2G} = \sigma_1\rho_{12} + \sigma_X\rho_{2X}$, we have $\rho_{2G} = ((\sigma_1\rho_{12} + \sigma_X\rho_{2X})/\sigma_G)$. In the same way, according to $\sigma_G\rho_{GV} = \sigma_1\rho_{1V} + \sigma_X\rho_{VX}$, we have $\rho_{GV} = ((\sigma_1\rho_{1V} + \sigma_X\rho_{VX})/\sigma_G)$. On the contrary, $N(t)$ is still a Poisson process with intensity λ, and $Z_G(t) = Z_X(t) + Z_1(t)$ is a normal distribution with mean $\mu_X + \mu_1$ and variance $\gamma_X^2 + \gamma_1^2$.

By using the orthogonal transformation, we have

$$\begin{cases} dG(t) = (r_d - k_1\lambda - k_X\lambda)G(t)\mathrm{d}t + \sigma_G G(t)\mathrm{d}W_G(t) + (e^{Z_G(t-)} - 1)G(t-)\mathrm{d}N(t), \\ \mathrm{d}V(t) = (r_d - k_V\lambda)V(t)\mathrm{d}t + \sigma_V\rho_{GV}V(t)\mathrm{d}B_G(t) + \sigma_V\sqrt{1-\rho_{GV}^2}\,V(t)\mathrm{d}B_V(t) + (e^{Z_V(t-)} - 1)V(t-)\mathrm{d}N(t), \\ \mathrm{d}S_2(t) = (r_d - k_2\lambda)S_2(t)\mathrm{d}t + \sigma_2\rho_{2V}S_2(t)\left(\rho_{GV}\mathrm{d}B_G(t) + \sqrt{1-\rho_{GV}^2}\,\mathrm{d}B_V(t)\right) + \sigma_2\sqrt{1-\rho_{2V}^2}\,S_2(t)\mathrm{d}B_2(t) + (e^{Z_2(t-)} - 1)S_2(t-)\mathrm{d}N(t), \end{cases} \tag{9}$$

where $\mathrm{d}B_G(t)$, $\mathrm{d}B_2(t)$, and $\mathrm{d}B_V(t)$ are independent Brownian motions under the risk-neutral probability measure P.

Define a new probability measure Q, which is equivalent to a probability measure P.

$$\Delta(t) = \frac{\mathrm{d}Q}{\mathrm{d}P} = \frac{E_t^P[S_2^{\alpha_2}(T)]}{E[S_2^{\alpha_2}(T)]}. \tag{10}$$

From the driving equation of risk assets $S_2(t)$ (9), the following equation can be obtained by transformation:

$$\begin{aligned} \mathrm{d}\ln S_2(t) &= \frac{\mathrm{d}S_2(t)}{S_2(t)} - \frac{1}{2}\frac{1}{S_2^2(t)}\mathrm{d}S_2(t)\cdot\mathrm{d}S_2(t) + (\ln S_2(t) - \ln S_2(t-))\mathrm{d}N(t) \\ &= \left(r_d - \frac{\sigma_2^2}{2} - k_2\lambda\right)\mathrm{d}t + \sigma_2\rho_{2v}\cdot\rho_{GV}\cdot\mathrm{d}B_G(t) + \sigma_2\rho_{2v}\sqrt{1-\rho_{GV}^2}\cdot\mathrm{d}B_V(t) + \sigma_2\sqrt{1-\rho_{2V}^2}\mathrm{d}B_2(t) \\ &\quad + \left[\ln\left(S_2(t-) + \left(e^{Z_2(t-)} - 1\right)S_2(t-)\right) - \ln S_2(t-)\right]\mathrm{d}N(t) \\ &= \left(r_d - \frac{\sigma_2^2}{2} - k_2\lambda\right)\mathrm{d}t + \sigma_2\rho_{2v}\rho_{GV}\mathrm{d}B_G(t) + \sigma_2\rho_{2v}\sqrt{1-\rho_{GV}^2}\cdot\mathrm{d}B_V(t) \\ &\quad + \sigma_2\sqrt{1-\rho_{2V}^2}\mathrm{d}B_2(t) + Z_2(t-)\mathrm{d}N(t). \end{aligned} \tag{11}$$

To integrate directly, we have

$$\begin{aligned} S_2^{\alpha_2}(t) = S_2^{\alpha_2}(0)\cdot\ \exp\Bigg\{&\alpha_2\left(r_d - \frac{\sigma_2^2}{2} - k_2\lambda\right)t + \alpha_2\sigma_2\rho_{2v}\rho_{GV}B_G(t) \\ &+ \alpha_2\sigma_2\rho_{2v}\sqrt{1-\rho_{GV}^2}\cdot B_V(t) + \alpha_2\sigma_2\sqrt{1-\rho_{2V}^2}\,B_2(t) \\ &+ \alpha_2\sum_k^{N(t)} Z_2(\tau(k))\Bigg\}. \end{aligned} \tag{12}$$

Then, to calculate its expectation, we have

$$\begin{aligned} E^P[S_2^{\alpha_2}(T)] = S_2^{\alpha_2}(0)\cdot\ \exp\Bigg[&\left(\alpha_2\left(r_d - \frac{\sigma_2^2}{2} - k_2\lambda\right) + \frac{\alpha_2^2\sigma_2^2}{2}\right) \\ &+ \lambda\left(e^{\alpha_2\mu_2 + (1/2)\alpha_2^2\gamma_2^2} - 1\right)\Bigg]T. \end{aligned} \tag{13}$$

Therefore, the new probability measure can be obtained:

$$\begin{aligned} \Delta(t) &= \frac{E_t^P[S_2^{\alpha_2}(T)]}{E[S_2^{\alpha_2}(T)]} = E^P\left[\frac{S_2^{\alpha_2}(T)}{E[S_2^{\alpha_2}(T)]}\,|\,F_t\right] \\ &= E\Bigg\{\exp\Bigg[-\frac{\alpha_2\sigma_2^2}{2}T + \alpha_2\sigma_2\rho_{2v}\cdot\rho_{GV}\cdot B_G(T) \\ &\quad + \alpha_2\sigma_2\rho_{2v}\sqrt{1-\rho_{GV}^2}\cdot B_V(T) + \alpha_2\sigma_2\sqrt{1-\rho_{2V}^2}\,B_2(T) \\ &\quad + \alpha_2\sum_k^{N(T)} Z_2(\tau(k)) - \lambda\left(e^{\alpha_2\mu_2 + (1/2)\alpha_2^2\gamma_2^2} - 1\right)T\Bigg]\,|\,F_t\Bigg\}. \end{aligned} \tag{14}$$

According to Girsanov theorem, $B_G^Q(t)$, $B_2^Q(t)$, and $B_V^Q(t)$ are Brownian motions under the probability measure Q, and

$$\begin{cases} B_G^Q(t) = B_G(t) - \alpha_2\sigma_2\rho_{2V}\rho_{GV}t, \\ B_2^Q(t) = B_2(t) - \alpha_2\sigma_2\sqrt{1-\rho_{2V}^2}\,t, \\ B_V^Q(t) = B_V(t) - \alpha_2\sigma_2\rho_{2V}\sqrt{1-\rho_{GV}^2}\,t. \end{cases} \tag{15}$$

On the contrary, after measure transformation, the discontinuity term also changes correspondingly, where $N^Q(t)$ is also the Poisson process, but its strength is $\lambda^Q = \lambda e^{\alpha_2\mu_2+(1/2)\alpha_2^2\gamma_2^2}$, $Z_{2,k}^Q$ is normal distribution which has mean $\mu_2+\alpha_2\gamma_2^2$ and variance γ_2^2, and $Z_{G,k}^Q$ and $Z_{V,k}^Q$ are normal distribution whose parameters are unchanged.

The value equation of the dual-currency power exchange options is

$$\begin{aligned} C^* &= e^{-rT}E\left[\left(\lambda_1 G^{\alpha_1}(T) - \lambda_2 S_2^{\alpha_2}(T)\right)^+\left(I_{\{V(T)\geq L^*\}} + (1-\alpha)\frac{V(T)}{L}I_{\{V(T)<L^*\}}\right)\right] \\ &= e^{-rT}E\left[\lambda_2 S_2^{\alpha_2}(T)\right]E\left[\frac{\lambda_2 S_2^{\alpha_2}(T)}{E\left[\lambda_2 S_2^{\alpha_2}(T)\right]}\left(\frac{\lambda_1 G^{\alpha_1}(T)}{\lambda_2 S_2^{\alpha_2}(T)} - 1\right)^+\left(I_{\{V(T)\geq L^*\}} + \frac{(1-\alpha)V(T)}{L}I_{\{V(T)<L^*\}}\right)\right]. \end{aligned} \tag{16}$$

That is to say,

$$\begin{aligned} C^* &= e^{-rT}E\left[\lambda_2 S_2^{\alpha_2}(T)\right]\cdot E^Q\left[\left(\frac{\lambda_1 G^{\alpha_1}(T)}{\lambda_2 S_2^{\alpha_2}(T)} - 1\right)^+\left(I_{\{V(T)\geq L^*\}} + (1-\alpha)\frac{V(T)}{L}I_{\{V(T)<L^*\}}\right)\right] \\ &= e^{-rT}E\left[\lambda_2 S_2^{\alpha_2}(T)\right]\cdot\sum_{n=0}^{\infty}E^Q\left[\left(\frac{\lambda_1 G^{\alpha_1}(T)}{\lambda_2 S_2^{\alpha_2}(T)} - 1\right)^+\left(I_{\{V(T)\geq L^*\}} + (1-\alpha)\frac{V(T)}{L}I_{\{V(T)<L^*\}}\right)\Big| N^Q(t) = n\right]\cdot P\left\{N^Q(t) = n\right\} \\ &= e^{-rT}E\left[\lambda_2 S_2^{\alpha_2}(T)\right]\cdot\sum_{n=0}^{\infty}\frac{(\widehat{\lambda}T)^n e^{-\widehat{\lambda}T}}{n!}E^Q\left[\left(\frac{\lambda_1 G^{\alpha_1}(T)}{\lambda_2 S_2^{\alpha_2}(T)} - 1\right)^+\cdot\left(I_{\{V(T)\geq L^*\}} + (1-\alpha)\frac{V(T)}{L}I_{\{V(T)<L^*\}}\right)\Big| Z^{(n)}\right] \\ &= e^{-rT}E\left[\lambda_2 S_2^{\alpha_2}(T)\right]\cdot\sum_{n=0}^{\infty}\frac{(\widehat{\lambda}T)^n e^{-\widehat{\lambda}T}}{n!}\cdot C_n, \end{aligned} \tag{17}$$

where C_n satisfy

$$\begin{aligned} C_n &= E^Q\left[\left(\frac{\lambda_1 G^{\alpha_1}(T,n)}{\lambda_2 S_2^{\alpha_2}(\mathrm{T},n)} - 1\right)\left(I_{\{(\lambda_1 G^{\alpha_1}(T,n)/\lambda_2 S_2^{\alpha_2}(\mathrm{T},n))\geq 1, V(T,n)\geq L^*\}} + (1-\alpha)\frac{V(T,n)}{L}I_{\{(\lambda_1 G^{\alpha_1}(T,n)/\lambda_2 S_2^{\alpha_2}(\mathrm{T},n))\geq 1, V(T,n)<L^*\}}\right)\right] \\ &= E^Q\left[\frac{\lambda_1 G^{\alpha_1}(T,n)}{\lambda_2 S_2^{\alpha_2}(\mathrm{T},n)}\cdot I_{\{(\lambda_1 G^{\alpha_1}(T,n)/\lambda_2 S_2^{\alpha_2}(\mathrm{T},n))\geq 1, V(T,n)\geq L^*\}}\right] - E^Q\left[I_{\{(\lambda_1 G^{\alpha_1}(T,n)/\lambda_2 S_2^{\alpha_2}(\mathrm{T},n))\geq 1, V(T,n)\geq L^*\}}\right] \\ &\quad + \frac{(1-\alpha)}{L}E^Q\left[\frac{\lambda_1 G^{\alpha_1}(T,n)}{\lambda_2 S_2^{\alpha_2}(\mathrm{T},n)}\cdot V(T,n)\cdot I_{\{(\lambda_1 G^{\alpha_1}(T,n)/\lambda_2 S_2^{\alpha_2}(\mathrm{T},n))\geq 1, V(T,n)<L^*\}}\right] \\ &\quad - \frac{(1-\alpha)}{L}E^Q\left[V(T,n)\cdot I_{\{(\lambda_1 G^{\alpha_1}(T,n)/\lambda_2 S_2^{\alpha_2}(\mathrm{T},n))\geq 1, V(T,n)<L^*\}}\right]. \end{aligned} \tag{18}$$

Next, the solutions of the three stochastic differential equations in (9) are calculated. According to the Girsanov theorem, the correlation between the $B_G^Q(t)$, $B_2^Q(t)$, and $B_V^Q(t)$ with the new probability measure Q and $B_G(t)$, $B_2(t)$, and $B_V(t)$ with the risk-neutral measure P is equation (15). By using the logarithmic transformation, we have

$$\ln G(t,n) = \ln G(0) + \left(r_d - \frac{\sigma_G^2}{2} - k_1\lambda - k_X\lambda + \alpha_2\sigma_2\sigma_G\rho_{2V}\rho_{GV}\right)t + \sigma_G B_G^Q(t) + \sum_{k=1}^{n} Z_G^Q(\tau(k)),$$

$$\ln V(t,n) = \ln V(0) + \left(r_d - \frac{\sigma_V^2}{2} - k_V\lambda + \alpha_2\sigma_V\sigma_2\rho_{2V}\right)t + \sigma_V\rho_{GV}B_G^Q(t) + \sigma_V\sqrt{1-\rho_{GV}^2}B_V^Q(t) + \sum_{k=1}^{n} Z_V^Q(\tau(k)),$$

$$\ln S_2(t,n) = \ln S_2(0) + \left(r_d - \frac{\sigma_2^2}{2} - k_2\lambda + \alpha_2\sigma_2^2\right)t + \sigma_2\rho_{2V}\rho_{GV}B_G^Q(t) + \sigma_2\rho_{2V}\sqrt{1-\rho_{GV}^2}B_V^Q(t) + \sigma_2\sqrt{1-\rho_{2V}^2}B_2^Q(t) + \sum_{k=1}^{n} Z_2^Q(\tau(k)),$$

$$\ln S_2(t,n) = \ln S_2(0) + \left(r_d - \frac{\sigma_2^2}{2} - k_2\lambda + \alpha_2\sigma_2^2\right)t + \sigma_2\rho_{2V}\rho_{GV}B_G^Q(t) + \sigma_2\rho_{2V}\sqrt{1-\rho_{GV}^2}B_V^Q(t) + \sigma_2\sqrt{1-\rho_{2V}^2}B_2^Q(t) + \sum_{k=1}^{n} Z_2^Q(\tau(k)),$$

$$\begin{aligned}\ln\frac{G^{\alpha_1}(T,n)}{S_2^{\alpha_2}(T,n)} &= \alpha_1\ln G(0) - \alpha_2\ln S_2(0) + \alpha_1\left(r_d - \frac{\sigma_G^2}{2} - k_1\lambda - k_X\lambda + \alpha_2\sigma_2\sigma_G\rho_{2V}\rho_{GV}\right)t - \alpha_2\left(r_d - \frac{\sigma_2^2}{2} - k_2\lambda + \alpha_2\sigma_2^2\right)t \\ &\quad + (\alpha_1\sigma_G - \alpha_2\sigma_2\rho_{2V}\rho_{GV})B_G^Q(t) - \alpha_2\sigma_2\rho_{2V}\sqrt{1-\rho_{GV}^2}B_V^Q(t) \\ &\quad - \alpha_2\sigma_2\sqrt{1-\rho_{2V}^2}B_2^Q(t) + \alpha_1\sum_{k=1}^{n} Z_G^Q(\tau(k)) - \alpha_2\sum_{k=1}^{n} Z_2^Q(\tau(k)),\end{aligned} \tag{19}$$

where the mathematical expectations of $\ln(G^{\alpha_1}(T,n)/S_2^{\alpha_2}(T,n))$ and $\ln V(T,n)$ are

$$\begin{cases} M_1 = E\left[\ln\dfrac{G^{\alpha_1}(T,n)}{S_2^{\alpha_2}(T,n)}\right] = \alpha_1\ln G(0) - \alpha_2\ln S_2(0) + \alpha_1\left(r_d - \dfrac{\sigma_G^2}{2} - k_1\lambda - k_X\lambda + \alpha_2\sigma_2\sigma_G\rho_{2V}\rho_{GV}\right)\cdot T - \alpha_2 \\ \quad \cdot\left(r_d - \dfrac{\sigma_2^2}{2} - k_2\lambda + \alpha_2\sigma_2^2\right)\cdot T + n\alpha_1\mu_G - n\alpha_2(\mu_2 + \alpha_2\gamma_2^2), \\ M_2 = E[\ln V(T,n)] = \ln V(0) + \left(r_d - \dfrac{\sigma_V^2}{2} - k_V\lambda + \alpha_2\sigma_V\sigma_2\rho_{2V}\right)\cdot T + n\mu_V. \end{cases} \tag{20}$$

Further, the variance and covariance of $\ln(G^{\alpha_1}(T,n)/S_2^{\alpha_2}(T,n))$ and $\ln V(T,n)$ are

$$\begin{cases} H_1 = \mathrm{Var}\left[\ln\dfrac{G^{\alpha_1}(T,n)}{S_2^{\alpha_2}(T,n)}\right] = \alpha_1^2\sigma_G^2T + \alpha_2^2\sigma_2^2T - 2\alpha_1\alpha_2\sigma_G\sigma_2\rho_{2V}\rho_{GV}T + n\alpha_1^2\gamma_G^2 + n\alpha_2^2\gamma_2^2, \\ H_2 = \mathrm{Var}[\ln V(T,n)] = \sigma_V^2\cdot T + n\gamma_V^2, \\ R_{12} = \mathrm{Cov}\left[\ln\dfrac{G^{\alpha_1}(T,n)}{S_2^{\alpha_2}(T,n)},\ \ln V(T,n)\right] = (\alpha_1\sigma_G\sigma_V\rho_{GV} - \alpha_2\sigma_2\sigma_V\rho_{2V})\cdot T. \end{cases} \tag{21}$$

Since $\ln(G^{\alpha_1}(T,n)/S_2^{\alpha_2}(T,n))$ and $\ln V(T,n)$ are two normal random variables with the above properties, for simplicity, it can be transformed into the following form:

$$\begin{cases} \ln \dfrac{G^{\alpha_1}(T,n)}{S_2^{\alpha_2}(T,n)} = M_1 + \sqrt{H_1}\,\xi_1, \\ \ln V(T,n) = M_2 + \sqrt{H_2}\,\xi_2, \end{cases} \tag{22}$$

where (ξ_1, ξ_2) is a two-dimensional standard normally distributed random variable with correlation coefficient $\rho = (R_{12}/\sqrt{H_1}\cdot\sqrt{H_2})$. As a result,

$$\begin{aligned} K_1 &= E^Q\left[\frac{\lambda_1 G^{\alpha_1}(T,n)}{\lambda_2 S_2^{\alpha_2}(T,n)}\cdot I_{\{(\lambda_1 G^{\alpha_1}(T,n)/\lambda_2 S_2^{\alpha_2}(T,n))\geq 1, V(T,n)\geq L^*\}}\right] \\ &= \frac{\lambda_1}{\lambda_2} e^{M_1+(1/2)H_1} N_2\left(\frac{M_1 - \ln(\lambda_2/\lambda_1) + H_1}{\sqrt{H_1}}, \frac{M_2 - \ln L^* + \rho\sqrt{H_1}\sqrt{H_2}}{\sqrt{H_2}}, \rho\right), \\ K_2 &= E^Q\left[I_{\{(\lambda_1 G^{\alpha_1}(T,n)/\lambda_2 S_2^{\alpha_2}(T,n))\geq 1, V(T,n)\geq L^*\}}\right] = N_2\left(\frac{M_1 - \ln(\lambda_2/\lambda_1)}{\sqrt{H_1}}, \frac{M_2 - \ln L^*}{\sqrt{H_2}}, \rho\right), \\ K_3 &= \frac{(1-\alpha)}{L} E^Q\left[\frac{\lambda_1 G^{\alpha_1}(T,n)}{\lambda_2 S_2^{\alpha_2}(T,n)}\cdot V(T,n)\cdot I_{\{(\lambda_1 G^{\alpha_1}(T,n)/\lambda_2 S_2^{\alpha_2}(T,n))\geq 1, V(T,n)< L^*\}}\right] \\ &= \frac{\lambda_1}{\lambda_2}\cdot\frac{(1-\alpha)}{L}\cdot e^{M_1+M_2+(1/2)H_1+(1/2)H_2+\rho\sqrt{H_1}\sqrt{H_2}}\cdot N_2\left(\frac{M_1 - \ln(\lambda_2/\lambda_1) + H_1 + \rho\sqrt{H_1}\sqrt{H_2}}{\sqrt{H_1}}, -\frac{\ln L^* - M_2 - \rho\sqrt{H_1}\sqrt{H_2} - H_2}{\sqrt{H_2}}, -\rho\right). \end{aligned} \tag{23}$$

Similarly,

$$\begin{aligned} K_4 &= \frac{(1-\alpha)}{L} E^Q\left[V(T,n)\cdot I_{\{(\lambda_1 G^{\alpha_1}(T,n)/\lambda_2 S_2^{\alpha_2}(T,n))\geq 1, V(T,n)< L^*\}}\right] \\ &= \frac{(1-\alpha)}{L} e^{M_2+(1/2)H_2} N_2\left(\frac{M_1 - \ln(\lambda_2/\lambda_1) + \rho\sqrt{H_1}\sqrt{H_2}}{\sqrt{H_1}},\right. \\ &\quad \left. -\frac{M_2 - \ln L^* + H_2}{\sqrt{H_2}}, -\rho\right), \end{aligned} \tag{24}$$

and finally, we get the pricing formula of the first kind of dual-currency power exchange options with jump risk. □

3.2. Solution of the Pricing Model of Type 2. For the power-exchange option model of the second kind of dual currency, the following conclusions can be obtained by applying the similar method.

Theorem 2. *Power exchange options with double risks of exchange rate and default under the second kind of jump risk have the following pricing formula:*

$$\begin{aligned} C^{**} &= e^{-r_d T}\lambda_2 E\left[X(T) S_2^{\alpha_2}(T)\right]\cdot\sum_{n=0}^{\infty}\frac{\left(\lambda^{\widehat{Q}}T\right)^n e^{-\lambda^{\widehat{Q}}T}}{n!} \\ &\quad\cdot\left(K_1' - K_2' + K_3' - K_4'\right) \\ &= \lambda_2 e^{-r_d T}\cdot X(0) S_2^{\alpha_2}(0)\cdot\exp\left\{\alpha_2\left(r_d - \frac{\sigma_X^2}{2} - \frac{\sigma_2^2}{2}\right.\right. \\ &\quad \left. - \rho_{2X}\sigma_2\sigma_X\right)T - \frac{1}{2}\alpha_2^2\left(\sigma_X^2 + 2\sigma_X\sigma_2\rho_{2V}\rho_{VX} + \sigma_2^2\right)T \\ &\quad + \alpha_2(-k_V\lambda - k_2\lambda)T + \lambda\left(e^{\mu_X+(1/2)\gamma_X^2} - 1\right)T \\ &\quad \left. + \lambda\left(e^{\alpha_2\mu_2+(1/2)\alpha_2^2\gamma_2^2} - 1\right)T\right\} \\ &\quad\cdot\sum_{n=0}^{\infty}\frac{\left(\lambda^{\widehat{Q}}T\right)^n e^{-\lambda^{\widehat{Q}}T}}{n!}\cdot\left(K_1' - K_2' + K_3' - K_4'\right), \end{aligned} \tag{25}$$

where

$$K_1' = \frac{\lambda_1}{\lambda_2} e^{M_1'+(1/2)H_1'} N_2\left(\frac{M_1' - \ln(\lambda_2/\lambda_1) + H_1'}{\sqrt{H_1'}}, \frac{M_2' - \ln L^* + \rho' \sqrt{H_1'}\sqrt{H_2'}}{\sqrt{H_2'}}, \rho'\right),$$

$$K_2' = N_2\left(\frac{M_1' - \ln(\lambda_2/\lambda_1)}{\sqrt{H_1'}}, \frac{M_2' - \ln L^*}{\sqrt{H_2'}}, \rho'\right),$$

$$K_3' = \frac{\lambda_1}{\lambda_2} \cdot \frac{(1-\alpha)}{L} \cdot e^{M_1'+M_2'+(1/2)H_1'+(1/2)H_2'+\widehat{\rho}\sqrt{H_1'}\sqrt{H_2'}} \cdot N_2\left(\frac{M_1' - \ln(\lambda_2/\lambda_1) + H_1' + \rho' \sqrt{H_1'}\sqrt{H_2'}}{\sqrt{H_1'}}, -\frac{\ln L^* - M_2' - \rho' \sqrt{H_1'}\sqrt{H_2'} - H_2'}{\sqrt{H_2'}}, -\rho'\right),$$

$$K_4' = \frac{(1-\alpha)}{L} \cdot e^{M_2'+(1/2)H_2'} \cdot N_2\left(\frac{M_1' - \ln(\lambda_2/\lambda_1) + \rho' \sqrt{H_1'}\sqrt{H_2'}}{\sqrt{H_1'}}, -\frac{M_2' - \ln L^* + H_2'}{\sqrt{H_2'}}, -\rho'\right). \tag{26}$$

4. Numerical Examples

According to the calculation results of two kinds of dual-currency power exchange options, we assign the related parameters in the pricing formula and simulate a numerical case to illustrate how some specific factors affect the price of power exchange options.

Firstly, we consider the first characteristic of power exchange options, the exponential factor of magnification, and analyze how the dual-currency power exchange options with exchange rate and default risk will react with the change of option time under the influence of exponential power. Secondly, in order to explore what kind of influence is taken by the jump risk, exchange rate risk, and default risk, respectively, we increase the special circumstances as the comparison sample: no jump risk fragile dual-currency power exchange option, without the fragility of the exchange rate risk jumping power exchange option, and without the risk of default dual-currency bouncing power exchange options, according to the two differences between them were analyzed and determine each factor's influence on power exchange option price direction and the amplitude of the influence. Finally, for the basic model of power exchange options with the dual risk of exchange rate and default with the jump process studied in this paper, the influence of the change of important parameters related to the jump process on the price of options is considered.

As a reference standard for numerical simulation test, Table 1 summarizes the assignment of selected basic parameters. The values of the following parameters are mainly derived from those of Wang et al. [12], which are commonly used in the literature by Bakshi et al. [21] and Christoffersen et al. [22]. Without loss of generality, we set the initial price of the underlying asset to be $S_1(0) = S_2(0) = 5.0$ and the domestic risk-free rate to be $r = 0.02$. Assume that the instantaneous volatility of the underlying asset to be $\sigma_1 = \sigma_2 = 0.15$ and the jump intensity to be $\lambda = 1$. Now, we should choose the parameters of the option seller's assets, and we assume instantaneous volatility $\sigma_V = 0.15$, the limit for a company to actually default is 3/4 of the initial value of the seller's assets $V(0)$; that is to say, in the following example, the default barrier $L^* = 7.5$, and Chen [23] found in the literature that the bond recovery rate of 9 different states was about 0.60, so we set the self-weight cost below to be $\alpha = 0.40$. Finally, the option expiration date is assumed to be $T = 1.0$. In Figures 1 and 2 and Table 1, we will change the parameter values accordingly to research the impact of exchange rate risk, counterparty risk, and jump risk on the option price. Other variables maintain the values listed in Table 1.

At the same time, to confirm the value of n, we should analyze the convergence of the series $\sum_{n=0}^{\infty}((\lambda^{\widehat{Q}}T)^n/n!)$. By using MATLAB, the following calculation is chosen for accuracy 0.001.

Next, we analyze the exponential sensitivity of power exchange options. The basic operations are set the expiration time as a variable on the premise that the data in the above parameter table remain unchanged. Change index term of power exchange options under the triple risk of exchange rate, volatility risk, counterparty default risk, and jump risk. Specifically, the exponential coefficients are set to be $\alpha_1 = \alpha_2 = 1; 1.5; 2; 2.5$. Then, observe the relative position of the four curves.

Figures 1 and 2, respectively, represent the price changes of power exchange options with different index terms related to expiration time T in two types of dual-currency power exchange options. From the trend, the option price increases with the expiration time T. However, due to the difference in the magnification factor of the index term, when the option α becomes larger, it is more sensitive to the expiration time T, which is what we understand the function of leverage in economic terms. In a benign financial environment, index power changes the structure of returns and effectively hedges risks. However, in a vicious financial environment, the higher the leverage, the more losses will be multiplied. If not timely controlled, the single high-leverage behavior can even cause irreconcilable harm to the entire financial market.

By comparing Figures 1 and 2, no matter which index value is selected, we find that, at time 0, the price of the power exchange option of the first type of dual currency is higher than the price of the power exchange option of the second type of dual currency. However, the second kind of

Table 1: Parameter assignment table.

Parameter	Assignment
Initial price $S_1(0)$	5.0
Constant coefficient λ_1	1
Index coefficient α_1	2
Volatility σ_1	0.15
Domestic risk-free rate r_d r_d	0.02
Initial rate X	1
Initial asset V	10
Deadweight loss α	0.4
Material default limit L^*	7.5
Creditor's rights L	7.5
Correlation coefficient ρ_{1V}	0.5
Maturity date T	1
Intensity of jump λ	1
Step of jump μ_X	0
Standard deviation γ_1	0.1
Standard deviation γ_2	0.1
Initial price $S_2(0)$	5.0
Constant coefficient λ_2	1
Index coefficient α_2	2
Volatility σ_2	0.15
Foreign risk-free rate r_f	0.03
Volatility σ_X	0.15
Volatility σ_V	0.15
Correlation coefficient ρ_{1X}	0.5
Correlation coefficient ρ_{VX}	0.2
Correlation coefficient ρ_{2X}	0.5
Correlation coefficient ρ_{2V}	0.5
Step of jump μ_1	0
Step of jump μ_2	0
Step of jump μ_V	0
Standard deviation γ_X	0.1
Standard deviation γ_V	0.1

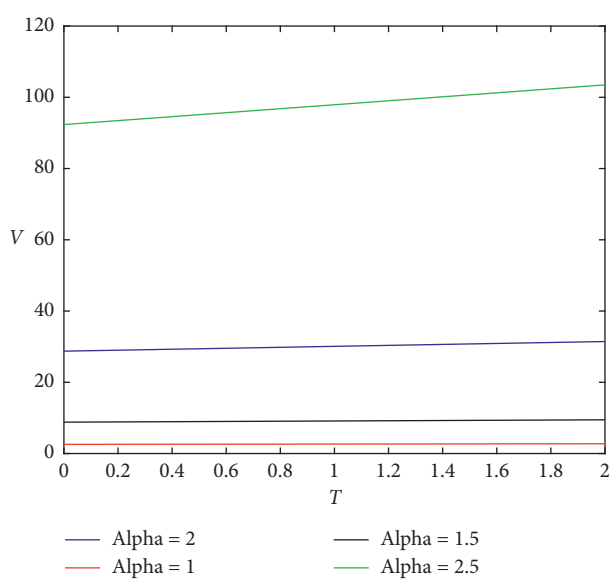

Figure 1: Exponential change of the first kind of dual-currency power exchange option price and expiration time.

the dual-currency power exchange option is more sensitive than the first kind of the dual-currency power exchange option.

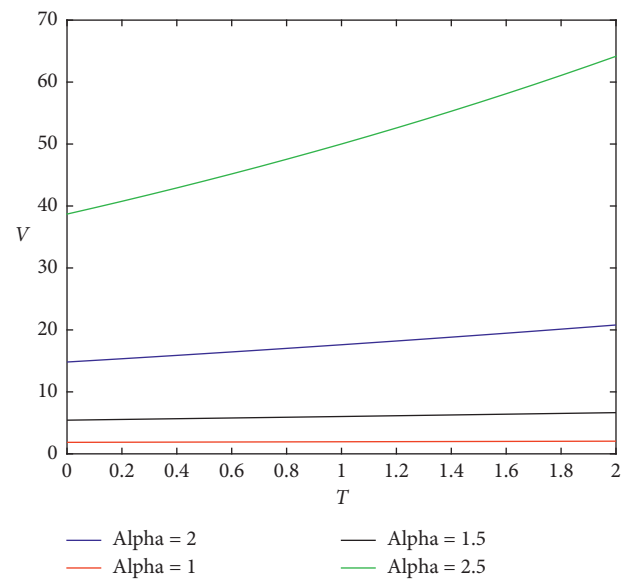

Figure 2: Exponential change of the second kind of dual-currency power exchange option price and expiration time.

Lines in Figures 3–6, black, green, red, and blue lines, represent the exchange rate, the risk of default, jumping, three kinds of standard power exchange option, no jump risk, power exchange option, without the risk of default power exchange option, and exchange rate risk of power exchange option of option value with the changing trend of related parameters.

Figures 3 and 4, respectively, represent the changing relationship between the value and expiration time of power swap options in two types of dual currencies. The overall trend is that, regardless of which of the three risks is removed, the option price increases with the expiration time.

In Figure 3, in the event of a market impact, compared with no jump risk, green line, black line, and red line, blue line all had greater growth, which is similar to our conjecture. As an example, in March 2018, when the trade war between China and the United States was launched, the market sentiment was depressed, and the global stock market fell, and the Shanghai stock index immediately fell below 3200. Many asset prices were suddenly impacted. Adding the same situation to our research when some unforeseen major events occur, it will not only cause the price of the underlying assets to soar or plummet but also take huge fluctuation to the value of the company, even cause frequent fluctuation of exchange rate. By the combined influence of these risk factors, the option price is bound to be higher than the option price without jump risk. A similar conclusion can be drawn from Figure 4, that is, the occurrence of major unexpected events will lead to the increase of option price.

Figures 5 and 6 show the impact of jump intensity λ on the option price. In terms of the overall trend, except for the nonjump risk model, the jump intensity all influences the other three models to some extent. However, despite the positive or negative impact of jump intensity on the option price, the impact range is almost zero. According to the

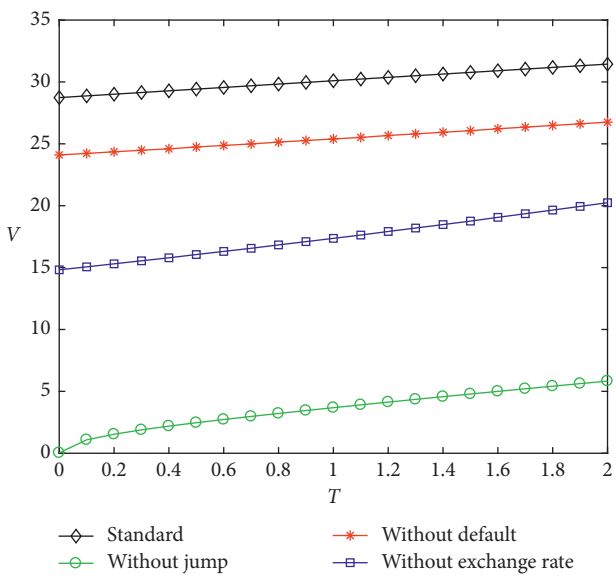

Figure 3: The relationship between the expiration time and the price of the power exchange option of the first kind in dual currency.

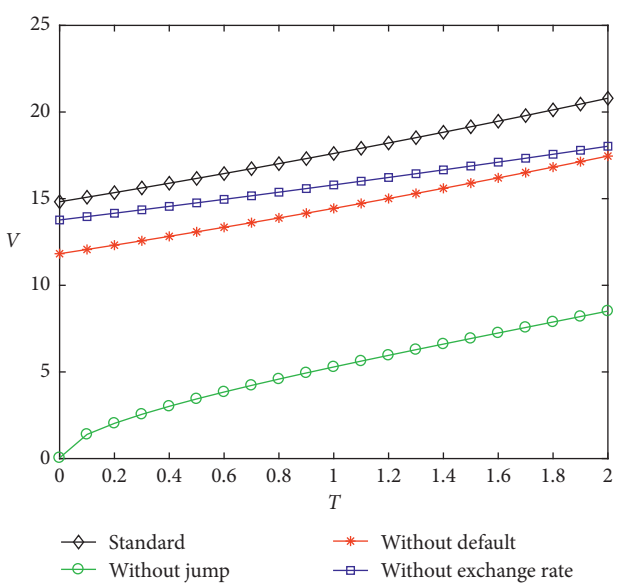

Figure 4: The relationship between the expiration time and the price of the power exchange option of the second kind in dual currency.

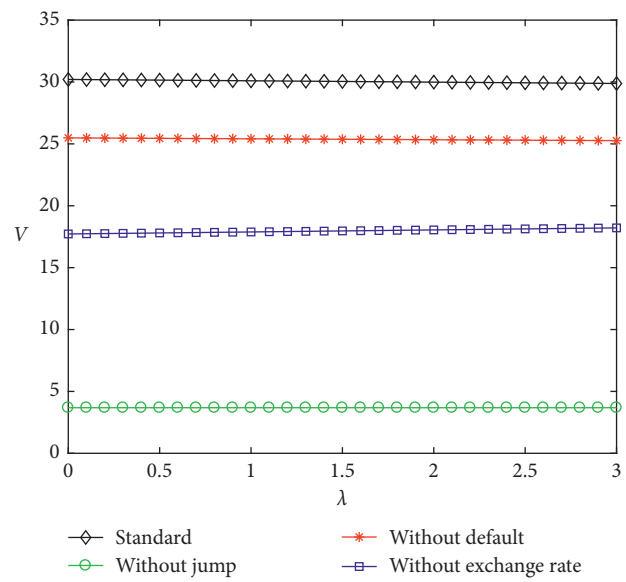

Figure 5: The relationship between jump strength and power exchange option price of the first kind in dual currency.

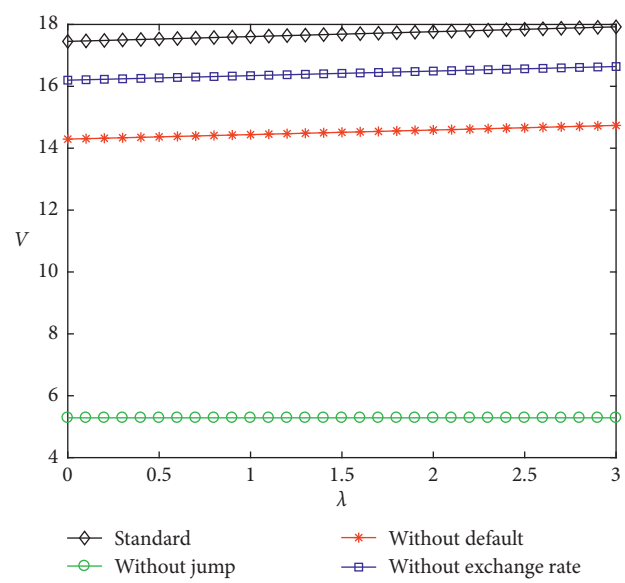

Figure 6: The relationship between jump strength and power exchange option price of the second kind in dual currency.

differences in Figures 5 and 6, for the first type of dual-currency option model, the jump intensity has a negative impact on the option price due to the change of exchange rate; for the second kind of dual-currency option model, the price of the power exchange option increases slightly with the increase of jump intensity.

To sum up, through numerical examples, we make some simple verification of the pricing formula and get the following conclusions:

(1) With the change of index term of power exchange option, the price of the power exchange option can more sensitively follow the change of relevant factors.

(2) Exchange rate risk, counterparty default risk, and jump risk all increase the price of power swap options, but their effects are different. The biggest impact is the jump risk, which is the impact of major events, which makes the price curve of power exchange options rise almost in parallel, which is consistent with the impact of major international relation events on cross-border transactions we analyzed.

(3) When other factors remain unchanged, the jump intensity makes the power exchange option price fluctuate up and down, but the fluctuation range is limited.

Data Availability

The numerical simulation data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

Authors' Contributions

The manuscript was written through the contributions of all authors. All authors read and approved the final manuscript.

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