


## Research Article

# U-Model Based Adaptive Neural Networks Fixed-Time Backstepping Control for Uncertain Nonlinear System

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Under U-model control design framework, a fixed-time neural networks adaptive backstepping control is proposed. The majority of the previously described adaptive neural controllers were based on uniformly ultimately bounded (UUB) or practical finite stable (PFS) theory. For neural networks control, it makes the control law as well as stability analysis highly lengthy and complicated because of the unknown ideal weight and unknown approximation error. Moreover, there has been very limited research focus on adaptive law for neural networks adaptive control in finite time. Based on fixed-time stability theory, a fixed-time bounded theory is proposed for fixed-time neural networks adaptive backstepping control. The most outstanding novelty is that fixed-time adaptive law for training weights of neural networks is proposed for fixed-time neural networks adaptive control. Furthermore, by combining fixed-time adaptive law and Lyapunov-based arguments, a valid fixed-time controller design algorithm is presented with universal approximation property of neural networks to ensure the system is fixed-time bounded, rather than PFS or UUB. The controller guarantees closed-loop system fixed-time bounded in the Lyapunov sense. The benchmark simulation demonstrated effectiveness and efficiency of the proposed approach.

## 1. Introduction

Recently, neural networks control has increasingly attracted attention and intensive research has been performed in adaptive law for training neural networks weights and application in different fields [1–3]. Neural network technique is a typical data-driven modelling method [4–6], which used measured data to find proper control in reversion of some expected closed-loop performance [7–9]. U-model control [10, 11] played an important role in some complex systems. U-model control, due to its capability to solve some complex problems as model separated design, provides a general way to separate system design process and control design process. U model control method makes control process explicitness and is easy to control. It provides a control direction to design the system controller. U model NNs control makes system control easy and clear based on the approximation ability of NNs.

The majority of the neural networks controllers previously used for nonlinear systems [12] are based on UUB theory and sliding mode schemes [13–15]. The conventional adaptive law for training neural networks and feedback control is linear feedback which makes the system exponential stable [16, 17] or exponentially bounded [18–20]. Finite time [21, 22] and fixed-time [14] stable results are more meaningful for uncertain nonlinear systems.

Motivated by the above critical analyses, fixed-time adaptive neural networks controller for uncertain nonlinear systems is proposed. We extend the prior works [23, 24] to the fixed-time case in which closed-loop systems are global bounded with fixed time. Fixed-time neural networks control is proposed in order to deal with convergence time of the neural networks control. The main contributions of this paper can be summarized as follows:

- (1) Fixed-time adaptive neural networks for uncertain nonlinear systems are proposed. As mentioned, this paper is the first study to propose convergence time as the fixed time for neural networks control.
- (2) For training neural networks weights, a new adaptive law is proposed to realize the fixed-time neural networks adaptive control for training neural network weights based on Lyapunov bounded theory.
- (3) U-model control technology, which is a model-independent design technology, is used to realize the model-independent control system design.

The rest of this paper is organized as follows. Section 2 gives problem formulation and preliminaries, including necessary inequality and some lemmas with necessary proof. In Section 3, a fixed-time bounded theory is proposed for fixed-time neural networks adaptive backstepping control based on U-model control. Based on fixed-time theory, a new fixed-time adaptive law is developed for training neural networks to control the the nonlinear system, and Lyapunov fixed-time bounded theory is used to guaranteeing the closed-loop system signals bounded in fixed time in Section 4. In Section 5, a bench test is proposed to indicate efficiency and effectiveness of the procedure. The conclusion is provided in Section 6.

## 2. Problem Description and Preliminaries

In this paper, a general dynamic system can be described as follows:

$$y^{(n)} + f_1(y)y^{(n-1)} + \dots + f_i(y)y^{(n-i)} + \dots + f_n(y)y = u, \quad (1)$$

where  $y \in R$  and  $u \in R$  are state variable and control input, respectively, and  $f_i(\cdot)$  is nonlinear with system state. This model is generally used in some areas, such as mechanical dynamic of the PMSM servo system.

To design the neural networks control, radial basis function (RBF) NN is adopted in order to approximate the continuous function  $F(x): \mathfrak{R}^n \rightarrow \mathfrak{R}$  over a compact set

$$F_{\text{NN}}(x, W) = W^T \Psi(x), \quad (2)$$

where  $x \in \Omega \subset \mathfrak{R}^n$  is neural networks input,  $W = [w_1, \dots, w_l]^T \in \mathfrak{R}^l$  is weight vector,  $\Psi(x) = [\psi_1(x), \dots, \psi_l(x)]^T$  is node vector, and element  $\psi_i(x)$  is Gaussian function in form of

$$\psi_i(x) = \exp\left[\frac{-(x - \mu_i)^T(x - \mu_i)}{\eta_i^2}\right], \quad i = 1, 2, \dots, l, \quad (3)$$

where  $\mu_i = [\mu_{i1}, \dots, \mu_{in}]^T$  is the center of the basis function and  $\eta_i$  is the scalar width of the Gaussian function.

The RBF NNs can be used to approximate any continuous function over a compact set  $\Omega_x \subset \mathfrak{R}^n$  as

$$F(x) = W^{*T} \Psi(x) + \varepsilon(x), \quad (4)$$

where  $\varepsilon(x)$  is the NN approximation error and  $W^*$  is the ideal NN weight which is given as

$$W^* = \arg \min_{W \in \mathfrak{R}^l} \left\{ \sup |F(x) - W^T \Psi(x)| \right\}, \quad (5)$$

where  $\widehat{W}$  is estimated weight and  $\widetilde{W} = \widehat{W} - W^*$ .

To design the fixed-time bounded theory, some lemmas are proposed based on a general nonlinear system:

$$\dot{x} = f(x), \quad (6)$$

where  $x$  is system state.

**Lemma 1** (see [25]). *Suppose that  $V(\cdot): R^n \rightarrow R_+ \cup \{0\}$  is a continuous radically unbounded function and the following two conditions hold:*

- (1)  $V(x) = 0 \iff x = 0$
- (2) Any solution  $x(t)$  of system (6) satisfies

$$\dot{V}(x(t)) \leq -aV^p(x(t)) - bV^q(x(t)), \quad (7)$$

for some  $a, b > 0$ ,  $0 \leq p < 1$ , and  $q > 1$ .

Then, the origin of system (6) can achieve fixed-time stability, and  $T_{\max} = (1/(a(1-p))) + (1/(b(q-1)))$ .

*Remark 1.* In Lemma 1, if  $p = 1 - (1/(2\mu))$  and  $q = 1 + (1/(2\mu))$ , where  $\mu \geq (1/2)$ , then the origin of system (6) can achieve fixed-time stability, and  $T_{\max} = ((\pi\mu)/\sqrt{ab})$ .

**Lemma 2.** For  $x_i \in R$ ,  $i = 1, 2, \dots, n$ ,  $q > 1$ ,  $0 < p < 1$ , then

$$\begin{aligned} \left( \sum_{i=1}^n |x_i| \right)^p &\leq \sum_{i=1}^n |x_i|^p \leq n^{1-p} \left( \sum_{i=1}^n |x_i| \right)^p, \\ n^{1-q} \left( \sum_{i=1}^n |x_i| \right)^q &\leq \sum_{i=1}^n |x_i|^q \leq \left( \sum_{i=1}^n |x_i| \right)^q. \end{aligned} \quad (8)$$

**Lemma 3** (Young's inequality). For any constant  $a, b \in \mathfrak{R}$ , the following inequality holds:

$$ab \leq \frac{1}{p} a^p + \frac{1}{q} b^q, \quad (9)$$

where  $p > 1$ ,  $q > 1$ , and  $(1/p) + (1/q) = 1$ .

## 3. Fixed-Time U-Model Control

In this section, a fixed-time bounded theory is proposed for fixed-time neural networks adaptive control based on U-model control.

**Theorem 1.** *Suppose that  $V(\cdot): R^n \rightarrow R_+ \cup \{0\}$  is a continuous radically unbounded function and the following two conditions hold:*

- (1)  $V(x) = 0 \iff x = 0$
- (2) Any solution  $x(t)$  of system (6) satisfies

$$\dot{V}(x(t)) \leq -aV^p(x(t)) - bV^q(x(t)) + c, \quad (10)$$

for some  $a, b > 0$ ,  $0 \leq p < 1$ ,  $q > 1$ , and  $p, q$  are odd rational number, which means numerator and denominator are both odd numbers.

Then states of system (6) can achieve fixed-time bounded, and the bound  $\xi$  is roots of the equation.

$$2^{p-1}a\xi^p + b\xi^q = c, \text{ fixed-time } T_{\max} = (1/2^{p-1}a(1-p)) + (1/b(q-1)).$$

*Proof.* Assume that  $V > \xi$ ; based on Lemma 2, we have

$$\begin{aligned} \dot{V} &\leq -aV^p - bV^q + c, \\ 2^{p-1}a\xi^p + b\xi^q &= c, \end{aligned} \quad (11)$$

and

$$-aV^p - bV^q + c = -aV^p - bV^q + 2^{p-1}a\xi^p + b\xi^q, \quad (12)$$

because

$$\begin{aligned} -aV^p + 2^{p-1}a\xi^p &\leq -2^{p-1}a(V - \xi)^p, \\ -bV^q + b\xi^q &\leq -b(V - \xi)^q. \end{aligned} \quad (13)$$

Therefore,

$$\dot{V} \leq -2^{p-1}a(V - \xi)^p - b(V - \xi)^q. \quad (14)$$

Let

$$V_\xi = V - \xi. \quad (15)$$

Then,

$$\dot{V}_\xi \leq -2^{p-1}aV_\xi^p - bV_\xi^q. \quad (16)$$

Based on Lemma 1,  $V_\xi$  is fixed-time stable and fixed time; therefore,  $V$  is fixed-time bounded with  $\xi$  and  $T_{\max} = (1/(2^{p-1}a(1-p))) + (1/(b(q-1)))$ , and if  $p = 1 - (1/(2\mu))$  and  $q = 1 + (1/(2\mu))$ , where  $\mu \geq (1/2)$ ,  $T_{\max} = ((\pi\mu)/\sqrt{2^{p-1}ab})$ .

The proof is completed.

For system (1), based on U-model technology, let

$$\begin{cases} x_1 = y - y_d, \\ x_i = y^{(i-1)} - y_d^{(i-1)}, \quad 2 \leq i \leq n. \end{cases} \quad (17)$$

Then, the system can be changed as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_i = x_{i+1}, \quad 2 \leq i \leq n-1, \\ \dot{x}_n = -f_1(x_1)x_n - \dots - f_i(x_1)x_{n-i+1} - f_n(x_1)x_1 + u, \\ y = x_1, \end{cases} \quad (18)$$

and then the system can be changed as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_i = x_{i+1}, \quad 2 \leq i \leq n-1, \\ \dot{x}_n = U, \\ y = x_1, \end{cases} \quad (19)$$

where

$$U = -f_1(x_1)x_n - \dots - f_i(x_1)x_{n-i+1} - f_n(x_1)x_1 + u, \quad (20)$$

which is a single-input single-output nonlinear system; U-model is used to design the control procedure.

In the first step,

$$z_1 = x_1, \quad (21)$$

and then we have

$$\dot{z}_1 = x_2, \quad (22)$$

and to design fixed-time control, choose the virtual control law

$$\alpha_1 = -a_1z_1^p - b_1z_1^q, \quad (23)$$

where for some  $a_1, b_1 > 0$ ,  $0 \leq p < 1$ ,  $q > 1$ , and  $p, q$  are odd rational numbers, which means numerator and denominator are both odd numbers; then,

$$\dot{z}_1 = -a_1z_1^p - b_1z_1^q + z_2, \quad (24)$$

where

$$z_2 = x_2 - \alpha_1. \quad (25)$$

Therefore, in the  $i$ th step ( $2 \leq i \leq n-1$ ),

$$\dot{z}_i = x_{i+1} - \dot{\alpha}_{i-1}, \quad (26)$$

and to design fixed-time control, choose the virtual control law

$$\alpha_i = -z_{i-1} - a_iz_i^p - b_iz_i^q + \dot{\alpha}_{i-1}, \quad (27)$$

where for some  $a_i, b_i > 0$ ,  $0 \leq p < 1$ ,  $q > 1$ , and  $p, q$  are odd rational numbers, which means numerator and denominator are both odd numbers; then,

$$\dot{z}_i = -z_{i-1} - a_iz_i^p - b_iz_i^q + z_{i+1}, \quad (28)$$

where

$$z_{i+1} = x_{i+1} - \alpha_i. \quad (29)$$

In the last stage, because

$$z_n = x_n - \alpha_{n-1}, \quad (30)$$

we have

$$\dot{z}_n = U - \dot{\alpha}_{n-1}, \quad (31)$$

and to design fixed-time control, choose the U-model control

$$U = -z_{n-1} - a_nz_n^p - b_nz_n^q + \dot{\alpha}_{n-1}, \quad (32)$$

where for some  $a_n, b_n > 0$ ,  $0 \leq p < 1$ ,  $q > 1$ , and  $p, q$  are odd rational numbers, which means numerator and denominator are both odd numbers; then,

$$\dot{z}_n = -z_{n-1} - a_n z_n^p - b_n z_n^q. \quad (33)$$

Under U-model control design framework and fixed-time theory, choose Lyapunov candidate functional

$$V = \frac{1}{2} \sum_{i=1}^n z_i^2, \quad (34)$$

and take time derivative of function (34) along with (24) and (28); (33) is derived as

$$\begin{aligned} \dot{V} &= -\sum_{i=1}^n a_i z_i^{p+1} - \sum_{i=1}^n b_i z_i^{q+1} \\ &\leq -\underline{a} \sum_{i=1}^n z_i^{p+1} - \underline{b} \sum_{i=1}^n z_i^{q+1}, \end{aligned} \quad (35)$$

where  $\underline{a} = \min(a_i)$ ,  $\underline{b} = \min(b_i)$ ,  $i = 1, 2, \dots, n$ ; based on Lemma 2,

$$\begin{aligned} -\sum_{i=1}^n (z_i^2)^{(1+p)/2} &\leq -\left(\sum_{i=1}^n z_i^2\right)^{(1+p)/2} \\ -\sum_{i=1}^n (z_i^2)^{(1+q)/2} &\leq -n^{(1-q)/2} \left(\sum_{i=1}^n z_i^2\right)^{(1+q)/2}. \end{aligned} \quad (36)$$

Therefore,

$$\dot{V} \leq -2^{p_1} \underline{a} V^{p_1} - 2^{q_1} n^{1-q_1} \underline{b} V^{q_1}, \quad (37)$$

where  $p_1 = ((1+p)/2)$ ,  $q_1 = ((1+q)/2)$ , and  $0 \leq p_1 < 1$ ,  $q_1 > 1$ .  $\square$

#### 4. Neural Networks Fixed-Time Control

In the last step of backstepping in equation (32), neural networks are used to approximate the nonlinear system

$$\dot{z}_n = -f_1(x_1)x_n - \dots - f_i(x_1)x_{n-i+1} - f_n(x_1)x_1 - \dot{\alpha}_{n-1} + u, \quad (38)$$

$$u^* = -z_{n-1} + f_1(x_1)x_n + \dots + f_i(x_1)x_{n-i+1} + f_n(x_1)x_1 + \dot{\alpha}_{n-1}, \quad (39)$$

$$-f_1(x_1)x_n - \dots - f_i(x_1)x_{n-i+1} - f_n(x_1)x_1 - \dot{\alpha}_{n-1} = W^{*T} \Psi(\bar{x}_n) + \varepsilon. \quad (40)$$

Choose adaptive law

$$\dot{\hat{W}} = \Gamma [z_n \Psi_n(\bar{x}_n) - a_\sigma \hat{W}^p - b_\sigma \hat{W}^q], \quad (41)$$

where  $\Gamma = \Gamma^T > 0$ , and  $a_\sigma > 0$ ,  $b_\sigma > 0$  are positive constant design parameters.

Finally, choose the controller as

$$u = -\hat{W}^T \Psi(\bar{x}_n) - z_{n-1} - \left(a_n + \frac{\zeta}{p+1}\right) z_n^p - \left(b_n + \frac{1-\zeta}{q+1}\right) z_n^q, \quad (42)$$

where  $0 \leq \zeta \leq 1$ ; then, the system

$$\dot{z}_n = -\tilde{W}^T \Psi(\bar{x}_n) - z_{n-1} - \left(a_n + \frac{\xi}{p+1}\right) z_n^p - \left(b_n + \frac{1-\xi}{q+1}\right) z_n^q + \varepsilon, \quad (43)$$

where  $\tilde{W} = \hat{W} - W^*$ .

**Theorem 2.** With regard to nonlinear system (1), the model dynamic is approximated by neural networks (40), with fixed-time adaptive law (41), with virtual control (23), (27), with controller (42), then the closed loop signal converge to a compact set with fixed-time

$$T_{\max} = \frac{1}{2^{p_2-1} a (1-p_2)} + \frac{1}{b (q_2-1)}. \quad (44)$$

*Proof.* Consider system (1) and Lemmas 1-3.

In  $i$ th ( $1 \leq i \leq n-1$ ) step, choose Lyapunov candidate functional

$$V_i = \frac{1}{2} z_i^2. \quad (45)$$

In the last step, choose Lyapunov candidate functional

$$V_n = \frac{1}{2} z_n^2 + \frac{1}{2} \tilde{W} \Gamma^{-1} \tilde{W}^T, \quad (46)$$

and then take time derivative of function (46) along with trajectory (41), and (43) is derived as

$$\begin{aligned} \dot{V}_n &= z_n \dot{z}_n + \tilde{W} \Gamma^{-1} \dot{\hat{W}}^T \\ &= -z_n \tilde{W}^T \Psi(\bar{x}_n) - z_{n-1} z_n - \left(a_n + \frac{\xi}{p+1}\right) z_n^p \\ &\quad - \left(b_n + \frac{1-\xi}{q+1}\right) z_n^q + z_n \varepsilon \\ &\quad + z_n \tilde{W} \Psi_n(\bar{x}_n) - a_\sigma \tilde{W} \hat{W}^p - b_\sigma \tilde{W} \hat{W}^q. \end{aligned} \quad (47)$$

Based on Lemma 3,

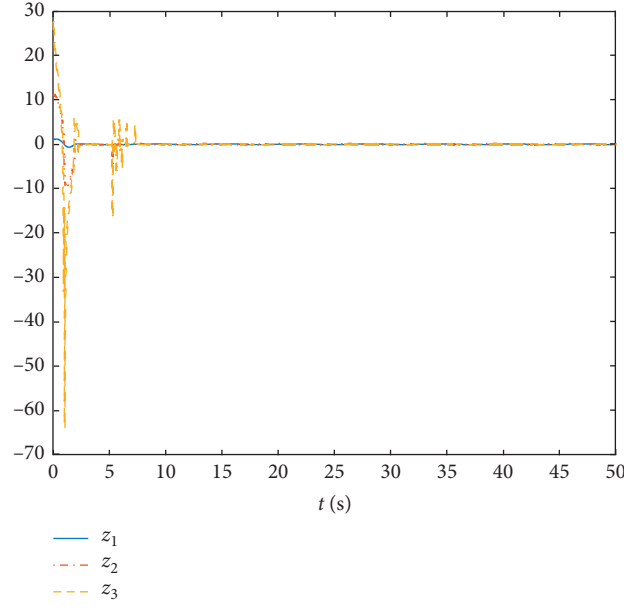


FIGURE 1: Trajectories of error system.

$$\begin{aligned}
 z_n \varepsilon &\leq \zeta \left( \frac{1}{p+1} z_n^{p+1} + \frac{p}{p+1} \varepsilon^{(p+1)/p} \right) + (1-\zeta) \left( \frac{1}{q+1} z_n^{q+1} + \frac{q}{q+1} \varepsilon^{(q+1)/q} \right) \\
 &- a_\sigma \tilde{W} \hat{W}^p \leq -c_p \tilde{W}^{(1+p)/2} \Gamma^{-1} \tilde{W}^{((1+p)/2)T} + a_p W^{*((1+p)/2)} W^{*((1+p)/2)T} \\
 &- b_\sigma \tilde{W} \hat{W}^q \leq -c_q \tilde{W}^{(1+q)/2} \Gamma^{-1} \tilde{W}^{((1+q)/2)T} + b_q W^{*((1+q)/2)} W^{*((1+q)/2)T},
 \end{aligned} \tag{48}$$

where  $0 \leq \zeta \leq 1$ ,  $c_p > 0$ ,  $c_q > 0$ ,  $a_p > 0$ ,  $b_q > 0$  exist.

$$\begin{aligned}
 \dot{V}_n &\leq -z_{n-1} z_n - \left( a_n + \frac{\zeta}{p+1} \right) z_n^p - \left( b_n + \frac{1-\zeta}{q+1} \right) z_n^q \\
 &+ \zeta \left( \frac{1}{p+1} z_n^{p+1} + \frac{p}{p+1} \varepsilon^{(p+1)/p} \right) + (1-\zeta) \\
 &\cdot \left( \frac{1}{q+1} z_n^{q+1} + \frac{q}{q+1} \varepsilon^{(q+1)/q} \right) \\
 &- c_p \tilde{W}^{(1+p)/2} \Gamma^{-1} \tilde{W}^{((1+p)/2)T} + a_p W^{*((1+p)/2)} W^{*((1+p)/2)T} \\
 &- c_q \tilde{W}^{(1+q)/2} \Gamma^{-1} \tilde{W}^{((1+q)/2)T} + b_q W^{*((1+q)/2)} W^{*((1+q)/2)T}.
 \end{aligned} \tag{49}$$

Therefore,

$$\begin{aligned}
 V &= \sum_{i=1}^n V_i, \\
 \dot{V} &\leq -\sum_{i=1}^n a_i z_i^{p+1} - \sum_{i=1}^n b_i z_i^{q+1} + \frac{p\zeta}{p+1} \varepsilon^{(p+1)/p} + \frac{q(1-\zeta)}{q+1} \varepsilon^{(q+1)/q} \\
 &- c_p \tilde{W}^{(1+p)/2} \Gamma^{-1} \tilde{W}^{((1+p)/2)T} + a_p W^{*((1+p)/2)} W^{*((1+p)/2)T} \\
 &- c_q \tilde{W}^{(1+q)/2} \Gamma^{-1} \tilde{W}^{((1+q)/2)T} + b_q W^{*((1+q)/2)} W^{*((1+q)/2)T} \\
 &\leq -aV^{p_2} - bV^{q_2} + c,
 \end{aligned} \tag{50}$$

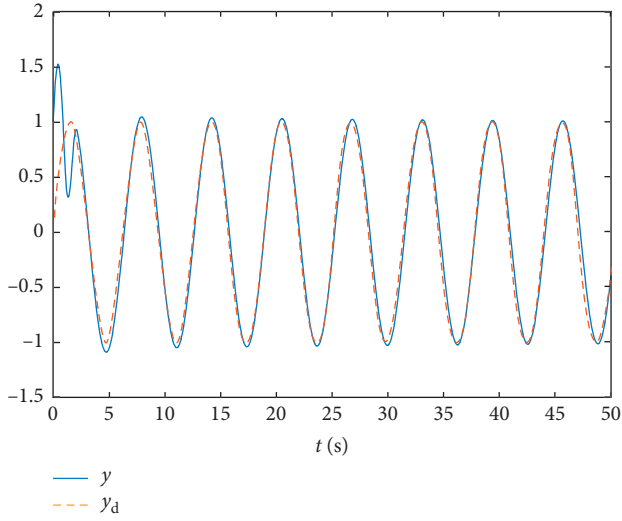
where

$$\begin{aligned}
 a &= 2^{(p+1)/2} \min(c_p, a_i, \quad i = 1, 2, \dots, n), \\
 b &= 2^{(q+1)/2} (n+1)^{(1-q)/2} \min(c_q, b_i, \quad i = 1, 2, \dots, n), \\
 c &= \frac{p\zeta}{p+1} \varepsilon^{\frac{p+1}{p}} + \frac{q(1-\zeta)}{q+1} \varepsilon^{(q+1)/q} + a_p W^{*((1+p)/2)} W^{*((1+p)/2)T} \\
 &\quad + b_q W^{*((1+q)/2)} W^{*((1+q)/2)T}, \\
 p_2 &= \frac{1+p}{2}, \\
 q_2 &= \frac{1+q}{2},
 \end{aligned} \tag{51}$$

and based on Lemma 3,  $V$  is bounded with fixed time. Therefore, it can be concluded that for all  $1 \leq i \leq n$ , the error signals  $z_i$ ,  $\tilde{W}_i$  are bounded with fixed time  $T_{\max} = (1/(2^{p_2-1}a(1-p_2))) + (1/(b(q_2-1)))$ . If  $p = 1 - (1/(2\mu))$  and  $q = 1 + (1/(2\mu))$ , where  $\mu \geq (1/2)$ , fixed-time  $T_{\max} = ((\pi\mu)/\sqrt{2^{p-1}ab})$ .

The proof is completed.  $\square$

*Remark 2.* For the virtual control in equation (26), to avoid singularity problem, we assume that  $|z_i| > \varepsilon$ , otherwise  $\dot{\alpha}_{i-1} = 0$ . Because this is bounded theory, the motivation is control  $|z_i| < \varepsilon$ .

FIGURE 2: Trajectories of  $y, y_d$ .

## 5. Simulation Example

A simulation has been performed for the nonlinear system in order to show the effectiveness and efficiency of the proposed approach.

$$y^{(3)} + (1 + y)y^{(2)} + (2 - y^2)y^{(1)} + y = u. \quad (52)$$

Based on U-model and neural networks technology, design the controller; the initial state is  $y(0) = 1$  and the reference output is  $y_d = \sin(t)$ ; then, based on U-model technology,

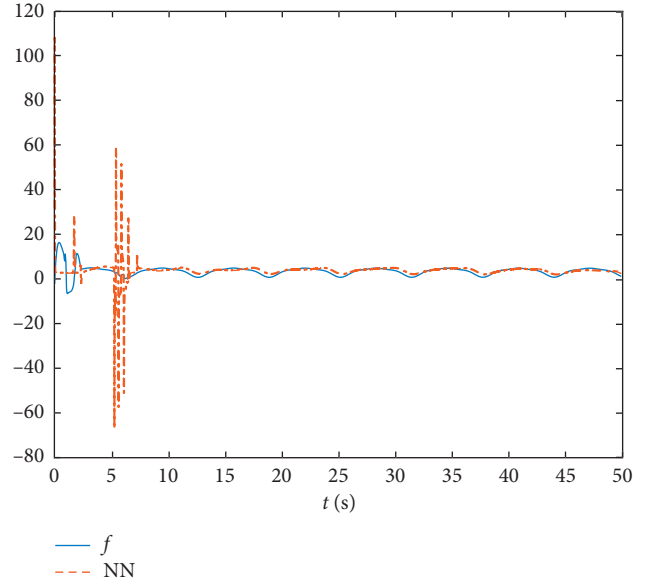
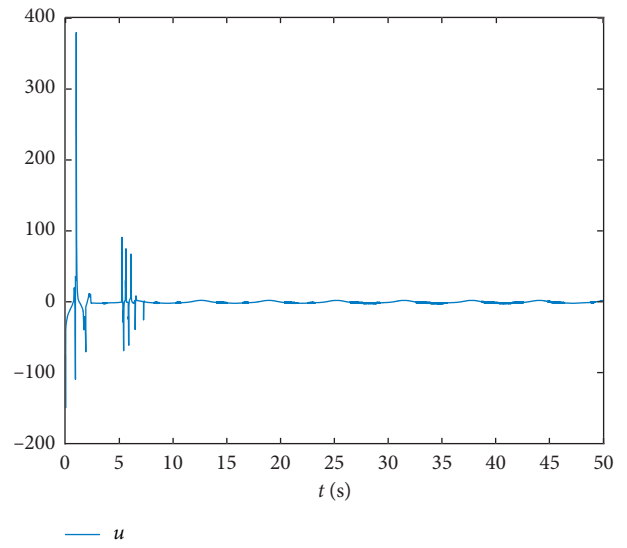
$$\begin{cases} x_1 = y - y_d, \\ x_2 = y^{(1)} - y_d^{(1)}, \\ x_3 = y^{(2)} - y_d^{(2)}, \end{cases} \quad (53)$$

and the system can be changed as

$$\begin{cases} \dot{x}_1 = x_2, \\ \dot{x}_2 = x_3, \\ \dot{x}_3 = f + u, \\ y = x_1, \end{cases} \quad (54)$$

where  $f = -(1 + x_1 + y_d)(x_3 + y_d^{(2)}) - (2 - (x_1 + y_d)^2)(x_2 + y_d^{(1)}) - x_1 - y_d$ .

The initial conditions of NN weights are chosen as zero and  $p = (1/3), q = 3$ . The motivation is to design the adaptive finite time neural tracking controller for a system such that all the system outputs follow the given reference signal  $y_d$  with finite time. To illustrate the ability of controller, Figures 1–4 show the better tracking performance. Figure 1 shows the states of error system convergence to origin point in finite time. Figure 2 shows the system output  $y$  and system reference output  $y_d$  and output tracked reference output quickly. Figure 3 shows the approximation of NNs, and Figure 4 shows the controller.

FIGURE 3: Trajectories of  $f, NN$ .FIGURE 4: Trajectory of control input  $u$ .

## 6. Conclusion

A fixed-time neural networks adaptive backstepping control is proposed under U-model control design framework. The proposed controller guarantees closed-loop system fixed-time bounded and not only uniformly ultimately bounded UUB or PFS. The benchmark simulation has well demonstrated effectiveness and efficiency of the proposed approach.

## Data Availability

No data were used to support this study.



## Conflicts of Interest

The authors declare that they have no conflicts of interest.

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