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Research Article

An Improved Nonhomogeneous Discrete Grey Model and Its Application

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In recent years, the nonhomogeneous grey model has received much attention owing to its flexibility and applicability of forecasting small samples. To improve further the prediction accuracy of the nonhomogeneous grey model, this paper is to introduce a new whitening equation with variable coefficient into the original nonhomogeneous grey model, which is abbreviated as ONGM$(1, 1, k, c)$. First of all, the detailed computational steps of the time response function of the novel model and the restored values of the raw data sequence are deduced through grey modelling techniques. Secondly, two empirical examples from the previous literature are conducted to prove the validity of the novel model. Finally, the novel model is applied to forecast natural gas demand of China, and the results show that the novel model has a better prediction performance compared with other commonly used grey models, including GM$(1, 1)$, DGM$(1, 1)$, NGM$(1, 1, k, c)$, and NGBM$(1, 1)$.

1. Introduction

Grey prediction model is one of the key branches in grey system theory [1], which has been widely used in various fields, including industry [2, 3], engineering [4, 5], and especially energy [6–8] on account of its abilities of sufficiently dealing with small sample-sized problems. Besides, many extensive models of the benchmark grey model have been developed in practical applications. For example, Xie and Liu [9] investigated the discrete grey model which is abbreviated as DGM$(1, 1)$; in their work, they also revealed the connection of DGM$(1, 1)$ to the traditional grey model. Kumar and Jain [10] utilized grey Markov, grey model with the rolling mechanism and singular spectrum analysis to forecast energy consumption in India. Hamzacebi et al. [11] presented an improved grey model through combining iterative and direct grey forecasting approaches in order to predict electricity demand of Turkey. Ding et al. [12] combined a new initial condition and rolling mechanism to improve the prediction performance of the traditional grey model and then employed the model to forecast electricity consumption of China. Chen et al. [13] put forward a grey approach to achieve time-series interval forecasting. Inspired by existing approaches to forecasting time-series intervals [13], Wu and Zhang [14] further optimized the grey method for forecasting time-series intervals. Ma and Liu [15] raised a novel time-delayed polynomial grey model and applied it to forecast natural gas consumption of China. Intharathirat et al. [16] used multivariate grey models to forecast municipal solid waste quantity in developing countries.

However, it should be noticed that these above models are all linear, which are unapplicable for nonlinear problems. As a result, Chen et al. [17] firstly propounded a novel nonlinear grey Bernoulli model by introducing the Bernoulli equation into the grey action quality. Afterward, Wang et al. [18] proposed an improved nonlinear grey Bernoulli model which is based on optimization of background value. Under the inspiration of the literature [18], Wu et al. [19] investigated a new approach to improve the prediction performance by simultaneously considering the new initial condition and dynamic background value. Nguyen et al. [20]

proposed a nonlinear grey Bernoulli model based on Fourier transformation. Ma et al. [21] proposed a novel nonlinear multivariable grey Bernoulli model to forecast tourism income of China, among them. It is worth noting the accumulation appeared in these models is all integer-order accumulation that impairs the prediction accuracy of forecasting models. For this, Wu et al. [22] originally proposed a grey model with fractional-order accumulation. Later, they also proposed a novel fractional multivariate grey model to forecast electricity consumption of Shandong Province [23]. Considering that computation of the existing fractional accumulation is complicated, therefore, Ma et al. [24] introduced a novel definition of conformable fractional accumulation, which is helpful in developing grey forecasting models.

Besides, the nonhomogeneous grey model, proposed by Cui et al. [25], also has received extensive attention owing to its ease of construction and flexibility. Chen and Yu [26] proposed a novel nonhomogeneous grey model in which the grey action quality was replaced by $bt + c$. Meanwhile, Ma et al. [27] investigated a novel kernel regularized nonhomogeneous grey model. Zeng et al. [28] developed a self-adaptive intelligence grey model. Based on the aforementioned knowledge, this paper introduces a new whitening equation with variable coefficient into the nonhomogeneous grey model to further enhance the prediction ability of the traditional nonhomogeneous grey model, which is abbreviated ONGM$(1,1,k,c)$. The novelties of this paper are drawn as follows: (i) a new whitening equation with variable coefficient is considered into the nonhomogeneous grey model; (2) the time response function and the restored values of the original series are deduced in detail; (3) the two empirical examples are used to validate the effectiveness of the novel model; and (4) the novel model is applied in natural gas demand of China.

The rest of this paper is organized as follows: Section 2 briefly depicts the NGM$(1,1,k,c)$ model. In Section 3, the modelling procedure of the novel model is discussed in detail. Section 4 provides two empirical examples to validate the novel model. Section 5 utilizes the novel model to forecast natural gas demand of China, and the main conclusions are listed in Section 6.

2. Description of the Traditional Nonhomogeneous Grey Model

To further improve the prediction performance of the NGM$(1,1)$ model, Chen and Yu presented the nonhomogeneous discrete grey model, namely, NGM$(1,1,k,c)$. The computational steps of NGM$(1,1,k,c)$ are described as follows.

Step 1. Assume a nonnegative series to be

$$X^{(0)} = \left(x^{(0)}(1), x^{(0)}(2), \ldots, x^{(0)}(n)\right), \tag{1}$$

and then the first-order accumulative generating operation $((1-\mathrm{AGO}))$ series is

$$X^{(1)} = \left(x^{(1)}(1), x^{(1)}(2), \ldots, x^{(1)}(n)\right), \tag{2}$$

where $x^{(0)}(k) = \sum_{i=1}^{k} x^{(0)}(1)$.

Step 2. The whitening equation of NGM$(1,1,k,c)$ can be given by

$$\frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t} + ax^{(1)}(t) = bt + c, \tag{3}$$

where a and $bt + c$ represent the development parameter and grey action quality, respectively.

Step 3. The discrete form of equation (3) is easily written as

$$x^{(0)}(k) + az^{(1)}(k) = bk + c, \tag{4}$$

where $z^{(1)}(k)$ is called the background value and $z^{(1)}(k) = 0.5\left(x^{(1)}(k) + x^{(1)}(k-1)\right)$.

Step 4. The model parameters a, b, and c can be estimated through the least square method; there is

$$(a,b,c)^T = \left(B^T B\right)^{-1} B^T Y, \tag{5}$$

where

$$B = \begin{pmatrix} -z^{(1)}(2) & 2 & 1 \\ -z^{(1)}(3) & 3 & 1 \\ \vdots & \vdots & \vdots \\ -z^{(1)}(n) & n & 1 \end{pmatrix}, Y = \left(x^{(0)}(2), x^{(0)}(3), \ldots, x^{(0)}(n)\right)^T. \tag{6}$$

Step 5. The time response function of equation (3) with $x^{(1)}(1) = x^{(0)}(1)$ is given by

$$\hat{x}^{(1)}(k) = \left(x^{(0)}(1) - \frac{b}{a} - \frac{c}{a} + \frac{b}{a^2}\right)e^{-a(k-1)} + \frac{b}{a}t + \frac{c}{a} - \frac{b}{a^2}. \tag{7}$$

Step 6. The restored value of $x^{(0)}(k), k = 2, 3, \cdots$, can be acquired by using the first-order inverse accumulative generating operation $(1-\mathrm{IAGO})$; there is

$$\hat{x}^{(0)}(k) = \hat{x}^{(1)}(k) - \hat{x}^{(1)}(k-1). \tag{8}$$

3. Presentation of the ONGM$(1,1,k,c)$ Model

Definition 1. Let $u > 0$; the whitening equation of a novel grey forecasting model is

$$\frac{\mathrm{d}x^{(1)}(t)}{\mathrm{d}t^u} + ax^{(1)}(t) = bt + c. \tag{9}$$

It is worth noting that equation (9) will be degenerated to equation (3) when $u = 1$. In other words, the novel model is a more general form of NGM$(1,1,k,c)$.

Theorem 1. *The discrete formula of the whitening equation of the novel model is*

$$x^{(0)}(k) + az_1^{(1)}(k) = b\left(\frac{u}{u+1}\left(k^{u+1} - (k-1)^{u+1}\right)\right) + c\left(k^u - (k-1)^u\right), \tag{10}$$

where $z_1^{(1)}(k) = 1/2\left(uk^{u-1} + u(k-1)^{u-1}x^{(0)}(k-1)\right)$.

Proof. According to equation (9), it is easily yielded

$$\frac{dx^{(1)}(t)}{dt} + aut^{u-1}x^{(1)}(t) = but^u + cut^{u-1}. \tag{11}$$

Integrating both sides of equation (11) over the interval $[k-1, k]$,

$$\int_{k-1}^{k} dx^{(1)}(t) + \int_{k-1}^{k} aut^{u-1}x^{(1)}(t)dt = \int_{k-1}^{k} but^u dt + \int_{k-1}^{k} cut^{u-1}dt. \tag{12}$$

Using the two-point trapezoidal formula, equation (12) becomes

$$x^{(0)}(k) + az_1^{(1)}(k) = b\left(\frac{u}{u+1}\left(k^{u+1} - (k-1)^{u+1}\right)\right) + c\left(k^u - (k-1)^u\right). \tag{13}$$

This completes the proof.

Similar to Section 2, the model parameters of the novel model can be given by

$$(a,b,c)^T = \left(H^T H\right)^{-1} H^T Y, \tag{14}$$

where

$$H = \begin{pmatrix} -z_1^{(1)}(2) & \frac{u}{u+1}\left(2^{u+1} - 1^{u+1}\right) & 2^u - 1^u \\ -z_1^{(1)}(3) & \frac{u}{u+1}\left(3^{u+1} - 2^{u+1}\right) & 3^u - 2^u \\ \vdots & \vdots & \vdots \\ -z_1^{(1)}(n) & \frac{u}{u+1}\left(n^{u+1} - (n-1)^{u+1}\right) & n^u - (n-1)^u \end{pmatrix}. \tag{15}$$

□

Theorem 2. *Given the model parameters a, b, and c of the novel model, the corresponding time response function can be given by*

$$x^{(1)}(t) = e^{-at^u}\left(e^a x^{(0)}(1) + b\int_1^k f(u,\tau)d\tau + \frac{c}{a}\left(e^{at^u} - e^a\right)\right). \tag{16}$$

Proof. Multiply both sides of equation (9) by e^{at^u}:

$$e^{at^u}\left(\frac{dx^{(1)}(t)}{dt} + aut^{u-1}x^{(1)}(t)\right) = e^{at^u}\left(but^u + cut^{u-1}\right). \tag{17}$$

That is,

$$\frac{de^{at^u}x^{(1)}(t)}{dt} = e^{at^u}\left(but^u + cut^{u-1}\right). \tag{18}$$

Integrating both sides of equation (18) over the interval $[1, t]$,

$$\int_1^t d\left(e^{a\tau^u}x^{(1)}(\tau)\right) = \int_1^t e^{a\tau^u}\left(bu\tau^u + cu\tau^{u-1}\right)d\tau. \tag{19}$$

Furthermore, by simplifying equation (19),

$$x^{(1)}(t) = e^{-at^u}\left(e^a x^{(0)}(1) + b\int_1^k f(u,\tau)d\tau + \frac{c}{a}\left(e^{at^u} - e^a\right)\right), \tag{20}$$

where $f(u,t) = e^{at^u}$.

In particular, $f(u,t)$ can be approximately calculated by the following equation:

$$\int_1^t f(u,\tau)d\tau \approx \sum_{i=1}^{1000} \triangle_i f(u,\varepsilon_i), \tag{21}$$

where $\triangle_i$ is taken as 0.001 for any i.

It is proved. □

4. Validation of the ONGM$(1,1,k,c)$ Model

This section provides two empirical examples from the previous literature to demonstrate the effectiveness of the novel model; the competitive models including the traditional grey model (GM(1, 1)), the discrete grey model (DGM(1, 1)), the nonlinear grey Bernoulli model (NGBM(1, 1)), and the NGM$(1,1,k,c)$ model are established thereof. To assess the prediction accuracies of these models, two statistical indices are used, which are the root mean square error (RMSE) and the mean absolute percentage error (MAPE):

$$\text{RMSE} = \sqrt{\frac{1}{n-1}\sum_{i=2}^{n}(e(i))^2}, \tag{22}$$

$$\text{MAPE} = \frac{1}{n-1}\sum_{i=2}^{n}\frac{|e(i)|}{x^{(0)}(i)} \times 100\%, \tag{23}$$

where $e(i)$ is the simulative error and $e(i) = \hat{x}^{(0)}(i) - x^{(0)}(i)$. The MAPE criteria of forecasting accuracies of these models are also given in Table 1.

Case 1. The data from paper [29] are taken as an example to verify the novel model, which describes wind turbine capacity of Europe from 2007 to 2017, the data from 2007 to 2015 are used to build prediction models, and the left two samples are used to examine the accuracies of these models. The prediction results by these models are given in Table 2,

Table 1: MAPE criteria for forecasting accuracy.

MAPE (%)	<10	10–20	20–50	>50
Forecasting ability	Excellent	Good	Reasonable	Weak

Table 2: The results of five grey models in wind turbine capacity of Europe (megawatts).

Year	Raw data	GM(1, 1) $a = -0.11$ $b = 58682.19$	DGM(1, 1) $\beta_1 = 1.12$ $\beta_2 = 62146.52$	NGBM(1, 1) $\gamma = 0.10$ $a = -0.11$ $b = 51933.42$	NGM(1, 1, k, c) $a = -0.04$ $b = 6967.69$ $c = 47718.09$	ONGM(1, 1, k, c) $u = 1.19$ $a = -0.04$ $b = 452.29$ $c = 46501.76$
2007	56748.89					
2008	64943.48	68733.07	68819.11	68053.13	62092.57	64959.03
2009	77019.99	76808.49	76910.91	76293.36	72082.88	76284.64
2010	86721.97	85832.70	85954.16	85366.11	82532.46	86889.65
2011	96603.13	95917.15	96060.72	95432.01	93462.41	97623.26
2012	109884.87	107186.43	107355.62	106629.26	104894.83	108856.78
2013	120994.68	119779.73	119978.58	119099.88	116852.81	120818.76
2014	133915.44	133852.61	134085.77	132997.36	129360.50	133684.27
2015	147637.65	149578.91	149851.68	148490.51	142443.19	147607.95
RMSE		1877.41	1880.05	1909.43	4325.85	586.52
MAPE (%)		1.58	1.54	1.79	4.22	0.44
2016	161939.87	167152.88	167471.36	165766.36	156127.30	162739.02
2017	178314.15	186791.62	187162.78	185032.76	170440.49	179229.35
RMSE		7037.15	7378.88	5467.26	6920.28	859.14
MAPE (%)		3.99	4.19	3.07	4.00	0.50

and curves of raw data and simulated and predicted values are graphed in Figure 1.

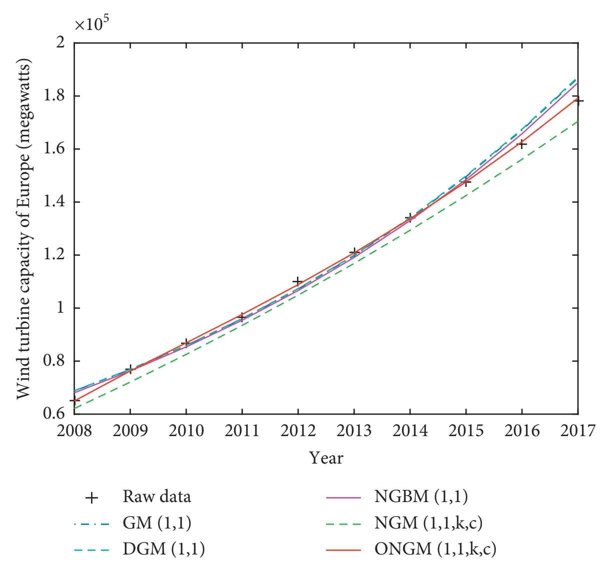

Figure 1: Curves of raw data and predicted values using the five grey system models.

Case 2. The data from paper [30] are taken as the second example which depicts the carbon dioxide emissions of India from 2000 to 2018. Similar to Case 1, the first 17 data are used to build the prediction models, and the other data are used to examine the prediction accuracies of these models. Table 3 lists the prediction results, and Figure 2 exhibits the curves of raw data and predictive values.

From the above computational results, the desired conclusions can be drawn as follows:

In Table 2, for simulated period, the RMSE values of competing models including GM(1, 1), DGM(1, 1), NGBM(1, 1), NGM(1, 1, k, c), and the proposed model are 1877.41, 1880.05, 1909.43, 4325.85, and 586.52; the MAPE values of these models are 1.58%, 1.54%, 1.79%, 4.22%, and 0.44%, respectively. For predicted period, the RMSE values of competing models are 7037.15, 7378.88, 5467.26, 6920.28, and 859.14; the MAPE values of these models are 3.99%, 4.19%, 3.07%, 4.00%, and 0.50%, respectively. It is seen that the proposed model outperforms among these models owing to its lowest RMSE value and MAPE value either in simulated period or in predicted period. In Figure 1, it is known that simulated and predicted values are more close to actual data.

In Table 3, for simulated period, the RMSE values of these models are 30.83, 30.86, 31.66, 25.37, and 19.83; the MAPE values of these models are 1.59%, 1.60%, 2.03%, 1.50%, and 0.99%, respectively. For predicted period, the RMSE values of these models are 105.93, 107.18, 83.74, 50.7323, and 35.25; the MAPE values of them are 4.42%, 4.47%, 3.48%, 1.93%, and 1.09%, respectively. It is obviously found that the proposed model has a better prediction performance with the lowest RMSE values and MAPE values both in simulated period and predicted period. From Figure 2, it is

Table 3: The results of five grey models in carbon dioxide emissions of India (million tons).

Year	Raw data	GM (1, 1) $a = -0.06$ $b = 867.30$	DGM (1, 1) $\beta_1 = 1.06$ $\beta_2 = 894.24$	NGBM (1, 1) $\gamma = 0.01$ $a = -0.06$ $b = 802.75$	NGM $(1, 1, k, c)$ $a = -0.04$ $b = 31.90$ $c = 834.26$	ONGM $(1, 1, k, c)$ $u = 0.73$ $a = -0.09$ $b = 84.41$ $c = 1011.42$
2000	962.5					
2001	970.3	969.25	969.62	956.09	934.76	970.10
2002	1021.9	1026.60	1027.00	1016.52	1002.39	1000.66
2003	1062.3	1087.34	1087.78	1078.93	1072.60	1060.32
2004	1116.6	1151.68	1152.15	1144.22	1145.44	1131.70
2005	1204.6	1219.83	1220.33	1212.86	1221.03	1209.57
2016	1252.5	1292.01	1292.55	1285.19	1299.46	1291.80
2007	1365.5	1368.46	1369.04	1361.51	1380.84	1377.38
2008	1466.9	1449.43	1450.05	1442.12	1465.28	1465.78
2009	1595.6	1535.20	1535.86	1527.29	1552.90	1556.70
2010	1661.0	1626.03	1626.79	1617.32	1643.81	1650.01
2011	1735.7	1722.24	1723.01	1712.50	1738.15	1745.61
2012	1849.2	1824.15	1824.98	1813.15	1836.03	1843.46
2013	1930.0	1932.09	1932.98	1919.60	1937.60	1943.56
2014	2083.3	2046.41	2047.36	2032.19	2042.96	2045.91
2015	2147.8	2167.50	2168.52	2151.29	2152.34	2150.54
2016	2234.2	2295.75	2296.85	2277.28	2265.80	2257.47
RMSE		30.83	30.86	31.66	25.37	19.83
MAPE (%)		1.59	1.60	2.03	1.50	0.99
2017	2316.9	2431.59	2432.77	2410.57	2383.53	2366.75
2018	2479.1	2575.47	2576.73	2551.58	2505.70	2478.42
RMSE		105.93	107.18	83.74	50.7323	35.25
MAPE (%)		4.42	4.47	3.48	1.93	1.09

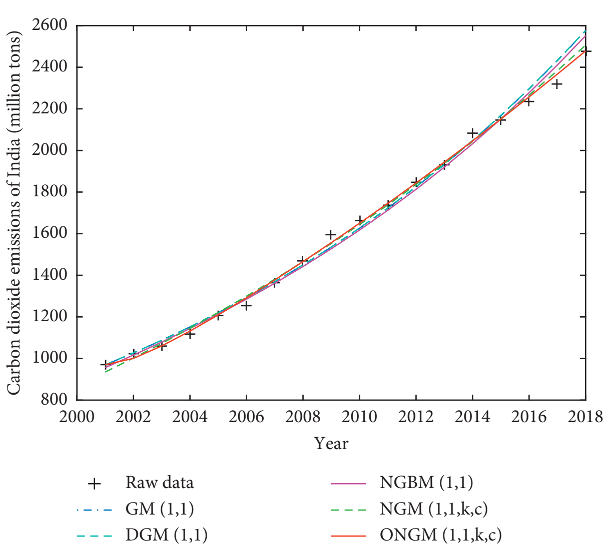

Figure 2: Curves of raw data and predicted values using the five grey system models.

easily seen that the predicted values of the carbon dioxide emissions of India are more close to actual data.

In summary, the proposed model outperforms among these competing models listed in this paper comparing the prediction accuracies of these models. Due to the feasibility of the proposed model, it will be used to predict China's natural gas demand in the next section.

5. Application

Natural gas plays an important role in the energy market on account of its high-quality, efficacy, and low-carbon feature. Therefore, development of natural gas has been written in the 13th Five Plan of China which is the most important official policy in China.

This paper collects the data of natural gas demand of China from 2001 to 2013 (cf. [28]), as shown in Table 4. The first ten data are used for building the novel model and other competing models, and the left 2 samples are used to examine the prediction accuracies of these models.

Furthermore, the simulated and predicted values of natural gas demand of China and statistical indices by the five grey models are listed in Table 5.

Ignoring the first item of predicted value, the simulated and predicted values are shown in Table 5, and these predicted values are graphed in Figure 3. It should be notable that the RMSE value of competitive models including GM (1, 1), DGM (1, 1), NGBM (1, 1), NGM $(1, 1, k, c)$, and the novel model is 30.83, 30.86, 31.66, 25.37, and 19.83; the MAPE values of these models are 1.59%, 1.60%, 2.03%, 1.50%, and 0.99%, respectively. What is more, the forecasting ability in predicted period is fact more important; for the predicted stage, the RMSE values of these models are 15.07, 15.30, 13.91, 3.40, and 2.80; the MAPE values of them are 13.19%, 13.39%, 12.18%, 2.24%, and 1.97%, respectively. It is obviously seen that the proposed model has best prediction performance than that of other grey system models. From the perspective of plotting curves of raw data and

Table 4: Natural gas demand of China from 2003 to 2013 (billion cubic meters).

Year	Raw data	Year	Raw data
2003	35.0	2009	85.2
2004	41.5	2010	94.8
2005	49.3	2011	103.1
2016	58.6	2012	107.2
2007	69.2	2013	119.3
2008	80.3		

Table 5: The results of five grey models in natural gas demand of China (billion cubic meters).

Year	Raw data	GM (1, 1) $a = -0.11$ $b = 9723.73$	DGM (1, 1) $\beta_1 = 1.12$ $\beta_2 = 10312.95$	NGBM (1, 1) $\gamma = 0.05$ $a = -0.10$ $b = 5987.40$	NGM (1, 1, k, c) $a = 0.08$ $b = 3477.01$ $c = 3169.31$	ONGM (1, 1, k, c) $u = 1.03$ $a = 0.07$ $b = 3155.37$ $c = 4964.02$
2003	35.0					
2004	41.5	46.18	46.29	45.74	35.15	40.69
2005	49.3	52.07 GM (1, 1)	52.19	51.73	45.04	50.32
2016	58.6	58.70	58.83	58.40	54.65	59.78
2007	69.2	66.19	66.32	65.87	63.98	68.95
2008	80.3	74.61	74.77	74.24	73.03	77.85
2009	85.2	84.12	84.30	83.66	81.83	86.50
2010	94.8	94.84	95.02	94.24	90.36	94.92
2011	103.1	106.92	107.13	106.16	98.65	103.13
RMSE		3.29	3.30	3.23	5.06	1.17
MAPE (%)		4.19	4.26	4.15	7.53	1.39
2012	107.2	120.55	120.77	119.55	106.70	111.14
2013	119.3	135.91	136.15	134.62	114.52	118.98
RMSE		15.07	15.30	13.91	3.40	2.80
MAPE (%)		13.19	13.39	12.18	2.24	1.97

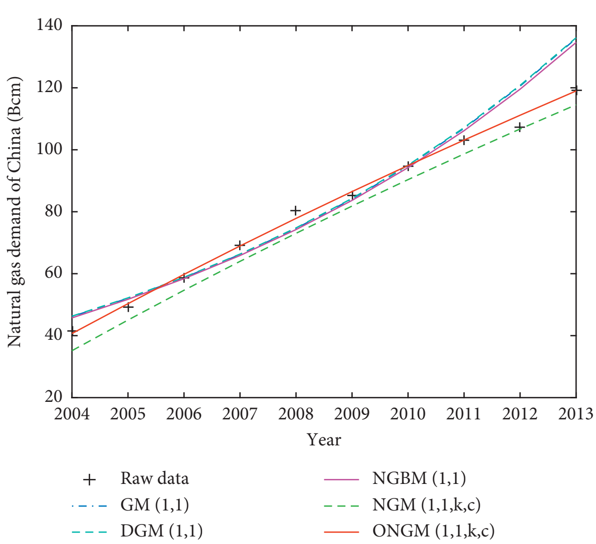

Figure 3: Curves of raw data and predicted values using the five grey system models.

simulated and predicted values, it is shown that the fitting values of the proposed model are more close to actual data. Therefore, the proposed model can improve the prediction performance significantly and should be regarded as an appreciate method to predict natural gas demand of China.

6. Conclusion

Aimed to improve further the prediction performance of the nonhomogeneous grey model, a new whitening equation with variable coefficient is introduced into the nonhomogeneous grey model; as a result of this paper, ONGM $(1, 1, k, c)$ is proposed. The main contributions of this paper can be summarized as follows:

(1) A new whitening equation with variable coefficient is introduced into the traditional nonhomogeneous grey model to further generalize the mathematical form of the nonhomogeneous grey model.

(2) By using grey modelling techniques, the time response function of the novel model and restored values of the original series are deduced in detail.

(3) Two empirical examples are used to prove the validity of the novel, and the novel model is applied to predict natural gas demand of China. The results indicate that the novel model has a better prediction performance compared with other competitive models.

Up to this point, the advantages of the proposed model have been discussed. However, there still exist some issues, which we should concentrate and solve. For instance, the ONGM $(1, 1, k, c)$ model is regarded as a kind of univariate model, that is, the other influential factors could be ignored in applications, and the multivariable model with shape parameter will be discussed in our future work; the fractional-order accumulation should also be introduced in this study to further enhance the prediction performance.

Data Availability

Raw data used for this work can be found in the existing literature.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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