Research Article

Fault Location of Active Distribution Networks Based on the Golden Section Method

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Aiming at the problem of fault location in distribution networks with distributed energy resources (DERs), a fault location method based on the concepts of minimum fault reactance and golden section is proposed in this paper. Considering the influence of distributed energy resource supply on fault point current in distribution networks, an improved trapezoidal iteration method is proposed for load flow analysis and fault current calculation. This method only needs to measure the synchronous current of the distributed energy resource and does not need to measure the voltage information. Therefore, the investment in equipment is reduced. Validation is made using the IEEE 34-node test feeder. The simulation results show that the method is suitable for fault location of distribution networks with multiple distributed generators. This method can accurately locate the faults of the active distribution network under different conditions.

1. Introduction

With the development of society, more and more DERs are connected to the distribution network, which leads to more and more applications of clean energy and brings huge economic benefits [1–3]. The integration of distributed generation (DG) and its synchronous operation with the power grid is one of the challenges we now face [4–6]. In this case, the supervision and management of power grid with self-healing function becomes more complex. Fast fault location is an important task to restore power supply when the distribution network fails. Although technology is developing rapidly, due to the increasing expectations of users, the fault location technology of the distribution network still needs improvement.

At present, the fault location methods for the active distribution network are based on the traveling wave method, artificial intelligence method, and impedance method. Traveling wave analysis is widely used. In [7–10], the traveling wave fault location method is introduced, but this method requires a higher sampling frequency and has many links, which has some impact on the overall reliability.

In [11], the evaluation function for the characteristics of the active distribution network was redefined, and a suitable immune algorithm was proposed. In [12], a radial basis function neural network model was proposed based on the fault information uploaded by the distribution network monitoring and data acquisition system. The improved algorithm is applied to fault location of the active distribution network. In [13], a fault location method based on the wavelet fuzzy neural network for postfault transient and steady-state measurements is proposed. Although the above-mentioned artificial intelligence-based fault location method has high performance, it requires a lot of data to learn and its accuracy is affected by the database. In [14–17], fault location is achieved by collecting synchronous current and voltage vectors. This method increases equipment investment because it requires measurement of synchronous current and voltage information. The impedance-based fault location method is one of the current research methods and has achieved good results in applications [18–21]. However, this method does not solve the problem of active distribution network fault location. In [22, 23], the effect of DG on the distribution network was simulated using a synchronous

machine, but only one DER was considered, and the effect of different DER penetration levels was not analyzed [24].

The fault location method proposed in this paper combines the golden section method and the trapezoidal iteration technique to analyze the fault location of the active distribution network under different operating conditions [25]. This method eliminates the potential relationship between the accuracy of fault location and search step size. In addition, this method only needs to collect the synchronous current vector of the DER, which reduces the equipment investment. It improves the accuracy of fault location and reduces costs.

2. Minimum Fault Reactance

The method proposed in this paper is based on the concept of minimum fault reactance [26]. Assuming that the fault points of the analysis line segment are random, the fault reactance equation can be obtained by calculating the fault reactance through the analysis circuit [27]. For each assumed fault distance, a fault reactance is calculated. This process is repeated on all line segments. Most line faults have impedance characteristics, so the fault location can be solved by solving the minimum fault reactance value. In the distribution network shown in Figure 1, there is a fault between nodes n and $n + 1$. Considering that the faulted line section is unknown, the downstream line segment of the first substation is initially considered.

As shown in Figure 2, by controlling the closing and opening of switches S_1, S_2, S_3, and S_4, the active distribution network can obtain different fault types. A generalized expression of the fault point voltage V_f is obtained by analyzing the data in Figure 2 [18] aswhere $[Z_{abc}]$ is the impedance matrix between nodes n and $n + 1$, $[Y_{abc}]$ is the admittance matrix, $[U]$ is the third-order identity matrix, $[V_n^f]$ is the fault voltage vector of the node n, $[I_n^f]$ is the fault current vector, and m is the normalized fault distance which is the ratio of the fault distance of the line to its length.

$$\left[V_f\right] = \left([U] + \frac{1}{2}m^2[Z_{abc}] \cdot [Y_{abc}]\right) \cdot \left[V_n^f\right] - m[Z_{abc}] \cdot \left[I_n^f\right].$$
(1)

Considering the influence of the downstream current, we use the following formula to calculate the fault current:

$$\left[I_f\right] = [I_u] + [I_d],$$
(2)

where $[I_u]$ is the upstream fault current vector and $[I_d]$ is the downstream fault current vector; $[I_u]$ is calculated by formula (3), and $[I_d]$ is calculated by using the improved trapezoidal iteration load flow calculation method:

$$[I_u] = -\left(m[Y_{abc}] + \frac{1}{4}m^3[Y_{abc}] \cdot [Z_{abc}] \cdot [Y_{abc}]\right)[V_n^f]$$

$$+ \left([U] + \frac{1}{2}m^2[Z_{abc}] \cdot [Y_{abc}] \cdot [I_n^f]\right).$$
(3)

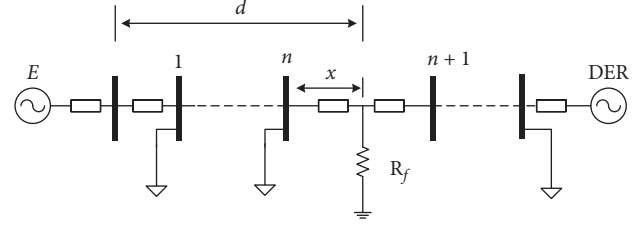A generalized expression of fault reactance obtained by formulae (1) and (2) is

FIGURE 1: Single-line diagram of the distribution network.

$$X_f = \text{Im}\left\{\frac{S_i \cdot V_{fi} - S_j \cdot V_{fj}}{S_i \cdot I_{fi}}\right\},$$
(4)

where V_{fi} is the voltage vector of the fault phase i and V_{fj} is the voltage vector of the fault phase j. When S_i and S_j are 1, it indicates that the phase has a fault; otherwise, they are 0. Im is the imaginary part of the solve operation. Table 1 shows the measured voltage and current vectors for solving the reactance of different fault types.

3. Golden Section Method

The primary task of the fault location method based on the minimum fault reactance value is to find the minimum reactance. The previous methods usually use a constant step length of Δm or a second derivative to solve the problem [28]. The golden section method proposed in this paper selects the test point by the principle of symmetry and keeping the reduction ratio [29]. By comparing the function values of the test points, the search interval is shortened to reach the minimum interval of the minimum point. The accuracy and speed of the search method are related to the step length. The golden section method takes fewer test points to determine the minimum fault reactance value, which improves the accuracy and speed of calculation.

The golden section method uses the golden section rule to select two symmetrical points in the search interval and then compares the function values at two points to determine a new search interval. In the fault location problem, $t \in [a, b]$, which indicates the change of the line fault distance. Select two golden points t_1 and t_2 in the interval $[a, b]$ and then use the golden section method to search the interval. The first search point t_1 takes 0.618 of $[a, b]$, and the second search point t_2 takes the symmetric point of t_1, that is, 0.382 of $[a, b]$, as shown in Figure 3.

Calculate the values of $X_{f(t_1)}$ and $X_{f(t_2)}$ and compare the sizes of the two values. If $X_{f(t_1)} < X_{f(t_2)}$, then t_1 is an effective point, so the interval $[a, t_2]$ is removed, the interval $[t_2, b]$ is reserved, and the new interval $[a_1, b_1] = [t_2, b]$:

$$t_1 = a + 0.618(b - a),$$
(5)

$$t_2 = a + 0.382(b - a).$$
(6)

Conversely, if $X_{f(t_1)} > X_{f(t_2)}$, then t_2 is the effective point, so the interval $[t_1, b]$ is removed, the interval $[a, t_1]$ is reserved, and the new interval $[a_1, b_1] = [a, t_1]$. It is judged whether the new interval is smaller than the allowable error ε. If the allowable error is not reached, the above search
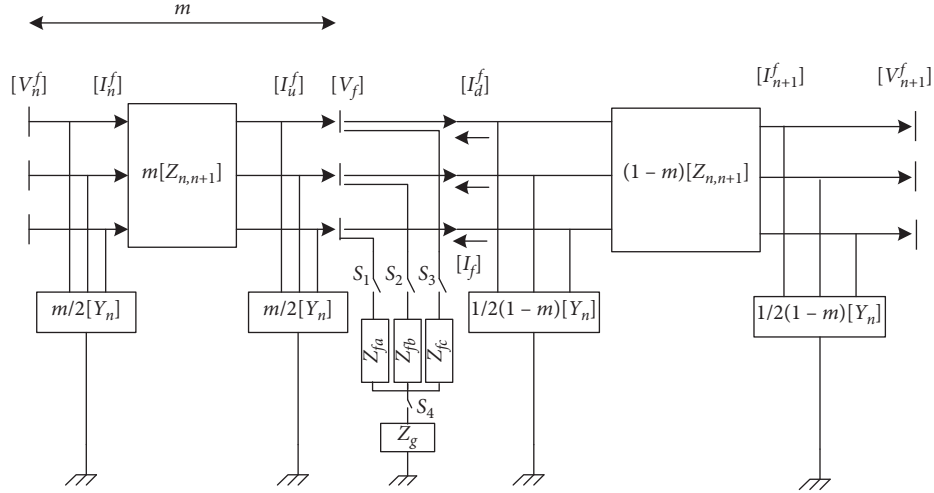
FIGURE 2: Distribution network fault model.

TABLE 1: Solve operation's fault reactance requirements.

Fault type	S_i	S_j	i phase (S_i V_{fi})		j phase (S_j V_{fj})		i phase fault current	
	1	0	a	V_{fa}	b, c	0	a	I_{fa}
Single-phase faults (a-g, b-g, c-g)	1	0	b	V_{fb}	a, c	0	b	I_{fb}
	1	0	c	V_{fc}	a, b	0	c	I_{fc}
	1	1	a	V_{fa}	b, b-c	V_{fb}	a	I_{fa}
Multiphase faults (a-b, b-c, c-a, a-b-g, b-c-g, c-a-g, a-b-c, a-b-c-g)	1	1	b	V_{fb}	c, a-c	V_{fc}	b	I_{fb}
	1	1	c	V_{fc}	a, a-b	V_{fa}	c	I_{fc}

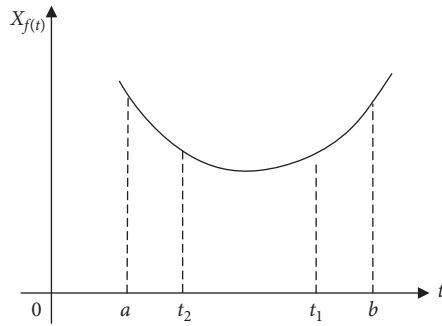

FIGURE 3: Golden section method.

process is continued in the new interval until the search interval $[a_i, b_i]$ is smaller than the allowable error ε:

$$\left|b_i - a_i\right| < \varepsilon. \tag{7}$$

The search is stopped when the allowable error is reached, and finally, the fault distance is determined according to the interval

$$m = \frac{b_i + a_i}{2}. \tag{8}$$

4. Active Distribution Network Fault Location

4.1. Establishing the Minimum Fault Reactance Model. The golden section method is used to model and analyze the fault location problem, and the fault location is transformed into the optimal solution problem. The optimization formula is as follows:

$$\begin{cases} f = \min\left|X_f(m)\right|, \\ m \in [0, 1]. \end{cases} \tag{9}$$

The objective function is given by the absolute value of formula (4), m is the normalized fault distance, and this function is used to solve the fault location. Since the faulted line is unknown, it is necessary to apply this method in each line of the distribution network.

4.2. Analysis Process. The flow diagram of the analysis method proposed in the paper is shown in Figure 4, which consists of four steps. The data processing for each step is as follows:

Step 1 (uploading data). Upload data of the active distribution network system, such as distribution system

topology, load, and line electrical parameters. In addition, there are the voltage and current vectors of the substation at fault time obtained by the Fourier transform and the synchronous current vector of the DER.

Step 2 (fault location). Calculate the fault current according to formula (2) and then calculate the fault reactance using formula (4). Assume that $[I_d] = -[I_n^p]$, $[I_n^p]$ is the prefault current of the node n. Formula (8) is used to calculate the fault distance of the line. The fault current is defined as a function of m, and then an iterative calculation is performed to update the downstream fault current $[I_d]$.

Step 3 (calculation of downstream fault current). Because the feeder of the active distribution network is radial, the traditional iterative method for power flow analysis has poor convergence in the active distribution network. Therefore, the improved trapezoidal iteration method is used to calculate the fault current of the radial power distribution system. As shown in Figure 5, the fault distance in the circuit is m, and then the downstream circuit of the fault point is analyzed. Assume that the system load is constant impedance during the fault and DER is the current source. The current is measured by the installed synchronous current device, and the downstream fault current value is obtained by an improved trapezoidal iteration method. This method has two processes: forward and backward [30].

In the forward process, the voltage vector [31] is calculated from the source node to the downstream node as follows:

$$\left[V_{n+1}^f\right] = [A] \cdot \left[V_n^f\right] - [B] \cdot \left[I_{n+1}^f\right]. \tag{10}$$

In the backward process, the current vector is calculated from the load node to the source node by using the voltage and circuit model derived from the most recent forward calculation as follows:

$$\left[I_n^f\right] = [c] \cdot \left[V_{n+1}^f\right] + [d] \cdot \left[I_{n+1}^f\right]. \tag{11}$$

The voltage and current vectors in formulae (10) and (11) are the values of nodes n and $n+1$ at fault. $[A]$, $[B]$, $[c]$, and $[d]$ are impedance matrices that vary for each system component type. Finally, a new source voltage is calculated based on the current.

When the algorithm is executed, in the first forward process, the voltages of all nodes of the system are initialized to the voltages of the fault points. In the backward process, the terminal current $[I_{n+1}^f]$ and the new source voltage are calculated using the voltage and circuit model obtained from the most recent forward calculation. System nodes can connect different components. When connected to parallel components, such as loads or capacitors, the current $[I_{n+1}^f]$ is calculated by the admittance matrix as

$$\left[I_{n+1}^f\right] = \left[Y_{n+1}^f\right] \cdot \left[V_{n+1}^f\right], \tag{12}$$

where $[Y_{n+1}^f]$ is the admittance matrix of the parallel components. When the DER is connected, the current $[I_{n+1}^f]$

is provided by the corresponding synchronous current-measuring device as

$$\left[I_{n+1}^f\right] = -\left[I_{\text{IED}_{n+1}}^f\right]. \tag{13}$$

Iterative search is done until the voltage error at the reference node is less than the specified T_{bfs}:

$$\left\| V_{\text{ref}}^f \right\| - \left\| V_f \right\| \le T_{\text{bfs}}. \tag{14}$$

Finally, when convergence is reached, the current at the reference node is taken as the fault current of the downstream circuit.

Step 4 (updating the voltage and current vectors of the downstream circuit). Steps 2 and 3 are used to analyze and calculate the line sections of the system. At this time, the calculation accuracy is improved by calculating the voltage and current of the downstream circuit nodes:

$$\begin{bmatrix} V_{n+1}^f \\ I_{n+1}' \end{bmatrix} = \begin{bmatrix} d_l & -b_l \\ -c_l & a_l \end{bmatrix} \cdot \begin{bmatrix} V_n^f \\ I_n^f \end{bmatrix}, \tag{15}$$

$$\left[I_{n+1}^f\right] = \left[I_{n+1}'\right] - \left[I_{L_{n+1}}^f\right], \tag{16}$$

where $[I_{n+1}']$ is the line current of the node $n+1$ without considering other branches, $[I_{n+1}]$ is the current at the node $n+1$ when considering other branches, and $[I_{L_{n+1}}]$ is the sum of the currents of the connection branches at the node $n+1$. For the node containing the DER branch, the current $[I_{n+1}]$ is

$$\left[I_{n+1}^f\right] = \left[I_{n+1}'\right] - \left[I_{L_{n+1}}^f\right] + \left[I_{\text{DER}}^f\right], \tag{17}$$

where $[I_{\text{DER}}]$ is the current flowing from the DER to the node $n+1$, which is calculated by step 3. If the fault is within the analyzed line section, the expression for the fault distance is

$$d = \sum_{i=1}^{n-1} L_i + M_i \cdot L_n, \tag{18}$$

where L_i is the length of the analyzed line i and M_i is the normalized fault length of the last line L_n, the value of which is calculated by formula (8).

5. Simulation Analysis

5.1. Simulation Modeling. PSCAD is used to build an IEEE 34-node test feeder system with different types of wires and loads. The working voltage level is 30 kV, and the total length of the line is 58 km. It is changed by adding three DERs with a penetration level of 10% in the system; the changed IEEE 34-node test feeder system is shown in Figure 6. The system is simulated and analyzed. Table 2 presents a summary of the tests realized. When the system fails, its fault data are obtained and the fault distance is calculated by the golden section method.
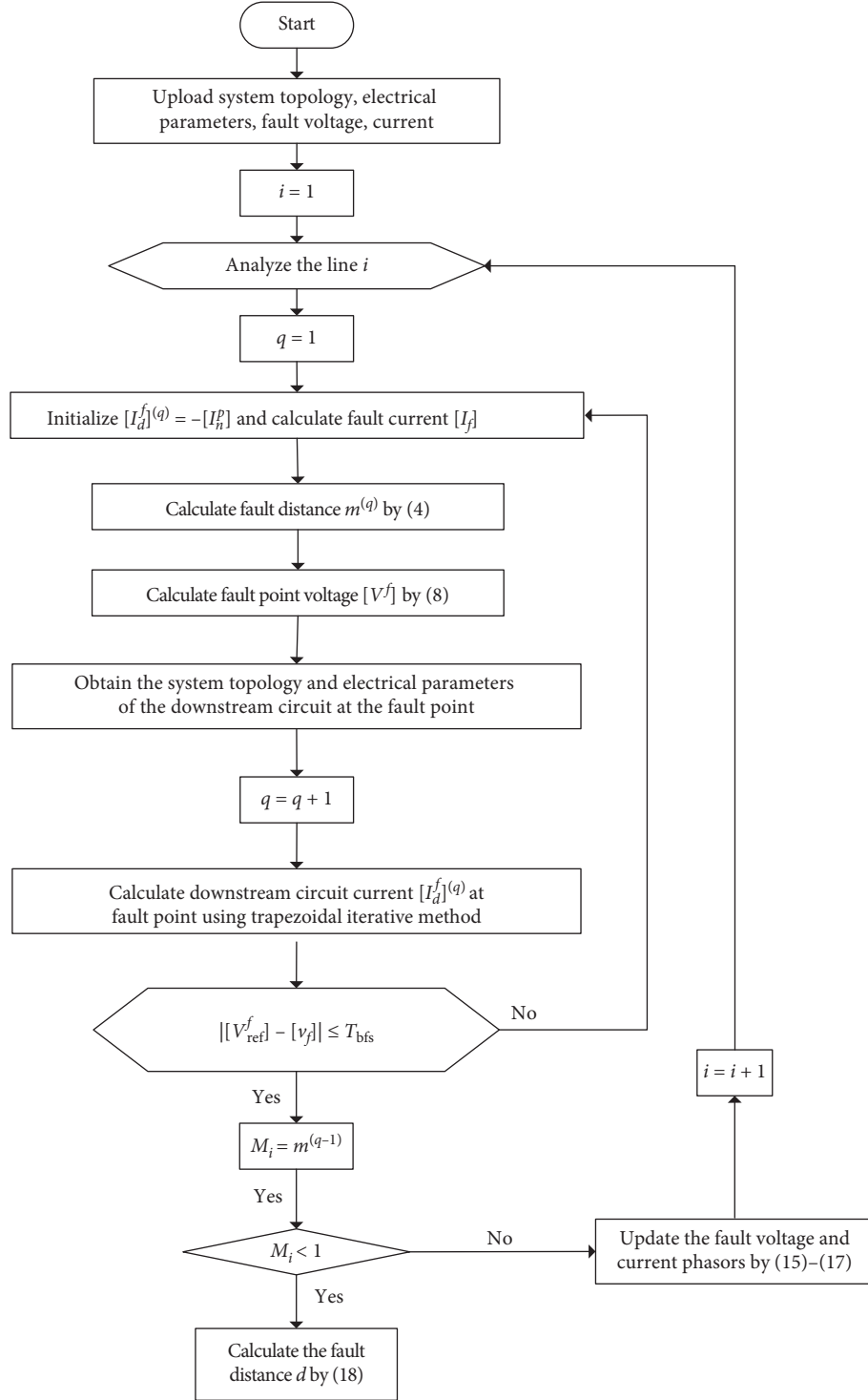
Figure 4: Flowchart of the algorithm.

5.2. Analysis of Results. The accuracy of the method is verified by analyzing the following scenarios. Considering the error between the calculated result and the actual value, the simulation result is evaluated by the following formula:

$$\% \, \text{error} = \left| \frac{d - d_{\text{rel}}}{d_{\text{tol}}} \right| \cdot \%, \tag{19}$$

where d is the calculated fault distance, d_{rel} is the actual fault distance, and d_{tol} is the total length of the line.

5.2.1. Effect of Fault Resistance. When analyzing the effect of fault resistance on the method, different fault types are considered, as shown in Figure 7. The results show that,
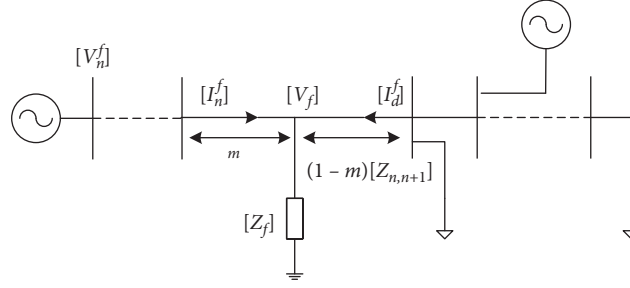
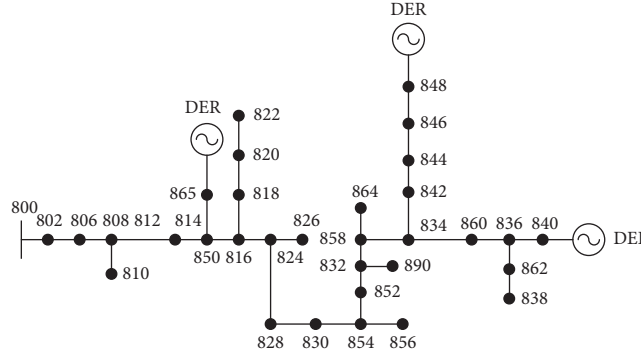FIGURE 5: Downstream circuit of the fault point.

FIGURE 6: IEEE 34-node system.

TABLE 2: Studied cases to validate the proposed method.

Test	Fault type	Description	Fault number
Fault resistance effect	All	Faults resistance between $0\,\Omega$ and $100\,\Omega$	2178
Fault distance effect	Single-phase faults, phase-to-phase faults	Fault resistance: $0\,\Omega$ and $100\,\Omega$	387
Load variation effect	Single-phase faults	Random load variation: 40–70%, 70–100%, 100–130%	129
DER penetration level	Single-phase faults	DER penetration level: 10%, 20%, 30%	129
Total faults			2823

under the influence of fault resistance, the maximum error of fault location is 0.16%, and this error occurs during single-phase ground fault. In the 58 km test feeder, the calculated average fault distance error of this method is about 25 m. This shows that the method has high accuracy of fault location.

In Figure 7, L-g is a single-phase ground fault, LL is a two-phase short-circuit fault, LL-g is a two-phase ground fault, LLL is a three-phase short-circuit fault, and LLL-g is a three-phase ground fault.

5.2.2. Effect of Fault Distance.
Figures 8 and 9 show the effect of fault distance on the proposed method.

It can be concluded from the analysis that the error increases slightly with increasing fault distance, as shown in Figures 8 and 9. And as the fault reactance value increases, this effect will be amplified but is still less than 0.14%. In the 58 km test feeder, the error is less than 80 m. Therefore, it is observed from the data above that the behavior of the method is little affected by the fault distance.

5.2.3. Effect of Load Variation.
Since the load in the active distribution network system varies greatly, this will have

some effect on the system. Therefore, it is necessary to analyze the effect of load variations on this method. As shown in Figure 10, the results show that the accuracy of the method is related to the value of the load. Because the load variation compensation method is not used in the simulation, the fault distance calculated by this method is smaller than the actual fault distance when the load level is low. This phenomenon occurs because the load conditions used by the proposed method are higher than the actual load conditions of the system, so the impedance observed by the proposed method from the substation is smaller. In contrast, for high load levels, the method calculates a fault distance that is higher than the actual fault distance. In this scenario, the error caused by the load variation is less than 0.18%. In the 58 km test feeder, the error is less than 104 m. This method still has good performance.

5.2.4. Effect of DER Penetration Level.
Three DERs with 10% penetration are added to the IEEE 34-node test feeder model. The effect of the DER penetration level on fault location of active distribution networks is analyzed. As shown in Figure 11, with the increase of the DER penetration level, the
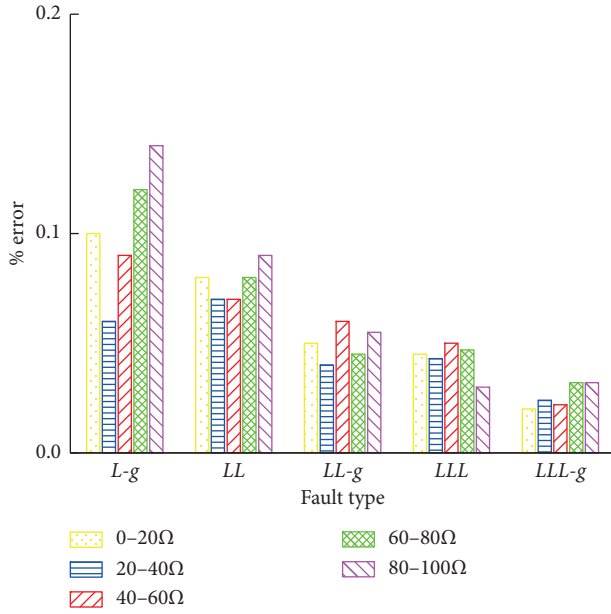
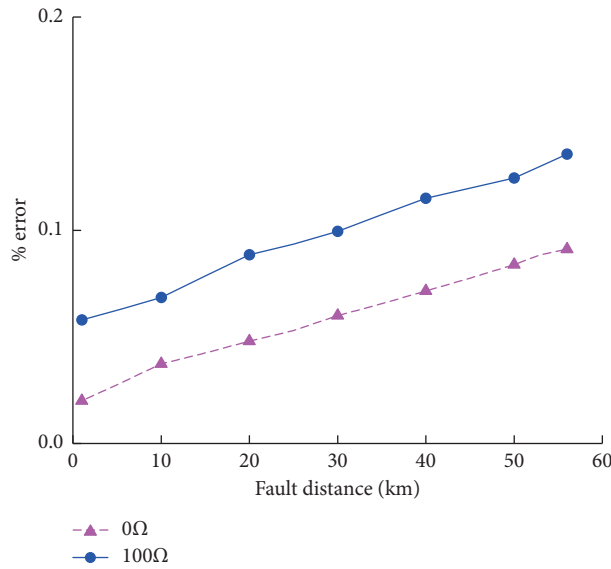FIGURE 7: Influence of fault types on fault location.

FIGURE 9: Performance curve for the proposed method considering the fault distance effect for faults BC.
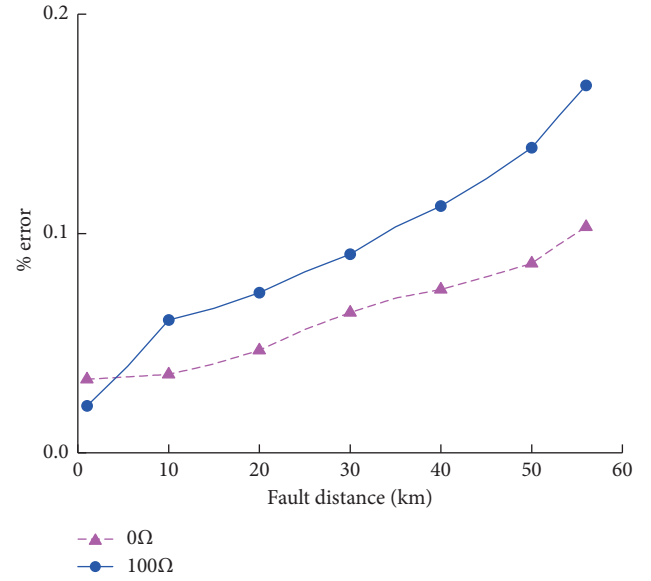

FIGURE 8: Performance curve for the proposed method considering the fault distance effect for faults A-g.
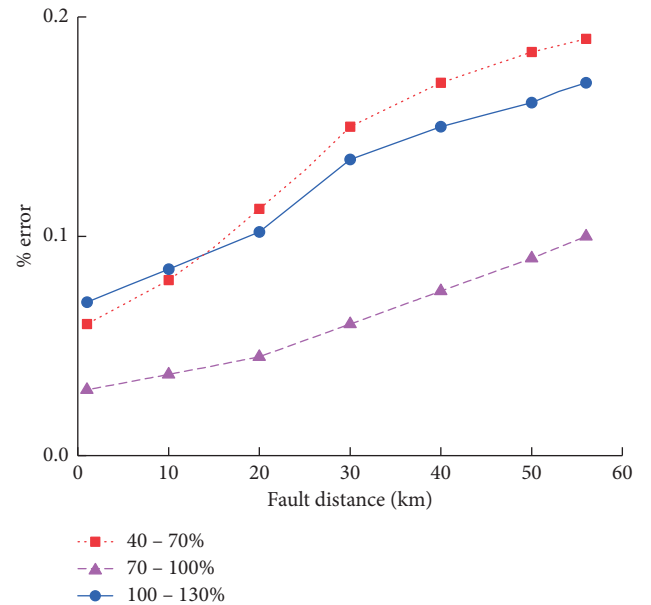
FIGURE 10: Influence of load variation on fault location.

calculated fault distance will be greater than the actual fault distance. This phenomenon is due to the increase in DER penetration, which leads to an increase in the error in the calculation of the current from the DER to the fault point. In this paper, an improved trapezoidal iteration method is used to calculate the current value, which reduces the error in calculating the current value. Thus, the error value is less than 0.15% in this scenario. In the 58 km test feeder, the error is less than 87 m. The method proposed in this paper still has high accuracy of fault location.

5.3. Comparison Test.
The two methods that have been implemented are compared with the proposed analytical

method: Nunes's method [21] and Bedoya-Cadena's method [27]. Nunes's method and Bedoya-Cadena's method are formulated considering that only one DER is connected to a network with a DER penetration level of 10%. Thus, for the comparison of the proposed method with these methods, the two DERs were disconnected. These methods were validated and compared using scenario 1, which evaluates the fault resistance effect. Test results are presented using the average error obtained for each fault type, as shown in Figure 12.

The test results show the better accuracy of our method when compared with the method proposed by Nunes and the method proposed by Bedoya-Cadena. The average error of the method proposed by Nunes is 2%, and the average error of the
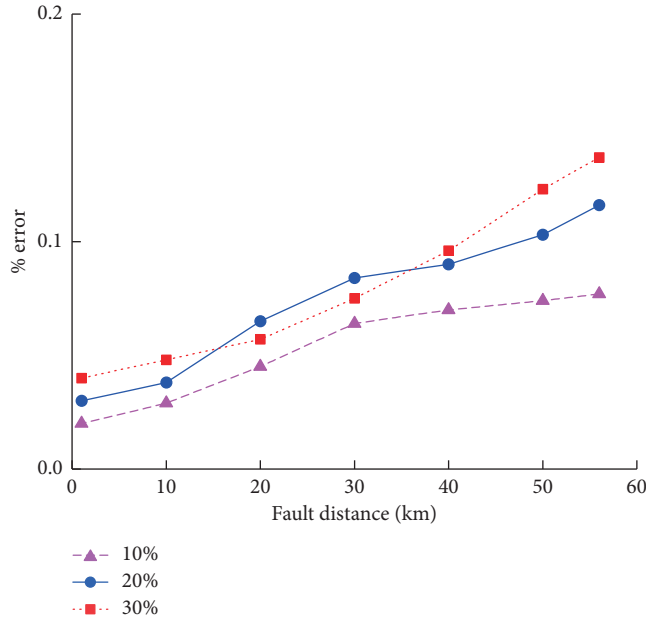
Figure 11: Influence of the DER penetration level on fault location.

Figure 13: Performance curve for the proposed method and comparison with Nunes's method and Bedoya-Cadena's method.

6. Conclusions

In this paper, a method for fault location of active distribution networks is proposed. This method uses the golden section method to search for the minimum fault reactance value and converts the fault location problem into a search minimum problem. Compared with the traditional fixed step search method, the golden section method proposed in this paper improves the search speed. According to the different loads of the nodes, an improved trapezoidal iterative power flow calculation method is proposed in this paper. This method improves the accuracy of power flow calculation and fault location. The simulation results show that this method has a high efficiency and robustness under different fault conditions and operating conditions. This method only needs to measure the synchronous current vector of the DER and does not need to measure the voltage vector. The fault location is realized by a simple method, which shows the potential of the method in practical applications.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.
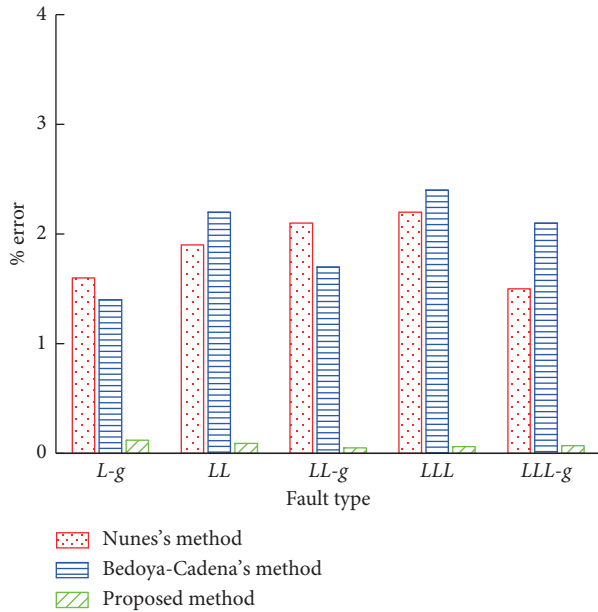
Acknowledgments

Figure 12: Error comparison of different methods in scenario 1.

method proposed by Bedoya-Cadena is 1.8%. However, the average error of the method proposed in this paper is 0.6%.

Finally, a test case is chosen to compare these two methods with the method proposed in this paper. Single-phase faults are with the resistance of 50 Ω. Figure 13 allows to understand why Nunes's method presents a low performance: it depends on the fault resistance and the location of the fault. This figure also shows that the performance of the proposed method is better than that of Bedoya-Cadena's method. The above is expected because the method proposed by Bedoya-Cadena is formulated using symmetrical components, and the unbalanced system is not considered in the formula derivation.
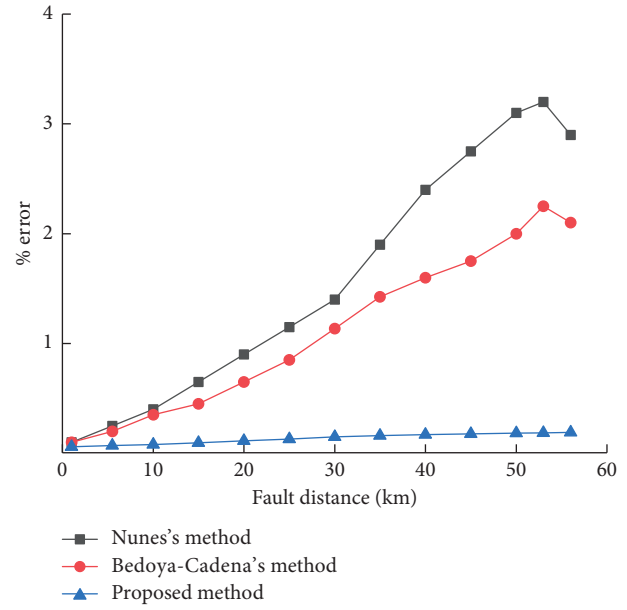
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