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Research Article

Rough-Number-Based Multiple-Criteria Group Decision-Making Method by Combining the BWM and Prospect Theory

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Multicriteria group decision-making (MCGDM) problems have been a research hotspot in recent years, and prospect theory is introduced to cope with the risk and imprecision in the process of decision-making. To guarantee the effectiveness of information aggregation and extend the feasibility of prospect theory, this paper proposes a novel decision-making approach based on rough numbers and prospect theory to solve risky and uncertain MCGDM problems. Firstly by combining rough numbers and the best-worst method (BWM), we construct a linear programming model to calculate rough criteria weights, which are defined by lower limitations and upper limitations. Then for the imprecision of value function and weighting function in prospect theory, we propose a novel method with the aid of combining rough numbers and prospect theory to handle the risk in decision-making problems. Finally, a numerical example involving investment is introduced to illustrate the application and validity of the proposed method.

1. Introduction

A multiple-criteria decision-making (MCDM) problem, as one of the most significant problems in the fields of management, economics, and engineering, is the process of selecting the optimal option in all possible alternatives according to diverse criteria. Since the complexity and variety of decision-making environments determine that opinions of different decision-makers should be taken into account, multiple-criteria group decision-making (MCGDM) becomes research hotspot and is widely used in practical problems [1], such as supplier selection [2, 3], emergency management [4, 5], and product development [6, 7]. Methods for solving MCDM problems, e.g., AHP [8], TOPSIS [9], VIKOR [10], and PROMETHEE [11], have been improved and applied to group decision-making problems.

The increase of imprecision and complexity in real-world problems leads to the fact that decision-makers might be unable to express personal preferences with numerical values, so some theories dealing with imprecision are introduced into MCGDM problems, especially the theory of fuzzy sets developed by Zadeh [12]. Classical methods have been extended to solve uncertain MCGDM problems based on fuzzy sets as well as their generations, such as fuzzy TOPSIS [13], triangular fuzzy AHP [14], intuitionistic fuzzy VIKOR [15], fuzzy prospect theory [16], intuitionistic fuzzy ELECTRE [17], and Pythagorean fuzzy PROMETHEE [18]. There are two defects which make it difficult to overcome these fuzzy methods: one is that the process of group information aggregation such as weighted average is mechanized, leading to the neglect of interaction among decision-makers, and another one is that the inherent subjectivity of membership function can easily result in the decision-making bias. Aiming at these defects, the rough-number method is proposed [19], which is based on the basic notion of approximates in rough set theory, an effective method to handle imprecision information developed by Pawlak [20]. A rough number can characterize imprecise information by means of rough boundary intervals bounded by the upper and lower limits, which can be directly computed from the raw data without any subjective adjustments, assumptions, or membership functions. Therefore, many researches combined traditional decision-making methods with rough numbers, resulting in extended models like rough AHP [7], rough VIKOR [21], rough DEMATEL [22], and rough MABAC [23].

As a classical MCDM method, the analytic hierarchy process (AHP) has been used widely for calculating the weights of criteria [7, 8, 14]. The AHP requires to compare the relative importance of each two criteria and obtain a comparison matrix, but due to the complexity of comparison procedures of the AHP, as well as the limitation of human cognition, the results obtained by the AHP always lack consistency in the pairwise comparison matrix; therefore, to improve the traditional AHP, Rezaei introduced a novel pairwise comparison idea and proposed the best-worst method (BWM) [24]. In the process of the BWM, decision-makers only need to compare each criterion with the best criterion and the worst criterion, rather than the comparisons between all the criteria. Therefore, the BWM yields two comparison vectors, and then the weights of criteria can be obtained by solving a mathematical programming model [24]. The BWM has been used widely in many areas, such as water scarcity management [25], supplier evaluation and selection [26, 27], quality assessment of scientific output [28], and sustainable architecture [29]. Some researchers have combined rough numbers with the BWM to handle the MCDM problems: Željko et al. proposed a rough BWM-SAW model to select wagons for the internal transport [30], a rough BWM-WASPAS model to determine the location selection for roundabout construction [31], and then a rough BWM-SERVQUAL model for quality assessment of scientific conferences [32]; Pamučar et al. integrated rough numbers and fuzzy sets, proposed interval-valued fuzzy-rough numbers (IVFRNs) to aggregate fuzzy evaluating values of the decision group, and presented an IVFRN-based BWM to obtain the weights of criteria [33]; and then Pamučar et al. proposed a BWM-WASPAS-MABAC model based on interval rough numbers to evaluate the third-party logistics provider [34]. All the models based on the rough BWM are based on the original BWM [24], which is nonlinear and may not obtain the unique solution of a mathematical programming model, resulting in a decision failure. Then, Rezaei modified the model and proposed a linear BWM, which is based on the same philosophy as the original model but yields a unique solution [35]. Therefore, in this paper, we intend to construct a linear rough BWM to obtain the weights of criteria.

The study for risk attitudes of decision-makers is another crucial aspect of decision-making problems, and many researchers introduced prospect theory to MCDM models. Prospect theory developed by Kahneman and Tversky [36] is a descriptive model of individual decision-making under condition of risk. Later, Tversky and Kahneman [37] developed the cumulative prospect theory, which captures psychological aspects of decision-making under risk. In the prospect theory, the outcomes are expressed by means of gains and losses from a reference alternative. The value function in prospect theory assumes an S-shape concave above the reference alternative, which reflects the aversion of risk in face of gains, and the convex part below the reference alternative reflects the propensity to risk in case of losses. Prospect theory has been an arisen behavioral model of decision-making under risk, and in order for the application in an uncertain environment, some research works have begun to explore the combination of prospect theory and imprecise information, such as prospect theory under the fuzzy environment [16], linguistic environment [38], interval type 2 fuzzy environment [39], and rough environment [40]. Unfortunately, the process is still at the primary stage: the imprecise information involved only includes fuzzy numbers and interval numbers, and although Fang et al. referred to the rough environment in [40], they did not explore the combination of rough numbers and prospect theory; proposed methods only concentrate on the imprecision of value function in prospect theory, while they ignore the imprecision of weighting function, and almost all the combined methods pay no attention to group decision-making problems. So it is essential to extend prospect theory to imprecise MCGDM problems.

In this paper, we introduce rough numbers to MCGDM models and combine the linear BWM and prospect theory to handle the risk and uncertain MCGDM problems. The rest of this paper is arranged as follows: In Section 2, we shortly describe some knowledge on methods and theories involved in this paper. In Section 3, we propose the rough-number-based MCGDM method based on the BWM and prospect theory, including processes of criteria weighting and alternative ranking. In Section 4, we present a practical example to illustrate the application and verify the feasibility and validity of this new method. In Section 5, some conclusions and directions for the future work are proposed.

2. Preliminaries

This section is composed of three subsections to review some preliminaries about the rough number, best-worst method, and prospect theory.

2.1. Rough Number. Inspired by rough set theory, rough number is first proposed by Zhai et al. [41] in order to handle subjective preferences of customers in quality function deployment. Similar to the notion of approximates in rough sets, a rough number is constructed by lower and upper limits, which determine a rough boundary interval to characterize imprecise information. While the rough number merely depends on original data without any prior knowledge, it can capture the experts' real perception effectively and aggregate every individual's preference into an objective and consistent group judgement. In this section, we review some basic definitions of rough number.

Definition 1 (see [41]). Suppose U is the universe containing all the objects and there are n classes expressed as $R = \{C_1, C_2, \ldots, C_n\}$. If they are ordered as $C_1 < C_2 < \cdots < C_n$, then for $\forall Y \in U, \forall C_i \in R$, the lower approximation ($\underline{\mathrm{Apr}}(C_i)$), upper approximation ($\overline{\mathrm{Apr}}(C_i)$), and boundary region ($\mathrm{Bnd}(C_i)$) of C_i can be defined as

$$\underline{\mathrm{Apr}}(C_i) = \bigcup \left\{ Y \in \frac{U}{R(Y) \le C_i} \right\}, \tag{1}$$

$$\overline{\mathrm{Apr}}(C_i) = \bigcup \left\{ Y \in \frac{U}{R(Y) \ge C_i} \right\}, \tag{2}$$

$$\begin{aligned}\mathrm{Bnd}(C_i) &= \bigcup \left\{ Y \in \frac{U}{R(Y) \ne C_i} \right\} \\ &= \left\{ Y \in \frac{U}{R(Y) > C_i} \right\} \bigcup \left\{ Y \in \frac{U}{R(Y) < C_i} \right\}.\end{aligned} \tag{3}$$

Definition 2 (see [41]). C_i can be expressed by a rough number $\mathrm{RN}(C_i)$, which is determined by its lower limit $\underline{\mathrm{RN}}(C_i)$ and upper limit $\overline{\mathrm{RN}}(C_i)$, which are expressed as

$$\underline{\mathrm{RN}}(C_i) = \frac{1}{M_{\mathrm{L}}} \sum R(Y) \mid Y \in \underline{\mathrm{Apr}}(C_i), \tag{4}$$

$$\overline{\mathrm{RN}}(C_i) = \frac{1}{M_{\mathrm{U}}} \sum R(Y) \mid Y \in \overline{\mathrm{Apr}}(C_i), \tag{5}$$

$$\mathrm{RBnd}(C_i) = \overline{\mathrm{RN}}(C_i) - \underline{\mathrm{RN}}(C_i), \tag{6}$$

$$\mathrm{RN}(C_i) = [C_i] = [\underline{\mathrm{RN}}(C_i), \overline{\mathrm{RN}}(C_i)], \tag{7}$$

where M_{L} and M_{U} are the number of objects contained in $\underline{\mathrm{Apr}}(C_i)$ and $\overline{\mathrm{Apr}}(C_i)$, respectively.

For a convenient expression, $\underline{\mathrm{RN}}(C_i)$, $\overline{\mathrm{RN}}(C_i)$, and $\mathrm{RN}(C_i)$ can be denoted as $\underline{C_i}$, $\overline{C_i}$, and $[C_i]$ for short, respectively.

According to the definition, the rough number is similar to the interval number in form, so with the aid of arithmetic operations of interval analysis [42], Zhai et al. proposed the operations of rough numbers [43].

Definition 3 (see [43]). Suppose $[a] = [\underline{a}, \overline{a}]$ and $[b] = [\underline{b}, \overline{b}]$ are two rough numbers and α is a real number, then the arithmetic operations of rough numbers can be expressed as

$$[a] \times \alpha = \alpha \times [a] = \begin{cases} [\alpha \times \underline{a}, \alpha \times \overline{a}], & \text{for } \alpha \ge 0, \\ [\alpha \times \overline{a}, \alpha \times \underline{a}], & \text{for } \alpha < 0, \end{cases} \tag{8}$$

$$[a] + [b] = \left[\underline{a} + \underline{b}, \overline{a} + \overline{b}\right], \tag{9}$$

$$[a] - [b] = \left[\underline{a} - \overline{b}, \overline{a} - \underline{b}\right], \tag{10}$$

$$\begin{aligned}[a] \times [b] = \big[&\min\left(\underline{a}\,\underline{b}, \underline{a}\,\overline{b}, \overline{a}\,\underline{b}, \overline{a}\overline{b}\right), \\ &\max\left(\underline{a}\,\underline{b}, \underline{a}\,\overline{b}, \overline{a}\,\underline{b}, \overline{a}\overline{b}\right)\big],\end{aligned} \tag{11}$$

$$\frac{[a]}{[b]} = [\underline{a}, \overline{a}] \times \left[\frac{1}{\overline{b}}, \frac{1}{\underline{b}}\right], \quad 0 \notin \left[\underline{b}, \overline{b}\right]. \tag{12}$$

Specifically, if $[a] > 0$ and $[b] > 0$, which means $\underline{a}$, $\overline{a}$, $\underline{b}$, and $\overline{b}$ are all greater than 0, then the multiplication operation (11) can be simplified as $[a] \times [b] = [\underline{a}\,\underline{b}, \overline{a}\overline{b}]$ and the division operation (12) can be simplified as $([a]/[b]) = [(\underline{a}/\overline{b}), (\overline{a}/\underline{b})]$.

To compare the values of different rough numbers, Zhai et al. proposed the ranking rules [41]. For any two rough numbers $\mathrm{RN}_1 = [\underline{\mathrm{RN}_1}, \overline{\mathrm{RN}_1}]$ and $\mathrm{RN}_2 = [\underline{\mathrm{RN}_2}, \overline{\mathrm{RN}_2}]$, there are five possible cases, which are shown in Figure 1. Denote M_1 and M_2 as the medians of RN_1 and RN_2, and the ranking rules can be easily explained as follows [41]:

(a) When $M_1 = M_2$,

  (i) If $\underline{\mathrm{RN}_1} = \underline{\mathrm{RN}_2}$ and $\overline{\mathrm{RN}_1} = \overline{\mathrm{RN}_2}$, then $\mathrm{RN}_1 = \mathrm{RN}_2$ (see Figure 1(a))

  (ii) If $\underline{\mathrm{RN}_1} < \underline{\mathrm{RN}_2}$ and $\overline{\mathrm{RN}_1} > \overline{\mathrm{RN}_2}$, then $\mathrm{RN}_1 > \mathrm{RN}_2$(see Figure 1(b))

(b) When $M_1 \ne M_2$,

  (i) If $M_1 < M_2$, then $\mathrm{RN}_1 < \mathrm{RN}_2$ (see Figure 1(c))

  (ii) If $M_1 > M_2$, then $\mathrm{RN}_1 > \mathrm{RN}_2$(see Figure 1(d) and (e))

2.2. Best-Worst Method. Pairwise comparison method, like the AHP, has been used widely in MCDM problems [44]. It shows the relative preferences between each two criteria, constructs a preference matrix, and provides a way to find the weights of criteria. As the complexity of comparison procedures and the limitation of human cognition, the pairwise comparison method faces an inevitable defect in practice, which is the lack of consistency of the pairwise comparison matrixes. Rezaei proposed a vector-based method called the best-worst method (BWM), deriving the weights of criteria based on pairwise comparisons in a different way. The steps of the BWM are described as follows [24]:

Step 1: the best (e.g., most important) and worst (e.g., least important) criteria are chosen among the criteria set $\{c_1, c_2, \ldots, c_n\}$, denoted as c_B and c_W, respectively.

Step 2: the preference of the best criterion over all the other criteria is determined using a number between 1 and 9, and the best-to-others vector can be expressed as $A_B = (a_{B1}, a_{B2}, \ldots, a_{Bn})$, where a_{Bi} indicates the preference of the best criterion c_B over the criterion c_j.

Step 3: the preference of all the other criteria over the worst criterion is determined using a number between 1 and 9, and the others-to-worst vector can be expressed as $A_W = (a_{1W}, a_{2W}, \ldots, a_{nW})^T$, where a_{iW} indicates the preference of the criterion c_j over the best criterion c_B.

Step 4: a nonlinear min-max mathematical programming problem is constructed as model (13), which can be transferred to the model (14), and the optimal

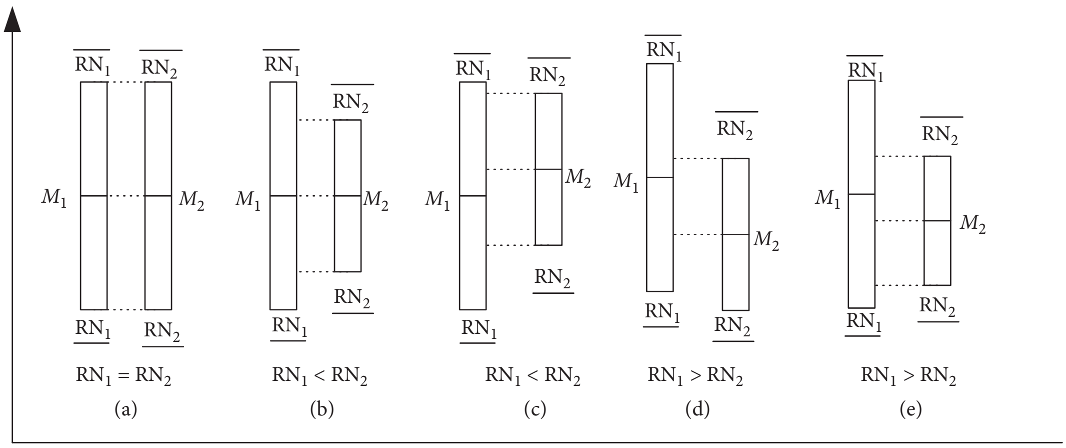

FIGURE 1: Ranking rules for rough numbers.

weights $(w_1^*, w_2^*, \ldots, w_n^*)$ and consistency index ξ are found by solving the model:

$$\begin{aligned} \min\text{-}\max_j \quad & \left\{ \left| \frac{w_B}{w_j} - a_{Bj} \right|, \left| \frac{w_j}{w_W} - a_{jW} \right| \right\}, \\ \text{s.t.} \quad & \sum_j w_j = 1, \\ & w_j \geq 0, \quad \text{for all } j, \end{aligned} \tag{13}$$

$$\begin{aligned} \min \quad & \xi, \\ \text{s.t.} \quad & \left| \frac{w_B}{w_j} - a_{Bj} \right| \leq \xi, \quad \text{for all } j, \\ & \left| \frac{w_j}{w_W} - a_{jW} \right| \leq \xi, \quad \text{for all } j, \\ & \sum_j w_j = 1, \\ & w_j \geq 0, \quad \text{for all } j. \end{aligned} \tag{14}$$

Compared with standard pairwise comparison methods such as the AHP, the BWM uses only integer numbers in describing preferences, reduces the times of comparisons, and most importantly provides more consistent and reliable results [24]. Due to the inconsistency of evaluation vectors and the nonlinearity of programming model (13), there may be multiple optimal solutions in some cases [35]. In order to obtain a unique solution, Rezaei improved the mathematical programming by transforming the objective function to the set $\{|w_B - a_{Bj}w_j|, |w_j - a_{jW}w_W|\}$. The programming model can be formulated as follows [35]:

$$\begin{aligned} \min\text{-}\max_j \quad & \left\{ \left| w_B - a_{Bj}w_j \right|, \left| w_j - a_{jW}w_W \right| \right\}, \\ \text{s.t.} \quad & \sum_j w_j = 1, \\ & w_j \geq 0, \quad \text{for all } j. \end{aligned} \tag{15}$$

Model (15) is equivalent to the following programming problem:

$$\begin{aligned} \min \quad & \xi^L \\ \text{s.t.} \quad & \left| w_B - a_{Bj}w_j \right| \leq \xi^L, \quad \text{for all } j, \\ & \left| w_j - a_{jW}w_W \right| \leq \xi^L, \quad \text{for all } j, \\ & \sum_j w_j = 1, \\ & w_j \geq 0, \quad \text{for all } j. \end{aligned} \tag{16}$$

Apparently, programming model (16) is a linear problem, so we can obtain a unique solution $(w_1^*, w_2^*, \ldots, w_n^*)$ and the consistency of comparisons ξ^L by solving problem (16).

2.3. Prospect Theory. Prospect theory, which was initially established by Kahneman and Tversky in 1979, can describe the actual decision behavior of decision-makers under risk and uncertainty [36]. Two core concepts of prospect theory are the value function and decision weighting function. Value function, reflecting the relationship between the decision-maker's subjective utility and the expected results, can be expressed as

$$v(x) = \begin{cases} \Delta x^{\alpha}, & \Delta x \geq 0, \\ -\lambda(-\Delta x)^{\beta}, & \Delta x \leq 0, \end{cases} \tag{17}$$

where Δx is the gain or loss of the outcome relative to the reference point: $\Delta x > 0$ for a gain, while $\Delta x < 0$ for a loss; α and β are adjustable coefficients determining the concavity and convexity of the value function, respectively, satisfying $0 < \alpha$ and $\beta < 1$; and λ is a parameter describing loss aversion and $\lambda > 1$. Regarding the adjustable coefficients α and β, the values are larger and the decision-maker is more prone to risk: when $\alpha = \beta = 1$, the decision-maker shows no change of risk preference for the gain and loss, the value function degenerates to utility function, and the utility $v(x)$ is linear to the variable Δx, which is depicted as the solid line in Figure 2; in contrast, when $\alpha < 1$ and $\beta < 1$, the decision-maker is sensitive to the gain and loss and the utility $v(x)$ is

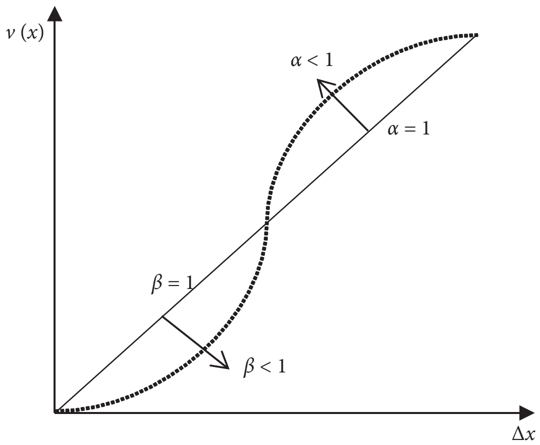

FIGURE 2: Value function of prospect theory with different α and β.

nonlinear to the variable Δx, which can be illustrated as the dashed line in Figure 2.

Tversky and Kahneman considered that decision weights are subjective judgements about the likelihood of occurrence, and they described the form of weighting function as [37]

$$\pi(p) = \begin{cases} \dfrac{p^{\gamma}}{\left(p^{\gamma} + (1-p)^{\gamma}\right)^{1/\gamma}}, & x \geq 0, \\ \\ \dfrac{p^{\delta}}{\left(p^{\delta} + (1-p)^{\delta}\right)^{1/\delta}}, & x \leq 0, \end{cases} \quad (18)$$

where γ and δ are the risk parameters for gains and losses, respectively, satisfying $0 < \gamma$ and $\delta < 1$.

Many empirical researches have explored the value of parameters in equations (17) and (18). According to Tversky and Kahneman [37], results are consistent with empirical data when $\alpha = \beta = 0.88$, $2.0 \leq \lambda \leq 2.5$, $\gamma = 0.61$, and $\delta = 0.72$. Abdellaoui [45] suggested that parameters in equation (17) should be $\alpha = 0.89$, $\beta = 0.92$, and $\lambda = 2.25$, and Gonzalez and Wu [46] considered that $\gamma = \delta = 0.74$ in equation (18).

Alternatives can be ranked by the prospect value resulting from the value function and weighting function, whose form is defined as

$$V = \sum v(\Delta x)\pi(p). \quad (19)$$

3. The Proposed Decision Model

This section proposes a new method for MCDGM problems. Firstly, we describe the MCDGM problem under risk and imprecision and then develop rough-number-based methods for criteria weighting and alternative ranking, respectively.

3.1. Framework of the Proposed Method. Suppose in an MCGDM problem that $A = \{A_1, A_2, \ldots, A_m\}$ is the set of all the alternatives and $C = \{c_1, c_2, \ldots, c_n\}$ is the set of criteria; $w = \{w_1, w_2, \ldots, w_n\}$ represents the weights of criteria, where w_j is the weight of c_j; the decision group is composed by h decision-makers, expressed as $E = \{e_1, e_2, \ldots, e_h\}$; there are l states of events in the problem, which can be represented as $S = \{S_1, S_2, \ldots, S_l\}$, with an occurrence probability p_t^k of the state S_t estimated by the decision-maker e_k; each decision-maker has an expectation for every criterion as the reference point in prospect theory, and the expectation vector of the decision-maker e_k under the state S_t can be denoted as $R_{kt} = \{r_1^{kt}, r_2^{kt}, \ldots, r_n^{kt}\}$. So the score table made by e_k can be expressed as in Table 1.

The framework of the proposed method is depicted in Figure 3.

3.2. Rough BWM for Criteria Weighting. As an advanced and efficient pairwise comparison method, the BWM has a special superiority to handle MCDM problems. For the subjectivity and imprecision in the criteria weighting procedure, this section proposes a new criteria weighting method by combining the rough-number method and BWM. The procedure of the rough BWM is described as follows:

Step 1: the best criterion c_B and the worst criterion c_W are determined by the decision group. Based on common rational cognition of individuals in the decision group, each could reach a consensus in choosing the best and worst criteria. If not, an extra criterion c_0 can be introduced as the best (or worst) criterion, which makes no difference in the results.

Step 2: comparison vectors of each decision-maker are determined. e_k can get the best-to-others vector expressed as $A_B^k = (a_{B1}^k, a_{B2}^k, \ldots, a_{Bn}^k)$ and the others-to-worst vector as $A_W^k = (a_{1W}^k, a_{2W}^k, \ldots, a_{nW}^k)$ using a number between 1 and 9.

Step 3: the integrated comparison vectors are constructed. The integrated best-to-others vector and integrated others-to-worst vector can be expressed as

$$\begin{aligned} A_B &= (a_{B1}, a_{B2}, \ldots, a_{Bn}), \\ A_W &= (a_{1W}, a_{2W}, \ldots, a_{nW}), \end{aligned} \quad (20)$$

where $a_{Bj} = \{a_{Bj}^1, a_{Bj}^2, \ldots, a_{Bj}^s\}$ and $a_{jW} = \{a_{jW}^1, a_{jW}^2, \ldots, a_{jW}^s\}$, in which a_{Bj} denotes the collection of preferences of c_B over c_j made by all the decision-makers and a_{jW} denotes the collection of preferences of c_j over c_W made by all the decision-makers.

Step 4: rough comparison vectors are constructed based on the rough-number method. According to equations (1)–(7), the preferences of each decision-maker e_k can be transformed to a rough number:

$$\begin{aligned} \mathrm{RN}\left(a_{Bj}^k\right) &= \left[\underline{a_{Bj}^k}, \overline{a_{Bj}^k}\right], \\ \mathrm{RN}\left(a_{jW}^k\right) &= \left[\underline{a_{jW}^k}, \overline{a_{jW}^k}\right], \end{aligned} \quad (21)$$

and then the rough sequences are formed as

Table 1: Score table of the decision-maker e_k.

Criteria		c_1				c_2				$\cdots$	c_n			
States		S_1	S_2	$\cdots$	S_l	S_1	S_2	$\cdots$	S_l	$\cdots$	S_1	S_2	$\cdots$	S_l
Alternatives	A_1	x_{11}^{k1}	x_{11}^{k2}	$\cdots$	x_{11}^{kl}	x_{12}^{k1}	x_{12}^{k2}	$\cdots$	x_{12}^{kl}	$\cdots$	x_{1n}^{k1}	x_{1n}^{k2}	$\cdots$	x_{1n}^{kl}
	A_2	x_{21}^{k1}	x_{21}^{k2}	$\cdots$	x_{21}^{kl}	x_{22}^{k1}	x_{22}^{k2}	$\cdots$	x_{22}^{kl}	$\cdots$	x_{2n}^{k1}	x_{2n}^{k2}	$\cdots$	x_{2n}^{kl}
	$\vdots$			$\vdots$				$\vdots$		$\ddots$			$\vdots$	
	A_m	x_{m1}^{k1}	x_{m1}^{k2}	$\cdots$	x_{m1}^{kl}	x_{m1}^{k1}	x_{m2}^{k2}	$\cdots$	x_{m2}^{kl}	$\cdots$	x_{mn}^{k1}	x_{mn}^{k2}	$\cdots$	x_{mn}^{kl}
Expectation		r_1^{k1}	r_1^{k2}	$\cdots$	r_1^{kl}	r_2^{k1}	r_2^{k2}	$\cdots$	r_2^{kl}	$\cdots$	r_n^{k1}	r_n^{k2}	$\cdots$	r_n^{kl}

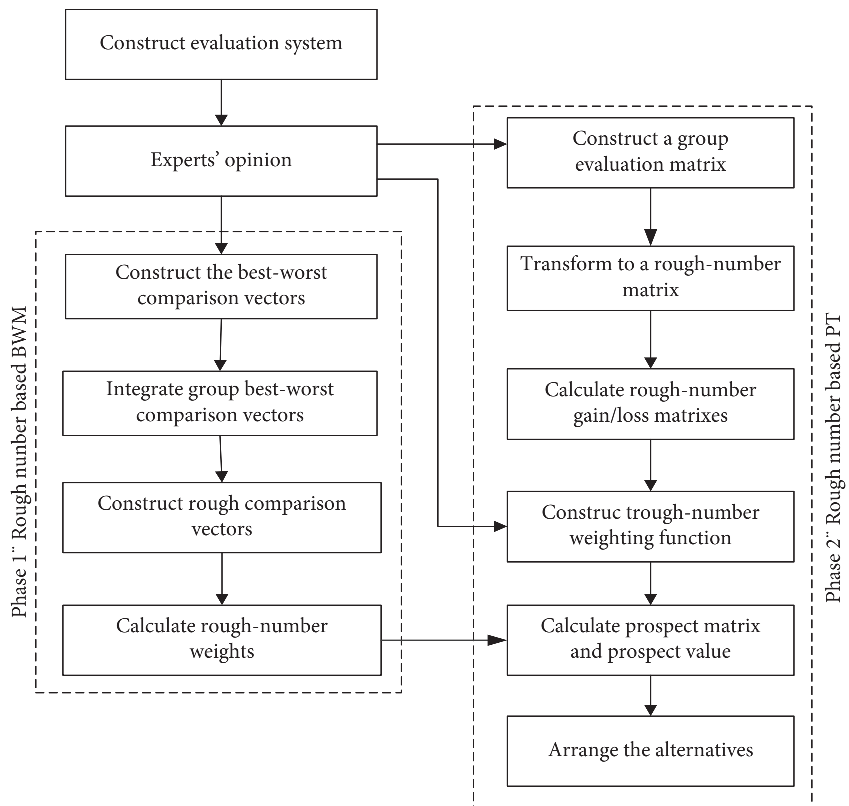

Figure 3: Framework of the proposed method.

$$\mathrm{RN}\left(\tilde{a}_{Bj}\right)=\left\{\left[\underline{a}_{Bj}^{1},\overline{a_{Bj}^{1}}\right],\left[\underline{a}_{Bj}^{2},\overline{a_{Bj}^{2}}\right],\ldots,\left[\underline{a}_{Bj}^{s},\overline{a_{Bj}^{s}}\right]\right\},$$
$$\mathrm{RN}\left(\tilde{a}_{jW}\right)=\left\{\left[\underline{a}_{jW}^{1},\overline{a_{jW}^{1}}\right],\left[\underline{a}_{jW}^{2},\overline{a_{jW}^{2}}\right],\ldots,\left[\underline{a}_{jW}^{s},\overline{a_{jW}^{s}}\right]\right\}. \tag{22}$$

So the integrated comparison vectors can be expressed as

$$\begin{aligned}\mathrm{RA}_B&=\left(\left[\underline{a}_{B1},\overline{a_{B1}}\right],\left[\underline{a}_{B2},\overline{a_{B2}}\right],\ldots,\left[\underline{a}_{Bn},\overline{a_{Bn}}\right]\right),\\ \mathrm{RA}_W&=\left(\left[\underline{a}_{1W},\overline{a_{1W}}\right],\left[\underline{a}_{2W},\overline{a_{2W}}\right],\ldots,\left[\underline{a}_{nW},\overline{a_{nW}}\right]\right),\end{aligned} \tag{23}$$

where $[\underline{a}_{Bj},\overline{a_{Bj}}]=(1/h)\sum_{k=1}^{s}[\underline{a}_{Bj}^{k},\overline{a_{Bj}^{k}}]$ and $[\underline{a}_{jW},\overline{a_{jW}}]=(1/h)\sum_{k=1}^{s}[\underline{a}_{jW}^{k},\overline{a_{jW}^{k}}]$.

Step 5: the rough weight of each criterion is calculated. Similar to the analysis in the BWM, the optimal rough weight for criteria is the one where, for each pair of $[w_B]/[w_j]$ and $[w_j]/[w_W]$, there are $[w_B]/[w_j]=[a_{Bj}]$ and $[w_j]/[w_W]=[a_{jW}]$, which can be rewritten as $[\underline{w_B}/\overline{w_j},\overline{w_B}/\underline{w_j}]=[\underline{a_{Bj}},\overline{a_{Bj}}]$ and $[\underline{w_j}/\overline{w_W},\overline{w_j}/\underline{w_W}]=[\underline{a_{jW}},\overline{a_{jW}}]$, respectively. To satisfy these conditions for all j, we should find a solution where the maximum absolute differences $|(\underline{w_B}/\overline{w_j})-\underline{a_{Bj}}|$, $|(\overline{w_B}/\underline{w_j})-\overline{a_{Bj}}|$, $|(\underline{w_j}/\overline{w_W})-\underline{a_{jW}}|$, and $|(\overline{w_j}/\underline{w_W})-\overline{a_{jW}}|$ for all j are minimized. To obtain a unique solution of the model, Rezaei improved the original BWM [35], where the conditions can be transferred to $|\underline{w_B}-\underline{a_{Bj}}\,\overline{w_j}|$, $|\overline{w_B}-\overline{a_{Bj}}\,\underline{w_j}|$, $|\underline{w_j}-\underline{a_{jW}}\,\overline{w_W}|$, and $|\overline{w_j}-\overline{a_{jW}}\,\underline{w_W}|$, respectively. So we can construct the programming problem as

$$\begin{aligned}
&\min\text{-}\max_{j} \quad \left\{ \left| \underline{w_B} - \underline{a_{Bj}}\, \overline{w_j} \right|, \left| \overline{w_B} - \overline{a_{Bj}}\, \underline{w_j} \right|, \left| \underline{w_j} - \underline{a_{jW}}\, \overline{w_W} \right|, \left| \overline{w_j} - \overline{a_{jW}}\, \underline{w_W} \right| \right\}, \\
&\text{s.t.} \quad \frac{1}{2} \sum_j \left(\underline{w_j} + \overline{w_j} \right) = 1, \\
&\qquad \overline{w_j} > \underline{w_j} \geq 0, \quad \text{for all } j,
\end{aligned} \tag{24}$$

where $(1/2)\sum_j (\underline{w_j} + \overline{w_j}) = 1$ is the normalization condition of rough-number vectors. Problem (24) can be transferred to the following problem:

$$\begin{aligned}
&\min \quad \zeta, \\
&\text{s.t.} \quad \left| \underline{w_B} - \underline{a_{Bj}}\, \overline{w_j} \right| \leq \zeta, \quad \text{for all } j, \\
&\qquad \left| \overline{w_B} - \overline{a_{Bj}}\, \underline{w_j} \right| \leq \zeta, \quad \text{for all } j, \\
&\qquad \left| \underline{w_j} - \underline{a_{jW}}\, \overline{w_W} \right| \leq \zeta, \quad \text{for all } j, \\
&\qquad \left| \overline{w_j} - \overline{a_{jW}}\, \underline{w_W} \right| \leq \zeta, \quad \text{for all } j, \\
&\qquad \frac{1}{2} \sum_j \left(\underline{w_j} + \overline{w_j} \right) = 1, \\
&\qquad \overline{w_j} > \underline{w_j} > 0, \quad \text{for all } j.
\end{aligned} \tag{25}$$

Solving problem (25), the optimal rough weight $([\underline{w_1}, \overline{w_1}], [\underline{w_2}, \overline{w_2}], \ldots, [\underline{w_n}, \overline{w_n}])$ is obtained.

In pairwise comparison methods, the consistency ratio plays a significant role to illustrate the consistency of evaluating values of criteria. Rezaei proposed the definition of consistency of criteria made by one decision-maker and built the consistency index (CI), as shown in Table 2, for the evaluating values between 1 and 9 [24]. To demonstrate the validity of integrated group information in the rough BWM, we define the consistency of integrated comparison as follows.

Definition 4. The integrated comparison is fully consistent if $[a_{Bj}] \times [a_{jW}] = [a_{BW}]$ for all j, where $[a_{Bj}]$, $[a_{jW}]$, and $[a_{BW}]$ are, respectively, the integrated preference of the best criterion over the criterion j, the integrated preference of the criterion j over the worst criterion, and the integrated preference of the best criterion over the worst criterion.

However, there is usually a situation that some pairs of criteria are not completely consistent in practical decision-making problems. Therefore to indicate how consistent an integrated comparison is, we discuss the consistency ratio of an integrated comparison as follows.

According to the discussion above, the maximum comparison value of a_{BW} identified by each decision-maker is 9, so the highest integrated rough value is $[a_{BW}] = [9, 9]$. Consistency decreases when there exists a difference between $[a_{Bj}] \times [a_{jW}]$ and $[a_{BW}]$, which means $[a_{Bj}] \times [a_{jW}] \neq [a_{BW}]$, and it is clear that the biggest difference occurs when $[a_{Bj}]$ and $[a_{jW}]$ have the maximum value which is equal to $[a_{BW}]$, leading to the value of ξ. The consistent condition can be rewritten as $([w_B]/[w_j]) \times ([w_j]/[w_W]) = ([w_B]/[w_W])$, and as the biggest difference occurs when assigning the maximum value to $[a_{Bj}]$ and $[a_{jW}]$, we should subtract the value ξ from $[a_{Bj}]$ and $[a_{jW}]$ and add it to $[a_{BW}]$. So we can obtain the equation as

$$\left([a_{Bj}] - \xi \right) \times \left([a_{jW}] - \xi \right) = \left([a_{BW}] + \xi \right). \tag{26}$$

As for the minimum consistency $[a_{Bj}] = [a_{jW}] = [a_{BW}]$, we obtain

$$\begin{aligned}
&\left([a_{BW}] - \xi \right) \times \left([a_{BW}] - \xi \right) = \left([a_{BW}] + \xi \right) \\
&\quad \Longrightarrow \xi^2 - \left(1 + 2[a_{BW}] \right)\xi + \left([a_{BW}]^2 - [a_{BW}] \right) = 0.
\end{aligned} \tag{27}$$

$[a_{BW}]$ is the integrated preference of the best criterion over the worst criterion, and in this paper, it is a rough number, which means $[a_{BW}] = [\underline{a_{BW}}, \overline{a_{BW}}]$. According to $\underline{a_{BW}} \leq \overline{a_{BW}}$, we can conclude that the integrated preference of the best criterion over the worst criterion cannot be greater than $\overline{a_{BW}}$. It is easy to find that the value of ξ is increasing in $a \in [1, 9]$ for the function $\xi^2 - (1 + 2a)\xi + (a^2 - a) = 0$, so we choose the upper limit $\overline{a_{BW}}$ to calculate the value of CI, which ensures the consistency ratio (CR) satisfying $\text{CR} \in [0, 1]$. So equation (27) can be transformed as

$$\xi^2 - \left(1 + 2\overline{a_{BW}} \right)\xi + \left(\overline{a_{BW}}^2 - \overline{a_{BW}} \right) = 0. \tag{28}$$

Solving equation (28) for different values of $\overline{a_{BW}}$, we can obtain the maximum possible values of ξ, which compose the consistency index of the rough BWM. As $[a_{BW}]$ is obtained by aggregating the evaluating information of

Table 2: Consistency index (CI) table.

a_{BW}	1	2	3	4	5	6	7	8	9
CI	0.00	0.44	1.00	1.63	2.30	3.00	3.73	4.47	5.23

decision-makers, $\overline{a_{BW}}$ is varied according to different values, so we cannot predefine the values of ξ. If there is an agreement among all the decision-makers about their preference for the best criterion over the worst, $[a_{BW}]$ is a crisp number (e.g., $\underline{a_{BW}} = \overline{a_{BW}}$) and belongs to $\{1, 2, \ldots, 9\}$ and then the values of ξ can be determined from the data in Table 2. Based on the value of ξ^* obtained by model (25) and values of CI, we can calculate the consistency ratio (CR) as

$$\mathrm{CR} = \frac{\xi^*}{\mathrm{CI}}. \tag{29}$$

3.3. Rough Prospect Theory for Alternative Evaluation. It is important to consider decision-makers' expectations for alternatives in the evaluation process. In prospect theory, expectations of decision-makers can be seen as references, relative to which gains and losses are obtained. This section combines the rough number and prospect theory to handle criteria values, criteria expectations, and probabilities of states and proposes a new method for alternative ranking and selection. The procedure is described as follows.

3.3.1. Step 1: Construction of the Group Rough Evaluation Matrix and Expectation Vector. The evaluation matrix D_k^t and expectation vector R_k^t from the decision-maker e_k under the state S_t can be expressed as

$$D_k^t = \begin{bmatrix} x_{11}^{kt} & x_{12}^{kt} & \cdots & x_{1n}^{kt} \\ x_{21}^{kt} & x_{22}^{kt} & \cdots & x_{2n}^{kt} \\ \vdots & \vdots & \ddots & \vdots \\ x_{m1}^{kt} & x_{m2}^{kt} & \cdots & x_{mn}^{kt} \end{bmatrix}, \tag{30}$$

$$R_k^t = \left(r_1^{kt} \;\; r_2^{kt} \;\; \ldots \;\; r_n^{kt} \right). \tag{31}$$

Evaluation matrixes and expectation vectors are aggregated, and equations (1)–(7) are used to obtain the group rough evaluation matrix and expectation vector under the state S_t as

$$D_t = \begin{bmatrix} \left[\underline{x_{11}^t}, \overline{x_{11}^t}\right] & \left[\underline{x_{12}^t}, \overline{x_{12}^t}\right] & \cdots & \left[\underline{x_{1n}^t}, \overline{x_{1n}^t}\right] \\ \left[\underline{x_{21}^t}, \overline{x_{21}^t}\right] & \left[\underline{x_{22}^t}, \overline{x_{22}^t}\right] & \cdots & \left[\underline{x_{2n}^t}, \overline{x_{2n}^t}\right] \\ \vdots & \vdots & \ddots & \vdots \\ \left[\underline{x_{m1}^t}, \overline{x_{m1}^t}\right] & \left[\underline{x_{m2}^t}, \overline{x_{m2}^t}\right] & \cdots & \left[\underline{x_{mn}^t}, \overline{x_{mn}^t}\right] \end{bmatrix}, \tag{32}$$

$$R_t = \left(\left[\underline{r_1^t}, \overline{r_1^t}\right] \;\; \left[\underline{r_2^t}, \overline{r_2^t}\right] \;\; \ldots \;\; \left[\underline{r_n^t}, \overline{r_n^t}\right] \right). \tag{33}$$

3.3.2. Step 2: Calculation of Group Gain and Loss Matrixes. Group evaluation values and expectations are both rough numbers, which are in the form of interval numbers, so we can calculate gains and losses by means of the relationship between interval numbers shown in Table 3. The group gain matrix G_t and group loss matrix L_t under the state S_t can be expressed as

$$G_t = \begin{bmatrix} G_{11}^t & G_{12}^t & \cdots & G_{1n}^t \\ G_{21}^t & G_{22}^t & \cdots & G_{2n}^t \\ \vdots & \vdots & \ddots & \vdots \\ G_{m1}^t & G_{m2}^t & \cdots & G_{mn}^t \end{bmatrix},$$
$$L_t = \begin{bmatrix} L_{11}^t & L_{12}^t & \cdots & L_{1n}^t \\ L_{21}^t & L_{22}^t & \cdots & L_{2n}^t \\ \vdots & \vdots & \ddots & \vdots \\ L_{m1}^t & L_{m2}^t & \cdots & L_{mn}^t \end{bmatrix}. \tag{34}$$

3.3.3. Step 3: Calculation of Group Rough Decision Weights. The probability vector of states given by the decision-maker e_k can be expressed as $(p_1^k, p_2^k, \ldots, p_l^k)$, where $p_t^k > 0$ and $\sum_t p_t^k = 1$. According to the rough-number method, all the probability vectors from the decision group can be transformed to a rough probability vector $([\underline{p_1}, \overline{p_1}], [\underline{p_2}, \overline{p_2}], \ldots, [\underline{p_l}, \overline{p_l}])$. Referring to the definition of consistency of interval probability initiated by Yager and Kreinovich [47], the following theorem proves that rough probability satisfies the property of consistency.

Theorem 1. *The rough probability vector* $([\underline{p_1}, \overline{p_1}], [\underline{p_2}, \overline{p_2}], \ldots, [\underline{p_l}, \overline{p_l}])$ *is consistent; in other words,* $\sum_t \underline{p_t} \leq 1$ *and* $\sum_t \overline{p_t} \geq 1$.

Proof. Denote $(p_t^1, p_t^2, \ldots, p_t^h)$ as the probability vector of the state S_t given by h decision-makers, and it can be transformed to a rough number $[\underline{p_t}, \overline{p_t}]$. For an easy and convenient expression, suppose $p_t^1 \leq p_t^2 \leq \cdots \leq p_t^h$. Let $(1/h)\sum_{k=1}^{h} p_t^k = m_t$, and according to equations (4) and (5), $\underline{p_t^1} = \overline{p_t^h} = m_t$. So $\underline{p_t^k} \leq \underline{p_t^h}$ and $\overline{p_t^k} \geq \overline{p_t^1}$ for $\forall p_t^k$, which lead to $\underline{p_t} = (1/h)\sum_{k=1}^{h} \underline{p_t^k} \leq m_t$ and $\overline{p_t} = (1/h)\sum_{k=1}^{h} \overline{p_t^k} \geq m_t$. Therefore, we can get

$$\sum_{t=1}^{l} \underline{p_t} = \sum_{t=1}^{l} \left(\frac{1}{h} \sum_{k=1}^{h} \underline{p_t^k} \right) \leq \frac{1}{h} \sum_{k=1}^{h} \sum_{t=1}^{l} p_t^k = 1,$$
$$\sum_{t=1}^{l} \overline{p_t} = \sum_{t=1}^{l} \left(\frac{1}{h} \sum_{k=1}^{h} \overline{p_t^k} \right) \geq \frac{1}{h} \sum_{k=1}^{h} \sum_{t=1}^{l} p_t^k = 1. \tag{35}$$

The theorem is proven.

The group rough decision weights of gain $[\pi_t^+] = [\underline{\pi_t^+}, \overline{\pi_t^+}]$ can be expressed as

Table 3: Gains and losses for all possible cases.

Types	Gains	Losses
a: $\overline{r_j} < \underline{x_{ij}}$	$0.5(\underline{x_{ij}} + \overline{x_{ij}}) - \overline{r_j}$	0
b: $\overline{x_{ij}} < \underline{r_j}$	0	$0.5(\underline{x_{ij}} + \overline{x_{ij}}) - \underline{r_j}$
c: $\underline{r_j} < \underline{x_{ij}} < \overline{r_j} < \overline{x_{ij}}$	$0.5(\overline{x_{ij}} - \overline{r_j})$	0
d: $\underline{x_{ij}} < \underline{r_j} < \overline{x_{ij}} < \overline{r_j}$	0	$0.5(\underline{x_{ij}} - \underline{r_j})$
e: $\underline{r_{ij}} < \underline{x_j} < \overline{x_{ij}} < \overline{r_j}$	0	0
f: $\underline{x_{ij}} < \underline{r_j} < \overline{r_{ij}} < \overline{x_j}$	$0.5(\overline{x_{ij}} - \overline{r_j})$	$0.5(\underline{x_{ij}} - \underline{r_j})$

$$\underline{\pi_t^+} = \frac{\left(\underline{p_t}\right)^\gamma}{\left[\left(\underline{p_t}\right)^\gamma + \left(1 - \underline{p_t}\right)^\gamma\right]^{1/\gamma}},$$
$$\overline{\pi_t^+} = \frac{(\overline{p_t})^\gamma}{\left[(\overline{p_t})^\gamma + (1 - \overline{p_t})^\gamma\right]^{1/\gamma}}. \quad (36)$$

And the group rough decision weights of loss $[\pi_t^-] = [\underline{\pi_t^-}, \overline{\pi_t^-}]$ can be expressed as

$$\underline{\pi_t^-} = \frac{\left(\underline{p_t}\right)^\delta}{\left[\left(\underline{p_t}\right)^\delta + \left(1 - \underline{p_t}\right)^\delta\right]^{1/\delta}},$$
$$\overline{\pi_t^-} = \frac{(\overline{p_t})^\delta}{\left[(\overline{p_t})^\delta + (1 - \overline{p_t})^\delta\right]^{1/\delta}}. \quad (37)$$

□

3.3.4. Step 4: Calculation of the Group Prospect Matrix. According to equation (34), the value matrixes for gain and loss under the state S_t are expressed as

$$V_t^+ = \left(\left(G_{ij}^t\right)^\alpha\right)_{m\times n},$$
$$V_t^- = \left(-\lambda\left(-L_{ij}^t\right)^\beta\right)_{m\times n}, \quad (38)$$

where α, β, and λ are parameters discussed in classical prospect theory.

Denote the prospect matrix by $V = ([V_{ij}])_{m\times n}$, where V_{ij} is the prospect value of the alternative A_i at the criterion c_j. The calculation formula of V_{ij} is expressed as

$$V_{ij} = \sum_{t=1}^{l} V_{ij}^{t+}[\pi_t^+] + \sum_{t=1}^{l} V_{ij}^{t-}[\pi_t^-]. \quad (39)$$

3.3.5. Step 5: Calculation of Prospect Values and Arranging the Alternatives. Standardizing the prospect matrix to eliminate the effect of diverse dimensions, we can obtain the standardized prospect matrix $V' = ([V'_{ij}])_{m\times n}$, where

$$\left[V'_{ij}\right] = \frac{\left[V_{ij}\right]}{V_j^*}, \quad V_j^* = \max_i\left\{\left|\underline{V_{ij}}\right|, \left|\overline{V_{ij}}\right|\right\}. \quad (40)$$

According to the rough weight obtained by the rough BWM, the overall prospect value U_i of the alternative A_i is calculated by

$$U_i = \sum_{j=1}^{n}\left[w_j\right] \times \left[V'_{ij}\right]. \quad (41)$$

By the ranking rules for rough numbers shown in Figure 1, alternatives are arranged according to their overall prospect values and the maximal one is selected as the optimal alternative.

4. An Illustrative Example

In this section, a practical MCGDM example in a risk environment is given to illustrate the feasibility and validity of the proposed method. The description of this example is as follows.

An investment company is planning to select an optimal alternative to invest a sum of money. After the initial screening, there remain six possible alternatives, which are an Internet company (A_1), a car company (A_2), two food companies (A_3 and A_4), an education company (A_5), and a chemical company (A_6). Four criteria, direct benefits (c_1), indirect benefits (c_2), social benefits (c_3), and environmental protection (c_4), are taken into account in order to evaluate the six alternatives and make the best choice. And there are three natural statuses according to the market forecast, including good (S_1), fair (S_2), and poor (S_3), which influence the evaluating values of alternatives in criteria. To obtain objective and comprehensive results, the investment company chooses five experts from different departments to constitute a decision committee, which is expressed as $E = \{e_1, e_2, \ldots, e_5\}$, and the committee needs to compare the relative importance of each criterion as well as the evaluating values of each alternative according to individual experiences.

4.1. Criteria Weighting. According to the discussion of the decision committee, c_1 and c_4 are determined as the best criterion and the worst criterion, respectively. Five decision-makers determine their comparison vectors by the scoring method in the BWM, and the results are collected in Tables 4 and 5.

Gathering the comparison vectors, we can obtain the integrated comparison vectors as

$$A_1 = (\{1,1,1,1,1\}, \{1,3,1,3,1\}, \{3,5,5,3,3\}, \{5,7,7,5,7\}),$$
$$A_4 = (\{5,7,7,5,7\}, \{3,3,5,3,3\}, \{1,1,3,1,1\}, \{1,1,1,1,1\}). \quad (42)$$

Integrated comparison vectors are transformed to rough comparison vectors as

Table 4: Scores of best-to-others vectors.

		c_1	c_2	c_3	c_4
c_1	e_1	1	1	3	5
	e_2	1	3	5	7
	e_3	1	1	5	7
	e_4	1	3	3	5
	e_5	1	1	3	7

Table 5: Scores of others-to-worst vectors.

	c_4				
	e_1	e_2	e_3	e_4	e_5
c_1	5	7	7	5	7
c_2	3	3	5	3	3
c_3	1	1	3	1	1
c_4	1	1	1	1	1

$$\begin{aligned} &\mathrm{RA}_1 = ([1,1],[1.32,2.28],[3.32,4.28],[5.72,6.68]),\\ &\mathrm{RA}_4 = ([5.72,6.68],[3.08,3.72],[1.08,1.72],[1,1]). \end{aligned} \tag{43}$$

A programming problem is constructed according to (21) as

$$\begin{aligned} &\min \quad \zeta,\\ &\text{s.t.} \quad \left|\underline{w_1} - 1.32\overline{w_2}\right| \le \zeta, \left|\overline{w_1} - 2.28\,\underline{w_2}\right| \le \zeta,\\ &\quad \left|\underline{w_1} - 3.32\overline{w_3}\right| \le \zeta, \left|\overline{w_1} - 4.28\,\underline{w_3}\right| \le \zeta,\\ &\quad \left|\underline{w_1} - 5.72\overline{w_4}\right| \le \zeta, \left|\overline{w_1} - 6.68\,\underline{w_4}\right| \le \zeta,\\ &\quad \left|\underline{w_2} - 3.08\overline{w_4}\right| \le \zeta, \left|\overline{w_2} - 3.72\,\underline{w_4}\right| \le \zeta,\\ &\quad \left|\underline{w_3} - 1.08\overline{w_4}\right| \le \zeta, \left|\overline{w_3} - 1.72\,\underline{w_4}\right| \le \zeta,\\ &\quad \frac{1}{2}\sum_j\left(\underline{w_j} + \overline{w_j}\right) = 1,\\ &\quad \overline{w_j} > \underline{w_j} > 0, \quad \text{for } j = 1,2,3,4. \end{aligned} \tag{44}$$

LINGO 16.0 is used to solve this problem, and we can obtain the unique solution as $\underline{w_1} = 0.464$, $\overline{w_1} = 0.528$, $\underline{w_2} = 0.242$, $\overline{w_2} = 0.333$, $\underline{w_3} = 0.117$, $\overline{w_3} = 0.147$, $\underline{w_4} = 0.083$, $\overline{w_4} = 0.086$, and $\xi = 0.025$.

Since c_1 and c_4 are the best criterion and the worst criterion, respectively, and $[a_{14}] = [5.72, 6.68]$, we use $\overline{a_{14}} = 6.68$ to calculate the value of CI. According to equation (28), we solve $\xi^2 - (1 + 2\times 6.68)\xi + (6.68^2 - 6.68) = 0$ and obtain CI = 3.499 and then CR = 0.025/3.499 = 0.007, which implies a very good consistency.

So the optimal rough weights can be denoted as

$$\overline{W} = ([0.464, 0.528], [0.242, 0.333], [0.117, 0.147], [0.083, 0.086]). \tag{45}$$

The rough weights of criteria are shown in Figure 4, which illustrates that the ranking of weights is $c_1 > c_2 > c_3 > c_4$.

4.2. Alternative Ranking. The evaluation values of alternatives in each criterion are linguistic variables {very poor, poor, fair, good, very good}, corresponding to the numerical variables {1, 3, 5, 7, 9}. Table 6 shows the evaluation scores of alternatives by the decision group, and with the aid of transforming them to rough numbers by equations (1)–(7), we can obtain the rough scores shown in Table 7. Then, the gain and loss matrixes under different risk statuses can be calculated according to the results in Table 3.

Gain and loss matrixes under the status S_1 are

$$\begin{aligned} G_1 &= \begin{bmatrix} 0 & 0 & 1.68 & 0.28\\ 0 & 0.295 & 0 & 0\\ 0.28 & 0.39 & 0 & 0.88\\ 0.28 & 0 & 0.99 & 2.08\\ 0.48 & 0 & 0 & 0.28\\ 0.28 & 0.295 & 2.48 & 0 \end{bmatrix},\\ L_1 &= \begin{bmatrix} -0.39 & -0.28 & 0 & 0\\ 0 & -0.28 & -0.48 & -1.28\\ 0 & 0 & -1.695 & 0\\ -0.259 & -0.48 & 0 & 0\\ 0 & -0.28 & 0 & -0.295\\ -0.259 & -0.72 & 0 & -2.08 \end{bmatrix}. \end{aligned} \tag{46}$$

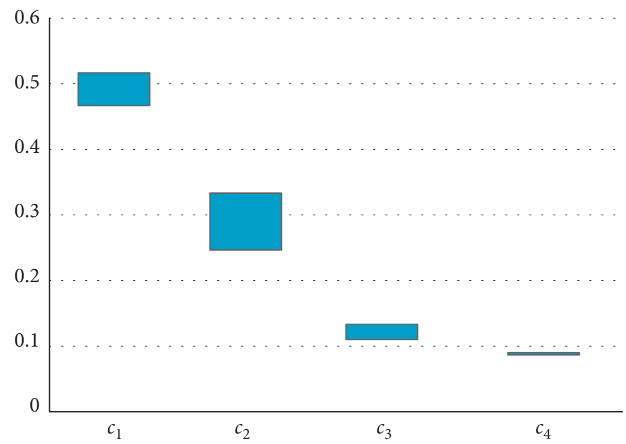

Figure 4: Rough weights of criteria.

Table 6: Scores and expectations of alternatives.

Criteria		c_1			c_2			c_3			c_4		
States		S_1	S_2	S_3	S_1	S_2	S_3	S_1	S_2	S_3	S_1	S_2	S_3
Alternatives	A_1	(7, 9, 7, 5, 7)	(9, 9, 7, 7, 7)	(5, 5, 7, 7, 5)	(5, 7, 7, 5, 7)	(3, 5, 5, 5, 7)	(3, 3, 1, 1, 3)	(7, 7, 9, 7, 7)	(3, 3, 5, 5, 7)	(3, 5, 3, 5, 5)	(5, 5, 7, 7, 5)	(3, 5, 3, 5, 5)	(1, 3, 1, 3, 3)
	A_2	(7, 7, 9, 7, 7)	(5, 3, 3, 7, 7)	(3, 3, 5, 7, 7)	(5, 7, 5, 9, 7)	(5, 7, 7, 5, 5)	(3, 1, 5, 5, 3)	(5, 5, 3, 5, 5)	(3, 1, 3, 3, 1)	(5, 7, 5, 7, 5)	(3, 5, 3, 5, 3)	(3, 3, 1, 3, 3)	(1, 1, 3, 5, 1)
	A_3	(9, 7, 7, 9, 7)	(7, 9, 5, 5, 9)	(3, 1, 3, 3, 5)	(7, 9, 7, 7, 5)	(5, 5, 7, 5, 5)	(3, 5, 5, 3, 5)	(3, 5, 1, 3, 5)	(1, 3, 1, 3, 1)	(5, 5, 5, 5, 7)	(7, 5, 7, 7, 7)	(7, 5, 5, 7, 5)	(5, 3, 3, 5, 3)
	A_4	(5, 7, 9, 9, 7)	(5, 5, 7, 5, 5)	(5, 3, 5, 5, 3)	(5, 5, 5, 7, 7)	(3, 7, 5, 7, 5)	(1, 5, 3, 5, 7)	(9, 7, 7, 5, 7)	(7, 5, 5, 5, 5)	(7, 7, 3, 1, 5)	(7, 7, 9, 7, 9)	(7, 7, 7, 7, 7)	(7, 5, 7, 7, 5)
	A_5	(9, 9, 7, 9, 7)	(7, 9, 5, 5, 5)	(5, 5, 3, 3, 7)	(7, 7, 7, 5, 5)	(5, 5, 7, 7, 5)	(3, 3, 5, 5, 3)	(7, 5, 5, 5, 7)	(5, 5, 3, 5, 5)	(3, 5, 3, 5, 5)	(7, 7, 3, 5, 5)	(7, 5, 3, 5, 5)	(5, 3, 3, 3, 5)
	A_6	(7, 5, 9, 9, 7)	(7, 7, 9, 7, 5)	(5, 5, 7, 5, 5)	(5, 3, 7, 7, 9)	(3, 1, 5, 7, 7)	(3, 3, 5, 5, 7)	(9, 9, 7, 9, 7)	(7, 7, 7, 7, 5)	(5, 5, 7, 5, 7)	(3, 3, 5, 3, 1)	(3, 1, 3, 1, 3)	(1, 1, 3, 3, 3)
Expectations		(7, 7, 7, 9, 7)	(5, 5, 5, 7, 5)	(5, 5, 5, 3, 3)	(7, 7, 5, 7, 7)	(5, 7, 5, 7, 5)	(3, 5, 3, 5, 3)	(5, 5, 7, 5, 5)	(5, 3, 5, 5, 5)	(5, 3, 5, 3, 3)	(5, 7, 5, 5, 5)	(5, 5, 3, 3, 5)	(3, 3, 1, 1, 3)

Table 7: Rough scores and expectations of alternatives.

Criteria		c_1			c_2			c_3			c_4		
States		S_1	S_2	S_3	S_1	S_2	S_3	S_1	S_2	S_3	S_1	S_2	S_3
Alternatives	A_1	[6.30, 7.70]	[7.32, 8.28]	[5.32, 6.28]	[5.72, 6.68]	[4.30, 5.70]	[1.72, 2.68]	[7.08, 7.72]	[3.72, 5.50]	[3.72, 4.68]	[5.32, 6.28]	[3.72, 4.68]	[1.72, 2.68]
	A_2	[7.08, 7.72]	[3.93, 6.07]	[4.67, 6.07]	[5.72, 7.51]	[5.32, 6.28]	[2.49, 4.28]	[4.28, 4.92]	[1.72, 2.68]	[5.32, 6.28]	[3.32, 4.28]	[2.28, 2.92]	[1.34, 3.12]
	A_3	[7.32, 8.28]	[5.93, 8.07]	[2.30, 3.70]	[6.30, 7.70]	[5.08, 5.72]	[3.72, 4.68]	[2.49, 4.28]	[1.32, 2.28]	[5.08, 5.72]	[6.28, 6.92]	[5.32, 6.28]	[3.32, 4.28]
	A_4	[6.49, 8.28]	[5.08, 5.72]	[3.72, 4.68]	[5.32, 6.28]	[4.49, 6.28]	[2.84, 5.51]	[6.30, 7.70]	[5.08, 5.72]	[3.04, 6.09]	[7.32, 8.28]	[7.00, 7.00]	[5.72, 6.68]
	A_5	[7.72, 8.68]	[5.34, 7.12]	[3.72, 5.51]	[5.72, 6.68]	[5.32, 6.28]	[3.32, 4.28]	[5.32, 6.28]	[4.28, 4.92]	[3.72, 4.68]	[4.49, 6.28]	[4.30, 5.70]	[3.32, 4.28]
	A_6	[6.49, 8.28]	[6.30, 7.70]	[5.08, 5.72]	[4.84, 7.51]	[3.04, 6.09]	[3.72, 5.51]	[7.72, 8.68]	[6.28, 6.92]	[5.32, 6.28]	[2.30, 3.70]	[1.72, 2.68]	[1.72, 2.68]
Expectations		[7.08, 7.72]	[5.08, 5.72]	[3.72, 4.68]	[6.28, 6.92]	[5.32, 6.28]	[3.32, 4.28]	[5.08, 5.72]	[4.28, 4.92]	[3.32, 4.28]	[5.08, 5.72]	[3.72, 4.68]	[1.72, 2.68]

Gain and loss matrixes under the status S_2 are

$$G_2 = \begin{bmatrix} 2.08 & 0 & 0.29 & 0 \\ 0.175 & 0 & 0 & 0 \\ 1.28 & 0 & 0 & 1.12 \\ 0 & 0 & 0.48 & 2.32 \\ 0.70 & 0 & 0 & 0.51 \\ 1.28 & 0 & 1.68 & 0 \end{bmatrix}, \quad L_2 = \begin{bmatrix} 0 & -0.51 & -0.28 & 0 \\ -0.575 & 0 & -2.08 & -1.12 \\ 0 & -0.12 & -2.48 & 0 \\ 0 & -0.415 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -1.14 & 0 & -1.52 \end{bmatrix}. \tag{47}$$

Gain and loss matrixes under the status S_3 are

$$G_3 = \begin{bmatrix} 1.12 & 0 & 0.2 & 0 \\ 0.695 & 0 & 1.52 & 0.22 \\ 0 & 0.2 & 1.12 & 1.12 \\ 0 & 0.615 & 0.905 & 3.52 \\ 0.415 & 0 & 0.2 & 1.12 \\ 0.72 & 0.615 & 1.54 & 0 \end{bmatrix}, \quad L_3 = \begin{bmatrix} 0 & -1.12 & 0 & 0 \\ 0 & -0.415 & 0 & -0.19 \\ -0.72 & 0 & 0 & 0 \\ 0 & -0.24 & -0.14 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{48}$$

The weight vector of states given by each decision-maker and group rough decision weights derived by equations (31)–(34) are described in Table 8. Here, we take the values of parameters given by Gonzalez and Wu in [46], so the rough decision weights of gain and loss are the same.

Taking $\alpha = 0.89$, $\beta = 0.92$, and $\lambda = 2.25$, the value matrixes for gain and loss under different states can be expressed as

$$V_1^+ = \begin{bmatrix} 0 & 0 & 1.587 & 0.322 \\ 0 & 0.337 & 0 & 0 \\ 0.322 & 0.433 & 0 & 0.893 \\ 0.322 & 0 & 0.991 & 1.919 \\ 0.520 & 0 & 0 & 0.322 \\ 0.322 & 0.337 & 2.244 & 0 \end{bmatrix}, \quad V_1^- = \begin{bmatrix} -0.946 & -0.698 & 0 & 0 \\ 0 & -0.698 & -1.145 & -2.824 \\ 0 & 0 & -3.656 & 0 \\ -0.649 & -1.145 & 0 & 0 \\ 0 & -0.698 & 0 & -0.732 \\ -0.649 & -1.663 & 0 & -4.414 \end{bmatrix},$$

$$V_2^+ = \begin{bmatrix} 1.919 & 0 & 0.332 & 0 \\ 0.212 & 0 & 0 & 0 \\ 1.246 & 0 & 0 & 1.106 \\ 0 & 0 & 0.520 & 2.115 \\ 0.728 & 0 & 0 & 0.549 \\ 1.246 & 0 & 1.587 & 0 \end{bmatrix}, \quad V_2^- = \begin{bmatrix} 0 & -1.211 & -0.698 & 0 \\ -1.352 & 0 & -4.414 & -2.497 \\ 0 & -0.320 & -5.189 & 0 \\ 0 & -1.002 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & -2.538 & 0 & -3.307 \end{bmatrix}, \tag{49}$$

$$V_3^+ = \begin{bmatrix} 1.106 & 0 & 0.239 & 0 \\ 0.723 & 0 & 1.452 & 0.260 \\ 0 & 0.239 & 1.106 & 1.106 \\ 0 & 0.649 & 0.915 & 3.065 \\ 0.457 & 0 & 0.239 & 1.106 \\ 0.746 & 0.649 & 1.469 & 0 \end{bmatrix}, \quad V_3^- = \begin{bmatrix} 0 & -2.497 & 0 & 0 \\ 0 & -1.002 & 0 & -0.488 \\ -1.663 & 0 & 0 & 0 \\ 0 & -0.605 & -0.369 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$

Table 8: Rough decision weights.

States	S_1	S_2	S_3
Probability vectors	(0.3, 0.4, 0.3, 0.3, 0.4)	(0.4, 0.4, 0.5, 0.4, 0.3)	(0.3, 0.2, 0.2, 0.3, 0.3)
Rough probability	[0.316, 0.364]	[0.365, 0.435]	[0.236, 0.284]
Rough weight vectors	[0.341, 0.375]	[0.376, 0.425]	[0.280, 0.317]

Then, the group prospect matrix is

$$V = \begin{bmatrix} [0.676, 0.844] & [-1.568, -1.393] & [0.436, 0.550] & [0.110, 0.121] \\ [-0.292, -0.189] & [-0.453, -0.392] & [-1.899, -1.590] & [-2.202, -1.956] \\ [0.051, 0.185] & [0.079, 0.118] & [-3.267, -2.847] & [1.030, 1.156] \\ [-0.134, -0.101] & [-0.865, -0.731] & [0.673, 0.779] & 2.308, 2.590 \\ [0.579, 0.649] & [-0.262, -0.238] & [0.067, 0.076] & [0.351, 0.455] \\ [0.544, 0.665] & [-1.406, -1.189] & [1.773, 1.982] & [-3.061, -2.749] \end{bmatrix}. \quad (50)$$

Also, the standardized prospect matrix is

$$V' = \begin{bmatrix} [0.207, 0.259] & [-0.480, -0.426] & [0.134, 0.168] & [0.034, 0.037] \\ [-0.090, -0.058] & [-0.142, -0.120] & [-0.581, -0.487] & [-0.674, -0.599] \\ [0.016, 0.057] & [0.024, 0.036] & [-1.000, -0.872] & [0.315, 0.354] \\ [-0.041, -0.031] & [-0.265, -0.224] & [0.206, 0.239] & [0.706, 0.793] \\ [0.177, 0.199] & [-0.080, -0.073] & [0.020, 0.023] & [0.108, 0.139] \\ [0.166, 0.204] & [-0.430, -0.364] & [0.543, 0.607] & [-0.937, -0.841] \end{bmatrix}. \quad (51)$$

Calculating the overall prospect values of each alternative according to rough criteria weights obtained in Section 4.1, we obtain

$$U = \begin{pmatrix} [-0.045, 0.062] \\ [-0.238, -0.163] \\ [-0.108, -0.029] \\ [-0.027, 0.035] \\ [0.067, 0.103] \\ [-0.083, 0.039] \end{pmatrix}. \quad (52)$$

The evaluating values of alternatives are depicted in Figure 5. According to the ranking rules of rough numbers shown in Figure 1, it is obvious to find that the ranking order of the alternatives is $A_5 > A_1 > A_4 > A_6 > A_3 > A_2$, so the best investment program of this company is A_5. The results of ranking order may change when we choose different values of parameters, while there are no regular changes of the final evaluating values in U with the changes of parameters because the evaluating values are comprehensive aggregation of gain and loss matrixes, and the variation tendencies of gain matrix and loss matrixes are offset, which results in an irregular change of the evaluating values. Therefore, it is crucial to determine appropriate values of parameters.

4.3. Discussion. In order to discuss the validity of the proposed method, we choose other decision-making techniques with reliable results to calculate the illustrative example and compare the ranks of alternatives. Three MCGDM methods chosen in this section are rough VIKOR [7], prospect theory [36], and fuzzy-number prospect theory [38], respectively. Rough VIKOR takes no account of the expectations of decision-makers, and the synthetic rough evaluating values of alternatives are obtained by the weighted mean model. The crisp values in classical prospect theory can be acquired with the aid of the average values of the decision group, and the fuzzy information in fuzzy-number prospect theory can be obtained by the transforming method from linguistic variables to trapezoidal fuzzy numbers [38]. The ranking results of alternatives derived from different models are shown in Table 9.

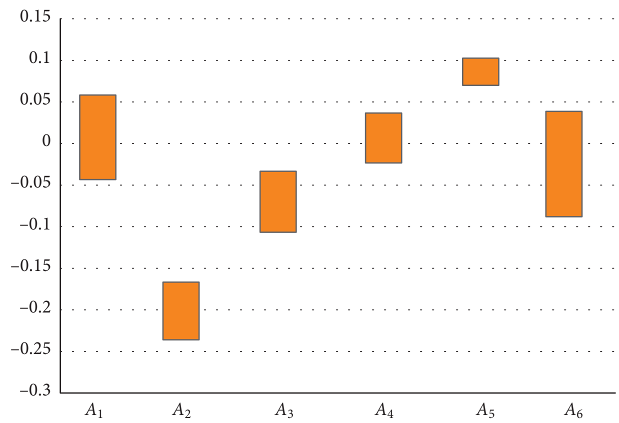

Figure 5: Evaluating results of alternatives.

Table 9: Comparison of the ranks of alternatives by different models.

Alter.	RVIKOR [6]	PT [36]	FPT [38]	RPT
A_1	1	2	2	2
A_2	6	6	6	6
A_3	5	5	5	5
A_4	4	3	3	3
A_5	3	1	1	1
A_6	2	4	4	4

It is easy to find that the results derived from methods based on prospect theory are all $A_5 > A_1 > A_4 > A_6 > A_3 > A_2$, while the rank by rough VIKOR is $A_1 > A_6 > A_5 > A_4 > A_3 > A_2$. The reason giving rise to this difference is that methods based on prospect theory pay attention to the expectations of decision-makers. With no expectations, A_1 performs well in each criterion and obtains a high score in comprehensive evaluation, while A_5 only holds the third place in the ranking. However, when considering the expectations of decision-makers, A_5 becomes the best alternative and A_1 falls to the second place. So it is crucial to pay more attention to the expectation, which could change the final result and influence the benefits of decision-makers.

The rough-number method based on prospect theory in this paper can acquire the same result as classical prospect theory and fuzzy prospect theory, which declares the validity of the novel model, and there are particular advantages of this new method:

(1) Compared with the traditional MCGDM decision-making methods which acquire group information by the ordinary weighted sum method or interval numbers, the proposed method constructs group preferences with the aid of rough numbers, a new tool to deal with uncertain and subjective information. The aggregated approach by rough numbers considers the relationships between original data from every decision-maker, which brings more accurate group judgements and decision results.

(2) The improvement of the BWM based on the rough number can be effectively applied to group decision-making problems with a high consistency, which reduces the risk for errors derived from tedious calculation and low consistency of traditional pairwise comparison methods such as the AHP. The process of weight calculation is much more accessible and reliable, and the results derived from group preferences are more objective. Besides, as a linear programming model, the proposed rough-number-based BWM in this paper is much easier to calculate and can obtain a unique solution for the weight vector, which reduces the probability of decision failure.

(3) The existing decision-making methods based on prospect theory in dealing with imprecision take advantage of interval numbers, fuzzy numbers, and linguistic variables, and there is no research in developing prospect theory with the aid of rough numbers. The transformations of criteria values, expectations of decision-makers, and even the risk probabilities by rough numbers contain all the group information and bring more objective and reliable decision results than other models. The improvement expands the range of the application of prospect theory.

5. Conclusion

With the rapid development and changes of the social and the economic environment, decision-makers always face multicriteria group decision-making problems with imprecision, risk, and subjectivity. To handle the realistic decision-making problems in a reasonable and effective way, this paper constructs a risky MCGDM method by combining rough numbers, BWM, and prospect theory, which is able to tackle subjectivity and imprecision. According to the process of decision-making, the proposed method can be divided into two stages. The first stage is to integrate the rough number and BWM to calculate the rough weights of criteria, and the second one is combining the rough number and prospect theory to arrange the alternatives. Then, a case study involving investment is introduced to illustrate the application of this new method, and the results verify the feasibility and validity. The proposed method constructs a linear programming model to obtain the weights of criteria and a psychological decision-making model to handle the subjectivity and risk of problems. It puts forward a new research direction in the MCGDM and can be applied to many practical group decision-making problems with various conditions. In terms of future research, we intend to consider the compensatory and interactive relationship between criteria, and the impact of changes of decision-makers and criteria on the final evaluation results in a dynamic procedure. The expansion of rough numbers to other MCDM methods and realistic problems is also an interesting and significant direction for further research.

Data Availability

All data used to support this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare no conflicts of interest.

Authors' Contributions

F.J. and X.W. performed the methodology. F.J. wrote the original draft, performed formal analysis, and acquired the fund. X.W. wrote, reviewed, and edited the paper.

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