A Theoretical Calculation Method of Influence Radius of Settlement Based on the Slices Method in Tunnel Construction

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At present, the empirical formula is used to calculate the influence radius of surface settlement and the width of settlement trough, which lacks theoretical support. Aiming at this problem, this paper derived the theoretical calculation formula for predicting the influence radius of formation settlement based on the slices method. Then, the expression of the width of settlement trough was obtained according to the relationship between the settlement influence radius and the settlement trough width. The rationality of the formula was verified by the Heathrow Express tunnel and the Green Park tunnel. Through analysis and discussion, it was found that in the clay stratum, the settlement calculation formula can more accurately predict the surface settlement, while there is a big error in predicting the stratum settlement within 4d near the tunnel vault. In the sand layer, the internal friction angle is less than 40°, and the reinforcement surface is applied to the unsupported face to reduce the radius of influence; in the clay formation, when the cohesion is less than 50 kPa, the influence radius can be reduced by applying reinforcement measure to the unsupported face.

1. Introduction

The underground excavation construction of urban subway tunnel will lead to the deformation of strata settlement and then affect the safety and stability of adjacent existing buildings within the settlement range of strata. If the formation is too large, it may cause damage to the building [1–5]. Therefore, it is of great engineering significance to predict the range of formation settlement and settlement caused by the underground excavation of urban subway tunnels.

At present, there are many methods for predicting surface settlement caused by urban subway tunnel construction, such as the empirical formula method [6–8], elastic strain method [9–13], air stress function method [14–18], stochastic medium theory [19–21], numerical simulation method [22, 23], and centrifugal test method [24, 25]. Although there are many methods for predicting surface settlement, the calculation of the influence radius of formation settlement is still mainly based on empirical formula data regression [26–29], and the settlement trough width is not theoretically determined. However, the empirical formula has a large degree of subjectivity. Engineers with different engineering conditions may get different results. When the regression data obtained from other projects are applied to a certain project, it is still not possible to accurately predict the settlement and influence radius of settlement due to the complexity of soil mass. The key to studying the law of settlement deformation is to determine the width of the settlement trough, which also is the focus of many scholars. Therefore, it is necessary to theoretically study the settlement deformation law of the overlying soil caused by the underground excavation construction of urban subway tunnels, so as to objectively predict the influence range of settlement and settlement.

Based on the aforementioned problems, this paper used the slices method of slope stability analysis to analyze the soil
of tunnel excavation disturbance range and derived the theoretical calculation formula of the influence radius of settlement. Then, the expression of the width of settlement trough was obtained according to the relationship between the settlement influence radius and the settlement trough width. The rationality of the formula was verified by the Heathrow Express tunnel and the Green Park tunnel. At the same time, the influence radius of the settlement of sand and clay stratum, frequently encountered in the construction of urban subway tunnels, as well as the measures to reduce the radius of settlement and maximum surface settlement are analyzed. The research methods and results of this paper provide a reference for the theoretical calculation of the influence radius of settlement caused by the underground excavation of urban subway tunnels.

2. Theoretical Calculation of Influence Radius of Settlement

At present, there are many theoretical studies on the failure modes of the arch crown and the face, for example, Janssen silos theory, rock failure mode, parabolic failure mode [30, 31], and logarithmic spiral mode [32]. However, in actual engineering, the failure mode of the excavation face is far from the analysis model above. In order to find out the true failure mode of the excavation face, Mair [33] used the centrifuge model test to prove that the failure surface of the cohesive soil layer is extended upward and towards two sides, and it is like a basin, narrow in the lower part and wide in the upper part. Selby [34] also shows that the failure surface of the sandy soil layer is narrow chimney shaped that directly develops to the surface of the tunnel from both sides. The actual failure mode of the excavation surface is consistent with the test results of Mair. In fact, for shallow tunnels in sand and clay, the effects of excavation will extend to the surface, failing to form a “natural arch.” After the tunnel is excavated, the rock and soil will collapse and move, which affects the surface and forms a collapsed area [35, 36]. The failure pattern is shown in Figure 1.

The excavation of the soil mass on the tunnel face will break the original stress balance state of surrounding soil mass, and the disturbed soil mass will deform towards the tunnel excavation face under the action of gravity. Due to construction methods and other factors, the volume of soil mass actually excavated is often larger than the volume of tunnel designed for excavation. The overexcavated part of the soil is called formation loss [17] (see Figure 1).

3. Peck Formula and the Width of the Settlement Trough

After the tunnel lining is closed into a ring, the overlying soil is layered and filled into the overexcavated part, which eventually causes surface settlement. Peck summarized a large number of measured data of surface settlement caused by tunnel construction and believed that surface settlement accords with normal distribution law [4] and proposed the famous Peck formula as follows:

\[
S_{(x)} = S_{\text{max}} \exp \left( -\frac{x^2}{2\ell^2} \right), \tag{1}
\]

where \( S_{(x)} \) is the settlement value of the distance from the surface of the tunnel to the central axis of the tunnel; \( S_{\text{max}} \) is the maximum surface settlement; and \( \ell \) is called the width of settlement trough (see Figure 1).

The Peck formula is needed to determine two parameters for the prediction of settlement: the ground loss \( (V_l) \) and the width of settlement trough \( (\ell) \). The ground loss \( (V_l) \) is generally related to the construction process, the nature of the formation soil, and so on and can be determined based on empirical values or pretest sections. The width of settlement trough \( (\ell) \) is related to the nature of the formation soil, the depth of the tunnel, and the radius of the tunnel. The literatures summarized the calculation formulas for various settlement trough width \( (\ell) \), which is summarized in Table 1.

Numerous experimental and measured data show that there is no simple linear relationship between the width coefficient of the formation settlement trough and the tunnel depth of the stratum. The deformation of the overlying soil caused by tunnel excavation in different soil layers is different [21, 22]. For clay layers with high cohesion, the disturbed soil is "pot" (see Figure 1) rather than inverted triangle shape. The complexity of the soil properties, the tunnel section size, the tunnel depth, and the construction method are different. Therefore, the expression of the width of settlement trough obtained by the researchers through the test or measured data is not uniform [6, 8, 29–33]. It is often only applied to a certain type or a certain area of the stratum, and there is little report on the theoretical solution of the width of the settlement trough.

Peck believed that ground volume \( V_l \) is equal to the volume of the area bounded by the surface settlement curve and the original surface level, without considering the factors of seepage and formation consolidation, and can be obtained by integrating formula (1).

\[
V_l = S_{\text{max}} \cdot \sqrt{2\pi i}, \tag{2}
\]

where \( V_l \) is numerically equal to the area of the pattern surrounded by the normal distribution curve \( S_{(x)} \) and the horizontal line of the surface.
In the stochastic medium theory, the horizontal distance \( r(z) \) from the tunnel center to the surface settlement edge is called the surface settlement influence radius. Settlement occurs within the radius of impact of settlement, and settlement outside this range is negligible. By comparing the stochastic medium theory and Peck formula, Han et al. gave the relationship between the influence radius of settlement and the width of settlement trough as follows [29]:

\[
r(\varphi) = \sqrt{2\pi i}.
\]

It can be seen from equations (2) and (3) that the area of the surface settlement trough is approximately equal to the area of the triangle with height \( S_{\text{max}} \) and base \( 2\sqrt{2\pi i} \). At the same time, it can be known from equation (3) that if the surface settlement influence radius \( r(\varphi) \) can be obtained, the influence range of the settlement and the width of settlement trough of the tunnel excavation can be obtained.

### 4. Calculation Method of the Influence Radius of Settlement Based on Slices Method

There is a certain similarity between the mechanism of disturbed soil deformation and the instability mechanism of slope in urban subway tunnel construction. The excavation of the soil in the tunnel face breaks the original stress balance state of the stratum, and the soil within a certain range above the tunnel is deformed or even destabilized under the action of gravity. According to the theory of loosening earth pressure, there are potential sliding surfaces in tunnel excavation [35, 36]. The formation process of this potential sliding surface is similar to that of slope stability analysis. Tunnel excavation is equivalent to unloading slope foot; because of the phenomenon of overexcavation, after the completion of tunnel support, there is still a gap between the supported soil and the supporting structure, and the soil will continue to deformation or move until it reaches an equilibrium state. The process of soil deformation or movement is simplified into the process of soil slices movement within the excavation disturbance range. Therefore, we can use the slope stability analysis method to analyze the surface deformation process of soil. The disturbance range of tunnel construction is limited, and the soil outside this range is not affected by construction disturbance. Therefore, as long as the position of the construction disturbance boundary surface can be determined, the horizontal distance from the tunnel center line to the disturbance boundary surface at different depths of the formation, that is, the influence radius of the settlement, can be calculated. Then, according to the relationship between the influence radius of settlement and the settlement trough width, the expression of the width of settlement trough can be obtained, and the settlement curves at different tunnel depths can be obtained.

The slices method is a commonly used method in the stability analysis of clay slopes. In view of the similarity between the mechanism of soil deformation caused by tunnel excavation and the mechanism of slope instability, this paper used slices method to determine the boundary of tunnel construction disturbance. In order to simplify the analysis process, the following two assumptions are made:

1. The soil is homogeneous, and the cohesion and internal friction angles of the various soil strips formed by the strips are the same. The vertical line at the center of the tunnel is the axis of symmetry for the deformation of soil mass.  
2. As long as a point in the soil mass slides, it is considered that the soil at that point has reached the failure, that is, the soil at the disturbance boundary is considered to be in a limit equilibrium state. Due to the symmetry of soil disturbed by tunnel construction, in order to simplify the calculation process, half of the soil is taken for analysis, as shown in Figure 2.

Assume that the disturbance boundary is a circular arc with the point \( O \) as the center and \( R \) as the radius (\( ab \) in Figure 2), and the point \( O \) of the arc center is directly above the centerline of the tunnel. The soil within the disturbance range is divided into \( n \) soil strips of equal width. If any of the \( i \)th soil strips (shaded soil strips in Figure 2) is used as the isolation object for force analysis, the force acting on the soil strips has the self-weight \( W_i \) and the road load \( Q_i \). The normal reaction force \( N_i \) and the tangential reaction force \( T_i \) on the bottom surface of the strip, the normal force \( E_{1i} \) and \( E_{2i} \), and the tangential forces \( F_{1i} \) and \( F_{2i} \) are shown in Figure 3. To simplify the calculation, it is assumed that the resultant force of \( E_{1i} \) and \( F_{1i} \) is opposite to the combined force of \( E_{2i} \) and \( F_{2i} \) and acts on the same straight line, and the two cancel each other out.

According to the static balance condition of the soil strip, there is

<table>
<thead>
<tr>
<th>Document source</th>
<th>Applicable conditions</th>
<th>Settlement trough width of ((i)) calculation formula</th>
<th>In accordance with</th>
</tr>
</thead>
<tbody>
<tr>
<td>Peck [6]</td>
<td>Various types of soil</td>
<td>(0.5(z/d)^{1-n} \times z), ((n = 0.8-1.0))</td>
<td>Measured statistics</td>
</tr>
<tr>
<td>Atewell [7]</td>
<td>British clay</td>
<td>0.5z</td>
<td>Measured and model tested</td>
</tr>
<tr>
<td>Han et al. [29]</td>
<td>Cohesive soil, soft clay, and hard clay in the London area</td>
<td>((1 - 0.02\varphi) \times z)</td>
<td>Measured statistics</td>
</tr>
<tr>
<td>Atkinson and Potts [37]</td>
<td>Compact superconsolidated clay</td>
<td>0.25((z + 0.5d))</td>
<td>Measured and model tested</td>
</tr>
<tr>
<td>Lee et al. [38]</td>
<td>Clay</td>
<td>0.25((1.5z + 0.25d))</td>
<td>Centrifuge test</td>
</tr>
<tr>
<td>O’Reilly [39]</td>
<td>Clay</td>
<td>(i = 0.43z + 1.1)</td>
<td>Measured data</td>
</tr>
<tr>
<td>Clough and Schmidt [40]</td>
<td>British clay</td>
<td>(0.5(z/d)^{0.2} \times z)</td>
<td>Measured statistics</td>
</tr>
</tbody>
</table>

### Table 1: Calculation of width parameters of surface settlement trough.
Overbreak volume
Tunnel boundary

\[T = \sum_{i=m}^{n} T_i = \sum_{i=m}^{n} (W_i + Q_i) \sin \alpha_i,\]

(8)

where \(m = r/d\) is the tunnel radius and \(d\) is the width of the bar.

With the continuous refinement of the scores, the aforementioned formula becomes the integral in the general sense.

\[T = \int_{r}^{R} R \frac{X}{R} \, dx,\]

\[= \int_{\theta_0}^{\theta_1} \gamma R \sin \theta \cos \theta d(\cos \theta) \int_{r}^{R} \frac{X}{R} \, dx,\]

\[= \int_{\theta_0}^{\theta_1} \gamma R^2 \sin^2 \theta \cos \theta d\theta - \int_{r}^{R} \frac{Y R^2 h^2}{3R} \, dx,\]

\[= \frac{\gamma (R^2 - r^2)^{3/2}}{2R} - \frac{Y R^2 h^2}{3R} \left(3R^2 - h^2 - 3r^2\right),\]

where \(\sin \theta_0 = h/R\), \(\cos \theta_1 = r/R\).

The total shear force produced over the entire disturbance boundary \(ab\) is

\[S = \sum_{i=m}^{n} S_i = \sum_{i=m}^{n} \left(c_i + \sigma_i \tan \phi_i\right) \cdot l_i,\]

\[S = \frac{c \cdot (R_1 - R_0) R + \int_{r}^{R} \gamma R \sin \theta \cos \theta d(R \cos \theta)}{R} \tan \phi d\theta,\]

\[= F_1 = S/T.\]
deformed by the tunnel construction disturbance, while soil mass outside the boundary is not affected by construction disturbance and remains static.

Therefore, the position of the construction disturbance boundary can be determined according to equations (6) and (8). Taking the intersection of the centerline of the tunnel and the ground as the coordinate origin and establishing the coordinate system with the horizontal direction as the x direction and the vertical direction as the z direction, the expression of the disturbance boundary can be expressed as

$$x^2 + (z + h)^2 = R^2,$$

where $h$ is the distance from the center of the arc to the ground.

According to formula (9), it is possible to determine the influence radius $r(z)$ of the settlement at different depths:

$$r(z) = x = \sqrt{R^2 - (z + h)^2}.$$

Then, according to formula (3), the width of settlement trough at different buried depths is determined, and then the settlement curves of the corresponding formation are obtained by formulas (1) and (2).

5. Case Study

In order to prove the rationality of the method, the theoretical calculation results and the measured data were compared with the Heathrow Express tunnel (sand layer) and the Green Park tunnel (clay formation). Stratum parameters of the two tunnels are shown in Table 2.

Figure 4(a) shows the comparison between the surface settlement curve calculated by the method and the measured settlement data in the sand layer. The surface settlement curve calculated in this paper is in good agreement with the measured surface settlement data, and the error is small. It is indicated that in the sand layer, the calculation method of surface settlement based on the slices method proposed in this paper has certain rationality. Figure 4(b) shows the comparison of the settlement values of the formations at different depths above the tunnel centerline calculated by the method and the measured data. Meanwhile, the settlement values of different tunnel depths above the tunnel center line are calculated by using the methods proposed by Mair et al. [8] and Han et al. [29]. The settlement calculation formula proposed in this paper can accurately reflect the variation trend of the settlement with the tunnel depth above the tunnel centerline, and the calculation results are in good agreement with the measured data. The method proposed by Mair and Han is more accurate for the prediction of surface settlement. However, as the depth of the stratum increases, the difference between the calculated results and the measured data is larger and larger, and the calculation results are significantly larger than the measured results. It is indicated that in the sand layer, the settlement calculation formula proposed in this paper more accurately reflects the variation trend of the settlement with the tunnel depth above the centerline of the tunnel compared with the other two methods.

Figure 5(a) shows the comparison between the land settlement curve calculated by the method and the measured data. As can be seen from the figure, the calculated results are in good agreement with the measured results. It is indicated that in the clay stratum, the calculation method of settlement based on the slices method proposed in this paper has certain rationality. Figure 5(b) shows the comparison between the settlement and the measured results at different depths above the tunnel centerline calculated by the method proposed by Mair et al. [8] and the method proposed by Han et al. [29]. In the clay stratum, this paper proposed that the settlement calculation formula can accurately predict the formation settlement outside the depth of 4d and predicted that there is a large error in the surface settlement within the 4d range, while the error of Mair and Han method is relatively small. It is indicated that in the clay stratum, the settlement calculation formula can predict the surface settlement and shallow stratum settlement more accurately and there is a big error in predicting the stratum settlement within 4d near the tunnel vault.

6. Analysis and Discussion

In order to further prove the rationality and feasibility of the formula proposed in this paper, this paper analyzed the parameters of sand and clay formations often encountered in urban subway construction.

The diameter of the tunnel is $d = 6, 8, 10, and 12 m$, and the ratio of tunnel depth to diameter is $z/d = 1, 1.5, 2.0, 2.5, and 3.0$. In the sand layer, the cohesion $c = 0$, the internal friction angle $\phi = 20^\circ, 30^\circ, 40^\circ, and 50^\circ$. The relationship between the influence radius of the settlement ($r(z)$) and the ratio of the tunnel depth to the diameter is shown in Figure 6(a); in clay formation, internal friction angle $\phi = 10^\circ$ and cohesion $c = 20 kPa, 30 kPa, 40 kPa$, and $50 kPa$. Figure 6(b) shows the relationship between $r(z)$ and $(z/d)$.

As shown in Figure 6(a), when the diameter of the tunnel is less than or equal to 8 m, the influence radius increases as $z/d$ increases. When the diameter of the tunnel is greater than or equal to 10 m, when $z/d < 1.5$, the radius of influence increases with the increase of $z/d$; when $1.5 < z/d < 2.0$, the influence radius decreases with the increase of $z/d$; and when $2.0 < z/d$, the radius of influence is zero. It is indicated that as the tunnel depth and internal friction angle increase, the disturbance caused by tunnel excavation will not develop to the ground surface. As shown in Figure 6(b), when the diameter of the tunnel is less than or equal to 6 m, the influence radius increases as $z/d$ increases. When the diameter of the tunnel is greater than 6 m, the radius of influence increases first with $z/d$ and then decreases to zero. It is indicated that as the tunnel depth and cohesion increase, the disturbance caused by tunnel excavation will not develop to the surface. The calculated influence radius is 0, which does not mean that there is no surface settlement, which means that the calculation formula in this paper is not applicable to this case. The sliding surface caused by tunnel excavation and developed to the surface is the precondition for deriving the formula. When the tunnel depth increases and the formation conditions are good, the tunnel
excavation disturbance will not affect the surface and forms a sliding surface, so the formula in this paper is not applicable.

In the construction process of the tunnel, in order to reduce the surface settlement, the excavation face is usually reinforced. Reinforcement of the excavation face is equivalent to applying a force to prevent the sliding of the soil strip where the tunnel is located and improving the stability of the slope. However, whether the reinforcement measures for the excavation face can reduce the radius of influence requires further analysis. The safety factor is used to characterize the size of the reinforcement, and the relationship between the safety factor and the influence radius is analyzed. When the safety factor of the slope takes different values, the relationship between the influence radius and the internal friction angle and cohesion is shown in Figure 7.

As shown in Figure 7(a), in the sand layer, the influence radius decreases as the internal friction angle increases and decreases as the safety factor increases. As shown in

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**Table 2: Formation parameters of the Heathrow Express and Green Park tunnels.**

<table>
<thead>
<tr>
<th>Name</th>
<th>r (m)</th>
<th>z₀ (m)</th>
<th>γ (kN/m³)</th>
<th>c (kPa)</th>
<th>φ (°)</th>
<th>R (m)</th>
<th>r₁(z₁) (m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Green Park tunnel [11]</td>
<td>2.07</td>
<td>29.4</td>
<td>19</td>
<td>175</td>
<td>—</td>
<td>34.1</td>
<td>34</td>
</tr>
</tbody>
</table>

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**Figure 4:** (a) Surface settlement curve and (b) stratum settlement above the center line, the Heathrow Express tunnel.

**Figure 5:** (a) Surface settlement curve and (b) stratum settlement above the center line, the Green Park tunnel.
Figure 7(b), in the clay formation, the influence radius decreases as the cohesion increases and decreases as the safety factor increases. It is indicated that in sand, when the internal friction angle is less than 40°, the reinforcement of the excavation face can reduce the radius of influence, and the radius of influence can also be reduced by increasing the internal friction angle of the formation. In clay formations, when the cohesion is less than 50kPa, the reinforcement of the excavation face can reduce the radius of influence, and the radius can also be reduced by increasing the cohesive of the formation. It can also be seen from Figure 7(a) that as the internal friction angle increases, the amplitude of the radius of influence decreases. In the sand layer, when the internal friction angle is greater than 40°, reinforcement measures are applied to the excavation face to reduce the influence radius. In the clay formation, when the cohesion is greater than 50 kPa, reinforcement measures are applied to the excavation face, and the radius of influence cannot be reduced.

7. Conclusions

Based on the slices method, the calculation formula for predicting the influence radius of surface settlement was established. The rationality of the formula was verified by engineering examples and discussion. The conclusions obtained are as follows:
(1) Based on the slices method, a formula for predicting the influence radius of the surface settlement and the width of the settlement trough was established.

(2) In the sand layer, the settlement calculation formula proposed in this paper more accurately reflects the variation trend of the settlement with the tunnel depth above the centerline of the tunnel compared with the other two methods. In the clay stratum, this paper proposed that the settlement calculation formula can predict ground settlement and shallow stratum settlement more accurately, and there is a big error in predicting the stratum settlement within 4d near the tunnel vault.

(3) In the sand layer, the internal friction angle is less than 40°, and the reinforcement applied to the excavation face can reduce the radius of influence; when the internal friction angle is greater than 40°, reinforcement measures for the excavation face cannot reduce the radius of influence. In the clay formation, when the cohesion is less than 50 kPa, the reinforcement for the excavation face can reduce the radius of influence; when the cohesion is greater than 50 kPa, the reinforcement surface is applied to the excavation face, and the radius of influence cannot be reduced.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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