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Research Article

Super-Efficiency Infeasibility in the Presence of Nonradial Measurement

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The original radial VRS super-efficiency model in DEA excludes the DMU under evaluation from the reference set. However, it must lead to the problem of infeasibility specifically when the DMU under consideration is at the extremity of the frontier. In this paper, a modified nonradial VRS super-efficiency model is established. The super-efficiency model in the presence of nonradial measurement still maintains some good properties, and the original radial VRS super-efficiency model infeasibility can also be detected through it. Our model with nonradial measurement can help decision makers allocate input resources and arrange production activities because it finds an efficient benchmark DMU, which is different from the reference DMU under radial measurement.

1. Introduction

Data envelopment analysis (DEA), first introduced by Charnes et al. [1] and extended by Banker et al. [2], is an effective nonparametric technique for measuring the relative efficiency of peer decision-making units (DMUs) with multiple inputs and outputs. Their initial models based upon the constant returns to scale (CRS) and variable returns to scale (VRS) are commonly referred to as the CCR model and the BCC model, respectively. They compute scalar efficiency scores with a range of zero to unity which indicate how efficient each DMU has performed as compared to other DMUs in converting inputs to outputs and determine efficient level or position for each DMU under evaluation among all DMUs. DMUs that obtain a score of unity are deemed as efficient and on the DEA (best-practice) frontier, while other DMUs are treated as inefficient. Nowadays, DEA has become a popular method without any prior complicated weight assumptions since its advent and has been rapidly applied in improving the performances of different kinds of entities engaged in different activities and contexts [3, 4], such as human resources planning [5, 6], fixed cost allocation [7, 8], and resource sharing [9, 10].

However, when some DMUs all get scores of unity, these efficient DMUs cannot be distinguished further through the CCR model alone. To break the tie of efficient DMUs and further enhance the discrimination power of DEA, Andersen and Petersen [11] proposed a new model according to the CCR model, which is called super-efficiency model where the DMU under evaluation is excluded from the reference set. It allows efficient DMUs to have efficiency scores larger than or equal to unity (under input-oriented super-efficiency model), and for inefficient DMUs, the super-efficiency model yields scores that are identical to those received from the CCR model. Analogously, based upon the variable returns to scale (VRS) model of Banker et al. [12], the VRS super-efficiency model can be obtained. But under the condition of VRS, the super-efficiency model may be infeasible when some efficient DMUs are under evaluation, while the super-efficiency model under CRS does not suffer the problem of infeasibility. In face of this trouble, much effort has been focused on solving the problem of VRS super-efficiency model's infeasibility.

Seiford and Zhu [13] indicate that infeasibility must occur in the case of the variable returns to scale (VRS) super-efficiency model and further provide the necessary and sufficient conditions for infeasibility of super-efficiency models. Lovell and

Rouse [14] assign a user-defined scaling factor to find a feasible solution for those efficient DMUs for which feasible solutions are unavailable in the VRS super-efficiency model. Chen [15] shows that in order to fully characterize the super-efficiency, both input-oriented and output-oriented super-efficiency DEA models are needed when infeasibility occurs. Chen [15] further points out that super-efficiency can be regarded as input saving/output surplus achieved by an efficient DMU. Cook et al. [16] develop a modified VRS super-efficiency model that yields optimal solutions and super-efficiency scores that characterize the extent of super-efficiency in both inputs and outputs. Lee et al. [17] develop a two-stage process to address the VRS infeasibility issue. In the first stage, they test whether a VRS super-efficiency model is infeasible by investigating the existence of output surplus (input saving) when infeasibility occurs in the input-oriented (output-oriented) VRS super-efficiency model. In the second stage, they proposed a modified VRS super-efficiency model to yield a super-efficiency score that characterizes both the radial efficiency and input saving/output surplus. Chen and Liang [18] further prove that the two-stage process can be solved in a single linear program. However, when a DMU has zero data, these models may still be infeasible. Thrall [19] and Zhu [20] point out that the CRS super-efficiency model can also be infeasible when an efficient DMU has zero input values. The same conclusion can be applied to non-CRS super-efficiency models. Lee et al. [21] first point out that zero output data will not lead to infeasibility of the output-oriented super-efficiency models developed in the studies of Cook et al. [16], Lee et al. [17], and Chen and Liang [18]. This is because the output side of the constraints can always be satisfied. Therefore, they only assume that some inputs are zero for some efficient DMUs. Then, they revise the model of Lee et al. [17], and the revised model will be feasible when zero data exist in inputs.

Nevertheless, these models mentioned above so far only consider radial efficiency. The current paper extends the work of Lee and Zhu [21] to nonradial measurement. And we find that the super-efficiency model in the presence of nonradial measurement still maintains some good properties. In fact, super-efficiency with nonradial measurement can also help decision makers more allocate input resources according to underlying preferences and resource availability.

The remainder of this paper is organized as follows: Section 2 looks back upon several radial super-efficiency models that had been established and the problem of super-efficiency infeasibility. In Section 3, we develop our super-efficiency model with nonradial measurement and demonstrate that the model has some good properties. Meanwhile, we point out the difference between radial and nonradial measurement. In Section 4, the newly developed approach is applied to a data set on the 15 Illinois strip mines. In the end, main conclusions are given in Section 5.

2. Radial Super-Efficiency Models

Suppose there are n DMUs $\{\mathrm{DMU}_j\ (j=1,2,\ldots,n)\}$. Each DMU_j consumes a set of m inputs, $x_{ij}\,(i=1,\ldots m)$, in the production of a set of s outputs, $y_{rj}\,(r=1,\ldots,s)$. Based upon the VRS model of Banker et al. [12], the input-oriented VRS super-efficiency model for efficient DMU_k can be expressed as

$$\begin{aligned}
\min\quad & \theta \\
\text{s.t.}\quad & \sum_{\substack{j=1\\ j\neq k}}^{n} \lambda_j x_{ij} \le \theta x_{ik}, \quad i=1,\ldots,m \\
& \sum_{\substack{j=1\\ j\neq k}}^{n} \lambda_j y_{rj} \ge y_{rk}, \quad r=1,\ldots,s \\
& \sum_{\substack{j=1\\ j\neq k}}^{n} \lambda_j = 1 \\
& \lambda_j \ge 0, \quad j\neq k,
\end{aligned} \tag{1}$$

where the DMU_k under consideration is excluded from the reference set. Obviously, one of the circumstances where model (1) is infeasible if the DMU under consideration has the largest outputs, regardless of the input values. In fact, as pointed out by Lee et al. [17], when the outputs of the evaluated DMU is outside the production possibility set spanned by the outputs of the remaining DMUs, the infeasibility of input-oriented super-efficiency will occur.

Consider the same simple numerical example given in the study of Lee et al. [17] in Table 1. Owing that $\mathrm{DMU_D}$ has the largest outputs and $\mathrm{DMU_E}$ has zero input data, they are both infeasible under model (1). Lee et al. [17] develop a new super-efficiency model and two-stage process to investigate the issue of the VRS super-efficiency infeasibility. Chen and Liang [18] establish a single model and integrate the two-stage process into an equivalent "one model" approach:

$$\begin{aligned}
\min\quad & \tau + M \times \sum_{r=1}^{s} \beta_r \\
\text{s.t.}\quad & \sum_{\substack{j=1\\ j\neq k}}^{n} \lambda_j x_{ij} \le (1+\tau) x_{ik}, \quad i=1,\ldots,m \\
& \sum_{\substack{j=1\\ j\neq k}}^{n} \lambda_j y_{rj} \ge (1-\beta_r) y_{rk}, \quad r=1,\ldots,s \\
& \sum_{\substack{j=1\\ j\neq k}}^{n} \lambda_j = 1 \\
& \lambda_j \ge 0,\ j\neq k, \beta_r \ge 0,
\end{aligned} \tag{2}$$

where M is a user-defined large positive number, and in the study of Cook et al. [16], M is set equal to 10^5. When all inputs are positive, model (1) is feasible if and only if all $\beta_r^* = 0$ in model (2). The super-efficiency score is defined as $1+\tau^* + 1/|R| \sum_{r\in R} 1/1-\beta_r^*$, where $R=\{r \mid \beta_r^* > 0\}$. However, owing to zero data $X_2 = 0$ in $\mathrm{DMU_E}$, as depicted in Figure 1, $\mathrm{DMU_E}$ remains infeasible under model (2).

Table 1: Data set of a simple example.

DMU	X_1	X_2	Y_1	Super-efficiency model (1)	Super-efficiency model (2)
A	2	1	1	1	1
B	1	2	1	1.4	1.4
C	1	4	2	Infeasible	3
D	2	3	1	0.6	0.6
E	3	0	1	Infeasible	Infeasible

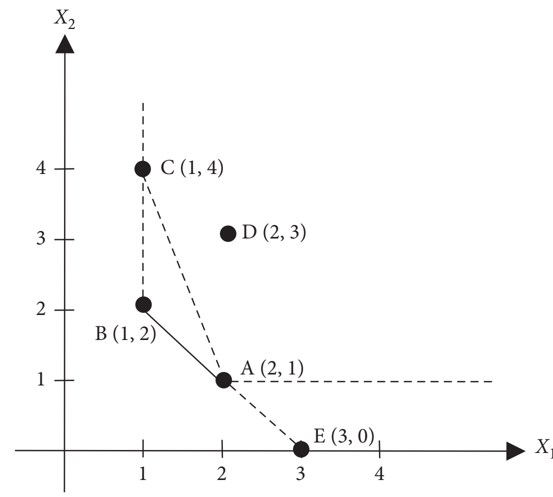

Figure 1: A simple example.

Subsequently, Lee et al. [21] consider the situation when zero input data exist. They propose model (3) to address this issue about zero input data and define the super-efficiency score as $1 + \tau^* + 1/|R|\sum_{r\in R} 1/1 - \beta_r^* + 1/|I|\sum_{r\in R} 1 + t_i^*/1$ when neither one of the sets R and I is empty, in which $R = \{r \mid \beta_r^* > 0\}$ and $I = \{i \mid t_i^* > 0\}$. They demonstrate that models (2) and (3) yield the same results when data are positive:

$$\begin{aligned}
\min \quad & \tau + M \times \left(\sum_{r=1}^{s} \beta_r + \sum_{i=1}^{m} t_i \right) \\
\text{s.t.} \quad & \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j x_{ij} - t_i x_i^{\max} \le (1+\tau) x_{ik}, \quad i = 1, \dots, m \\
& \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j y_{rj} \ge (1-\beta_r) y_{rk}, \quad r = 1, \dots, s \\
& \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j = 1 \\
& \lambda_j \ge 0,\ j \neq k, t_i \ge 0, \beta_r \ge 0, \tau \text{ is unrestricted},
\end{aligned} \tag{3}$$

where $x_i^{\max} = \max_{1\le k\le n}\{x_{ik}\}$. The radial scaling factor $1 + \tau^*$, super-efficiency score, and more details of results by using model (3) that are computed by Lee et al. [21] are listed in Table 2. Here, $\mathrm{DMU_E}$ can be feasible under model (3).

3. A Modified Nonradial Super-Efficiency Model

In the real production processes, decision makers tend to arrange production plan reasonably according to possession of the resources and production capability. Investment of input resources must be consistent with resource availability and underlying preferences (for example, decision makers may be more willing to use cheaper to help lower costs). Radial scaling with inputs, i.e., decreasing inputs, proportionally is not always met in the actual production processes. Therefore, it is necessary and meaningful to incorporate nonradial measurement into super-efficiency model at some point. So, we modify model (3) to the situation of nonradial measurement. Consider the following model for DMU_k:

$$\begin{aligned}
\min \quad & \frac{1}{m}\sum_{i=1}^{m} \tau_i + M \times \left(\sum_{r=1}^{s} \beta_r + \sum_{i=1}^{m} t_i \right) \\
\text{s.t.} \quad & \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j x_{ij} - t_i x_i^{\max} \le (1+\tau_i) x_{ik}, \quad i = 1, \dots, m \\
& \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j y_{rj} \ge (1-\beta_r) y_{rk}, \quad r = 1, \dots, s \\
& \sum_{\substack{j=1 \\ j\neq k}}^{n} \lambda_j = 1 \\
& \lambda_j \ge 0,\ j \neq k, 0 \le t_i \le 1, \beta_r \ge 0, \tau_i \text{ is unrestricted}.
\end{aligned} \tag{4}$$

Under nonradial measurement, we permit that each input element could be scaled down by an exclusive scaling factor, $1 + \tau_i$ $(i = 1, \dots, m)$, instead of the common proportional scaling factor $1 + \tau$ for all inputs in model (3). Obviously, model (4) still works when zero input data exist. Based on model (4), some good properties can be derived as follows.

Theorem 1. *In model (4),* $\tau_i^* \ge -1$ *and* $1/m \sum_{i=1}^{m} \tau_i^* \ge -1$.

Proof. Note that $\sum_{j=1, j\neq k}^{n} \lambda_j = 1$. Therefore, $\sum_{j=1, j\neq k}^{n} \lambda_j x_{ij} - t_i x_i^{\max} \le \sum_{j=1, j\neq k}^{n} \lambda_j x_i^{\max} - t_i x_i^{\max} = (1 - t_i) x_i^{\max}$. Also, there exists $0 \le t_i \le 1$; thus, $\sum_{j=1, j\neq k}^{n} \lambda_j x_{ij} - t_i x_i^{\max} \ge 0$. So, when τ_i is large enough, $1 + \tau_i^* \ge 0$ must exist; then, $\tau_i^* \ge -1$. Furthermore, $1/m \sum_{i=1}^{m} \tau_i^* \ge -1$. □

Table 2: Results under model (3).

DMU	$1+\tau^*$	Input saving	Output surplus	Super-efficiency	t_1^*	t_2^*	β_1^*
A	1	0	0	1	0	0	0
B	1.4	0	0	1.4	0	0	0
C	1	0	2	3	0	0	0.5
D	0.6	0	0	0.6	0	0	0
E	0.666667	1.25	0	1.916667	0	0.25	0

Theorem 2. *Model (4) is identical with model (3) when $\tau_1 = \tau_2 = \cdots = \tau_m$.*

In other words, when $\tau_1 = \tau_2 = \cdots = \tau_m$, the objective function values of model (3) and model (4) are identical.

Theorem 3. *Model (4) is always feasible.*

Proof. According to the proof for feasibility of model (3) of Lee et al. [21], model (3) is always feasible. Suppose $\overline{\lambda_j}, \overline{t_i}, \overline{\beta_i}, \overline{\tau}$ is a feasible solution of model (3), let $\tau_1 = \tau_2 = \cdots = \tau_m = \overline{\tau}$, then, $\overline{\lambda_j}, \overline{t_i}, \overline{\beta_i}, \tau_1, \tau_2, \ldots, \tau_m$ is just a feasible solution to model (4). In other words, the feasible region of model (4) is not empty. Thus, model (4) is always feasible. □

Theorem 4. *Model (1) is infeasible if and only if*

(i) Some $\beta_r^* > 0$ $(r = 1, \ldots, s)$ when $x_i^{\max} = x_{ik}$, where β_r^* is the optimal solution in model (4)

(ii) Some $\beta_r^* > 0$ $(r = 1, \ldots, s)$ or some $t_i^* > 0$ $(i = 1, \ldots, m)$ when $x_i^{\max} \neq x_{ik}$, where β_r^*, t_i^* is the optimal solution in model (4)

Proof

(i) When $x_i^{\max} = x_{ik}$, the first constraint condition in model (4) becomes $\sum_{j=1, j\neq k}^{n} \lambda_j x_{ij} \leq (1 + \tau_i + t_i) x_{ik}$, $i = 1, \ldots, m$. Let $\tau_i' = \tau_i + t_i$, then model (4) could turn out to be

$$
\begin{aligned}
\min \quad & \frac{1}{m}\sum_{i=1}^{m} \tau_i' + M \times \sum_{r=1}^{s} \beta_r \\
\text{s.t.} \quad & \sum_{\substack{j=1\\ j\neq k}}^{n} \lambda_j x_{ij} \leq (1 + \tau_i') x_{ik}, \quad i = 1, \ldots, m \\
& \sum_{\substack{j=1\\ j\neq k}}^{n} \lambda_j y_{rj} \geq (1 - \beta_r) y_{rk}, \quad r = 1, \ldots, s \\
& \sum_{\substack{j=1\\ j\neq k}}^{n} \lambda_j = 1 \\
& \lambda_j \geq 0,\ j \neq k, 0 \leq t_i \leq 1, \beta_r \geq 0, \tau_i \text{ is unrestricted.}
\end{aligned}
\tag{5}
$$

Based upon the studies of Cook et al. [16], Lee et al. [17] and Chen and Liang [18], the proposition is obviously true, i.e., model (1) is infeasible if and only if some $\beta_r^* > 0 \, (r = 1, \ldots, s)$ in model (4).

(ii) When $x_i^{\max} \neq x_{ik}$, $\beta_r^* = 0$ $(r = 1, \ldots, s)$ and $t_i^* = 0$ $(i = 1, \ldots, m)$ in model (4) indicate that model (1) is feasible. So, model (1) is infeasible, and there must be some $\beta_r^* > 0 \, (r = 1, \ldots, s)$ or some $t_i^* > 0 \, (i = 1, \ldots, m)$ in model (4). On the contrary, if model (1) is feasible, this means $\beta_r^* = 0 \, (r = 1, \ldots, s)$ and $t_i^* = 0 \, (i = 1, \ldots, m)$ is a feasible solution to model (4), which directly contradict the fact that some $\beta_r^* > 0$ or $t_i^* > 0$ is the optimal solution in model (4). □

Theorem 5. *Results of model (4) is not larger than that of model (3).*

Proof. Adding the constraint condition of $\tau_1 = \tau_2 = \cdots = \tau_m$ to model (4), model (4) has become model (3). So, the feasible region of model (3) is smaller than model (4). A relaxation in one of the constraints may yield a better optimal solution inside the new polyhedron in space. Therefore, the result of model (4) is not larger than that of model (3). □

Theorem 6. *Models (4) and (3) yield the same amount of input saving and output surplus when zero input data exist.*

Proof. For any $i' \in (1, 2, \ldots, m)$, suppose DMU_k has zero input data $x_{i'k} = 0$, we have $(1 + \tau_i) x_{ik} = 0$. Due to $\tau_i \geq -1$ and the minimization of objective function, there must exist $\tau_i^* = -1$. So, the first constraint condition $\sum_{j=1, j\neq k}^{n} \lambda_j x_{ij} - t_i x_i^{\max} \leq (1 + \tau_i) x_{ik}$ in model (4) will be $\sum_{j=1, j\neq k}^{n} \lambda_j x_{ij} - t_i x_i^{\max} \leq 0$, which is identical with that in model (3) when $x_{i'k} = 0$.

Theorem 6 also indicates the way where the DMU under evaluation in model (4) is projected to the frontier formed by other DMUs is the same as that in model (3) when the DMU has zero input values.

The super-efficiency model (4) with nonradial measurement of ours also determines an efficient referent (benchmark) DMU that is on the frontier formed by the remaining DMUs. In order to fully characterize the super-efficiency by input saving/output surplus achieved by an efficient DMU if it exists when the VRS super-efficiency feasibility is present, and based upon Cook et al. [16] and Lee et al. [17, 21], we also denote $R = \{r \mid \beta_r^* > 0\}$ and $I = \{i \mid t_i^* > 0\}$. Then input savings index and output savings index can be defined in the following manner:

Table 3: Results under model (4).

DMU	$1 + 1/m\sum_{i=1}^{m} \tau_i^*$	Input saving	Output surplus	Super-efficiency	τ_1^*	τ_2^*	t_1^*	t_2^*	β_1^*
A	0.75	0	0	0.75	0.5	−1	0	0	0
B	1.25	0	0	1.25	1	−0.5	0	0	0
C	0.75	0	2	2.75	0	−0.5	0	0	0.5
D	0.58335	0	0	0.58335	−0.5	−0.3333	0	0	0
E	0.33349	1.25	0	1.58349	−0.3333	−1	0	0.25	0

$$\hat{i} = \begin{cases} 0, & \text{if } I = \phi, \\ \dfrac{\sum_{i\in I}\left(x_i^{\max} + t_i^* x_i^{\max}/x_i^{\max}\right)}{|I|} = \dfrac{\sum_{i\in I}\left(1 + t_i^*\right)}{|I|} & \text{if } I \neq \phi, \end{cases}$$

$$\hat{o} = \begin{cases} 0, & \text{if } R = \phi, \\ \dfrac{\sum_{r\in R}\left(1/1 - \beta_r^*\right)}{|R|}, & \text{if } R \neq \phi, \end{cases} \tag{6}$$

where $|R|$ and $|I|$ are the cardinality of the sets R and I, respectively.

Unlike super-efficiency score of Lee et al. [21], we put $1 + 1/m\sum_{i=1}^{m} \tau_i^* \geq 0$ as the nonradial efficiency. Then, the super-efficiency score which could also characterize the super-efficiency in both inputs and outputs can be defined as

$$\breve{\theta} = 1 + \frac{1}{m}\sum_{i=1}^{m} \tau_i^* + \hat{i} + \hat{o}. \tag{7}$$

The efficiency measure $\breve{\theta}$ consists of three parts: the nonradial efficiency $1 + 1/m\sum_{i=1}^{m} \tau_i^*$, the input saving index $\hat{i}$, and the output surplus index $\hat{o}$. As described by Lee et al. [21], when the set $I = \{i \mid t_i^* > 0\}$ is not empty, the input savings index $1/|I|\sum_{i\in I}(1 + t_i^*/1)$ reflects how far the DMU_k is below the dashed horizontal efficient boundary (see the dashed line through A in Figure 1). And the output surplus index $1/|R|\sum_{r\in R}(1/1 - \beta_r^*)$ reflects how far the DMU_k is above the dashed efficient boundary.

We use Figure 1 to illustrate the difference from the method of Lee et al. [21], mainly including the different projection mode to the (best-practice) frontier formed by other DMUs and the different nonradial super-efficiency result obtained. For instance, for DMU_B, the efficient frontier is made up of broken lines CA and AE. DMU_B is on the left of the frontier which implies that its super-efficiency should be greater than 1. DMU_B achieves radial efficiency of 1.4 and gets its projective point (efficient referent DMU) at $(1.4, 2.8)(= 1.4 \times (1, 2))$ on the frontier through scaling down proportionally under radial measurement. While under nonradial measurement, DMU_B gets its projective point (efficient referent DMU) at $(2, 1)(= (1 + 1) \times 1, (1 - 0.5) \times 2)$; that is to say, DMU_B takes DMU_A as its benchmark under nonradial measurement. Meanwhile, DMU_B gains its nonradial efficiency of $1.25(= 1 + 1/2(1 - 0.5))$. For DMU_C, on account that it has the largest input $X_2 = 4$ and $\beta_1^* = 0.5 > 0$ in model (4), it is infeasible under model (1), but it is on the frontier and gets radial efficiency of 1 under model (3). It gets its projective point on the frontier at $(1, 2)(= (1 + 0) \times 1, (1 - 0.5) \times 4)$ under nonradial measurement, i.e., DMU_C puts DMU_B as its best-practice benchmark. Now, DMU_C earns its nonradial efficiency of $0.75(= 1 + 1/2(0 - 0.5))$. DMU_C has the same amount of output surplus of 2 under both measurements which equals the distance from DMU_B to DMU_C. Finally, for DMU_E, it has zero input data $X_2 = 0 \neq X_2^{\max}$ and $t_2^* = 0.25 > 0$ in model (4), it is also infeasible under model (1). However, it reaches the efficient referent (benchmark) DMU_A in the same type of way and derives the same input saving of $1.25(= 1 + 0.25/1)$ under both measurements. Concretely, DMU_E goes through point (2, 0) on X-axis by radial or nonradial scaling and then moves upwards by 1 ($t_i^* x_i^{\max} = 1$). But it has different radial efficiency of 0.666667 and nonradial efficiency of 0.33349. Table 3 shows more specific details and final super-efficiency result of model (4) for each DMU under nonradial measurement. □

4. An Empirical Example

We apply our approach to the data set which contains a set of 15 Illinois strip mines in Lee et al. [21]. The data set is shown in Table 4, which has only one output of tons of coal produced and eight inputs. Of the inputs, one is labor in thousand miner days, and the other variables include three capital variables (K1, K2, and K3) and four geological variables (T1, D1, T2, and D2). In Table 4, K1 = bucket capacity of draglines, K2 = dipper capacity of power shovels, K3 = earth moving capacity of wheel excavators, T1 = thickness of first (upper) seam mined, D1 = depth to first seam mined, T2 = thickness of second (lower) seam mined, and D2 = depth to second seam mined. In this particular data set, there are many zero inputs.

Table 5 shows super-efficiency score and solutions details of each DMU under four models, respectively. But the very first point which needs to be made is that we set $M = 10^4$, here, to seek its optimal solutions of all DMUs under model (4) with nonradial measurement. Because under nonradial measurement, it should adjust the value of user-defined large positive number M to suit for the specific data set to find its optimal solution. From Table 5, we discover that there are 5 DMUs that are infeasible under the original super-efficiency model (1). From Theorem 4 and the solution details of model (4), we know that DMU_1, DMU_2, and DMU_4 have nonzero output surplus because of the existence of $\beta_1^* > 0$ and DMU_{10} and DMU_{13} have nonzero input saving because of the existence of $t_2^* > 0$ and $t_3^* > 0$, respectively. For example, DMU_{10} is infeasible under model (1) because the second

Table 4: Illinois strip mines.

DMU	Labor (1000 miner days)	K1 (cubic yards)	K2 (cubic yards)	K3 (cubic yards per hour)	T1 (feet)	1/D1 (D1 in feet)	T2 (feet)	1/D2 (D2 in feet)	Output (1000 tons)
1	98.5	142	245	0	6	0.016	4.3	0.012	3264
2	96.5	30	215	0	6	0.016	0	0	3065
3	57.6	18	105	0	5.6	0.026	4.2	0.016	2275
4	59.2	160	0	0	5.9	0.025	3.7	0.011	1978
5	57.6	200	0	0	8	0.022	3.5	0.011	1833
6	49.9	27	85	0	4.5	0.019	0	0	1218
7	53.5	143	65	0	6	0.01	0	0	928
8	34	70	65	12	6	0.02	5	0.01	919
9	39.6	67.5	40	0	6.5	0.013	0	0	777
10	51.3	0	145	0	3.2	0.019	0	0	745
11	74.2	110	65	0	2.1	0.014	0	0	742
12	24	25	65	0	4.4	0.012	0	0	488
13	26.5	58	0	0	3	0.014	0	0	407
14	43.1	70	0	0	6.5	0.012	0	0	402
15	20.7	236	0	0	5.7	0.01	0	0	396

Table 5: Illinois strip mines super-efficiency score (SE score).

DMU	Model (1)	Model (2)		Model (3)			Model (4)		
		SE score	Solution details	$1+\tau^*$	SE score	Solution details	$1+1/m\sum_{i=1}^{m}\tau_i^*$	SE score	Solution details
1	Infeasible	2.064927	$\beta_1^* = 0.060968$	1	2.064927	$\beta_1^* = 0.060968$	0.508564	1.573490	$\beta_1^* = 0.060968$
2	Infeasible	3.70392	$\beta_1^* = 0.60261$	1.1875	3.70392	$\beta_1^* = 0.60261$	0.468745	2.985165	$\beta_1^* = 0.60261$
3	1.547346	1.547346		1.547346	1.547346		0.735467	0.735467	
4	Infeasible	2.435037	$\beta_1^* = 0.073306$	1.35593	2.435037	$\beta_1^* = 0.073306$	0.800606	1.879711	$\beta_1^* = 0.073306$
5	1.073871	1.073871		1.073871	1.073871		0.67373	0.67373	
6	1.159819	1.159819		1.159819	1.159819		0.613154	0.613154	
7	1.14681	1.14681		1.14681	1.14681		0.47557	0.47557	
8	0.937467	0.937467		0.937467	0.937467		0.485029	0.485029	
9	1.001438	1.001438		1. 001438	1.001438		0.51114	0.51114	
10	Infeasible	Infeasible		0.42488	2.532909	$t_2^* = 0.10805$	0.404706	1.512756	$t_2^* = 0.10805$
11	1.554315	1.554315		1.554315	1.554315		0.501842	0.501842	
12	1.478827	1.478827		1.478827	1.478827		0.681564	0.681564	
13	Infeasible	3.1794104	$\beta_1^* = 0.012285$	1.1664	3.168001	$t_3^* = 0.001648$	0.732485	1.734133	$t_3^* = 0.001648$
14	1.127483	1.127483		1.127483	1.127483		0.383954	0.383954	
15	1.376085	1.376085		1.376085	1.376085		0.431534	0.431534	

input gets input savings of 25.49864 ($= t_2^* x_2^{\max} = 0.108045 \times 236$) compared with the factual second input amount of zero. That is the reason why DMU_{10} is infeasible under model (1) and model (2). For DMU_{13}, it gets output surplus of 5 ($= \beta_1^* y_1^1 = 0.012285 \times 407$) under model (2), so it is infeasible under model (1). However, under model (3) and model (4), output surplus is eliminated and input saving of the third input with the value of 0.40368 ($= t_3^* x_3^{\max} = 0.001648 \times 245$) comes into being. The result under model (4) with nonradial measurement is smaller than that of model (3) for each DMU. Using model (3) and model (4), the same amount of input saving or output surplus derived under both radial and nonradial measurement can also be seen in Table 5.

5. Conclusions

Based upon the previous work, the current paper extends radial super-efficiency model to the circumstances of nonradial measurement and establishes a modified nonradial super-efficiency model. We find that the super-efficiency model with nonradial measurement still maintains some good properties and the original radial VRS super-efficiency model can also be detected whether it is infeasible through our nonradial super-efficiency model. In the actual production processes, due to underlying preferences and resource availability, super-efficiency model with nonradial measurement can help decision makers seek an efficient benchmark different from the reference under radial measurement. It is beneficial for decision makers to allocate input resources, make production plan, and arrange production activities. This paper only discusses the input-oriented nonradial model, for output-oriented situation, and it can be discussed in the same way.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

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