In view of the incipient fault characteristics are difficult to be extracted from the raw bearing fault signals, an incipient bearing fault diagnosis method based on parameter-adaptive variational mode decomposition (VMD) is proposed. The beetle antennae search (BAS) algorithm is adopted to seek for the optimal combination of the VMD parameters. The reciprocals of the calculated kurtosis values of intrinsic mode functions (IMFs) decomposed via VMD are employed as a fitness function in the searching process. The optimal mode number and the quadratic penalty term of VMD are adaptively set after the search. Afterwards, a vibration signal is decomposed into a set of IMFs using the parameter-adaptive VMD, and the IMF with the maximal kurtosis value is selected as the sensitive one. The selected IMF is further analyzed by Hilbert envelope demodulation. The resulting envelope spectrum can show the significant fault impulse characteristics which are highly helpful to diagnose incipient bearing faults. The kurtosis and the proportion of fault energy are introduced as the input vector of the extreme learning machine (ELM). Comparisons have been conducted via ELM to evaluate the performance by using EMD and the fixed-parameter VMD. The experimental results demonstrate that the proposed method is more effective in extracting the incipient bearing fault characteristics.

1. Introduction

Bearing is one of the most critical components in rotating machinery. The presence of defects in the bearing may lead to noise, vibration, or even system breakdown. For this reason, the bearing fault diagnosis has received considerable attention in the past decades.

It has become a widely acceptable method to assess the bearing performance by studying the vibration signals. As a result, this method has been adopted in many fields, such as bearing fault diagnosis [1–3], performance evaluation [4, 5], and equipment health monitoring [6–8].

However, the bearing vibration signals are ordinarily nonstationary and may be polluted by noise interference [6, 9]. Additionally, the measured fault characteristic frequencies usually could not exactly match the theoretically values due to geometry imperfection, speed variation, or load variation [10], and so on. All of these factors perplex the extraction of fault characteristics from the raw signals.

Bearing fault diagnosis is mainly applied in two fields which are the quality control in bearing manufactories and the condition monitoring of the bearing in service, respectively. This paper is based on the former application and focuses on the fault diagnosis techniques. In this case, the bearing faults are mostly characterized as incipient faults which mean that the faults are slight or tiny, and the characteristics are not obvious. It gets difficult to accurately recognize and diagnose the incipient faults. At the same time, the measurements suffer little negative influence introduced by other mechanical transmission parts such as gears. Bearing manufactories usually complete bearing vibration measurement on the dedicated vibration tester. The measurement conditions are that the outer ring of the bearing is stationary, the inner ring rotates together with the shaft at the specified speed, and the specified load is applied at the same time.

Empirical mode decomposition (EMD) [11, 12] is a well-known adaptive time-frequency processing method and can
recursively decompose signals without the prior knowledge. A large number of scholarly articles covering a variety of applications have been published. However, EMD suffers from some shortcomings, such as the limited mathematical understanding, mode mixing, pseudocomponents, and end effects [9].

Variational mode decomposition (VMD) is a novel and self-adaptive signal decomposition algorithm proposed by Dragomiretskiy and Zosso in 2014 [13]. In comparison with the recursive EMD method, VMD can decompose a signal into an ensemble of band-limited intrinsic mode functions (IMFs) synchronously. The literature [13–15] compared the performance of VMD and EMD and deduced that VMD outperformed EMD with regard to the noise robustness and the effectiveness of feature extraction.

However, a significant drawback of VMD is that the parameters, namely, the mode number M and the quadratic penalty term α, need to be set in advance. Unreasonable preset parameters may induce the loss of effective modes or the mixing of different components [16] and affect the subsequent feature extraction results. M should be specified based on the number of different frequency components contained in the raw signals, while α, namely, the mode frequency bandwidth control parameter, is determined based on each central frequency of the mode. They are correlated with each other. A large number of modes may result in redundant VMD information. Correspondingly, a small number of modes may lead to mode mixing in the VMD results. α is related to the performance of suppressing noise interference. As α decreases, the bandwidth of the mode tends to be wide, and the VMD results may involve more background noise. But, the VMD results are likely to be distorted if the bandwidth is too narrow [14]. Since the two parameters need to be specified in advance without any prior knowledge about the raw signals, it is difficult to guarantee the accuracy of the VMD results. Accordingly, seeking for the optimum of M and α that matches with the analyzed signals is the key to the VMD method.

In order to overcome the critical drawback, some studies have been conducted. Lian et al. [17] chose M according to a series of indicators including permutation entropy, extreme value in the frequency domain, kurtosis criterion, and energy loss coefficient. Liu et al. [18] estimated M by using the minimum redundancy maximum relevance. The above two methods in [17, 18] both optimized the value of M but neglected the influence of α on decomposition results. Based on the grasshopper optimization algorithm, Zhang et al. [19] proposed a method to obtain the optimum of M and α. Wang et al. [20] used multi-objective particle swarm optimization (MOPSO) to obtain the optimum of M and α, in which the symbol dynamic entropy and the power spectral entropy were selected as the objective function of MOPSO. In [16], an improved adaptive genetic algorithm was proposed to optimize the two parameters of VMD. The performance of several decomposition methods for the simulation signal was compared, and authors considered that the decomposition effect of VMD preceded those of other methods. In [19], the effective value range of α was preassigned in the interval of [1000, 10,000]. However, with separate optimization algorithm, our experimental results described in the following as well as the literature [16] indicate that the optimal α may lie out of the interval stated in [19], respectively.

Compared with the above key parameters, the noise tolerance τ and the convergence tolerance level ε have little influence on the decomposition results; thus, the default values in the original VMD method are usually adopted [19].

In this paper, a novel optimization algorithm called beetle antennae search (BAS) is employed to estimate the optimal M and α. BAS is a nature-inspired algorithm developed by Jiang and Li [21]. In [21], the global optimization performance of BAS was benchmarked on the Michalewicz function and the Goldstein-Price function, in which the numerical results validated the efficacy of this algorithm.

2. BAS Principles

BAS is a nature-inspired algorithm to solve the optimization problems, which mimics the detecting and searching behaviors of long-horn beetles. A beetle wobbles its two antennae to detect the odour while preying or finding mates, i.e., the beetle explores nearby area randomly using a pair of antennae. When the antenna in one side receives a higher concentration of odour, the beetle would turn to the direction towards the same side; otherwise, it would turn to the other side. Searching behavior of beetles may be formulated in a way which is associated with an objective function to be optimized.

The position of the beetle can be expressed as a vector \( x \) at \( t \)-th time instant \( (t = 1, 2, \ldots) \). \( f(x) \) is denoted as a fitness function which indicates the concentration of odour at position \( x \). The maximal value of \( f(x) \) corresponds to the source point of the odour, namely, the optimal solution of the function.

The random direction of searching behavior can be modeled as follows:

\[
\overrightarrow{b} = \frac{\text{rnd}(c, 1)}{\|\text{rnd}(c, 1)\|}
\]

where \( \text{rnd}(\cdot) \) presents a random function and \( c \) denotes the dimensions of the position. The barycenter coordinates of both right-hand and left-hand antennae can be then described in the following equation:

\[
\begin{align*}
\overrightarrow{x_r} &= \overrightarrow{x} + s \overrightarrow{b}, \\
\overrightarrow{x_l} &= \overrightarrow{x} - s \overrightarrow{b},
\end{align*}
\]

where \( \overrightarrow{x_r} \) and \( \overrightarrow{x_l} \) stand for the barycenter coordinate of the right-hand antenna and that of the left-hand antenna, respectively, and \( s \) is the sensing length. The value of \( s \) should be large enough to cover an appropriate searching area for fear of falling into local minimum points at the beginning and then attenuate with the iterations.

Considering the searching behavior, the iterative model of detecting behavior can be formulated as follows:

\[
\overrightarrow{x} = \overrightarrow{x}^{t-1} + s \overrightarrow{b} \text{ sign}(f(\overrightarrow{x_r}) - f(\overrightarrow{x_l})),
\]
where \( \text{sign}(\cdot) \) represents a sign function and \( \delta \) is the step size of searching. In consideration of convergence speed, \( \delta \) follows a decreasing function of \( t \). The initialization of \( \delta \) should be equivalent to the sensing length \( s \). As examples, the sensing length and the step size may be updated as follows:

\[
\begin{align*}
    s' &= 0.95s^{t-1} + 0.01, \\
    \delta' &= 0.95\delta^{t-1}. 
\end{align*}
\]

As aforementioned, the BAS algorithm can achieve efficient optimization without the acquisition of functional expression or the gradient of function. Compared with the particle swarm optimization algorithm [22, 23], the BAS algorithm only employs one individual, i.e., one beetle; thus, the computation is reduced.

### 3. Parameter-Adaptive VMD Method Based on BAS

In this section, a parameter-adaptive VMD method based on BAS for incipient bearing fault diagnosis is introduced. The basic idea of the proposed method is to seek for the optimal \( M \) and \( \alpha \) by using the BAS algorithm.

Kurtosis criterion [24–26] has been developed based on the resonance demodulation technique. As a dimensionless index, the kurtosis can be used as a measure of impact components in the signals. The impact impulses caused by defect will increase the kurtosis obviously. Dong [27] introduced the spectral \( L_p/L_q \) norm and deduced that the kurtosis could be explained as a special case of the spectral \( L_p/L_q \) norm. Dong drew a conclusion that the spectral \( L_p/L_q \) norm could be used for characterizing the repetitive impulses in the bearing vibration signals.

The bearing faults, in view of the application described in this paper, are mostly characterized by surface defects such as pitting and spot, which account for more than 90% of all faults. In our experiments, the kurtosis is effective for the diagnosis of bearing faults and can be used to identify the conditions of the bearings. In consideration of this advantage, the reciprocal of kurtosis values calculated from modes may be adopted as the fitness function to optimize VMD parameters. The optimization objective is to search for the mode with the maximal kurtosis, which indicates the most obvious fault information is contained in the screening mode.

\[
f(x) = \min_{[M, \alpha]} \left\{ \frac{1}{K_i} \right\},
\]

s.t. \( M = 3, 4, \ldots, 8 \).

After obtaining the optimal parameters of VMD, the vibration signals are decomposed into a set of IMFs via VMD, and the mode with the maximal kurtosis is selected as the sensitive one. The selected mode may be further analyzed by Hilbert envelope demodulation to estimate whether the bearing is defective.

The following steps detail the procedures of the proposed fault diagnosis method:

1. **Step 1:** calculate the fault characteristic frequencies according to the geometry of the bearing and its rotational speed.
2. **Step 2:** in order to optimize \( M \) and \( \alpha \), initialize the vector \( x' = \{M_1, \alpha_1\} \). In this paper, \( M \) is preset as an integer in the interval of \([3, 8]\), and the initial values of \( M_1 \) and \( \alpha_1 \) are equal to 3 and 2000, respectively.
3. **Step 3:** with each \( x \), decompose the vibration signals into a set of IMFs via VMD. Compute the kurtosis values of all IMFs, and specify the reciprocal of the maximal kurtosis as the value of \( f(x) \).
4. **Step 4:** model the normalized random searching direction \( b \) according to equation (1). Calculate \( x_1 \) and \( x_2 \) with the aid of equation (2), where \( x'_i = \{M_i, \alpha_i\} \) and \( x'_i = \{M_i, \alpha_i\} \).
5. **Step 5:** decompose the vibration signals via VMD by using \( x_1 \) and \( x_2 \) separately. And then calculate the respective \( f(x_1) \) and \( f(x_2) \).
6. **Step 6:** update \( x' \) by solving equation (3).
7. **Step 7:** reiterate Steps 4–6 through 100 loops to acquire the optimal \( M \) and \( \alpha \).
8. **Step 8:** decompose the vibration signals into a set of IMF components via VMD by using the optimal \( M \) and \( \alpha \), and choose the IMF with the maximal kurtosis.

The flowchart of the proposed method is illustrated in Figure 1.

The proposed method is applied to the low-noise deep groove ball bearing 6203. The geometry of bearing 6203 is listed in Table 1. More details about the calculation of the fault characteristic frequencies can be found in [28]. The measurement conditions are in accordance with those described in Section 1.

### 4. Applications on Bearing Fault Diagnosis

The experimental setup is a self-developed bearing vibration tester, as shown in Figure 2. Vibration signals are sampled via two vibration velocity sensors. A data acquisition card PCI-6143 is used to sample the conditioned vibration signals. The sampling rate and the sampling numbers are set to 45 KHz and 8192, respectively. The spindle speed is 1800 r/min.

#### 4.1. Incipient Inner Raceway Fault

Figure 3 shows a time-domain vibration signal of bearing 6203 with the incipient inner raceway fault. As shown, it is difficult to find periodic impact impulses in the time-domain waveform.

The above vibration signal is processed by the proposed method. \( \tau \) and \( \epsilon \) in the VMD method are both set to the default values, i.e., \( \tau = 0 \) and \( \epsilon = 1 \times 10^{-7} \). The optimal \( M \) and \( \alpha \) acquired by BAS are 8 and 380, respectively. Decomposition results of the signal via VMD with the optimal parameters are presented in Figure 4. The kurtosis values (\( K \)) of IMFs are calculated and listed in Table 2. The maximal kurtosis appears in IMF3, namely, the IMF3 has the most observable impact composition.
As shown in Figure 4, despite IMF3 is still polluted by a little noise, significant time-domain impulses can be found. For further evaluating the fault characteristic frequency of the inner raceway, IMF3 is demodulated by Hilbert envelope analysis. The consequent envelope spectrum is shown in Figure 5.

It can be observed that after processed by the proposed method, 5 spectrum peaks are highlighted in the envelope spectrum of IMF3. These 5 spectrum peaks correspond to the fault characteristic frequency $f_i$ (148.3 Hz) and its harmonics. The evident features suggest the inner raceway of the bearing is defective.

As shown in Figure 4, IMF3 selected by the proposed method demonstrates distinct impact impulses in time-domain waveform. In contrast, mode4 illustrated in Figure 6 and IMF2 illustrated in Figure 8 both exhibit more noise and less obvious impact impulses. Comparing Figures 5, 7, and 9, it can be observed that, in Figure 5, the fault characteristic frequency of inner raceway $f_i$ and its harmonics are highlighted in the envelope spectrum. A fundamental tone at 148.3 Hz can be clearly found in both Figure 7 and 9, but the higher harmonics like the third and the fourth components are smeared or completely obscured by noise in two envelope spectra.

For the sake of comparison, the signal shown in Figure 3 is decomposed by means of EMD. The results are illustrated in Figure 6. As shown, faint time-domain impulses can be found in mode4 and mode3. The kurtosis values of the modes are calculated and listed in Table 3. Similarly, mode4 with the maximal kurtosis value is chosen and demodulated by Hilbert envelope analysis. The corresponding envelope spectrum is shown in Figure 7.

In order to further validate the effectiveness of the proposed method in extracting the bearing fault characteristics, the fixed-parameter VMD method is adopted to process the signal shown in Figure 3. Drawing on the experiences in the literature [15], $M$ is set to 4, while $\alpha$, $\tau$, and $\varepsilon$ are all set to the default values in original VMD, i.e., $\alpha = 2000$, $\tau = 0$, and $\varepsilon = 1 \times 10^{-7}$. Likewise, decomposition results of the same signal are illustrated in Figure 8, and the kurtosis values of all IMFs are listed in Table 4.

Following the same ideas, IMF2 with the maximal kurtosis value is chosen and demodulated by Hilbert envelope analysis. The result is presented in Figure 9.

Table 1: Information of the experimental bearing 6203.

<table>
<thead>
<tr>
<th>Geometric parameters</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Outer race diameter, $d_o$</td>
<td>40 mm</td>
</tr>
<tr>
<td>Inner race diameter, $d_i$</td>
<td>17 mm</td>
</tr>
<tr>
<td>Rolling element diameter, $d$</td>
<td>6.747 mm</td>
</tr>
<tr>
<td>Number of the rolling elements, $z$</td>
<td>8</td>
</tr>
<tr>
<td>Pitch diameter, $D$</td>
<td>29 mm</td>
</tr>
<tr>
<td>Contact angle, $\theta$</td>
<td>0°</td>
</tr>
<tr>
<td>Spindle rotational frequency, $f_s$</td>
<td>30 Hz</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Fault characteristic frequencies</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Inner raceway, $f_i$</td>
<td>148 Hz</td>
</tr>
<tr>
<td>Outer raceway, $f_o$</td>
<td>92 Hz</td>
</tr>
<tr>
<td>Rolling element, $f_b$</td>
<td>122 Hz</td>
</tr>
</tbody>
</table>

As shown in Figure 4, despite IMF3 is still polluted by a little noise, significant time-domain impulses can be found. For further evaluating the fault characteristic frequency of the inner raceway, IMF3 is demodulated by Hilbert envelope analysis. The consequent envelope spectrum is shown in Figure 5.

According to the experiences in the literature [15], $M$ is set to 4, while $\alpha$, $\tau$, and $\varepsilon$ are all set to the default values in original VMD, i.e., $\alpha = 2000$, $\tau = 0$, and $\varepsilon = 1 \times 10^{-7}$. Likewise, decomposition results of the same signal are illustrated in Figure 8, and the kurtosis values of all IMFs are listed in Table 4.

Following the same ideas, IMF2 with the maximal kurtosis value is chosen and demodulated by Hilbert envelope analysis. The result is presented in Figure 9.
Figure 3: A time-domain vibration signal of bearing 6203 with the incipient inner raceway fault.

Figure 4: Parameter-adaptive VMD results of the signal with the incipient inner raceway fault.

Table 2: Kurtosis values of the decomposed IMFs via parameter-adaptive VMD.

<table>
<thead>
<tr>
<th>IMF</th>
<th>IMF1</th>
<th>IMF2</th>
<th>IMF3</th>
<th>IMF4</th>
<th>IMF5</th>
<th>IMF6</th>
<th>IMF7</th>
<th>IMF8</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>2.81</td>
<td>3.74</td>
<td>11.06</td>
<td>3.79</td>
<td>2.86</td>
<td>2.89</td>
<td>3.18</td>
<td>3.08</td>
</tr>
</tbody>
</table>
Figure 5: Envelope spectrum of the IMF3 decomposed via parameter-adaptive VMD.

Figure 6: EMD results of the vibration signal with the incipient inner raceway fault.

Table 3: Kurtosis values of the decomposed modes via EMD.

<table>
<thead>
<tr>
<th>Mode</th>
<th>Mode1</th>
<th>Mode2</th>
<th>Mode3</th>
<th>Mode4</th>
<th>Mode5</th>
<th>Mode6</th>
<th>Mode7</th>
<th>Mode8</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>2.11</td>
<td>2.85</td>
<td>3.73</td>
<td>5.17</td>
<td>3.46</td>
<td>3.04</td>
<td>3.12</td>
<td>3.30</td>
</tr>
</tbody>
</table>
Figure 7: Envelope spectrum of mode 4 decomposed via EMD.

Figure 8: Fixed-parameter VMD results of the vibration signal with the incipient inner raceway fault.

Table 4: Kurtosis values of the decomposed IMFs via fixed-parameter VMD.

<table>
<thead>
<tr>
<th>IMF</th>
<th>IMF1</th>
<th>IMF2</th>
<th>IMF3</th>
<th>IMF4</th>
</tr>
</thead>
<tbody>
<tr>
<td>$K$</td>
<td>2.82</td>
<td>3.89</td>
<td>2.90</td>
<td>2.11</td>
</tr>
</tbody>
</table>
The above experiments demonstrate that the proposed parameter-adaptive VMD outperforms EMD and the fixed-parameter VMD with regard to noise robustness and effectiveness in extracting weak fault characteristics contained in bearing vibration signals. Meanwhile, the experiments also indicate the VMD parameters have appreciably impacts on decomposition results. The performance of VMD with inappropriate parameters may be inferior to that of EMD.

4.2. Incipient Outer Raceway Fault. Figure 10 illustrates a time-domain vibration signal with the incipient outer raceway fault. The optimal $M$ and $\alpha$ sought by BAS are 5 and 130, respectively. Decomposition results of the signal via VMD with the optimal parameters are displayed in Figure 11. The kurtosis values of the resulting IMFs are calculated and listed in Table 5.

As shown in Figure 11, distinct time-domain impulses are exhibited in IMF2, and meanwhile, the maximal kurtosis value appears in IMF2. In view of the aforementioned analysis, IMF2 is separately selected for further envelope analysis via Hilbert transform. The result is displayed in Figure 12.

The fault characteristic frequency of outer raceway $f_o$ (93.4 Hz) and its harmonics have been clearly demonstrated in Figure 12. The distinct spectrum peaks indicate the outer raceway of the bearing is defective.

For comparison, the signal shown in Figure 10 is processed via EMD. Decomposed mode4 with the maximal kurtosis value 10.15 and its envelope spectrum are separately displayed in Figures 13 and 14.

The signal shown in Figure 10 is decomposed by fixed-parameter VMD. The key parameters of VMD are the same as those mentioned above, i.e., $M$ and $\alpha$ are set to 4 and 2000, respectively.

IMF2 has the maximal kurtosis value 3.13 in the decomposition results. The time-domain waveform of IMF2 and its corresponding envelope spectrum are separately illustrated in Figures 15 and 16.

Comparing the above experimental results, the following conclusions may be drawn: both parameter-adaptive VMD and EMD can demonstrate clear impact impulses in proper decomposition components, as shown in Figures 11 and 13. The corresponding envelope spectra shown in Figures 12 and 14 highlight the fault characteristic frequency of outer raceway $f_o$, and its harmonics, which clearly indicate the outer raceway of the bearing is defective. By contrast, the results decomposed by parameter-adaptive VMD exhibit more dominant and clearer fault characteristic information, while sensitive mode4 decomposed by EMD involves more noise interference.

It can be observed that IMF2 decomposed by fixed-parameter VMD shows little sign of impact impulses but strong noise interference. And it is hard to distinguish the fault characteristic components in the corresponding envelope spectrum displayed in Figure 16. The analysis results indicate that inappropriate parameters of VMD may not identify the fault characteristic frequency.

4.3. Incipient Rolling Element Fault. Figure 17 illustrates a time-domain vibration signal with the incipient rolling element fault. The optimal $M$ and $\alpha$ sought by BAS are 6 and 170, respectively. Using parameter-adaptive VMD, the vibration signal is processed to acquire the IMF components, and the result is displayed in Figure 18. The kurtosis values of the resulting IMFs are calculated and listed in Table 6.

The envelope spectrum of IMF2 with maximal kurtosis value is displayed in Figure 19. As shown, the raw vibration signal is contaminated by heavy noise, and the weak fault characteristics are buried by other interference frequency components. Nevertheless, satisfactory results can be acquired by the proposed method. As illustrated in Figure 19, the fault characteristic frequency of rolling element $f_b$ (126.3 Hz) along with its second and third harmonics can be easily recognized.

The signal shown in Figure 17 is processed by EMD, in which decomposed mode4 has the maximal kurtosis value 10.73. Mode4 and its envelope spectrum are displayed in Figures 20 and 21, respectively.

Comparing Figure 21 with Figure 19, it can be noted that the fault characteristic frequency $f_b$ acquired by EMD may be roughly identified, but the envelope spectrum of mode4 is mixed with a large amount of noise. It may have something to do with mode mixing effects in the EMD method.

Figure 22 illustrates the time-domain waveform of IMF2 with the maximal kurtosis value 3.06, which is decomposed by fixed-parameter VMD. The envelope spectrum of IMF2 is illustrated in Figure 23.

It is difficult to identify the fault characteristic components in the envelope spectrum shown in Figure 23. The existence of serious mode mixing leads to the selected IMF
Figure 10: Time-domain vibration signal of bearing 6203 with the incipient outer raceway fault.

Figure 11: Parameter-adaptive VMD results of the vibration signal with the incipient outer raceway fault.

Table 5: Kurtosis values of the decomposed IMFs via parameter-adaptive VMD.

<table>
<thead>
<tr>
<th>IMF</th>
<th>IMF1</th>
<th>IMF2</th>
<th>IMF3</th>
<th>IMF4</th>
<th>IMF5</th>
</tr>
</thead>
<tbody>
<tr>
<td>K</td>
<td>2.90</td>
<td>15.49</td>
<td>2.92</td>
<td>2.45</td>
<td>2.23</td>
</tr>
</tbody>
</table>

Figure 12: Envelope spectrum of IMF2 decomposed via parameter-adaptive VMD.
containing heavy noise and other frequency components. The strong interference masks the useful fault characteristic information and results in the failure of bearing incipient fault diagnosis.

For further comparison, an extreme learning machine (ELM) [29–31] classifier is adopted to estimate the proposed method. Considering that the energy at the fault characteristic frequencies will increase if the bearing is faulty, the proportion of fault energy (PFE) is firstly introduced. As aforementioned, the measured fault characteristic frequencies may be slightly different from the theoretical ones in the envelope spectrum. Therefore, the calculated fault energy is calculated as follows:

\[ PFE = \frac{\sum_{i=1}^{n} E_i}{\sum_{i=1}^{n} E_{th}} \]

where \( E_i \) is the energy of the measured frequency component at frequency \( f_i \), and \( E_{th} \) is the theoretical energy at frequency \( f_{th} \).

**Figure 13**: Decomposed mode 4 obtained by EMD from the incipient outer raceway fault signal.

**Figure 14**: Envelope spectrum of mode 4 obtained by EMD.

**Figure 15**: Decomposed IMF 2 obtained by fixed-parameter VMD from the incipient outer raceway fault signal.

**Figure 16**: Envelope spectrum of IMF 2 obtained by fixed-parameter VMD.
characteristic frequency $f$ is updated to a narrow band frequency range $[f - \Delta f, f + \Delta f]$, where $\Delta f$ is set to 5 Hz. The fault energy $E_1$ can be calculated by the following equation:

$$E_1 = \sum_{j=f-5}^{f+5} f_j^2. \quad (6)$$

The frequency range $[0.5f, 1.5f]$ is chosen to calculate the signal energy $E_{ia}$ to avoid the influences derived from other frequency bands.

$$E_{ia} = \sum_{k=0.5f}^{1.5f} f_k^2. \quad (7)$$
The ratio of \( R_1 = \frac{E_1}{E_{1a}} \) is calculated to evaluate the proportion of the 1st harmonic fault energy. Similarly, the 2nd harmonic fault energy \( E_2 \) is calculated. Considering that the deviation between the theoretical 2nd harmonic and the measured one will increase further, the 2nd harmonic \( 2f \) is updated to \([2f - 2\Delta f, 2f + 2\Delta f]\), where \( \Delta f \) is still set to 5 Hz.
\[ E_2 = \sum_{j=2 f^{-10}}^{2 f+10} f_j^2. \]  
(8)

The frequency range \([1.5f, 2.5f]\) is chosen to calculate the signal energy \(E_{2a}\):

\[ E_{2a} = \sum_{k=1.5f}^{2.5f} f_k^2. \]  
(9)

As shown in Table 8, the fault diagnosis accuracy of the proposed method is higher than that of the other two methods. Compared with the proposed method and EMD, the testing accuracy of the fixed-parameter VMD is much lower, which indicates that \(M\) and \(\alpha\) will greatly affect the VMD results.

### 5. Conclusions

The VMD method can completely decompose the raw vibration signals into a set of IMFs from low frequency to high frequency. The mode number \(M\) and the quadratic penalty term \(\alpha\) of VMD will have considerable influence on decomposition results. The key to the VMD method lies in seeking for the optimal combination of \(M\) and \(\alpha\) that matches with the analyzed signals.

In this paper, the BAS optimization algorithm is employed to adaptively estimate the optimal \(M\) and \(\alpha\). The optimized parameters can guarantee the availability of VMD.

The proposed parameter-adaptive VMD method has been applied in the field tests for low-noise deep groove ball bearing 6203. Comparisons have also been conducted to evaluate the performances by using the proposed method, EMD, and the fixed-parameter VMD. Three cases studies demonstrate that the proposed method outperforms EMD and the fixed-parameter VMD in suppressing the noise interference and highlighting the weak fault characteristic frequency information contaminated by heavy noise.

**Table 7:** The PFEs obtained by different methods.

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Inner raceway fault (f_i)</th>
<th>Outer raceway fault (f_o)</th>
<th>Rolling element fault (f_b)</th>
</tr>
</thead>
<tbody>
<tr>
<td>The proposed method</td>
<td>0.496</td>
<td>0.497</td>
<td>0.993</td>
</tr>
<tr>
<td>EMD</td>
<td>0.256</td>
<td>0.268</td>
<td>0.524</td>
</tr>
<tr>
<td>Fixed-parameter VMD</td>
<td>0.151</td>
<td>0.263</td>
<td>0.414</td>
</tr>
</tbody>
</table>

**Table 8:** The testing results of ELM.

<table>
<thead>
<tr>
<th>Methodology</th>
<th>Normal</th>
<th>Inner raceway fault</th>
<th>Outer raceway fault</th>
<th>Rolling element fault</th>
<th>Total</th>
<th>Testing accuracy</th>
</tr>
</thead>
<tbody>
<tr>
<td>The proposed method</td>
<td>2/20</td>
<td>3/20</td>
<td>1/20</td>
<td>0/20</td>
<td>6/80</td>
<td>92.5%</td>
</tr>
<tr>
<td>EMD</td>
<td>2/20</td>
<td>5/20</td>
<td>2/20</td>
<td>0/20</td>
<td>9/80</td>
<td>88.75%</td>
</tr>
<tr>
<td>Fixed-parameter VMD</td>
<td>4/20</td>
<td>9/20</td>
<td>5/20</td>
<td>5/20</td>
<td>23/80</td>
<td>71.25%</td>
</tr>
</tbody>
</table>
Data Availability
The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest
The authors declare that they have no conflicts of interest.

References