

## Research Article

# An Improved Prediction Model Combining Inverse Exponential Smoothing and Markov Chain

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On the basis of the triple exponential smoothing prediction model, this paper introduces the reverse prediction idea and establishes the reverse triple exponential smoothing model by setting parameters such as threshold value and iteration times and reasonably correcting its initial value. This method can effectively reduce the error of early prediction value. At the same time, aiming at the problem that the predicting advantage of the reverse triple exponential smoothing model weakens in the later period, Markov theory is introduced to correct its error value, and an improved prediction model combining inverse exponential smoothing and Markov chain is further established. The improved model combines the advantages of index model trend prediction and Markov fluctuation prediction, and the prediction accuracy and stability of the model are significantly improved through case tests.

## 1. Introduction

National defense expenditure is an important component of national financial expenditure. It is the source of funds and important support for national defense and military construction. It reflects the economic level of a country's investment in national defense construction and embodies national defense policy and national defense strategy [1]. Predicting a country's national defense expenditure is not only helpful to analyze the trend of the country's national defense and military construction but also helpful to analyze the relationship between its national defense expenditure and economic growth. Therefore, it is of far-reaching practical significance to make a reasonable prediction of national defense expenditure. In this paper, a new model is established on the basis of exponential smoothing model to effectively predict defense expenditure.

Exponential smoothing method is a time series analysis and prediction method. This method predicts the future trend according to the current situation and data by calculating the smoothing value of the index and combining with a reasonable time series prediction model [2–4]. The

exponential smoothing method can be divided into single exponential smoothing method, double exponential smoothing method, and triple exponential smoothing method according to exponential times. Among them, the triple exponential smoothing method is often used to fit and predict nonlinear time series and has achieved good prediction effect especially in short-term and medium-term prediction of nonlinear time series, with small error fluctuation range and strong credibility. At present, it has been widely used in the fields of public transportation passenger volume prediction [5, 6], economic output value prediction [7, 8], spare parts prediction [9, 10], wind speed prediction of wind farms [11, 12], building displacement prediction [13, 14], GPS PWV prediction [15], Docker container resource load prediction [16], etc. Qi and Huo proposed a single exponential smoothing model based on self-adaptation. By introducing the approximate dynamic programming method and combining with the actual traffic flow data, the exponential smoothing coefficient was optimized to make it update automatically with the prediction process, thus ensuring the real-time accuracy of the prediction [17]. Wang et al. proposed an adaptive dynamic cubic exponential

smoothing prediction method. In this method, the carpet search method is used, the best smoothing coefficient is obtained according to the principle of minimum sum of squares error, and the prediction effect of the model is verified by an example of wind speed data [18]. Mi et al. proposed a short-term power load forecasting method based on improved exponential smoothing gray model. This method combined the exponential smoothing model and gray model and used the 0.618 method to search for the best smoothing coefficient, which achieved good prediction effect [19]. Liu et al. proposed a new short-medium satellite clock error prediction algorithm based on the modified exponential smoothing method, improved the weighted parameters in ES, and proposed the dynamic weighted parameters based on the sliding window. The gray scale model (GM) is introduced to learn the prediction error of DES, which improves the prediction accuracy of the algorithm [20].

In this paper, the triple exponential smoothing predicting model is applied to the field of defense expenditure prediction. On the basis of the triple exponential smoothing model, the error trend and fluctuation of the initial data are fully considered, the reverse predicting idea and Markov state transition matrix are introduced to correct its data fluctuation, and a reverse triple exponential smoothing model based on Markov correction is established. After example verification, the new model has higher prediction accuracy in the field of national defense expenditure than the traditional triple exponential smoothing model.

## 2. Triple Exponential Smoothing Model

Exponential smoothing model is a weighted average model that uses dynamic weight coefficients to weigh the original data. And the biggest characteristic of this method is that it focuses on the influence of recent data on the prediction model [21]. In other words, the more recent the data, the greater the weight coefficient and the smaller the weight coefficient of the earlier data. The triple exponential smoothing method is to add another exponential smoothing on the basis of the first exponential smoothing and the second quadratic exponential smoothing. By estimating the parameters of the quadratic curve model, the nonlinear time series can be adjusted to eliminate irregular disturbances and random errors. It is suitable for numerical prediction of quadratic curve trend of original data.

*2.1. Traditional Triple Exponential Smoothing Model.* Let the time series be

$$X = \{x_1, x_2, x_3, \dots, x_n\}, \quad (1)$$

where  $x_i$  is the time series data at time  $i$ ,  $x_1$  is the first group of time series data,  $x_2$  is the second group of time series data,  $x_3$  is the third group of time series data, and  $x_n$  is the  $n$ th group of time series data.

Single exponential smoothing series:

$$S^1 = \{S_0^1, S_1^1, S_2^1, \dots, S_n^1\}. \quad (2)$$

Double exponential smoothing series:

$$S^2 = \{S_0^2, S_1^2, S_2^2, \dots, S_n^2\}. \quad (3)$$

Triple exponential smoothing series:

$$S^3 = \{S_0^3, S_1^3, S_2^3, \dots, S_n^3\}. \quad (4)$$

Among them,  $S_t^1$ ,  $S_t^2$ , and  $S_t^3$  are exponential smoothing values ( $t = 1, 2, \dots, n$ );  $S_0^1$ ,  $S_0^2$ , and  $S_0^3$  are exponential smoothing initial values, which generally take the first original value or the average of the previous original values. According to the initial value of exponential smoothing and the original time series, the exponential smoothing value at the following time is determined. The recurrence formulas are as follows:

$$\begin{aligned} S_t^1 &= \alpha x_t + (1 - \alpha)S_{t-1}^1, \\ S_t^2 &= \alpha S_t^1 + (1 - \alpha)S_{t-1}^2, \\ S_t^3 &= \alpha S_t^2 + (1 - \alpha)S_{t-1}^3. \end{aligned} \quad (5)$$

Among them,  $\alpha$  is called smoothing coefficient ( $0 < \alpha < 1$ ). Existing literatures usually use MSE, MAE, AARE minimum principle, or artificial subjective test to determine the reasonable value of  $\alpha$ . The larger  $\alpha$ , the higher the emphasis on new data in the prediction, the greater the role of new data, the higher the sensitivity of prediction results, and the better the ability to adapt to new levels. The smaller  $\alpha$ , the higher the emphasis on old data in the prediction, the more conservative the prediction results, and the slower the response to the changes of the actual data, and lag is easy to occur [22, 23]. In this paper, the AARE minimum principle is used to determine the value of  $\alpha$ . Parameters  $a_t$ ,  $b_t$ , and  $c_t$  are usually calculated by using exponential smoothing values. The parameter estimation formulas are as follows:

$$\begin{aligned} a_t &= 3S_t^1 - 3S_t^2 + S_t^3, \\ b_t &= \frac{\alpha}{2(1 - \alpha)^2} [(6 - 5\alpha)S_t^1 - (10 - 8\alpha)S_t^2 + (4 - 3\alpha)S_t^3], \\ c_t &= \frac{\alpha^2}{2(1 - \alpha)^2} [S_t^1 - 2S_t^2 + S_t^3]. \end{aligned} \quad (6)$$

The quadratic parabola model is established by using parameters  $a_t$ ,  $b_t$ , and  $c_t$ .  $T$  is used to predict the future value at  $t + T$ :

$$y_{t+T} = a_t + b_t T + c_t T^2. \quad (7)$$

Generally speaking, when the number of time series of original data is large (i.e., for data with  $>25$  items), the triple exponential smoothing method takes  $S_0^1 = S_0^2 = S_0^3 = x_1$ . When the original data is 25 items or less, the average value of previous periods of data is often taken as the initial value of exponential smoothing. However, in general, the selection of initial values has great subjectivity, which will cause certain error influence on the prediction trend of the later

model [24]. Although this error often has little influence in the medium- and long-term prediction composed of a large amount of data, it cannot be ignored in the short-term prediction. Therefore, this paper introduced the reverse prediction. Due to the unreasonable selection of initial values, the error of future predicted values may be large. Therefore, a reverse cubic exponential smoothing model is established to solve this problem.

**2.2. Reverse Triple Exponential Smoothing Model.** The inverse triple exponential smoothing model is based on the triple exponential smoothing model and uses the idea of reverse prediction to correct the initial value of exponential smoothing. Firstly, the traditional triple exponential smoothing model is used to predict and obtain  $n + 1$  to  $n + j$  predicted values ( $j$  is selected according to needs; usually  $j \leq n$ , and this paper only takes the  $n + 3$  value as an example). And at this time, the initial value selected by the model is the first value of the original data. Then, the obtained  $n + 1$ ,  $n + 2$ , and  $n + 3$  predicted values and  $n - 1$  actual values are used for reverse prediction to obtain the first 3 reverse predicted values of the original data. According to the quadratic parabola trend of triple exponential smoothing model, if this set of data fitting effect is good, the initial value obtained by reverse prediction is closely related to the first three items of actual data. And it has similar numerical values. The modified initial value can be obtained by weighting the predicted initial value and the initial value of the original data. The specific steps are as follows.

*Definition 1.* Let  $X_1^{(0)}$  be that original sequence of single reverse prediction:

$$X_1^{(0)} = (x_1^{(0)}(1), x_1^{(0)}(2), \dots, x_1^{(0)}(n)). \quad (8)$$

(i) *Step 1.* The traditional triple exponential smoothing model is used to fit the original data containing  $n$  items, and the prediction can be obtained:

$$\hat{y}(n + 1), \hat{y}(n + 2), \hat{y}(n + 3). \quad (9)$$

(ii) *Step 2.* Establish a reverse prediction:

$$\begin{cases} x_1^{(0)}(k) = \hat{y}(n + 4 - k), & k = 1, 2, 3, \\ x_1^{(0)}(k) = x(n + 4 - k), & k = 4, 5, \dots, n. \end{cases} \quad (10)$$

(iii) *Step 3.* Establish a triple exponential smoothing model for the newly established reverse single prediction original sequence  $X_1^{(0)}$  to obtain

$$\hat{x}_1^{(0)}(n + 1), \hat{x}_1^{(0)}(n + 2), \hat{x}_1^{(0)}(n + 3). \quad (11)$$

(iv) *Step 4.* The first three initial values are weighted and combined to obtain an initial value  $C^1(k)$  after single reverse prediction correction:

$$C^1(k) = \eta x^{(0)}(k) + (1 - \eta) \hat{x}_1^{(0)}(k), \quad (k = 1, 2, 3), \quad (12)$$

where  $\eta$  is the adjustment factor (usually 0.5).

(v) *Step 5.* Set the threshold index.

According to the actual demand, the corresponding threshold value is set, and the fitting accuracy of the new model is judged by using different accuracy test indexes. If it meets the requirements, the model can be used for prediction. If it does not meet the requirements, the next reverse prediction is carried out until the initial value  $C^n(k)$  meets the accuracy requirements after  $n$  iterations, and then the corresponding prediction is carried out by using the model.

(vi) *Step 6.* Output the predicted value.

The improved triple exponential smoothing model is established by using the modified initial values  $C^n(k)$ , and the predicted values can be obtained:

$$\hat{y}_{C^n}^{(0)}(t + T). \quad (13)$$

### 3. Improved Prediction Model Combining Inverse Exponential Smoothing and Markov Chain

The Markov model is a random time series analysis method, which predicts the future state of things by studying different states and state transition probability matrices of things. It has high scientific accuracy and adaptability [14, 25–27]. This method requires less historical data and only needs the recent data and information of the predicted object to make prediction. It has better error correction effect for data with large random fluctuation. Markov theory is a branch of stochastic process, which is a method to predict future system development according to the transition probability between states. The Markov prediction model is a random and variable mathematical process; the core of modeling is to master the law of system state transition. The basic idea of the Markov probabilistic prediction model is to analyze the current situation of the system and use Markov chain to solve the probability of a particular state to which the system may change in the future.

Assuming time series  $\{X(t), t \in T\}$  and the observable data in time series are discrete, we divide the range of error values into  $r$  intervals. So, this time series has  $r$  states  $E = \{E_1, E_2, \dots, E_r\}$ . Suppose that the probability of the sequence  $X(t)$  in state  $E_i$  is  $a_i(t) = P\{x_t = i\}$ . When the probability of sequence  $X(t)$  to be transferred from the first state  $E_i$  to another state  $E_j$  is  $p_{ij}$ , then  $p_{ij} = P(x_{t+i} = j | x_t = i)$ ; when the time series value at time  $t + 1$  is only related to the transition probability and time series value at time  $t$ , then this time series becomes a time series with Markov property.

Based on the inverse triple exponential smoothing model, Markov's state transition probability matrix is used

to predict the error fluctuation, thus correcting the error and further improving the prediction accuracy of the model. The specific steps are as follows:

- (i) *Step 1.* According to the nature of the state transition and full probability formula, deduce the equation of Markov chain.

$$a_i(t+1) = \sum_{j=1}^r a_i(t)p_{ij},$$

$$\begin{cases} \sum_{i=1}^r a_i(t) = 1, & t \in T, \\ p_{ij} \geq 0, \\ \sum p_{ij} = 1, \end{cases} \quad (14)$$

where  $a_i(t) = P\{x_i = i\}$  is the probability of the sequence  $X(t)$  in state  $E_i$  and  $p_{ij}$  is the probability of sequence  $X(t)$  to be transferred from the first state  $E_i$  to another state  $E_j$ .

- (ii) *Step 2.* State interval division.

According to the prediction results of the inverse triple exponential smoothing model, the error between it and the actual value is calculated, the corresponding interval threshold is set, and the error is divided into several intervals. The error selection here usually adopts relative error or actual residual error [28].

- (iii) *Step 3.* Calculate the initial probability.

Assuming that the definition domain of time series  $\{X(t), t \in T\}$  is  $X$ , we divide this definition domain into  $r$  states  $E = \{E_1, E_2, \dots, E_r\}$  according to certain requirements. For this time series  $\{X(t), t \in T\}$ , all we know is the transition state observed previously, and the transition state of the last term is unknown. Calculating the initial probability requires the number of  $E_i$  state data in the state of the previous  $n-1$  data. Suppose that  $M_i$  data are in the state; the occurrence frequency of state  $E_i$  is

$$p_i = \frac{M_i}{n-1}. \quad (15)$$

- (iv) *Step 4.* Construct Markov state transition matrix and calculate the transition probability.

In this paper, the transition probability of error from one state to another is defined as the state transition probability matrix. For example, the number of transitions from state  $E_i$  to state  $E_j$  is  $m_{ij}$ , and the total number of transitions from state  $E_i$  is  $M_i$ ; then, the probability of transitions from state  $E_i$  to state  $E_j$  is  $p_{ij} = (m_{ij}/M_i)$ , and a state transition probability matrix  $P$  is constructed [29, 30]:

$$P = \begin{pmatrix} p_{11} & p_{12} & \cdots & p_{1n} \\ p_{21} & p_{22} & \cdots & p_{2n} \\ \vdots & \vdots & \ddots & \vdots \\ p_{n1} & p_{n2} & \cdots & p_{nn} \end{pmatrix}. \quad (16)$$

After predicting backward  $n$  times from the current state, the transition probability matrix correspondingly becomes  $P^n$ .

- (v) *Step 5.* Error correction.

According to the actual situation of the observed data falling into the state  $E_i$ , it is also clear that the  $i$ th row element of  $P$  is  $p_{ij} = \max\{p_{i1}, p_{i2}, \dots, p_{in}\}$  because the probability of transition to  $E_j$  may be greater than other states, so the final transition state can be predicted. And according to Step 4, after the state transition probability matrix is determined, the next state that is most likely to occur can be predicted, and the error interval of the predicted state can be grasped so that the error of this state can be reasonably estimated. Let us define the error to be  $\Delta y_{t+T}$ . In this paper, the median value of the range of state values is used to correct Markov error.

- (vi) *Step 6.* Output the corrected predicted value.

$$y_{\text{after}}^{\wedge} = y_{\text{before}}^{\wedge} + \Delta y_{t+T}. \quad (17)$$

The overall flowchart of building the model is shown in Figure 1. First is the selection stage of the research object. The trend analysis, seasonal analysis, periodic analysis, and other methods are adopted to identify the time series and determine the applicable model. In this paper, the exponential smoothing model is selected according to the data instance. Secondly, the traditional exponential smoothing model is studied. Different principles are adopted to determine the relevant parameters, which are substituted into the recursive formula to calculate the coefficient of the exponential smoothing model. The third is the improvement stage of exponential smoothing model. The inverse prediction method is used to determine the number of iterations and other parameters, and the initial value is corrected so as to output the inverse triple exponential smoothing initial value correction model. In the fourth part, an improved inverse exponential smoothing and Markov combination prediction model is established. On the basis of inverse triple exponential smoothing model, Markov theory is introduced to correct the fluctuation error by dividing the state interval and establishing the probability transition matrix. Finally, in the stage of verification and analysis, the accuracy of the traditional exponential smoothing model, inverse triple exponential smoothing model, and improved prediction model combining inverse exponential smoothing and Markov chain was tested, respectively, and the prediction effects were analyzed to output the optimal prediction model.

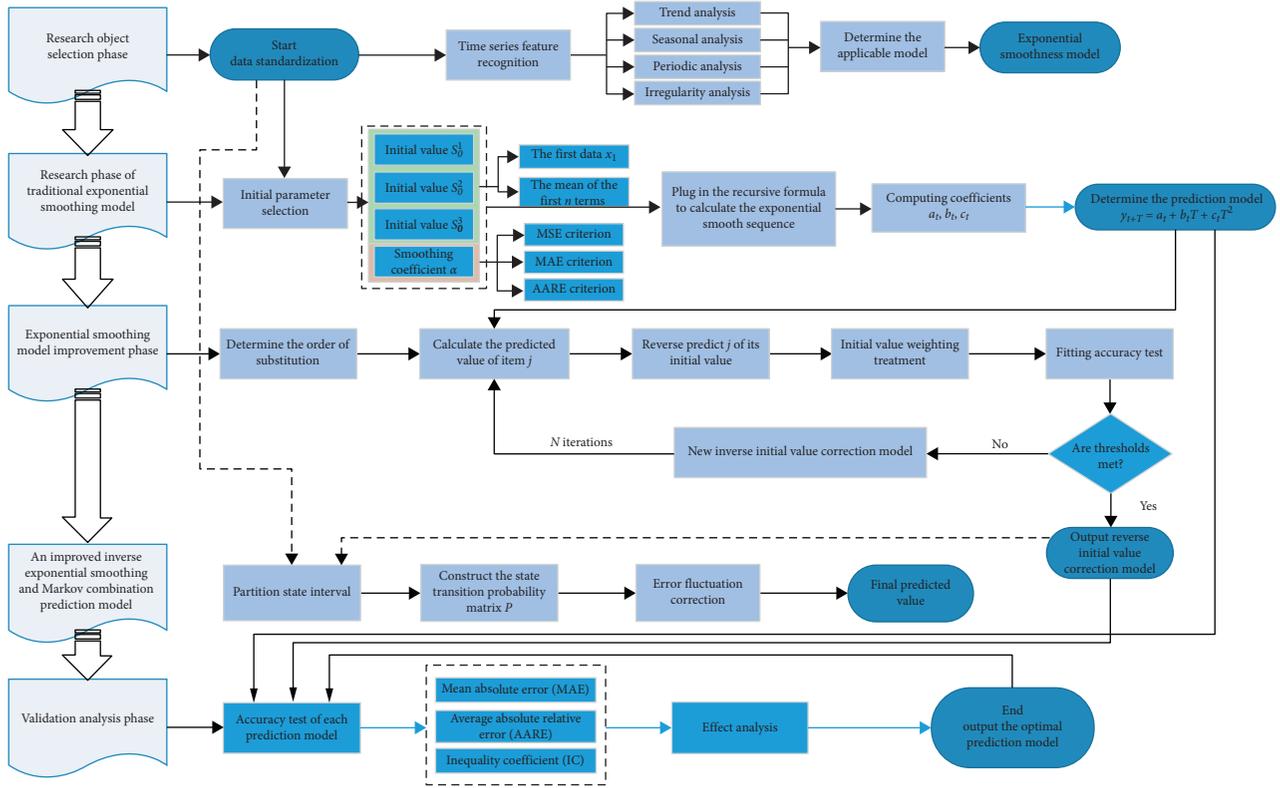


FIGURE 1: Overall model flowchart.

## 4. Instance Validation

**4.1. Data Selection.** This paper selects India's defense expenditure data from 1990 to 2017 (Table 1) to verify the prediction accuracy of the model. The self-fitting of MATLAB data shows that this group of data shows an obvious quadratic parabola trend (Figure 2), and its growth trend is obvious and relatively stable, which meets the data requirements of the triple exponential smoothing model. Among them, India's defense expenditure data from 1990 to 2012 are taken as the original time series, and India's defense expenditure data from 2013 to 2017 are taken as the predicted test data.

**4.2. Data Inspection Method.** At present, the commonly used data inspection methods mainly include the following ( $y_j$  represents the actual value of the  $j$  th time series data, and  $\hat{y}_j$  represents the predicted value of the  $j$  th time series data).

(1) Mean absolute error (MAE):

$$MAE = \frac{1}{n} \sum_{j=1}^n |e_j| = \frac{1}{n} \sum_{j=1}^n |\hat{y}_j - y_j|. \quad (18)$$

The smaller the MAE, the higher the prediction accuracy.

(2) Average absolute relative error (AARE):

$$AARE = \frac{1}{n} \sum_{j=1}^n \left| \frac{\hat{y}_j - y_j}{y_j} \right|. \quad (19)$$

The accuracy division range of AARE is shown in Table 2.

AARE is usually expressed as a percentage. The smaller the value, the higher the prediction accuracy.

(3) Inequality coefficient (IC):

$$IC = \frac{\sqrt{(1/n) \sum_{j=1}^n (\hat{y}_j - y_j)^2}}{\sqrt{(1/n) \sum_{j=1}^n \hat{y}_j^2} + \sqrt{(1/n) \sum_{j=1}^n y_j^2}}. \quad (20)$$

The value of IC is between 0 and 1. The closer to 1, the worse the prediction accuracy, and the closer to 0, the higher the prediction accuracy.

**4.3. Prediction Results and Analysis of Results.** The data of India's defense expenditure from 1990 to 2012 are substituted into the traditional triple exponential smoothing prediction model, and the initial value is set as the average of the first three data:  $S_0^1 = S_0^2 = S_0^3 = ((x_1 + x_2 + x_3)/3)$ . The  $\alpha$  value is determined by AARE minimum principle. The traditional triple exponential smoothing model is used to output the corresponding fitting value from 1990 to 2012

TABLE 1: India's defense expenditure (1990–2017) (hundred million rupees).

Year	Defense expenditure (hundred million rupees)
1990	1875.57
1991	1989.42
1992	2130.22
1993	2645.63
1994	2833.00
1995	3273.12
1996	3588.35
1997	4354.92
1998	5106.19
1999	6274.99
2000	6469.72
2001	7029.45
2002	7216.66
2003	7739.66
2004	9648.66
2005	10350.30
2006	11019.10
2007	11904.20
2008	15175.60
2009	19932.90
2010	21456.00
2011	23733.80
2012	25730.60
2013	28459.70
2014	31943.60
2015	33228.20
2016	39667.30
2017	42350.60

Note. All data are from the official website of the Stockholm international peace research institute: <https://sipri.org/databases/milex>. And all data are denominated in rupee, India's official currency.

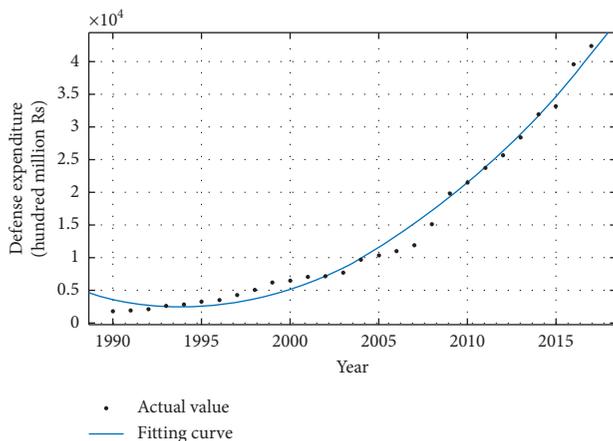


FIGURE 2: Quadratic fitting trend chart of original data.

and the predicted value from 2013 to 2017. Then, the first three initial values are reversed by using the 2013–2015 predicted value to establish a reverse triple exponential smoothing model, and the corresponding fitting value and the 2013–2017 predicted value are output. The fitting values of the two models are shown in Table 3 and Figure 3, and the predicted values are shown in Table 4 (model I refers to the

triple exponential smoothing model and model II refers to the reverse triple exponential smoothing model).

From Table 5, it can be seen that the MAE and AARE indexes of model II are lower and the IC indexes are higher than those of model I. According to the fitting data of the two models from 1991 to 2012, it can be found that the errors of the first three data are too large, which seriously affects the evaluation of indexes. This is because the reverse prediction model carries out reverse prediction correction for the first three initial values, while for the original data less than or equal to three items, it does not meet the reverse prediction conditions, resulting in excessive error of the first three fitting values. Therefore, the first three fitting data are removed to make a precision comparison (see Table 6).

As can be seen from Table 6, after removing the first three fitting values, the three indexes of model II are better than those of model I. It should be emphasized that the cubic exponential smoothing model in this paper is established based on dynamic data. Therefore, when predicting the leading period number  $T=1, 2, 3, 4,$  and  $5,$  respectively, the model comprehensively considers the error results of the fitting value and the predicted value and calculates the optimal smoothing coefficient  $\alpha$  so as to generate the corresponding parameter values  $a_{2012}, b_{2012},$  and  $c_{2012},$  respectively (predicting the benchmark year is 2012), as shown in Table 7.

According to models I and II, India's defense spending from 2013 to 2017 is predicted, respectively, and compared with actual data (Table 4), and the prediction precision of model I and model II is compared (Table 8).

It can be seen from Tables 4 and 8 that the prediction effect of model II was better than that of model I from 2012 to 2017, with the mean absolute error (MAE) reduced by 12.49%, the average absolute relative error (AARE) reduced by 14.89%, and the inequality coefficient (IC) reduced by 6.67%. Overall, the prediction accuracy is improved compared with the traditional exponential smoothing prediction model. However, for specific data, it can be seen that the absolute error of reverse triple exponential smoothing model II is obviously reduced on the predicted values from 2013 to 2015, but for the predicted data from 2016 to 2017, the error has basically not changed.

Therefore, on the basis of model II, Markov theory is introduced to further correct the error. According to the calculation results of model II, its relative error is divided into several states, and according to Markov theory, the concentration degree of error range is divided so that each interval meets the objective law of state change [31–34]. The standard of state interval division is shown in Table 9. According to the relative error of the fitting values obtained in model II, we can divide them into different states, as shown in Table 10.

Since the state transition is random, probability must be used to describe the possibility of the state transition, that is, the state transition probability [35, 36]. According to the relevant probability calculation formula in Section 3 (*Improved Prediction Model Combining Inverse Exponential Smoothing and Markov Chain*), the single state transition probability matrix is constructed as follows:

TABLE 2: Reference criteria for evaluating prediction accuracy (AARE).

AARE value range	Below 10%	10%–20%	20%–50%	More than 50%
Prediction model accuracy	High accuracy	Good accuracy	Feasible	Not feasible

TABLE 3: Fitting values of model I and model II.

Year	Actual value	Model I	Absolute error	Relative error (%)	Model II	Absolute error	Relative error (%)
1990	1875.57	—	—	—	—	—	—
1991	1989.42	1851.00	138.42	6.96	1444.22	545.20	27.40
1992	2130.22	1958.14	172.08	8.08	1704.37	425.85	19.99
1993	2645.63	2164.25	481.38	18.20	2112.12	533.51	20.17
1994	2833.00	2825.12	7.88	0.28	2774.19	58.81	2.08
1995	3273.12	3160.86	112.26	3.43	3238.32	34.80	1.06
1996	3588.35	3668.46	80.11	2.23	3799.97	211.62	5.90
1997	4354.92	4042.42	312.50	7.18	4115.12	239.80	5.51
1998	5106.19	4899.57	206.62	4.05	4979.33	126.86	2.48
1999	6274.99	5825.04	449.95	7.17	5885.66	389.33	6.20
2000	6469.72	7207.07	737.35	11.40	7094.16	624.44	9.65
2001	7029.45	7458.92	429.47	6.11	7382.91	353.46	5.03
2002	7216.66	7833.70	617.04	8.55	7724.44	507.78	7.04
2003	7739.66	7837.46	97.80	1.26	7685.59	54.07	0.70
2004	9648.66	8200.85	1447.81	15.01	8307.23	1341.43	13.90
2005	10350.30	10365.27	14.97	0.14	10533.85	183.55	1.77
2006	11019.10	11456.27	437.17	3.97	11537.29	518.19	4.70
2007	11904.20	12113.08	208.88	1.75	12087.04	182.84	1.54
2008	15175.60	12912.67	2262.93	14.91	13167.58	2008.02	13.23
2009	19932.90	16628.87	3304.03	16.58	16945.34	2987.56	14.99
2010	21456.00	22717.92	1261.92	5.88	23371.58	1915.58	8.93
2011	23733.80	25095.92	1362.12	5.74	25232.82	1499.02	6.32
2012	25730.60	27141.58	1410.98	5.48	27004.24	1273.64	4.95

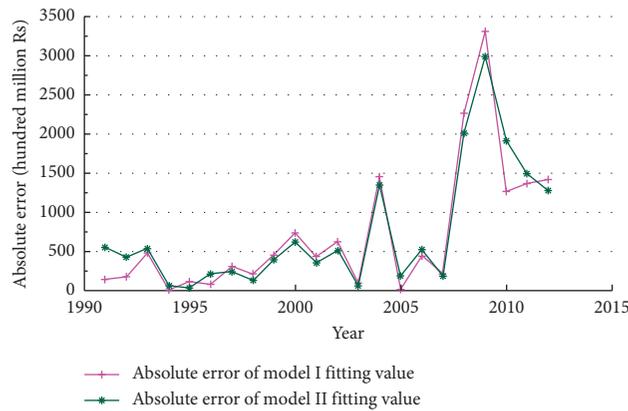


FIGURE 3: Absolute error of fitting values for model I and model II.

TABLE 4: Predicted values of model I and model II.

Year	Actual value	Model I	Absolute error	Relative error (%)	Model II	Absolute error	Relative error (%)
2013	28459.70	28787.64	327.94	1.15	28506.25	46.55	0.16
2014	31943.60	32033.60	90.00	0.28	32030.02	86.42	0.27
2015	33228.20	35239.83	2011.63	6.05	34696.09	1467.89	4.42
2016	39667.30	36704.50	2962.80	7.47	36703.42	2963.88	7.47
2017	42350.60	41150.43	1200.17	2.83	41146.20	1204.40	2.84

TABLE 5: Comparison of fitting value accuracy between model I and model II (including the first three data).

	MAE	AARE	IC
Model I	706.98	7.02	0.0453
Model II	727.97	8.34	0.0441

TABLE 6: Comparison of fitting value accuracy between model I and model II (excluding the first three data).

	MAE	AARE	IC
Model I	776.94	6.37	0.0451
Model II	763.73	6.10	0.0435

TABLE 7: Values of parameters  $\alpha$ ,  $a_{2012}$ ,  $b_{2012}$ , and  $c_{2012}$  in model I and model II.

$T$	Model I				Model II			
	$\alpha$	$a_{2012}$	$b_{2012}$	$c_{2012}$	$\alpha$	$a_{2012}$	$b_{2012}$	$c_{2012}$
1	0.40	26035.37	2684.58	67.69	0.45	25942.50	2521.59	42.16
2	0.35	26114.59	2795.02	82.24	0.35	26113.87	2793.91	82.08
3	0.35	26114.59	2795.02	82.24	0.40	26035.14	2684.14	67.61
4	0.45	25942.56	2521.73	42.19	0.45	25942.50	2521.59	42.16
5	0.40	26035.37	2684.58	67.69	0.40	26035.14	2684.14	67.61

TABLE 8: Prediction precision comparison between model I and model II.

	MAE	AARE	IC
Model I	1318.51	3.56	0.0240
Model II	1153.83	3.03	0.0224

TABLE 9: Status interval division.

Status	Interval
E1	(-30%, -10%]
E2	(-10%, 0]
E3	(0, 5%]
E4	(5%, 10%]

$$P = \begin{pmatrix} \frac{1}{2} & \frac{1}{6} & \frac{1}{6} & \frac{1}{6} \\ \frac{1}{4} & \frac{3}{4} & 0 & 0 \\ \frac{1}{3} & 0 & \frac{2}{3} & 0 \\ \frac{1}{7} & \frac{2}{7} & \frac{1}{7} & \frac{3}{7} \end{pmatrix}. \tag{21}$$

As can be seen from Table 10, the data in 2012 are in state E3, and according to the state transition matrix, it can be seen that the predicted data in 2013 are most likely to be in state E3. The median value of the state value range is taken for Markov error correction, and combined with the prediction value of reverse triple exponential

TABLE 10: Status of model II fitting values.

Year	Number	Relative error (%)	Belonging status
1991	1	-27.40	E1
1992	2	-19.99	E1
1993	3	-20.17	E1
1994	4	-2.08	E2
1995	5	-1.06	E2
1996	6	5.90	E4
1997	7	-5.51	E2
1998	8	-2.48	E2
1999	9	-6.20	E2
2000	10	9.65	E4
2001	11	5.03	E4
2002	12	7.04	E4
2003	13	-0.70	E2
2004	14	-13.90	E1
2005	15	1.77	E3
2006	16	4.70	E3
2007	17	1.54	E3
2008	18	-13.23	E1
2009	19	-14.99	E1
2010	20	8.93	E4
2011	21	6.32	E4
2012	22	4.95	E3

Note. Positive and negative relative errors should be considered in the table.

smoothing model, the final prediction value for 2013 is 2786.298 billion rupees.

Similarly, the  $n$  times state transition probability matrix  $P^n$  can be obtained according to the one-time state transition probability matrix  $P$ . After the transition matrix is determined, the possible states in 2013–2017 can be predicted according to the state in 2012, and then the final prediction values of an improved reverse exponential smoothing and Markov combination prediction model (set as model III) for 2013–2017 can be obtained, respectively, as shown in Table 11.

TABLE 11: Predicted values of an improved prediction model combining inverse exponential smoothing and Markov chain (model III).

Year	Model III predicted value
2013	27862.98
2014	31386.76
2015	34052.83
2016	39276.48
2017	43719.26

TABLE 12: Comparison of prediction accuracy of models I, II, and III.

	MAE	AARE (%)	IC
Model I	1318.51	3.56	0.0240
Model II	1153.83	3.03	0.0224
Model III	747.53	2.11	0.0115

According to the three indexes of MAE, AARE, and IC, the prediction accuracy of the three models are compared, as shown in Table 12 and Figures 4–6.

From the analysis of Table 12 and Figures 4–6, it can be seen that the prediction accuracy of model III is significantly improved compared with models I and II, and its MAE, AARE, and IC are greatly reduced. The MAE index of model III is 43.30% lower than model I, the AARE index of model III is 40.76% lower than model I, and the IC index of model III is 52.03% lower than model I. For specific data, the prediction error of model III increased from 2013 to 2014, but its prediction error from 2015 to 2016 decreased significantly compared with that of models I and II, and the overall curve was closer to the actual value.

### 5. Conclusion

In this paper, triple exponential smoothing model I, inverse triple exponential smoothing model II, and improved prediction model III combining inverse exponential smoothing and Markov chain are established, respectively. Taking India’s defense expenditure data from 1990 to 2012 as the original time series and the data from 2013 to 2017 as the unknown test values, this paper draws the following conclusions through comparative analysis:

- (1) The quadratic curve fitting trend of India’s defense expenditure data from 1990 to 2017 is good. Verified by models I, II, and III, the average relative error of the fitting values is below 10%, and the average relative error of the predicted values is below 4%, with high accuracy, which shows that it is more reasonable to use the triple exponential smoothing model to predict India’s defense expenditure.
- (2) Compared with model I, the overall prediction accuracy of model II is improved. Its MAE is relatively reduced by 12.49%, AARE is relatively reduced by 14.89%, and IC is relatively reduced by 6.67%, but its prediction advantage is weakened in the later period.
- (3) Model III has the highest prediction accuracy. Compared with model I, its MAE, AARE, and IC are

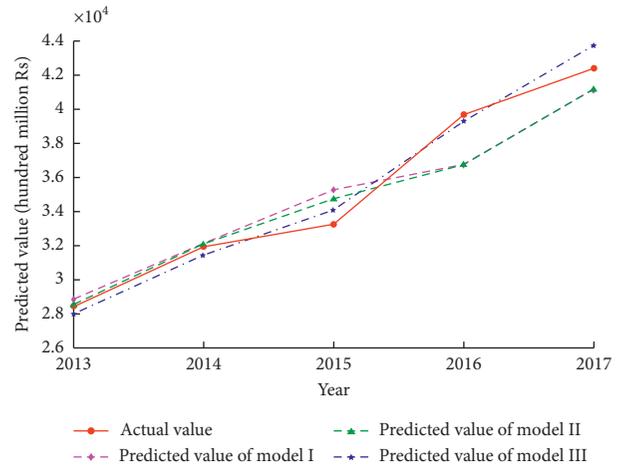


FIGURE 4: Predicted and actual values of models I, II, and III.

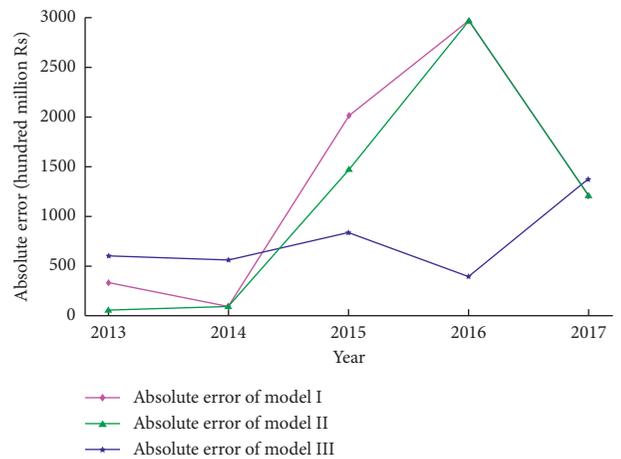


FIGURE 5: Absolute errors of models I, II, and III.

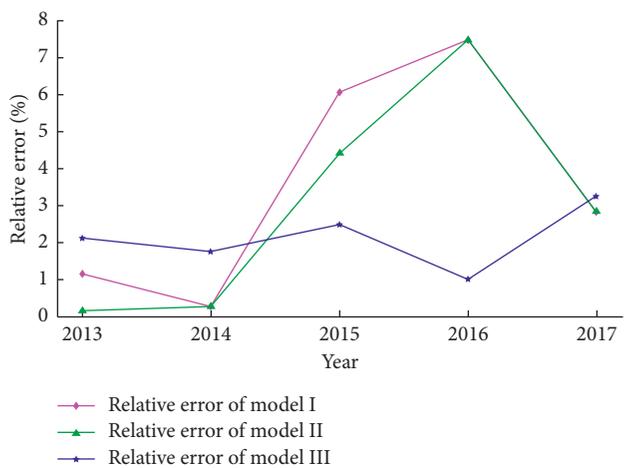


FIGURE 6: Relative errors of models I, II, and III.

reduced by 43.30%, 40.73%, and 52.08% respectively. Compared with model II, its MAE, AARE, and IC are reduced by 35.21%, 30.36%, and 48.66%,

respectively. On the basis of making full use of time series to predict the trend, the *improved prediction model combining inverse exponential smoothing and Markov chain* uses Markov theory to predict the fluctuation, thus obviously reducing the error fluctuation range and making the predicted value closer to the actual value and the prediction effect more stable.

This paper only predicts the national defense expenditure from the perspective of time series, but in fact, national defense expenditure will inevitably be affected by multidimensional factors such as economic development, military construction, and national defense policies [37, 38]. In the following research, if the time series and multidimensional influencing factors can be integrated to establish a prediction model, it will more effectively support the prediction decision.

### Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

### Conflicts of Interest

The authors declare that they have no conflicts of interest.

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### References

- [1] K. Zhang, B. Liu, D. Huang, and X. Ren, "Empirical study on the relationship between national defense expenditure, economic growth and resident consumption in China," *Journal of Naval Engineering University*, vol. 28, no. 5, pp. 69–74, 2016.
- [2] Z. Cao, Y. Liu, and J. Dong, "Predict of railway passenger volume based on triple exponential smoothing," *Railway Transportation and Economy*, vol. 40, no. 11, pp. 49–53, 2018.
- [3] G. Feng, L. Chen-Yu, Z. Bin, and Z. Su-Qin, "Spares consumption combination forecasting based on genetic algorithm and exponential smoothing method," in *Proceedings of the 2012 Fifth International Symposium on Computational Intelligence and Design*, pp. 198–201, Hangzhou, China, October 2012.
- [4] K. Y. Chan, T. S. Dillon, J. Singh, and E. Chang, "Neural-network-based models for short-term traffic flow forecasting using a hybrid exponential smoothing and levenberg-marquardt algorithm," *IEEE Transactions on Intelligent Transportation Systems*, vol. 13, no. 2, pp. 644–654, 2012.
- [5] D. Shi, S. Wang, Y. Cai, and L. Chen, "Stochastic predictive energy management of power split hybrid electric bus for real-world driving cycles," *IEEE Access*, vol. 6, pp. 61700–61713, 2018.
- [6] Y. Li, H. He, and J. Peng, "An adaptive online prediction method with variable prediction horizon for future driving cycle of the vehicle," *IEEE Access*, vol. 6, pp. 33062–33075, 2018.
- [7] H. Wang and H. Wang, "GDP predict based on exponential smoothing method and regression analysis," *Economic Research Journal*, vol. 35, no. 7, pp. 1–6, 2018.
- [8] Li. Qian, "Predict of the development trend of regional income difference of rural residents-based on double exponential smoothing and ARMA model," *Journal of Central University of Finance and Economics*, vol. 7, pp. 78–82, 2014.
- [9] X. Dong, Y. Chen, Z. Cai, and W. Zhang, "Exponential smoothing prediction method for subsequent spare parts based on rough set theory correction," *Systems Engineering and Electronic Technology*, vol. 40, no. 4, pp. 833–838, 2018.
- [10] J. Cao, H. Du, X. Chen, and Q. Wang, "Prediction of armored equipment consumption based on smooth index simulation optimization," *Journal of System Simulation*, vol. 25, no. 8, pp. 1961–1965, 2013.
- [11] Y. Zhang, H. Sun, and Y. Guo, "Wind power prediction based on PSO-SVR and grey combination model," *IEEE Access*, vol. 7, pp. 136254–136267, 2019.
- [12] B. Zhou, X. Ma, Y. Luo, and D. Yang, "Wind power prediction based on LSTM networks and nonparametric kernel density estimation," *IEEE Access*, vol. 7, pp. 165279–165292, 2019.
- [13] G. Duan, R. Niu, Y. Zhao, K. Zhang, and D. Yao, "Prediction of rainfall-induced landslides based on dynamic exponential smoothing model," *Journal of Wuhan University (Information Science Edition)*, vol. 41, no. 7, pp. 958–962, 2016.
- [14] J. Lu and F. Xu, "Study on landslide predict model based on exponential smoothing method and regression analy," *Journal of Wuhan University of Technology*, vol. 33, no. 10, pp. 88–91, 2011.
- [15] S. Manandhar, S. Dev, Y. H. Lee, and S. Winkler, "Predicting GPS-based PWV measurements using exponential smoothing," in *Proceedings of the 2019 USNC-URSI Radio Science Meeting (Joint with AP-S Symposium)*, Atlanta, GA, USA, July 2019.
- [16] Y. Xie, M. Jin, Z. Zou et al., "Real-time prediction of docker container resource load based on a hybrid model of ARIMA and triple exponential smoothing," *IEEE Transactions on Cloud Computing*, In press.
- [17] C. Qi and Z. Hou, "Application of adaptive single exponential smoothing method in short-term traffic flow predict," *Control Theory and Application*, vol. 29, no. 4, pp. 465–469, 2012.
- [18] G. Wang, S. Wang, H. Liu et al., "Wind speed prediction of wind farms based on adaptive dynamic cubic exponential smoothing method," *Power System Protection and Control*, vol. 42, no. 15, pp. 117–122, 2014.
- [19] J. Mi, L. Fan, X. Duan, and Y. Qiu, "Short-term power load forecasting method based on improved exponential smoothing grey model," *Mathematical Problems in Engineering*, vol. 2018, Article ID 3894723, 11 pages, 2018.
- [20] Q. Liu, X. Chen, Y. Zhang, Z. Liu, C. Li, and D. Hu, "A novel short-medium term satellite clock error prediction algorithm based on modified exponential smoothing method," *Mathematical Problems in Engineering*, vol. 2018, Article ID 7486925, 7 pages, 2018.
- [21] M. Akpınar and N. Yumusak, "Day-ahead natural gas forecasting using nonseasonal exponential smoothing methods," in *Proceedings of the 2017 IEEE International Conference on Environment and Electrical Engineering and 2017 IEEE Industrial and Commercial Power Systems Europe (EEEIC/I&CPS Europe)*, pp. 1–4, Milan, Italy, June 2017.
- [22] J. Lian and L. He, "Research on production prediction based on exponential smoothing method," in *Proceedings of the 2018 9th International Conference on Information Technology in Medicine and Education (ITME)*, pp. 961–963, Hangzhou, China, October 2018.

- [23] W. Setiawan, E. Juniati, and I. Farida, "The use of triple exponential smoothing method (winter) in forecasting passenger of PT Kereta Api Indonesia with optimization alpha, beta, and gamma parameters," in *Proceedings of the 2016 2nd International Conference on Science in Information Technology (ICSITech)*, pp. 198–202, Balikpapan, Indonesia, October 2016.
- [24] L. Zhang, Z. Mu, and C. Sun, "Remaining useful life prediction for lithium-ion batteries based on exponential model and particle filter," *IEEE Access*, vol. 6, pp. 17729–17740, 2018.
- [25] N. V. Malyshkina and F. L. Mannering, "Markov switching multinomial logit model: an application to accident-injury severities," *Accident Analysis & Prevention*, vol. 41, no. 4, pp. 829–838, 2009.
- [26] L. R. Rabiner, "A tutorial on hidden Markov models and selected applications in speech recognition," *Proceedings of the IEEE*, vol. 77, no. 2, pp. 257–286, 1989.
- [27] L. Rabiner and B. Juang, "An introduction to hidden Markov models," *IEEE ASSP Magazine*, vol. 3, no. 1, pp. 4–16, 1986.
- [28] D. Li, H. Xu, D. Liu et al., "Application of improved grey Markov model in flight accident rate prediction," *Chinese Journal of Safety Sciences*, vol. 19, no. 9, pp. 53–57, 2009.
- [29] Y. Li, L. Lei, and M. Yan, "Mobile user location prediction based on user classification and Markov model," in *Proceedings of the 2019 International Joint Conference on Information, Media and Engineering (IJCIME)*, pp. 440–444, Osaka, Japan, December 2019.
- [30] D. Zhao, Y. Gao, Z. Zhang, Y. Zhang, and T. Luo, "Prediction of vehicle motion based on Markov model," in *Proceedings of the 2017 International Conference on Computer Systems, Electronics and Control (ICCSEC)*, pp. 205–209, Dalian, China, December 2017.
- [31] Y. Qi, Y. Yang, Z. Feng, and X. Zhao, "Predict method of urban public transport passenger volume based on grey theory and Markov model," *Journal of China Highway*, vol. 26, no. 6, pp. 169–175, 2013.
- [32] H. Rui, Q. Wu, H. Yuan, Z. Feng, and W. Zhu, "Predict method of highway passenger volume based on exponential smoothing method and Markov model," *Journal of Transportation Engineering*, vol. 13, no. 4, pp. 87–93, 2013.
- [33] Y. Wang, Z. Zhang, H. Liu, and H. Ma, "Prediction of equipment consumption based on optimized grey Markov," *Logistics Technology*, vol. 34, no. 1, pp. 158–160, 2015.
- [34] T. Xu, A. Jin, J. Zhang, and Z. Li, "Decision-making model of equipment condition maintenance based on Markov," *Journal of Artillery Launching and Control*, vol. 39, no. 3, pp. 90–94, 2018.
- [35] G. Zhang, Y. Wang, and W. Fan, "Research on dynamic decision-making model of equipment condition maintenance monitoring interval using Markov chain," *Journal of Sichuan Military Engineering*, vol. 36, no. 4, pp. 81–84, 2015.
- [36] P. Lv, Z. Yuan, L. Yang, and K. Yang, "BP neural network-markov predict model for ship traffic volume," *Journal of Shanghai Maritime University*, vol. 38, no. 2, pp. 17–28, 2017.
- [37] T. Niu, L. Zhang, S. Wei, B. Zhang, and B. Zhang, "Study on a combined prediction method based on BP neural network and improved Verhulst model," *Systems Science & Control Engineering*, vol. 7, no. 3, pp. 36–42, 2019.
- [38] C. Yuan, Y. Zhang, N. Xu, and J. Xu, "Grey relational analysis on the relation between China's gdp and defense expenditure," in *Proceedings of the 2017 International Conference on Grey Systems and Intelligent Services (GSIS)*, pp. 106–110, Stockholm, Sweden, August 2017.