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Research Article

Optimization of the Accelerated Degradation Test Plan for Electrical Connector Contact Pairs Based on a Nonlinear Wiener Process

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Accelerated degradation test is an effective method to evaluate the reliability of products with long life and high reliability. The performance of most products fluctuates randomly in the degradation process, so it is suitable to use Wiener process. At present, the diffusion coefficient is regarded as constant in Wiener process, while the drift coefficient is related to stress. However, in practice, the amplitude of product performance fluctuation increases with the increase of stress level, which is not constant. Therefore, for the nonlinear Wiener case where both the drift coefficient and the diffusion coefficient are stress dependent, this paper studies the constant-stress accelerated degradation test theories and methods. Taking the contact pairs of electrical connectors as the research object, the minimum variance of reliable life estimate under normal stress is taken as the target. After determining the censored time at each stress level, the test stress level, the sample distribution ratio at each stress level, and the test interval at the one-third power scale of time are taken as design variables. The test plan under 3, 4, and 5 stress levels is optimized and compared with the general test plan. The influence of the difference between high and low stress levels on the evaluation accuracy is analyzed. Finally, the sensitivity analysis of parameters shows that the optimization plan has good robustness, and the change of stress quantity has little influence on the robustness of the plan.

1. Introduction

With the rapid development of modern science and technology, many products with high reliability and long life have emerged in the fields of machinery, electronics, aerospace, etc., which makes it more difficult for engineers to obtain enough failure data in a reasonable and effective time. Therefore, an accelerated degradation test (ADT) method is developed, which studies the product performance degradation process and extrapolates the reliability measure of products by statistical analysis of the degradation data and thus solves the reliability evaluation problem under lack of failure data or no failure data. At present, the research on accelerated degradation test focuses on performance degradation models, statistical analysis methods for degradation data, and optimization design for accelerated degradation test.

The most widely used performance degradation models are Wiener processes and Gamma processes. Wiener process is suitable for nonmonotonic degradation of product performance and has been widely studied and applied in the analysis of degraded data [1]. Whitmore [2] established a degradation model that takes into account the randomness of performance degradation and measurement error. The randomness of performance degradation is described by Wiener process, and the randomness of measurement error is described by normal distribution. Tseng et al. [3, 4] established an acceleration model of LEDs under step stress and multiple stresses based on Wiener process, respectively. Ye [5] considered the feature of high reliability of products

with high degradation rate and high volatility and proposed a Wiener degradation model with linear relationship between diffusion coefficient and drift coefficient. On the basis of YE, Feng [6] proposed a degradation model, which is more consistent with product degradation data by considering the influence of measurement error. The Gamma process is used to describe the monotonic degradation of performance. Zhao [7] proposed a degradation modeling method based on Gamma process in the situation where the product degradation is caused by the continuously cumulative damages. Wang [8] studied parameter estimation method of nonhomogeneous Gamma process with random effects and proposed a semiparametric pseudolikelihood estimation method.

The statistical analysis of degradation data usually includes the parameter estimation and reliability index estimation. There are many estimation methods for unknown parameters of models, such as least squares (LSE), Bayesian (BAYES) estimation, maximum likelihood estimation (MLE), and expectation maximum (EM) algorithms. Liu [9] and Wang [10] solved unknown parameters of degradation path model with random effect by combining LSE and MLE. About the related works using maximum likelihood estimation methods, the reader can refer to [11–13]. Si [11], Ye [12], and Wang [13] estimated the parameters of degradation model by using MLE. Zhang [14] trained hyperparameters in the inverse problem models by using MLE. Robinson [15] and Gebraeel [16] estimated mixed effect model parameter by using the Bayesian estimation method. Li [17] proposed a grid-based EM method, which can learn online and classify signal patterns; the computational efficiency had greatly improved compared with traditional EM method. Chen [18] proposed a method that combines EM algorithm and unscented Kalman filter for estimating the accurate white noise process.

The objective of the optimal design of accelerated degradation test plan is to obtain as much degradation information as possible under the shortest test time and the smallest test cost to improve the reliability evaluation accuracy. The optimal design of the accelerated degradation test plan mainly focuses on the situation that the product performance degradation model obeys the Wiener process and the Gamma process. When the performance degradation model obeys the linear Wiener process, the related work can be referred to [19, 20] With the minimum cost as the optimization goal, Tang [19] proposed a step accelerated degradation test plan considering the evaluation accuracy. With the minimum asymptotic variance of the maximum likelihood estimation of the qth quantile of the lifetime distribution as goal, Heonsang et al. [20] optimized the design of constant accelerated degradation test. When the product performance degradation process is as nonlinear Wiener process, Liao and Tseng [4] provided the optimal step-stress accelerated degradation test plans by minimizing the asymptotic variance of the estimated 100pth percentile of the product's lifetime distribution under constraints on the total cost. When the performance degradation model obeys the Gamma process, some reference materials can be found in [21, 22]. Tseng et al. [21] provided the optimal step-stress accelerated degradation test plans by minimizing the approximate variance of the estimated MTTF of the lifetime distribution of the product under constraints on the total cost. Tsai et al. [22] provided the optimal design plans for degradation tests based on a Gamma degradation process with random effects by minimizing the asymptotic variance of the estimate of the 100pth percentile of the lifetime distribution of the product under the constraint that the total experimental cost does not exceed a prespecified budget.

As a basic electromechanical component for transmitting signals and electric energy on the system, the various electrical connector is widely used in the system; meanwhile, its reliability directly determines the reliability of the system equipment; therefore, it is important to correctly evaluate the electrical connector reliability. With the progress of electrical connector process technology, its reliability is higher and has longer lifetime. As present, the accelerated degradation test method usually is used to evaluate electrical connector reliability index. In the test, the entire electrical connector is used as a test sample. The reliability index of the electrical connector is evaluated based on the performance degradation data obtained. If electrical connector model specification is changed, we need to carry out an accelerated degradation test of this model specification. This will consume a lot of manpower, material resources, and financial resources; therefore, the accelerated degradation test of electrical connector component is developed. No type of electrical connector will be omitted by the combination of component type spectrum. The literature [23] indicates that the lifetime of an electrical connector is primarily determined by the contact pair performance. Therefore, if electrical connector contact pairs are considered as the accelerated test object, it not only solves reliability assessment problem of various types of electrical connector but also saves costs and improves efficiency.

The contact resistance growth process of contact pairs of electrical connectors obeys the nonlinear Wiener process and has certain fluctuation. In addition, in most of the above accelerated test optimization design studies, only the drift coefficient is related to the stress in the nonlinear Wiener process model, and the diffusion coefficient is a constant. The literature [23] shows that, for most electronic products, the fluctuation range of the performance parameters increases with the increase of the stress level; that is, the diffusion coefficient is not constant. Therefore, the assumption of a constant diffusion coefficient will affect the evaluation accuracy of the test. As such, with the contact pair of a certain type of aerospace electrical connector as test object and the minimum variance of product reliable life maximum likelihood estimation value at normal stresses as goal, this paper studies the constant-stress accelerated degradation test theories and methods; meanwhile, this paper also optimizes the constant-stress accelerated degradation test plan of contact pairs of electrical connectors.

The rest of this article is as follows. Section 2 presents the statistical model of reliability and the design criterion of accelerated degradation test plan. Section 3 describes the optimization theory of accelerated degradation test plan design. Section 4 establishes the mathematical model to

optimize the accelerated degradation test plan design. Through a case study of the contact pairs of certain type electrical connector, Section 5 illustrates the proposed optimal design method and compared the optimal design plan with traditional test plan. Section 6 analyzes the sensitivity of the parameters. Section 7 concludes this article.

2. Statistical Model of Reliability for Contact Pairs of Electrical Connectors

2.1. Contact Performance Degradation Model for Contact Pairs of Electrical Connectors. An aerospace electrical connector transmits electrical signals via contact spots on the contact pair surface. The contact performance gradually degrades due to temperature stress during long-term storage. Due to the fact that the growth of oxidation corrosive substances on the surface of contact spots obeys a nonlinear Wiener process, macroscopically, the degradation process of contact resistance obeys the nonlinear Wiener process [23]; namely,

$$r(t) = r_0 + \mu\tau(t) + \sigma W(\tau(t)), \tag{1}$$

where $r(t)$ represents the contact resistance at time t; r_0 represents the initial contact resistance, u is the drift coefficient, σ is the diffusion coefficient, $W(\tau(t))$ denotes Brownian motion with mean 0 and variance $\tau(t)$, and, according to the law of oxidation corrosive substances growth obeying the cubic law and both diffusion term and drift term are related to the law of oxidation corrosive substances growth; therefore literature [24] indicates that $\tau(t) = t^{1/3}$(refer to Appendix B for the derivation process).

If the failure mechanism of the electrical connector remains unchanged at different temperatures stress, its specific degradation model can be expressed as follows:

(1) The product degradation process is independent and obeys the nonlinear Wiener process

(2) The relationship between the drift and diffusion coefficients in the Wiener process model and the temperature stress satisfies the following equations:

$$\mu = \exp(a + b\varphi(x)), \tag{2}$$

$$\sigma = \exp(\gamma_0 + \gamma_1\varphi(x)), \tag{3}$$

where a, b, γ_0, and γ_1 are unknown parameters, $x = 1000/(273.15 + T)$ denotes the temperature stress level after conversion, $\varphi(x) = (x - x_0)/(x_M - x_0)$, x_M, x_0, respectively, represent the highest and normal temperature stresses, and T is the ambient temperature (°C)

2.2. Lifetime Distribution Model for Contact Pairs of Electrical Connectors. For degradation failure products, when the performance of a contact pair degrades with time and reaches the failure threshold D, the product fails. Thus, the time when the performance parameter is first taken to reach failure threshold D is defined as the lifetime of the product. When the degradation process follows the Wiener process, the product lifetime follows the inverse Gaussian distribution; that is, the lifetime distribution of the contact pairs of electrical connectors is

$$F_e(t) = \Phi\left(\frac{\mu\tau(t) - D + r_0}{\sqrt{\sigma^2\tau(t)}}\right) + \exp\left\{\frac{2(D - r_0)\mu}{\sigma^2}\right\} \cdot \Phi\left(-\frac{\mu\tau(t) + D - r_0}{\sqrt{\sigma^2\tau(t)}}\right). \tag{4}$$

2.3. Guidelines for Test Design. In order to facilitate the optimal design of constant-stress accelerated degradation test plan for contact pairs of electrical connectors, the following assumptions are given:

(1) During the test, all samples should be tested simultaneously. The censored time for each test stress level is τ_i ($i = 1, \cdots, M$, where M is the number of test stress levels) unless all samples fail.

(2) In the degradation test, the highest stress x_M and the normal stress x_0 are given in advance. The highest stress level is determined based on the criterion that the product failure mechanism does not change [25].

(3) To ensure that sufficient degradation data are obtained for each test stress level, the sample size for each stress level n_i cannot be less than a certain minimum value $n_{\min}$. According to the literature [26], in order to ensure statistical accuracy, the sample size at each stress level should not be less than 5, so the minimum value of the sample size is taken as $n_{\min} = 5$.

(4) In order to ensure sufficient degradation of contact performance in a certain period of time, the acceleration coefficient at the lowest test stress level cannot be smaller than a certain value.

2.4. Guidelines for Optimization of Degradation Test Plans. In order to ensure the accuracy of the product lifetime estimation under certain test time and test cost, with the minimum variance of product pth quantile lifetime estimation value at normal stress x_0 as goal, the constant-stress accelerated degradation test plan is optimized.

2.5. Method of Lifetime Estimation. The maximum likelihood estimation method (MLE) is a statistical method widely used to process accelerated degradation test data. Estimators obtained by the MLE have many excellent properties such as being asymptotically unbiased, asymptotically normal, and asymptotically efficient, and they are easy to be computed. For large sample sizes, the standard deviation of the MLE is the smallest compared to those of other estimation methods. This approach is also preferred to other estimation methods when the sample size is small. Therefore, in the process of optimization of degradation test plan, MLE is used to calculate the pth quantile of the product lifetime distribution and its variance. In addition, prior to

the degradation test plan optimization, the estimated values of the degradation model parameters a, b, γ_0, and γ_1 must be known. These can only be estimated from experience or preliminary test data. Thus, the test plan determined in this manner may not be optimal, but, undoubtedly, it is always better to determine the test plan based on these roughly estimated model parameters than blindly.

3. MLE Estimation Theory for Degradation Test Plan Design

3.1. Standardization of Test Stress. To facilitate the statistical analyses of the data, simplify the model, and make the conclusions more general, the test stress level x_i is standardized as follows:

$$\zeta_i = \frac{x_i - x_0}{x_M - x_0} \text{ where } \zeta_i = \varphi(x_i), \quad i = 1, 2, \ldots, M. \tag{5}$$

Thus,

$$0 = \zeta_0 < \zeta_1 < \zeta_2 < \cdots < \zeta_M = 1. \tag{6}$$

3.2. Sample Likelihood Function Based on the Nonlinear Wiener Process. It is assumed that, at the ith test stress level $\zeta_i\,(i = 1, 2, \cdots, M)$, n_i contact pairs are subjected to the accelerated degradation test. For the jth $(j = 1, 2, \cdots, n_i)$ contact pair, a contact resistance increment $\Delta r_{ijk} = r_{ijk} - r_{ij(k-1)}$. $k\,(k = 2, \cdots, l)$ denotes the number of measurements.

As per the reliability statistical model, the contact resistance increment follows a normal distribution; that is,

$$\Delta r_{ijk} = r_{ijk} - r_{ij(k-1)} \sim \left(\mu_i \Delta t_{ijk}^{(1/3)}, \sigma_i^2 \Delta t_{ijk}^{(1/3)}\right), \tag{7}$$

where $\Delta t_{ijk}^{(1/3)} = t_{ijk}^{(1/3)} - t_{ijk-1}^{(1/3)}$, $\mu_i = \exp(a + b\zeta_i)$, and $\sigma_i = \exp(\gamma_0 + \gamma_1 \zeta_i)$.

Thus, the corresponding log-likelihood function can be obtained.

$$L_i = \sum_{j=1}^{n_i} \sum_{k=1}^{l_i} \left\{ -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln\left(\Delta t_{ijk}^{(1/3)}\right) - \gamma_0 - \gamma_1 \zeta_i - \frac{\left(\Delta r_{ijk} - \exp(a + b\zeta_i)\Delta t_{ijk}^{(1/3)}\right)^2}{2\exp(2\gamma_0 + 2\gamma_1 \zeta_i)\Delta t_{ijk}^{(1/3)}} \right\}. \tag{8}$$

For a total of M stress levels, the log-likelihood function becomes

$$L = \sum_{i=1}^{M} \sum_{j=1}^{n_i} \sum_{k=1}^{l_i} \left\{ -\frac{1}{2}\ln(2\pi) - \frac{1}{2}\ln\left(\Delta t_{ijk}^{(1/3)}\right) - \gamma_0 - \gamma_1 \zeta_i - \frac{\left(\Delta r_{ijk} - \exp(a + b\zeta_i)\Delta t_{ijk}^{(1/3)}\right)^2}{2\exp(2\gamma_0 + 2\gamma_1 \zeta_i)\Delta t_{ijk}^{(1/3)}} \right\}. \tag{9}$$

3.3. Information and Covariance Matrix. According to the MLE theory, the variance and covariance matrix of the degradation model parameters a, b, γ_0, and γ_1 are as follows:

$$\sum = \begin{bmatrix} \mathrm{Var}(\widehat{a}) & \mathrm{Cov}(\widehat{a}, \widehat{b}) & \mathrm{Cov}(\widehat{a}, \widehat{\gamma}_0) & \mathrm{Cov}(\widehat{a}, \widehat{\gamma}_1) \\ \mathrm{Cov}(\widehat{b}, \widehat{a}) & \mathrm{Var}(\widehat{b}) & \mathrm{Cov}(\widehat{b}, \widehat{\gamma}_0) & \mathrm{Cov}(\widehat{b}, \widehat{\gamma}_1) \\ \mathrm{Cov}(\widehat{\gamma}_0, \widehat{a}) & \mathrm{Cov}(\widehat{\gamma}_0, \widehat{b}) & \mathrm{Var}(\widehat{\gamma}_0) & \mathrm{Cov}(\widehat{\gamma}_0, \widehat{\gamma}_1) \\ \mathrm{Cov}(\widehat{\gamma}_1, \widehat{a}) & \mathrm{Cov}(\widehat{\gamma}_1, \widehat{b}) & \mathrm{Cov}(\widehat{\gamma}_1, \widehat{\gamma}_0) & \mathrm{Var}(\widehat{\gamma}_1) \end{bmatrix}. \tag{10}$$

According to the MLE theory, the variance and covariance matrix Σ are the inverse matrix of the Fisher information matrix F; that is, $\Sigma = F^{-1}$. The information matrix is obtained by taking the mathematical expectation of negative second partial derivative of the log-likelihood function L with respect to the degradation model parameters a, b, γ_0, and γ_1 and is given by (refer to Appendix A for the derivation process),

$$F = \sum_{i=1}^{M} n_i \begin{bmatrix} A(\zeta_i)C_{ijk} & \zeta_i A(\zeta_i)C_{ijk} & 0 & 0 \\ \zeta_i A(\zeta_i)C_{ijk} & \zeta_i^2 A(\zeta_i)C_{ijk} & 0 & 0 \\ 0 & 0 & 2l & 2l\zeta_i \\ 0 & 0 & 2l\zeta_i & 2l\zeta_i^2 \end{bmatrix}, \tag{11}$$

where $A(\zeta_i) = (\exp(2a + 2b\zeta_i)/\exp(2\gamma_0 + 2\gamma_1\zeta_i))$ and $C_{ijk} = \sum_{k=1}^{l} \Delta t_{ijk}^{(1/3)}$.

3.4. Asymptotic Variance of Function Estimation. From equation (4), the pth quantile $\widehat{t}_p(\zeta_i)$ of the lifetime distribution under stress ζ_i is a function of parameters r_0, D, a, b, γ_0, and γ_1. The literature [27] indicates that the pth quantile $\widehat{t}_p(\zeta_i)$ under stress ζ_i can be approximated as

$$\hat{t}_P(\zeta_i) \approx \tau^{-1}\left(\frac{\left(z_P \exp(\hat{\gamma}_0 + \hat{\gamma}_1\zeta_i) + \sqrt{z_P^2 \exp(2\hat{\gamma}_0 + 2\hat{\gamma}_1\zeta_i) + 4\exp(\hat{a} + \hat{b}\zeta_i)(D - r_0)}\right)^2}{4\exp(2\hat{a} + 2\hat{b}\zeta_i)}\right), \tag{12}$$

where τ^{-1} is inverse function of τ and z_p is the pth quantile of standard normal distribution.

Since the maximum likelihood estimation has asymptotic normality, its asymptotic variance under a normal stress ζ_0 is

$$\mathrm{Var}(\hat{t}_p(\zeta_0)) = h\sum h', \tag{13}$$

where

$$h = \left(\frac{\partial\hat{t}_p(\zeta_0)}{\partial a}, \frac{\partial\hat{t}_p(\zeta_0)}{\partial b}, \frac{\partial\hat{t}_p(\zeta_0)}{\partial\gamma_0}, \frac{\partial\hat{t}_p(\zeta_0)}{\partial\gamma_1}\right). \tag{14}$$

Let

$$w = \sqrt{z_p^2 \exp(2\gamma_0) + 4\exp(a)(D - r_0)}, \tag{15}$$

and then

$$\begin{aligned}
\frac{\partial\hat{t}_p(\zeta_0)}{\partial a} &= \frac{3(z_p\exp(\gamma_0) + w)^5(D - r_0)}{16\exp(5a)w} - \frac{3(z_p\exp(\gamma_0) + w)^6}{32\exp(6a)},\\
\frac{\partial\hat{t}_p(\zeta_0)}{\partial b} &= 0,\\
\frac{\partial\hat{t}_p(\zeta_0)}{\partial\gamma_0} &= \frac{3(z_p\exp(\gamma_0) + w)^5\left(z_p\exp(\gamma_0) + \left(\left(z_p^2\exp(2\gamma_0)\right)/w\right)\right)}{32\exp(6a)},\\
\frac{\partial\hat{t}_p(\zeta_0)}{\partial\gamma_1} &= 0.
\end{aligned} \tag{16}$$

4. Mathematical Model to Optimize the Accelerated Degradation Test Plan Design

4.1. Objective Function. For an accelerated degradation test with the stress level number of M $(0 < \zeta_1 < \zeta_2 < \cdots < \zeta_M = 1)$ and a total of n contact pairs, $n\pi_i$ samples are allocated to each stress level and π_i is the ratio of samples input for each stress level. Thus, the Fisher information matrix F can be expressed as

$$F = n\sum_{i=1}^{M}\begin{bmatrix} \pi_i A(\zeta_i)C_{ijk} & \pi_i\zeta_i A(\zeta_i)C_{ijk} & 0 & 0\\ \pi_i\zeta_i A(\zeta_i)C_{ijk} & \pi_i\zeta_i^2 A(\zeta_i)C_{ijk} & 0 & 0\\ 0 & 0 & 2\pi_i l & 2\pi_i l\zeta_i\\ 0 & 0 & 2\pi_i l\zeta_i & 2\pi_i l\zeta_i^2 \end{bmatrix}. \tag{17}$$

From equations (12) and (13), the asymptotic variance of the pth quantile of the MLE lifetime estimation for an electrical connector contact pairs at normal stress ζ_0 is

$$\mathrm{Var}(\hat{t}_p(\zeta_0)) = hF^{-1}h' = (V_M/n), \tag{18}$$

where V_M is the variance factor when the number of stress levels is M.

As per the provisions of the test method, before the plan optimization, the highest temperature stress level T_M, normal temperature stress level T_0, test censored time τ_i at each stress level, and failure threshold D are given. Moreover, if the sample size n is also given, the model parameters r_0, a, b, γ_0, and γ_1 can also be obtained through preliminary tests. Consequently, under the principle that the variance of the pth quantile of the lifetime distribution at use stress level is minimum, when the number of stress levels is M, equation (17) shows that the variance factor V_M is the objective function of the optimal design.

4.2. Selection of Design Variables. Under the conditions that the model parameter estimation values r_0, a, b, γ_0, and γ_1 are known, if the test censored time τ_i at each stress level, the highest stress ζ_M, normal stress level ξ_0, and the objective function are completely determined by the remaining $M - 1$ stress levels $\xi_1, \xi_2, \cdots, \xi_{M-1}$ and the allocation ratio of samples $\pi_1, \pi_2, \cdots, \pi_M$ and test interval in the one-third power scale of time $f_1, f_2, \cdots, f_M$ at each stress level, therefore the design variable for the optimal design is

$$\begin{aligned}
&\zeta_1, \zeta_2, \cdots, \zeta_{M-1},\\
&\pi_1, \pi_2, \cdots, \pi_M,\\
&f_1, f_2, \cdots, f_M,
\end{aligned} \tag{19}$$

where the test censored time τ_i refers to the time when the test stops at each stress level, while the test interval f_i $(i = 1, \cdots, M)$ refers to the interval of periodic inspection.

4.3. Determination of Constraints

(1) According to the fourth criterion of the test plan design, the standard test stress level should satisfy the following:

$$0 = \zeta_0 < \zeta_1 < \zeta_2 < \cdots < \zeta_M = 1. \quad (20)$$

(2) To ensure the acceleration of test and obtain more information about product degradation in the effective time, and according to the fourth criterion of the test plan design and the preliminary results, the acceleration factor $\tau_{m/0}$ should satisfy the following:

$$\tau_{m/0} = \hat{t}_p(\zeta_0)/\hat{t}_p(\zeta_m) \geq 32. \quad (21)$$

where the acceleration factor $\tau_{m/0}$ is defined as the ratio of the pth quantile of the lifetime under the lowest test stress level ζ_m relative to its counterpart at use stress levelζ_0.

(3) The allocation ratio of samples $\pi_1, \pi_2, \cdots, \pi_M$ at each stress level must satisfy the following condition:

$$\sum_{i=1}^{M} \pi_i = 1 \text{ and } 0 < \pi_i < 1, \quad (i = 1, 2, \cdots, M-1). \quad (22)$$

(4) According to the third criterion of the test plan design, the total sample size input for the test is n, and the sample size at each stress level should not be smaller than the minimum value $n_{\min}$. Therefore, the sample size under each stress is the following:

$$n_i = \max[n_{\min}, n\pi_i]. \quad (23)$$

(5) On the one hand, in order to ensure that the sample has a certain degradation trend, the test time in the test chamber should not be less than 30 minutes; that is to say, the test interval of each stress on the scale of one-third of the time should not be less than 0.79 h; on the other hand, in order to obtain more reliability information of the product, the test time shall be no less than 6 times as much as possible. According to the previous test results [28], the electrical connector did not fail in 1209 hours at 105°C. In order to obtain enough sample performance data before the test censored time, the censored time under each stress level of this test is set as 5000 hours. Therefore, it is concluded that the test interval of each stress on the scale of one-third of time should not be higher than 2.85 h. In conclusion, the range of test interval f_i at one-third time scale under each stress level is

$$0.79 \leq f_i \leq 2.85; \quad i = 1, 2, \cdots, M. \quad (24)$$

where f_i is the test interval under each stress level and M is the number of stress levels.

(6) In order to ensure that the contact pair of electrical connector has enough degradation in each test, the following conditions should be met in each test under each stress:

$$\Delta r \geq 0.013(D - r_0). \quad (25)$$

4.4. Determination of Sample Size. Based on the MLE theory, under normal stress level, the estimation value $\hat{t}_p(\zeta_0)$ of the pth quantile at a certain time asymptotically follows a normal distribution with mean $t_p(\zeta_0)$ and variance $\mathrm{Var}[t_p(\hat{\zeta}_0)]$; that is,

$$\hat{t}_p(\zeta_0) \sim N\left(t_p(\zeta_0), \mathrm{Var}\left[\hat{t}_p(\zeta_0)\right]\right). \quad (26)$$

If $t_p(\zeta_0)$ is replaced by $\hat{t}_p(\zeta_0)$, the upper and lower confidence limits, respectively, for $\hat{t}_p(\zeta_0)$ at the confidence level ϕ are

$$\left[t_p(\zeta_0)\right]_U = \hat{t}_p(\zeta_0) + K_\phi (V_M/n)^{1/2}, \quad (27)$$

$$\left[t_p(\zeta_0)\right]_L = \hat{t}_p(\zeta_0) - K_\phi (V_M/n)^{1/2}, \quad (28)$$

where K_ϕ is the $(1+\phi)/2$-th quantile of the standard normal distribution at the confidence level ϕ. Under the condition that the probability is ϕ, when $\hat{t}_p(\zeta_0)$ is required to be within the range of $t_p(\zeta_0) \pm W$ (where $t_p(\zeta_0)$ denotes the true value), the sample size is the following:

$$n = V_M\left(K_\phi/W\right)^2. \quad (29)$$

5. Optimal Design of Accelerated Degradation Test Plan for Electrical Connector Contact Pairs Based on the Nonlinear Wiener Process

5.1. Determination of Test Parameters

5.1.1. Failure Threshold and Test Stress Level. In this paper, the contact pair of a certain type of aerospace electrical connector are considered as the test object. Test stresses must satisfy the principle that the failure mechanism at test stress levels is consistent with that at normal stress level. As per the fact that the failure of aerospace electrical connectors is contact failure (contact failure threshold, $D = 5\,m\Omega$) at a test temperature stress level of 158°C [28], we thereby conclude that the highest temperature stress level is $T_M = 158$°C, while the corresponding stress after conversion is $x_M = 2.3194$, and the standard stress is $\zeta_M = 1$. The results of the investigation on the actual storage profile of electrical connectors indicate that the normal temperature is $T_0 = 30$°C, while the converted stress is $x_0 = 3.2987$ and the standard stress is $\xi_0 = 0$.

5.1.2. Initial Estimation of Degradation Model Parameters. Based on the results of the statistical analysis of the preliminary test data [24], the rough estimated values of the parameters of the reliability statistical model for the contact pairs of certain type electrical connector are obtained:

$$\begin{aligned} r_0 &= 2.63, \\ a &= -12.0033, \\ b &= 10.2968, \\ \gamma_0 &= -2.7057, \\ \gamma_1 &= 1.8391. \end{aligned} \tag{30}$$

5.2. The Optimal Design Plan of Stress Level $M = 3, 4, 5$. Let $p = 0.05$; that is, reliability $R = 0.95$. The variance factor V_M of the product lifetime maximum likelihood estimation value under a normal stress level is taken as the objective function. The lowest stress level ξ_1 and the allocation ratio of samples $\pi_1, \pi_2, \cdots, \pi_M$ and test interval at the one-third power scale of time $f_1, f_2, \cdots, f_M$ are used as the design variables; meanwhile, equal reciprocal interval of temperature method is used to calculate stress level. Therefore, the intermediate stress level can be calculated according to formula (31). The accelerated degradation test plan for the contact pair of certain type of electrical connector is optimized. The optimized test plans for stress levels of $M = 3, 4, 5$ are shown in Tables 1–3:

$$\zeta_i = \frac{(M-i)\zeta_1 + (i-1)\zeta_M}{M-1}, \quad i = 2, \cdots, M-1. \tag{31}$$

If the confidence level and confidence width are defined as $\phi = 80\%$ and $2W = 0.4$, respectively, then $K_\phi = 1.28155$. Therefore, the total numbers of samples for optimum test plans with stress levels of $M = 3, 4, 5$ are as follows: $n_{M=3} = 37$, $n_{M=4} = 42$, and $n_{M=5} = 48$.

From Tables 1–3, it can be found that, in the optimum test plan of stress level $M = 3$, 4, 5, the estimation accuracy decreases gradually with the increase of the number of stress levels, and, in each optimum test plan, the allocation ratio of sample distribution under the minimum stress level is larger than that under other stress levels. It shows that the minimum stress has greater influence on the estimation accuracy of the objective function than other stress levels.

Although the estimation accuracy of the three stress levels is the best, from the perspective of the robustness and effectiveness of the test, the engineering design can choose an optimal design with a stress level of $M = 4$.

5.3. Comparison of Optimal Test Plan and Traditional Test Plan. The optimal test plan for stress level $M = 3, 4, 5$ is shown in Tables 1–3. For comparison, a traditional test plan with stress level $M = 3, 4, 5$ from literature [23] is employed. The test interval at each stress level is equal and allocation of samples is the same at each stress level in the traditional test plan. The results of Tables 4–6 are as follows:

(i) The contrast results of test plan with stress level $M = 3$ show the following: the sample size of the traditional test plan $n = 42$, and the total sample of the optimal test plan with stress level $M = 3$ is reduced by a factor of 1.14 compared to the traditional test plan. When the traditional test plan reaches the accuracy of the optimal test plan, the test time of the traditional test plan at each stress is 6313 h; the test time of the optimal test plan with stress level $M = 3$ is reduced by a factor of 1.26 compared to the traditional test plan.

(ii) The contrast results of test plan with stress level $M = 4$ show the following: the sample size of the general test plan $n = 51$, and the total sample of the optimal test plan with stress level $M = 4$ is reduced by a factor of 1.2 compared to the traditional test plan. When the traditional test plan reaches the accuracy of the optimal test plan, the test time of the traditional test plan at each stress is 6566 h; the test time of the optimal test plan with stress level $M = 4$ is reduced by a factor of 1.31 compared to the traditional test plan.

(iii) The contrast results of test plan with stress level $M = 5$ show the following: the sample size of the traditional test plan $n = 56$, and the total sample of the optimal test plan with stress level $M = 5$ is reduced by a factor of 1.17 compared to the traditional test plan. When the traditional test plan reaches the accuracy of the optimal test plan, the test time of the traditional test plan at each stress is 6459 h; the test time of the optimal test plan with stress level $M = 5$ is reduced by a factor of 1.29 compared to the traditional test plan.

5.4. Analysis of the Influence of the Lowest Stress Level and the Difference of High and Low Stress Levels on Variance Factor V_M. The allocation ratio of sample for the lowest stress level is higher than others in Tables 1–3; this result indicates that influence of the lowest stress level on variance factor V_M is higher than other stress levels. Therefore, this section analyzes how the lowest stress level T_1 influences variance factor V_M. Taking the traditional test plan of four stress levels as example, this paper chooses the lowest stress level T_1 as 100°C, 105°C, 110°C, and 115°C; meanwhile, due to the preliminary test result, the highest stress level is 158°C in the above. In order to explore how the variance factor changes under different values of the highest stress level, the highest stress level T_2 is 125°C, 135°C, 145°C as a comparison; according to the equal interval formula, the middle stress level $\xi_i = ((M-i)\xi_1 + (i-1)\xi_M/M-1)$, $i = 1, 2, \ldots, M-1$. The results of each stress combination are listed in Table 7. The results of Table 7 are as follows:

(i) Variance factor V_M is minimum at $T_1 = 100°C, T_2 = 158°C$. The variance factor V_M is maximum at $T_1 = 100°C, T_2 = 125°C$

(ii) Figure 1 shows that the variance factor V_M increases with the increase of the lowest stress level T_1 when T_2 is constant and T_1 is increased from 100°C to 115°C

(iii) Variance factor V_M decreases with the increase of the highest stress level T_2 when T_1 is constant

Table 1: The optimal design plan of stress level $M = 3$.

The optimal design plan of stress level $M = 3$				
Temperature stress T (°C)	Sample distribution ratio	Test frequency ($t^{(1/3)}$)	Specific test time t (h)	Variance factor $V_M \times 10^{18}$
119	0.571	1.9	6.86 54.88 185.22 ⋮ 5000	0.89
138	0.132	0.81	0.53 4.24 14.31 ⋮ 4908.33	
158	0.296	0.81	0.53 4.24 14.31 ⋮ 4908.33	

Table 2: The optimal design plan of stress level $M = 4$.

The optimal design plan of stress level $M = 4$				
Temperature stress T (°C)	Sample distribution ratio	Test frequency ($t^{(1/3)}$)	Specific test time t (h)	Variance factor $V_M \times 10^{18}$
119	0.545	1.9	6.86 54.88 185.22 ⋮ 5000	1.02
132	0.125	0.9	0.729 5.8 19.68 ⋮ 5000	
144	0.132	0.85	0.61 5.49 16.47 ⋮ 4880	
158	0.197	0.81	0.53 4.24 14.31 ⋮ 4908.33	

(iv) Figures 2 and 3 indicate that the variance factor V_M increases with the decrease of the value of $T_2 - T_1$; therefore, the engineering test should choose the large difference value of T_2 and T_1 in reasonable range

6. Analysis of Parameter Sensitivity

Sensitivity is expressed as a dimensionless index; it reflects the extent to which the output value of the model changes with small changes in parameters. In view of several key parameters (a, b, γ_0, and γ_1) of the reliability statistical model of electrical connector contact pairs, this paper explores how the change of parameters influences the object function value. The values of the other three parameters are fixed; one of the parameter values needs to be changed. The change range is taken as ±5%, ±10%, and ±15%.

6.1. Analysis of Parameter Sensitivity with Four Stress Levels. When the number of stress levels is 4, the fluctuation results of object function value change with parameters are listed in Table 8 and Figures 4–6. τ_1, τ_2, τ_3, and τ_4 represent the errors of parameters a, b, γ_0, and γ_1, respectively. The results are as follows.

According to Table 8 and Figures 4–6, when the error range of the parameter is ±5%, ±10%, and ±15%, the change

Table 3: The optimal design plan of stress level $M = 5$.

The optimal design plan of stress level $M = 5$				
Temperature stress T (°C)	Sample distribution ratio	Test frequency ($t^{(1/3)}$)	Specific test time t (h)	Variance factor $V_M \times 10^{18}$
			6.86	
			54.88	
119	0.483	1.9	185.22	
			⋮	
			50000	
			1.19	
			9.52	
128	0.127	1.06	32.13	
			⋮	
			4874.24	
			0.61	
			4.88	
138	0.13	0.85	16.47	1.17
			⋮	
			4880	
			0.53	
			4.24	
148	0.127	0.81	14.31	
			⋮	
			4908.33	
			0.53	
			4.24	
158	0.134	0.81	14.31	
			⋮	
			4908.33	

Table 4: $M = 3$: constant stress accelerated degradation test plan.

The optimal test plan				The traditional test plan			
Temperature stress T (°C)	Sample distribution ratio	Test frequency ($t^{(1/3)}$)	Variance factor $V_M \times 10^{18}$	Temperature stress T (°C)	Sample distribution ratio	Test frequency ($t^{(1/3)}$)	Variance factor $V_M \times 10^{18}$
119	0.571	1.9		105	0.333	2.29	
138	0.132	0.81	0.89	130	0.333	2.29	1.03
158	0.296	0.81		158	0.333	2.29	

Table 5: $M = 4$: constant stress accelerated degradation test plan.

The optimal test plan				The traditional test plan			
Temperature stress T (°C)	Sample distribution ratio	Test frequency ($t^{(1/3)}$)	Variance factor $V_M \times 10^{18}$	Temperature stress T (°C)	Sample distribution ratio	Test frequency ($t^{(1/3)}$)	Variance factor $V_M \times 10^{18}$
119	0.545	1.9		105	0.25	2.29	
132	0.125	0.9	1.02	121	0.25	2.29	1.23
144	0.132	0.85		139	0.25	2.29	
158	0.197	0.81		158	0.25	2.29	

amount of variance factor is very small, so it shows that the robustness of this optimization plan is good.

6.2. Analysis of Parameter Sensitivity with Three Stress Levels. In order to explore whether the optimization plan still has a good robustness when the number of stresses changes, the optimization plan of 3 stress levels is explored when the error is ±10%. The results of Table 9 and Figure 7 show that the optimization plan of 3 stress levels is the same as the optimization plan of 4 stress levels, which has a good robustness. This shows that the robustness of the plan will not change greatly with the change of stress quantity.

Table 6: $M = 5$: constant stress accelerated degradation test plan.

The optimal test plan				The traditional test plan			
Temperature stress T(°C)	Sample distribution ratio	Test frequency ($t^{(1/3)}$)	Variance factor $V_M \times 10^{18}$	Temperature stress T(°C)	Sample distribution ratio	Test frequency ($t^{(1/3)}$)	Variance factor $V_M \times 10^{18}$
119	0.483	1.9		105	0.2	2.29	
128	0.127	1.06		117	0.2	2.29	
138	0.13	0.85	1.17	130	0.2	2.29	1.36
148	0.127	0.81		143	0.2	2.29	
158	0.134	0.81		158	0.2	2.29	

Table 7: The test results under T_1 and T_2 combination.

T_1 (°C)	T_2 (°C)	$V_M \times 10^{18}$
100	158	0.96
105		1.22
110		1.59
115		2.11
100	145	1.36
105		1.84
110		2.56
115		3.72
100	135	1.97
105		2.87
110		4.42
115		7.38
100	125	3.35
105		5.62
110		10.74
115		25.88

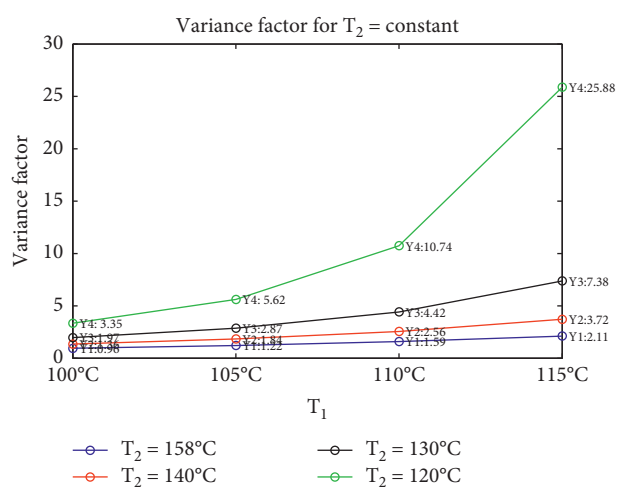

Figure 1: V_M changes with T_1 when T_2 is constant.

7. Conclusions

According to the criterion of the minimum variance of product reliable lifetime MLE value under the reliability of 0.95 at use stress level, with the test stress levels, the allocation ratio of sample, and test interval at each stress level as design variables, this paper establishes a mathematical model for a constant-stress accelerated degradation test plan optimization design when the performance degradation model obeys a nonlinear Wiener process. The accuracy of product reliable lifetime estimation value at normal stress level is improved by optimizing test plan; meanwhile, the number of test samples and the test times are reduced. Taking the accelerated degradation test of contact pairs of

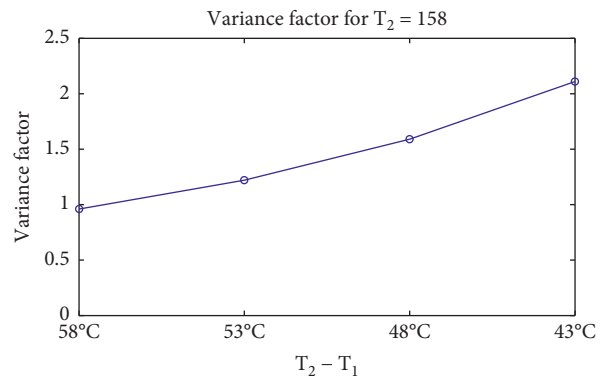

Figure 2: V_M changes with $T_2 - T_1$ when T_2 is 158°C.

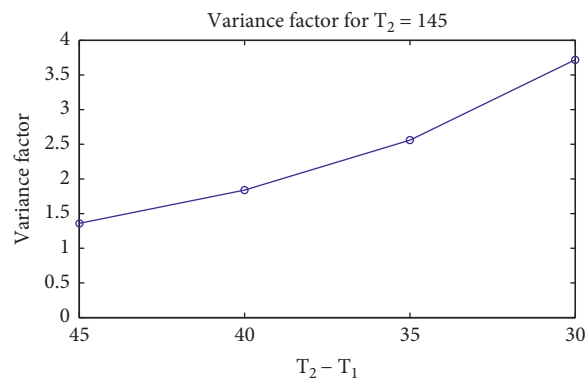

Figure 3: V_M changes with $T_2 - T_1$ when T_2 is 145°C.

Table 8: Analysis of parameter sensitivity.

The fluctuation range is ±5%					The fluctuation range is ±10%					The fluctuation range is ±15%				
τ_1 (%)	τ_2 (%)	τ_3 (%)	τ_4 (%)	$V_M \times 10^{18}$	τ_1 (%)	τ_2 (%)	τ_3 (%)	τ_4 (%)	$V_M \times 10^{18}$	τ_1 (%)	τ_2 (%)	τ_3 (%)	τ_4 (%)	$V_M \times 10^{18}$
+5	0	0	0	1.03	+10	0	0	0	1	+15	0	0	0	1.08
−5	0	0	0	1.05	−10	0	0	0	1.05	−15	0	0	0	0.99
0	+5	0	0	0.97	0	+10	0	0	0.98	0	+15	0	0	0.99
0	−5	0	0	0.974	0	−10	0	0	1.21	0	−15	0	0	0.98
0	0	+5	0	1.04	0	0	+10	0	1.03	0	0	+15	0	1.03
0	0	−5	0	0.99	0	0	−10	0	1.11	0	0	−15	0	0.98
0	0	0	+5	0.98	0	0	0	+10	0.99	0	0	0	+15	1.15
0	0	0	−5	1.04	0	0	0	−10	1.06	0	0	0	−15	0.97

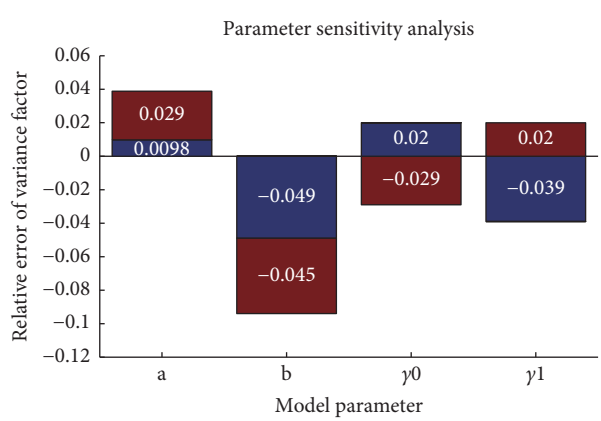

Figure 4: Parameter sensitivity analysis with a fluctuation range of ± 5%.

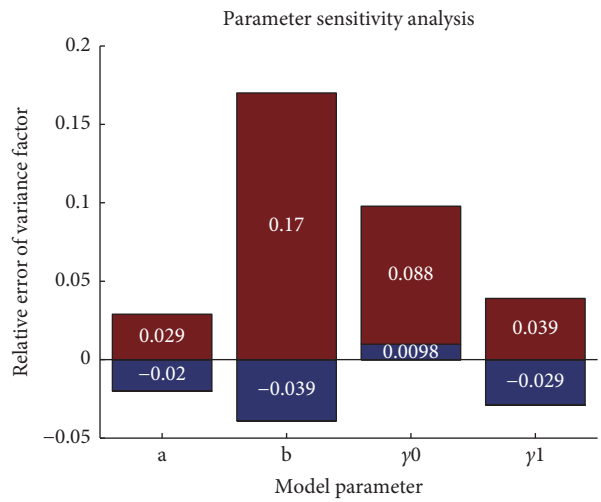

Figure 5: Parameter sensitivity analysis with a fluctuation range of ± 10%.

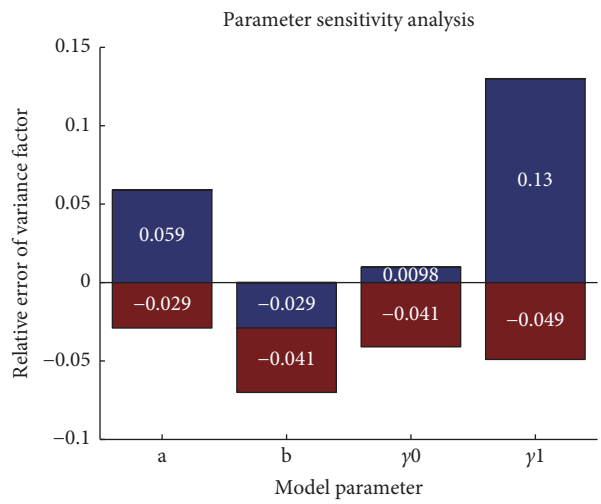

Figure 6: Parameter sensitivity analysis with a fluctuation range of ±15%.

Table 9: Analysis of parameter sensitivity with fluctuation range of ±10%.

τ_1 (%)	τ_2 (%)	τ_3 (%)	τ_4 (%)	$V_M \times 10^{18}$	Optimal (V_M)
+10	0	0	0	0.85	
−10	0	0	0	0.89	
0	+10	0	0	0.89	
0	−10	0	0	0.86	0.89
0	0	+10	0	0.92	
0	0	−10	0	0.85	
0	0	0	+10	0.9	
0	0	0	−10	0.89	

electrical connectors subjected to nonlinear wiener process as an example, the optimization result of test plan with stress levels of 3, 4, and 5 shows that, at the same confidence level, the estimation accuracy of optimal test plan is improved compared to the traditional test plan; meanwhile, the total

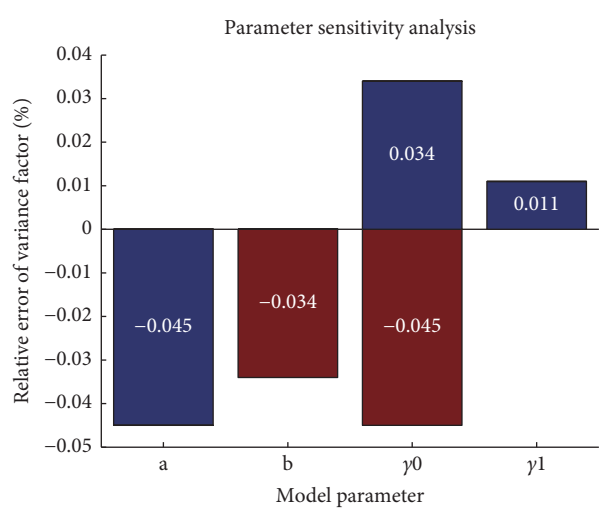

Figure 7: Parameter sensitivity analysis with a fluctuation range of ±10%.

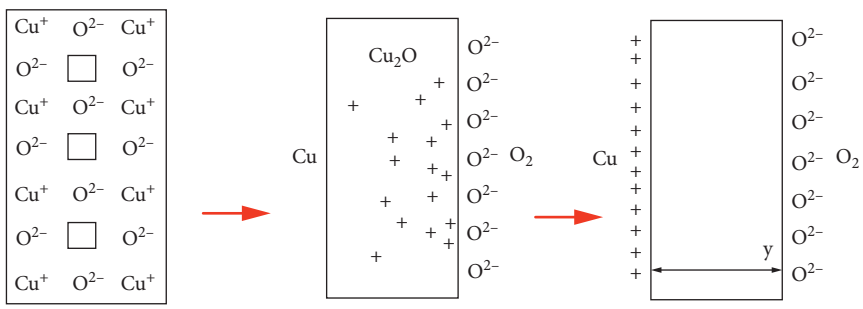

Figure 8: Oxide film growth mechanism.

number of samples is also reduced. The estimation accuracy with stress level of 3 is improved by a factor of 1.16 and the total sample size is reduced by a factor of 1.14. The estimation accuracy with stress level of 4 is improved by a factor of 1.21 and the total sample size is reduced by a factor of 1.2. The estimation accuracy with stress level of 5 is improved by a factor of 1.16 and the total sample size is reduced by a factor of 1.17. At the same estimation accuracy, the optimal design test plan time is shortened. The optimal design test plan time with stress level of 3 is reduced by a factor of 1.26. The optimal design test plan time with stress level of 4 is reduced by a factor of 1.31. The optimal design test plan time with stress level of 5 is reduced by a factor of 1.29. The effect of the lowest stress level on the variance factor V_M is analyzed. The results show that the variance factor V_M increases with the increase of the lowest stress level; meanwhile, the variance factor V_M increases with the decrease of the difference between the highest stress and the lowest stress. Therefore, the engineering test should choose the large difference value of T_2 and T_1 in reasonable range. When the error range of parameter is 5%−15%, the change of the estimation accuracy of the optimization plan is very small, the plan has good robustness, and the change of the stress quantity has little effect on the plan's robustness.

Appendix

A. The Derivation of the Parameter 1/3

As the picture shows, the diffusion rate of Cu^+ ions in the oxide film replaces the electron mobility rate as a controlling factor for the growth rate of the oxide film Cu^+ ions diffusing outward through the electric double layer of the oxide film to the surface of the film layer and reacts with O^{2-} ions adsorbed on the surface to form Cu_2O; this promotes oxide film growth. Therefore, the growth rate of the oxide film depends on the copper ion vacancy flow i_{Cu^+}; meanwhile, i_{Cu^+} is proportional to the electric field strength E and the Cu^+ ions vacancy concentration:

$$\frac{dy}{dt} \propto i_{Cu^+} \propto En_{Cu^+}. \tag{A.1}$$

It is known from the principle of electrical neutrality that the concentration of Cu^+ is proportional to the concentration of O^{2-} adsorbed on the surface of the oxide:

$$n_{Cu^+} = kn_{O^{2-}}. \tag{A.2}$$

Electric field strength E is

$$E = \frac{4\pi e n_o}{\varepsilon}, \tag{A.3}$$

where ε is the dielectric constant of the oxide.

Since the electric field strength decreases as the oxide film grows, the electric field intensity is inversely proportional to the thickness y of the oxide film; n_{cu^+} is also inversely proportional to the thickness of the oxide film; that is,

$$En_{Cu^+} \propto \frac{1}{y^2}. \tag{A.4}$$

Then,

$$\frac{dy}{dt} \alpha \frac{1}{y^2}. \tag{A.5}$$

By integrating the upper formula, the oxide growth is cubic (Figure 8).

$$y = k_3 t^{(1/3)}. \tag{A.6}$$

B. The First Partial Derivatives of L_i with respect to Each Parameter are Given by

$$\frac{\partial L_i}{\partial a} = \sum_{j=1}^{n_i}\sum_{k=1}^{l_i}\left\{\frac{\left(\Delta r_{ijk} - \exp(a+b\zeta_i)\Delta t_{ijk}^{(1/3)}\right)\exp(a+b\zeta_i)}{\exp(2\gamma_0 + 2\gamma_1\zeta_i)}\right\},$$

$$\frac{\partial L_i}{\partial b} = \sum_{j=1}^{n_i}\sum_{k=1}^{l_i}\left\{\frac{\left(\Delta r_{ijk} - \exp(a+b\zeta_i)\Delta t_{ijk}^{(1/3)}\right)\exp(a+b\zeta_i)}{\exp(2\gamma_0 + 2\gamma_1\zeta_i)}\zeta_i\right\},$$

$$\frac{\partial L_i}{\partial \gamma_0} = \sum_{j=1}^{n_i}\sum_{k=1}^{l_i}\left\{-1 + \frac{\left(\Delta r_{ijk} - \exp(a+b\zeta_i)\Delta t_{ijk}^{(1/3)}\right)^2}{\exp(2\gamma_0 + 2\gamma_1\zeta_i)\Delta t_{ijk}^{(1/3)}}\right\},$$

$$\frac{\partial L_i}{\partial \gamma_1} = \sum_{j=1}^{n_i}\sum_{k=1}^{l_i}\left\{\left(-1 + \frac{\left(\Delta r_{ijk} - \exp(a+b\zeta_i)\Delta t_{ijk}^{(1/3)}\right)^2}{\exp(2\gamma_0 + 2\gamma_1\zeta_i)\Delta t_{ijk}^{(1/3)}}\right)\zeta_i\right\}. \tag{B.1}$$

The second partial derivatives of L_i with respect to each parameter are given by

$$\frac{\partial^2 L_i}{\partial a^2} = \sum_{j=1}^{n_i}\sum_{k=1}^{l_i}\left\{\frac{\left(\Delta r_{ijk} - \exp\left(a + b\zeta_i\right)\Delta t_{ijk}^{(1/3)}\right)\exp\left(a + b\zeta_i\right)}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)} - \frac{\exp\left(2a + 2b\zeta_i\right)\Delta t_{ijk}^{(1/3)}}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)}\right\},$$

$$\frac{\partial^2 L_i}{\partial a\,\partial b} = \sum_{j=1}^{n_i}\sum_{k=1}^{l_i}\left\{\frac{\left(\Delta r_{ijk} - \exp\left(a + b\zeta_i\right)\Delta t_{ijk}^{(1/3)}\right)\exp\left(a + b\zeta_i\right)}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)} - \frac{\exp\left(2a + 2b\zeta_i\right)\Delta t_{ijk}^{(1/3)}\cdot\zeta_i}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)}\right\},$$

$$\frac{\partial^2 L_i}{\partial a\,\partial\gamma_0} = \sum_{j=1}^{n_i}\sum_{k=1}^{l_i}\left\{-2\frac{\left(\Delta r_{ijk} - \exp\left(a + b\zeta_i\right)\Delta t_{ijk}^{(1/3)}\right)\exp\left(a + b\zeta_i\right)}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)}\right\},$$

$$\frac{\partial^2 L_i}{\partial a\,\partial\gamma_1} = \sum_{j=1}^{n_i}\sum_{k=1}^{l_i}\left\{-2\frac{\left(\Delta r_{ijk} - \exp\left(a + b\zeta_i\right)\Delta t_{ijk}^{(1/3)}\right)\exp\left(a + b\zeta_i\right)\cdot\zeta_i}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)}\right\},$$

$$\frac{\partial^2 L_i}{\partial b^2} = \sum_{j=1}^{n_i}\sum_{k=1}^{l_i}\left\{\frac{\left(\Delta r_{ijk} - \exp\left(a + b\zeta_i\right)\Delta t_{ijk}^{(1/3)}\right)\exp\left(a + b\zeta_i\right)\cdot\zeta_i^2}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)} - \frac{\exp\left(2a + 2b\zeta_i\right)\Delta t_{ijk}^{(1/3)}\cdot\zeta_i^2}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)}\right\},$$

$$\frac{\partial^2 L_i}{\partial b\,\partial\gamma_0} = \sum_{j=1}^{n_i}\sum_{k=1}^{l_i}\left\{-2\frac{\left(\Delta r_{ijk} - \exp\left(a + b\zeta_i\right)\Delta t_{ijk}^{(1/3)}\right)\exp\left(a + b\zeta_i\right)\cdot\zeta_i}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)}\right\},$$

$$\frac{\partial^2 L_i}{\partial b\,\partial\gamma_1} = \sum_{j=1}^{n_i}\sum_{k=1}^{l_i}\left\{-2\frac{\left(\Delta r_{ijk} - \exp\left(a + b\zeta_i\right)\Delta t_{ijk}^{(1/3)}\right)\exp\left(a + b\zeta_i\right)\cdot\zeta_i^2}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)}\right\},$$

$$\frac{\partial^2 L_i}{\partial\gamma_0^2} = \sum_{j=1}^{n_i}\sum_{k=1}^{l_i}\left\{-2\frac{\left(\Delta r_{ijk} - \exp\left(a + b\zeta_i\right)\Delta t_{ijk}^{(1/3)}\right)^2}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)\Delta t_{ijk}^{(1/3)}}\right\},$$

$$\frac{\partial^2 L_i}{\partial\gamma_0\partial\gamma_1} = \sum_{j=1}^{n_i}\sum_{k=1}^{l_i}\left\{-2\zeta_i\frac{\left(\Delta r_{ijk} - \exp\left(a + b\zeta_i\right)\Delta t_{ijk}^{(1/3)}\right)^2}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)\Delta t_{ijk}^{(1/3)}}\right\},$$

$$\frac{\partial^2 L_i}{\partial\gamma_1^2} = \sum_{j=1}^{n_i}\sum_{k=1}^{l_i}\left\{-2\zeta_i^2\frac{\left(\Delta r_{ijk} - \exp\left(a + b\zeta_i\right)\Delta t_{ijk}^{(1/3)}\right)^2}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)\Delta t_{ijk}^{(1/3)}}\right\},$$

$$E\left(\frac{\partial^2 L_i}{\partial a^2}\right) = \frac{\exp\left(2a + 2b\zeta_i\right)}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)}\sum_{j=1}^{n_i}\sum_{k=1}^{l_i}\Delta t_{ijk}^{(1/3)} = \frac{\exp\left(2a + 2b\zeta_i\right)}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)}n_i\sum_{k=1}^{l_i}\Delta t_{ijk}^{(1/3)} = n_i A\left(\zeta_i\right)C_{ijk},$$

$$E\left(\frac{\partial^2 L_i}{\partial a\,\partial b}\right) = \frac{\exp\left(2a + 2b\zeta_i\right)\zeta_i}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)}\sum_{j=1}^{n_i}\sum_{k=1}^{l_i}\Delta t_{ijk}^{(1/3)} = \frac{\exp\left(2a + 2b\zeta_i\right)\zeta_i}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)}n_i\sum_{k=1}^{l_i}\Delta t_{ijk}^{(1/3)} = n_i\zeta_i A\left(\zeta_i\right)C_{ijk},$$

$$E\left(\frac{\partial^2 L_i}{\partial a\,\partial \gamma_0}\right) = 0,$$

$$E\left(\frac{\partial^2 L_i}{\partial a\,\partial \gamma_1}\right) = 0,$$

$$E\left(\frac{\partial^2 L_i}{\partial b^2}\right) = \frac{\exp\left(2a + 2b\zeta_i\right)\zeta_i^2}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)} \sum_{j=1}^{n_i}\sum_{k=1}^{l_i} \Delta t_{ijk}^{(1/3)} = \frac{\exp\left(2a + 2b\zeta_i\right)\zeta_i^2}{\exp\left(2\gamma_0 + 2\gamma_1\zeta_i\right)} n_i \sum_{k=1}^{l_i} \Delta t_{ijk}^{(1/3)} = n_i\zeta_i^2 A\left(\zeta_i\right)C_{ijk},$$

$$E\left(\frac{\partial^2 L_i}{\partial b\,\partial \gamma_0}\right) = 0,$$

$$E\left(\frac{\partial^2 L_i}{\partial b\,\partial \gamma_1}\right) = 0,$$

$$E\left(\frac{\partial^2 L_i}{\partial \gamma_0^2}\right) = 2l_i n_i,$$

$$E\left(\frac{\partial^2 L_i}{\partial \gamma_0 \partial \gamma_1}\right) = 2l_i n_i \zeta_i,$$

$$E\left(\frac{\partial^2 L_i}{\partial \gamma_1^2}\right) = 2l_i n_i \zeta_i^2. \tag{B.2}$$

Since

$$E\left(-\frac{\partial^2 L}{\partial \theta^2}\right) = E\left(-\sum_{i=1}^{M}\frac{\partial^2 L_i}{\partial \theta^2}\right) = \sum_{i=1}^{M} E\left(-\frac{\partial^2 L_i}{\partial \theta^2}\right),$$

$$E\left(-\frac{\partial^2 L}{\partial \theta_1 \partial \theta_2}\right) = E\left(-\sum_{i=1}^{M}\frac{\partial^2 L_i}{\partial \theta_1 \partial \theta_2}\right) = \sum_{i=1}^{M} E\left(-\frac{\partial^2 L_i}{\partial \theta_1 \partial \theta_2}\right). \tag{B.3}$$

Then, the Fisher information matrix is given as in equation (11).

Data Availability

The data used to support the findings of this study are available from the corresponding author upon reasonable request.

Conflicts of Interest

The authors declare no conflicts of interest with respect to the research, authorship, and/or publication of this article.

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