

Research Article

Event-Triggered Dissipative Filter Design for Semi-Markovian Jump Systems with Time-Varying Delays

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This paper deals with the event-triggered dissipative filtering problem for semi-Markovian jump systems with time-varying delays. The purpose is to design a filter which guarantees that the filtering error system is not only stochastically stable but also satisfies dissipativity. First, based on Lyapunov–Krasovskii theory and matrix integral inequality, a sufficient condition is established for the existence of such a filter. Then, the codesign method of event-triggered matrices and the desired filter parameters is proposed. Finally, two numerical examples are given to illustrate the advantages and validity of the scheme developed in this paper.

1. Introduction

Over the past decades, Markovian jump systems (MJSs) have attracted significant attention by the research community mainly due to the fact that they have a powerful framework to model practical systems, such as economy [1], robotics [2], mechanics [3], and solar thermal receivers [4]. However, among the aforementioned works, the jump time of a Markov chain is a time-homogeneous exponential random variable. This result in the MJSs has many limitations in applications. To overcome this obstacle, a more wider class of systems with general sojourn time distributions, named semi-Markovian jump systems (SMJSs), have been studied in recent years. To mention a few, the problem of stability and stabilization for semi-Markovian jump systems was addressed in [5–7]. The controller design problem for semi-Markovian jump systems was investigated in [8–11]. In [12, 13], the filtering problem for semi-Markovian jump systems was considered.

It is known that time-delay phenomenon is commonly observed in real-world applications and usually deteriorates the stability of a control system [14–22]. As a consequence, increasing attention has been devoted to studying of SMJSs with time delays. For instance, the problem of finite-time H_∞ synchronization for SMJSs with time-varying delays was reported in [23, 24]. The resilient estimation problem for

nonlinear descriptor SMJSs was probed in [25], where the T-S fuzzy model and time-varying delays are taken into account. In the case, when the actuator or sensor failures appear in the SMJSs, the reliable controller or filter was designed for delayed SMJSs in [26, 27], respectively. By applying a piecewise analysis method, a stochastic stability condition was established in [28], where the time-varying delays are dependent on system modes. However, the stability condition in [28] may be conservative because Jensen's inequality was used to bound the integral term, and thus there still exists some room to improve. This is the first motivation of this paper.

On the other hand, communication resource saving is also a significant issue for networked systems because of the bandwidth resource becomes more and more limited as the complexity of the network increase [29]. It has been shown that the event-triggered scheme is an effective way to solve this issue. Compared with a time-triggered scheme, not all of sampled data packets in the event-triggered scheme are transmitted through a communication network. Only when an event condition is satisfied, the sampled data packets are transmitted [30]. As a result, the unnecessary communication resources are saved in the event-triggered scheme. In recent years, the event-triggered scheme has been utilized in the controller and filter design problem for various dynamic systems [31–36]. As for SMJSs, the event-triggered leader-

following consensus for a multiagent system with semi-Markovian jumping parameters was studied in [37]. In the case of discrete-time systems, an H_∞ filter was designed in [38] for a class of semi-Markovian jumping neural networks, where the quantized measurements was used in order to save the network resource, although the dissipative filtering problem for SMJSs was discussed in [39], where time-varying delays were not be considered. Therefore, the event-triggered filtering problem for SMJSs with time-varying delays should be further investigated. This is also a motivation to this paper.

Based on the above mentioned discussions, this paper considers the dissipative filter design problem for SMJSs with time-varying delays by using the event-triggered scheme. The main contributions of this paper can be highlighted as follows: (1) the dissipative filter design problem for SMJSs in the presence of time-varying delays is investigated by applying the event-triggered scheme. (2) Based on Lyapunov–Krasovskii theory and matrix integral inequality, a new stochastic stability criterion for SMJSs is established. (3) The method developed in this paper will be helpful for some further development involving this topic, such as nonfragile control and state estimation.

The rest part of the paper is organized as follows. In Section 2, we describe the problem formulation. Section 3 presents the event-triggered filter design scheme for the underlying system. Numerical examples and simulation results which demonstrate the effectiveness and advantages of the proposed scheme are provided in Section 4. The conclusion of this paper is made in Section 5.

Notations. Throughout this paper, a real symmetric matrix $X > 0$ ($X \geq 0$) means that X is positive definite (respectively, positive semidefinite) matrix; X^T represents transpose of the matrix X ; \mathbb{R}^n and $\mathbb{R}^{n \times m}$ stand for n -dimensional Euclidean space and set of all $n \times m$ real matrices; I_n denotes the n -dimensional identity matrix; 0_n and $0_{n \times m}$ denote the n -dimensional zero matrix and $n \times m$ -dimensional zero matrix; $\text{sym}\{X\}$ denotes $X + X^T$; $\text{diag}\{\cdot\}$ stands for a block-diagonal matrix. The notation $*$ is used as an ellipsis for terms that are induced by symmetry; $L_2[0, \infty)$ is the space of the square-integrable vector function over $[0, \infty)$; $|\cdot|$ denotes the Euclidean norm for vectors. Matrix dimensions, if not explicitly stated are assumed to be compatible for algebraic operations.

2. Problem Statement and Preliminaries

Given a probability space $\{\Omega, \mathcal{F}, P\}$, we consider the following semi-Markovian jump systems with time-varying delays:

$$\begin{cases} \dot{x}(t) = A(\lambda_t)x(t) + A_d(\lambda_t)x(t - \tau(t)) + B(\lambda_t)\omega(t), \\ y(t) = C(\lambda_t)x(t) + C_d(\lambda_t)x(t - \tau(t)) + D(\lambda_t)\omega(t), \\ z(t) = L(\lambda_t)x(t), \\ x(t) = \varphi(t), \quad t \in \{-\tau_2, \dots, 0\}, \end{cases} \quad (1)$$

where $x(t) \in \mathbb{R}^n$ is the system state vector; $y(t) \in \mathbb{R}^p$ is the measurement output; $z(t) \in \mathbb{R}^q$ is the control output; and $\omega(t) \in \mathbb{R}^r$ is the disturbance input, which belongs to $L_2[0, \infty)$; $\varphi(t)$ is a initial condition. The process $\{\lambda_t\}$ is a time-homogeneous semi-Markov stochastic process, which takes value in a finite state space $\mathcal{S} = \{1, 2, \dots, s\}$. The evolution of λ_t is governed by the following probability transition:

$$\Pr\{\lambda_{t+h} = j \mid \lambda_t = i\} = \begin{cases} \phi_{ij}(h)h + o(h), & i \neq j, \\ 1 + \phi_{ii}(h)h + o(h), & i = j, \end{cases} \quad (2)$$

where $h > 0$, $\lim_{h \rightarrow 0} o(h)/h = 0$, and $\phi_{ij}(h)$ is the transition rate from mode i at time t to mode j at time $t + h$ when $i \neq j$ and

$$\phi_{ii}(h) = -\sum_{j=1}^s \phi_{ij}(h). \quad (3)$$

Remark 1. It is worth noting that when the transition rate $\phi_{ij}(h)$ of SMJSs is independent of h , that is, $\phi_{ij}(h) = \phi_{ij}$, then the SMJSs reduce to the common MJSSs, which have been investigated extensively in [1, 2, 18, 40, 41].

In system (1), $\tau(t)$ denotes the time-varying delay, which satisfies

$$\tau_1 \leq \tau(t) \leq \tau_2, \quad 0 \leq \dot{\tau}(t) \leq \mu < 1. \quad (4)$$

To simplify the notation, for each $\lambda_t = i \in \mathcal{S}$, the matrices $A(\lambda_t)$, $A_d(\lambda_t)$, $B(\lambda_t)$, $C(\lambda_t)$, $C_d(\lambda_t)$, $D(\lambda_t)$, and $L(\lambda_t)$ will be denoted by A_i , A_{di} , B_i , C_i , C_{di} , D_i , and L_i , respectively.

In this paper, the sampler is time-driven with a sampling period $g > 0$. In the traditional method, the sampled data should be sent to the filter in every sampling period, and this would lead to need of more band resource. To solve this issue, we introduce an event-triggered processor between the sampler and filter to determine whether or not the current sampled data should be transmitted to the filter. It is assumed that the latest released output is t_k , then, the next released instant t_{k+1} is determined by the following condition [37, 38]:

$$\begin{aligned} t_{k+1} = \inf\{t > t_k \mid [y(t) - y(t_k)]^T \Upsilon_i [y(t) - y(t_k)] \\ \geq \rho_i y(t)^T \Upsilon_i y(t)\}, \end{aligned} \quad (5)$$

where $\rho_i \in (0, 1)$ and $\Upsilon_i > 0$ are event-triggered parameters.

For $t \in [t_k, t_{k+1})$, we define the following function:

$$\delta(t) = y(t) - y(t_k). \quad (6)$$

Then, by the event-triggered condition (5), we obtain

$$\rho_i y(t)^T \Upsilon_i y(t) - \delta(t)^T \Upsilon_i \delta(t) > 0. \quad (7)$$

This paper aims to design a filter as follows:

$$\begin{cases} \dot{x}_f(t) = A_f(\lambda_t)x_f(t) + B_f(\lambda_t)y(t_k), \\ z_f(t) = C_f(\lambda_t)x_f(t), \end{cases} \quad t \in [t_k, t_{k+1}), \quad (8)$$

where $x_f(t) \in \mathbb{R}^n$ is the filter state; $z_f(t) \in \mathbb{R}^q$ is the filter output; and $A_f(\lambda_t)$, $B_f(\lambda_t)$, and $C_f(\lambda_t)$ are the filter parameters to be designed. For each $\lambda_t = i \in \mathcal{S}$, the matrices $A_f(\lambda_t)$, $B_f(\lambda_t)$, and $C_f(\lambda_t)$ will be denoted by A_{fi} , B_{fi} , and C_{fi} .

Let $\xi(t) = [x(t) \ x_f(t)]^T$ and $e(t) = z(t) - z_f(t)$. Then, the filtering error system resulting from (1) and (8) is given by

$$\begin{cases} \dot{\xi}(t) = \tilde{A}_i \xi(t) + \tilde{A}_{di} x(t - \tau(t)) + \tilde{B}_{\delta i}(t) \delta(t) + \tilde{B}_{\omega i} \omega(t), \\ e(t) = \tilde{L}_i \xi(t), \end{cases} \quad (9)$$

where

$$\begin{aligned} \tilde{A}_i &= \begin{bmatrix} A_i & 0 \\ B_{fi} C_i & A_{fi} \end{bmatrix}, \\ \tilde{A}_{di} &= \begin{bmatrix} A_{di} \\ B_{fi} C_{di} \end{bmatrix}, \\ \tilde{B}_{\delta i} &= \begin{bmatrix} 0 \\ -B_{fi} \end{bmatrix}, \\ \tilde{B}_{\omega i} &= \begin{bmatrix} B_i \\ B_{fi} D_i \end{bmatrix}, \\ \tilde{L}_i &= [L_i \quad -C_{fi}]. \end{aligned} \quad (10)$$

Throughout this paper, the following definitions will be adopted.

Definition 1 (see [45]). System (1) with $\omega(t) = 0$ is said to be stochastically stable if the following inequality holds for any initial mode λ_0 and initial condition φ :

$$\lim_{t \rightarrow \infty} \mathcal{E} \left\{ \int_0^t x(t)^T x(t) dt \mid x(0), \lambda_0 \right\} < \infty. \quad (11)$$

Definition 2 (see [9, 43]). For given matrices \mathcal{Z} , \mathcal{Y} , \mathcal{X} with $\mathcal{Z} \leq 0$ and \mathcal{X} is symmetric, the filtering error system (9) is said to be dissipative, if there exist $\theta > 0$, such that for any $\mathcal{T} > 0$ and zero initial condition, the following inequality holds:

$$\mathcal{E} \left\{ \int_0^{\mathcal{T}} J(t) dt \right\} \geq \theta \mathcal{E} \left\{ \int_0^{\mathcal{T}} \omega(t)^T \omega(t) dt \right\}, \quad (12)$$

where $J(t) = e(t)^T \mathcal{Z} e(t) + 2e(t)^T \mathcal{Y} \omega(t) + \omega(t)^T \mathcal{X} \omega(t)$.

Noting that $\mathcal{Z} \leq 0$, we can deduce that there always exists matrix $\tilde{\mathcal{Z}} \geq 0$, such that $\mathcal{Z} = -\tilde{\mathcal{Z}}$.

The main purpose of this paper is to design a filter in the form of (8), such that the following conditions hold:

- (1) When $\omega(t) = 0$, the filtering error system (9) is stochastically stable
- (2) When $\omega(t) \neq 0$ and $\omega(t) \in L_2[0, \infty)$, under zero initial condition, the filtering error system (9) is dissipative in the sense of Definition 2

To realize this purpose, we need to introduce the following lemmas.

Lemma 1 (see [44]). For any matrix $Z > 0$, scalars a, b with $b > a$, the following inequality holds:

$$(b-a) \int_a^b \dot{x}(\alpha)^T Z \dot{x}(\alpha) d\alpha \geq \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}^T \begin{bmatrix} Z & 0 \\ 0 & 3Z \end{bmatrix} \begin{bmatrix} \omega_1 \\ \omega_2 \end{bmatrix}, \quad (13)$$

where $\omega_1 = x(b) - x(a)$ and $\omega_2 = x(b) + x(a) - 2/(b-a) \int_a^b x(\alpha) d\alpha$.

The following lemma is a direct result of [45].

Lemma 2 (see [46]). For a given matrix $Z > 0$, scalar $\varepsilon \in (0, 1)$, then there exist matrices Y_1 and Y_2 , such that the following inequality holds:

$$\begin{bmatrix} \frac{1}{\varepsilon} Z & 0 \\ 0 & \frac{1}{1-\varepsilon} Z \end{bmatrix} \geq \text{sym} \{ Y_1 [I_n \quad 0_n] + Y_2 [0_n \quad I_n] \} - \varepsilon Y_1 Z^{-1} Y_1^T - (1-\varepsilon) Y_2 Z^{-1} Y_2^T. \quad (14)$$

Remark 2. It is well recognized that the reciprocally convex combination inequality in [47] is a powerful tool to handle the time-varying delays when analysing the stability of a system with time-varying delays. It has been shown that the inequality in Lemma 2 is a more accurate inequality than the reciprocally convex combination inequality; thus, the stability criteria derived by Lemma 2 is expected to be less conservative.

3. Main Results

In this section, we carry out the analysis of stochastic stability and dissipativity for the resulting filtering error system (9). Then, on the basis of these results, the desired filter design scheme for system (1) is established.

Theorem 1. For given scalars $\tau_1, \tau_2, \mu, 0 < \rho_i < 1$ and matrices $\mathcal{Z}, \mathcal{Y}, \mathcal{X}$ with $\mathcal{Z} \leq 0$ and \mathcal{X} is symmetric, the filtering error system (9) is stochastically stable with dissipative, if there exist matrices $P_i > 0, Q_{1i} > 0, Q_{2i} > 0, Q_{3i} > 0, S > 0, R_i > 0, U > 0, Z_i > 0, V > 0$, and $Y_i > 0$, any matrices Y_1 and Y_2 , such that the following inequalities hold, for each $i \in \mathcal{S}$:

$$\sum_{j=1}^s \phi_{ij}(h) (Q_{1j} + Q_{3j}) - S < 0, \quad (15)$$

$$\sum_{j=1}^s \phi_{ij}(h) (Q_{2j} + Q_{3j}) - S < 0, \quad (16)$$

$$\sum_{j=1}^s \phi_{ij}(h)Q_{3j} - S < 0, \quad (17)$$

$$\tau_1 \sum_{j=1}^s \phi_{ij}(h)R_j - U < 0, \quad (18)$$

$$\tau_{12} \sum_{j=1}^s \phi_{ij}(h)Z_j - V < 0, \quad (19)$$

$$\begin{bmatrix} \Phi_i(\tau_1) & \Psi^T Y_2 \\ * & -\tilde{Z}_i \end{bmatrix} < 0, \quad (20)$$

$$\begin{bmatrix} \Phi_i(\tau_2) & \Psi^T Y_1 \\ * & -\tilde{Z}_i \end{bmatrix} < 0, \quad (21)$$

where

$$\begin{aligned} \Phi_i(\tau(t)) &= W_{2i}^T P_i W_1(\tau(t)) + W_1(\tau(t))^T P_i W_{2i} \\ &+ W_1(\tau(t))^T \left(\sum_{j=1}^s \phi_{ij}(h) P_j \right) W_1(\tau(t)) \\ &+ e_1^T (Q_{1i} + Q_{2i} + Q_{3i} + \tau_2 S) e_1 \\ &- e_3^T Q_{1i} e_3 - (1 - \mu) e_4^T Q_{2i} e_4 \\ &- e_5^T Q_{3i} e_5 + W_{si}^T \left(\tau_1^2 R_i + \frac{1}{2} \tau_1^2 U + \tau_{12}^2 Z_i + \frac{1}{2} (\tau_2^2 - \tau_1^2) V \right) W_{si} \\ &- \Psi^T \text{sym} \{ Y_1 [I_{2n} \ 0_{2n}] + Y_2 [0_{2n} \ I_{2n}] \} \Psi - W_3^T \tilde{R}_i W_3 \\ &+ \rho_i (C_i e_1 + C_{di} e_4 + D_i e_{10})^T \\ &\cdot Y_i (C_i e_1 + C_{di} e_4 + D_i e_{10}) - e_9^T Y_i e_9 \\ &- W_0^T \tilde{L}_i^T \mathcal{X} \tilde{L}_i W_0 - 2W_0^T \tilde{L}_i^T \mathcal{Y} e_{10} - e_{10}^T \mathcal{X} e_{10}, \end{aligned}$$

$$W_{si} = [A_i \ 0_{n,2n} \ A_{di} \ 0_{n,5n} \ B_i],$$

$$\tilde{W}_{si} = [\tilde{A}_i \ 0_{2n,n} \ \tilde{A}_{di} \ 0_{2n,4n} \ \tilde{B}_{di} \ \tilde{B}_{wi}],$$

$$W_0 = \text{col}\{e_1, e_2\}, \quad \tau_{12} = \tau_2 - \tau_1,$$

$$W_1(\tau(t)) = \text{col}\{W_0, \tau_1 e_6, (\tau(t) - \tau_1) e_7 - (\tau_2 - \tau(t)) e_8\},$$

$$W_{2i} = \text{col}\{\tilde{W}_{si}, e_1 - e_3, e_3 - e_5\},$$

$$W_3 = \text{col}\{e_1 - e_3, e_1 + e_3 - 2e_6\},$$

$$W_4 = \text{col}\{e_3 - e_4, e_3 + e_4 - 2e_7\},$$

$$W_5 = \text{col}\{e_4 - e_5, e_4 + e_5 - 2e_8\},$$

$$e_j = [0_{n,(j-1)n} \ I_n \ 0_{n,(8-j)n+p+r}], \quad j = 1, \dots, 8,$$

$$e_9 = [0_{p,8n} \ I_p \ 0_{p,r}],$$

$$e_{10} = [0_{r,8n+p} \ I_r],$$

$$\Psi = \text{col}\{W_4, W_5\},$$

$$\tilde{Z}_i = \text{diag}(Z_i, 3Z_i),$$

$$\tilde{R}_i = \text{diag}(R_i, 3R_i).$$

Proof. We define the following augmented vector:

$$\zeta(t) = \text{col} \left\{ \xi(t), \int_{t-\tau_1}^t x(\alpha) d\alpha, \int_{t-\tau_2}^{t-\tau_1} x(\alpha) d\alpha \right\}. \quad (23)$$

Then, we choose a Lyapunov-Krasovskii functional candidate for system (9) as follows:

$$V(\xi_t, \lambda_t, t) = \sum_{j=1}^4 V_j(t), \quad (24)$$

where $\xi_t = \xi(t + \sigma)$, $\sigma \in [-\tau_2, 0]$, and

$$V_1(t) = \zeta(t)^T P_i \zeta(t),$$

$$\begin{aligned} V_2(t) &= \int_{t-\tau_1}^t x(\alpha)^T Q_{1i} x(\alpha) d\alpha + \int_{t-\tau(t)}^{t-\tau_1} x(\alpha)^T Q_{2i} x(\alpha) d\alpha \\ &+ \int_{t-\tau_2}^t x(\alpha)^T Q_{3i} x(\alpha) d\alpha + \int_{-\tau_2}^0 \int_{t+\beta}^t x(\alpha)^T S x(\alpha) d\alpha, \end{aligned}$$

$$\begin{aligned} V_3(t) &= \tau_1 \int_{-\tau_1}^0 \int_{t+\beta}^t \dot{x}(\alpha)^T R_i \dot{x}(\alpha) d\alpha d\beta \\ &+ \int_{-\tau_1}^0 \int_{\theta}^0 \int_{t+\beta}^t \dot{x}(\alpha)^T U \dot{x}(\alpha) d\alpha d\beta d\theta, \end{aligned}$$

$$\begin{aligned} V_4(t) &= \tau_{12} \int_{-\tau_2}^{-\tau_1} \int_{t+\beta}^t \dot{x}(\alpha)^T Z_i \dot{x}(\alpha) d\alpha d\beta \\ &+ \int_{-\tau_2}^{-\tau_1} \int_{\theta}^0 \int_{t+\beta}^t \dot{x}(\alpha)^T V \dot{x}(\alpha) d\alpha d\beta d\theta. \end{aligned} \quad (25)$$

Let \mathcal{L} be the weak infinitesimal generator of the random process $\{\xi_t, \lambda_t\}$ and $\eta(t) = \text{col} \{x(t)x_f(t), x(t - \tau_1), x(t - \tau(t)), x(t - \tau_2), 1/\tau_1 \int_{t-\tau_1}^t x(\alpha) d\alpha, 1/(\tau(t) - \tau_1) \int_{t-\tau(t)}^{t-\tau_1} x(\alpha) d\alpha, 1/(\tau_2 - \tau(t)) \int_{t-\tau_2}^{t-\tau(t)} x(\alpha) d\alpha, \delta(t), \omega(t)\}$. Then, for each $i \in \mathcal{S}$, following the similar lines as in [13, 27], we obtain

$$\begin{aligned} \mathcal{L}V_1(t) &= \dot{\zeta}(t)^T P_i \zeta(t) + \zeta(t)^T P_i \dot{\zeta}(t) \\ &+ \zeta(t)^T \left(\sum_{j=1}^s \phi_{ij}(h) P_j \right) \zeta(t) \\ &= \eta(t)^T \left[W_{2i}^T P_i W_1(\tau(t)) + W_1(\tau(t))^T P_i W_{2i} \right. \\ &\quad \left. + W_1(\tau(t))^T \sum_{j=1}^s \phi_{ij}(h) P_j W_1(\tau(t)) \right] \eta(t). \end{aligned} \quad (26)$$

(22)

By means of (15)–(17), one has

$$\begin{aligned}
 \mathcal{L}V_2(t) &\leq x(t)^T(Q_{1i} + Q_{2i} + Q_{3i} + \tau_2 S)x(t) - x(t - \tau_1)^T Q_{1i} x(t - \tau_1) \\
 &\quad - (1 - \mu)x(t - \tau(t))^T Q_{2i} x(t - \tau(t)) - x(t - \tau_2)^T Q_{3i} x(t - \tau_2) \\
 &\quad + \int_{t-\tau_1}^t x(\alpha)^T \left(\sum_{j=1}^s \phi_{ij}(h) Q_{1j} + \sum_{j=1}^s \phi_{ij}(h) Q_{3j} - S \right) x(\alpha) d\alpha \\
 &\quad + \int_{t-\tau(t)}^{t-\tau_1} x(\alpha)^T \left(\sum_{j=1}^s \phi_{ij}(h) Q_{2j} + \sum_{j=1}^s \phi_{ij}(h) Q_{3j} - S \right) x(\alpha) d\alpha \\
 &\quad + \int_{t-\tau_2}^{t-\tau(t)} x(\alpha)^T \left(\sum_{j=1}^s \phi_{ij}(h) Q_{3j} - S \right) x(\alpha) d\alpha \\
 &\leq \eta(t)^T \left[e_1^T (Q_{1i} + Q_{2i} + Q_{3i} + \tau_2 S) e_1 - e_3^T Q_{1i} e_3 - (1 - \mu) e_4^T Q_{2i} e_4 - e_5^T Q_{3i} e_5 \right] \eta(t).
 \end{aligned} \tag{27}$$

In terms of (18), we deduce that

According to Lemma 1, one has

$$\begin{aligned}
 \mathcal{L}V_3(t) &= \tau_1^2 \dot{x}(t)^T R_i \dot{x}(t) - \tau_1 \int_{t-\tau_1}^t \dot{x}(\alpha)^T R_i \dot{x}(\alpha) d\alpha \\
 &\quad + \tau_1 \int_{-\tau_1}^0 \int_{t+\beta}^t \dot{x}(\alpha)^T \sum_{j=1}^s \phi_{ij}(h) R_j \dot{x}(\alpha) d\alpha d\beta \\
 &\quad + \frac{1}{2} \tau_1^2 \dot{x}(t)^T U \dot{x}(t) - \int_{-\tau_1}^0 \int_{t+\beta}^t \dot{x}(\alpha)^T U \dot{x}(\alpha) d\alpha d\beta \\
 &\leq \dot{x}(t)^T \left(\tau_1^2 R_i + \frac{1}{2} \tau_1^2 U \right) \dot{x}(t) - \tau_1 \int_{t-\tau_1}^t \dot{x}(\alpha)^T R_i \dot{x}(\alpha) d\alpha.
 \end{aligned} \tag{28}$$

$$\begin{aligned}
 -\tau_1 \int_{t-\tau_1}^t \dot{x}(\alpha)^T R_i \dot{x}(\alpha) d\alpha &\leq - \begin{bmatrix} x(t) - x(t - \tau_1) \\ x(t) + x(t - \tau_1) - \frac{2}{\tau_1} \int_{t-\tau_1}^t x(\alpha) d\alpha \end{bmatrix}^T \begin{bmatrix} R_i & 0 \\ 0 & 3R_i \end{bmatrix} \times \begin{bmatrix} x(t) - x(t - \tau_1) \\ x(t) + x(t - \tau_1) - \frac{2}{\tau_1} \int_{t-\tau_1}^t x(\alpha) d\alpha \end{bmatrix} \\
 &= -\eta(t)^T \begin{bmatrix} e_1 - e_3 \\ e_1 + e_3 - 2e_6 \end{bmatrix}^T \tilde{R}_i \begin{bmatrix} e_1 - e_3 \\ e_1 + e_3 - 2e_6 \end{bmatrix} \eta(t) \\
 &= -\eta(t)^T W_3^T \tilde{R}_i W_3 \eta(t).
 \end{aligned} \tag{29}$$

Then, it follows from (28) and (29) that

Now, $\mathcal{L}V_4(t)$ is obtained as

$$\mathcal{L}V_3(t) \leq \eta(t)^T \left[W_{si}^T \left(\tau_1^2 R_i + \frac{1}{2} \tau_1^2 U \right) W_{si} - W_3^T \tilde{R}_i W_3 \right] \eta(t). \tag{30}$$

$$\begin{aligned}
 \mathcal{L}V_4(t) &= \tau_{12}^2 \dot{x}(t)^T Z_i \dot{x}(t) - \tau_{12} \int_{t-\tau_2}^{t-\tau_1} \dot{x}(\alpha)^T Z_i \dot{x}(\alpha) d\alpha + \tau_{12} \int_{-\tau_2}^{-\tau_1} \int_{t+\beta}^t \dot{x}(\alpha)^T \left(\sum_{j=1}^s \phi_{ij}(h) Z_j \right) \dot{x}(\alpha) d\alpha d\beta \\
 &\quad + \frac{1}{2} (\tau_2^2 - \tau_1^2) \dot{x}(t)^T V \dot{x}(t) - \int_{-\tau_2}^{-\tau_1} \int_{t+\beta}^t \dot{x}(\alpha)^T V \dot{x}(\alpha) d\alpha d\beta.
 \end{aligned} \tag{31}$$

Splitting the following integral into two terms, we obtain

$$\begin{aligned} -\tau_{12} \int_{t-\tau_2}^{t-\tau_1} \dot{x}(\alpha)^T Z_i \dot{x}(\alpha) d\alpha &= -\tau_{12} \int_{t-\tau(t)}^{t-\tau_1} \dot{x}(\alpha)^T Z_i \dot{x}(\alpha) d\alpha \\ &\quad - \tau_{12} \int_{t-\tau_2}^{t-\tau(t)} \dot{x}(\alpha)^T Z_i \dot{x}(\alpha) d\alpha. \end{aligned} \quad (32)$$

By Lemma 1, it can be verified that

$$\begin{aligned} &-\tau_{12} \int_{t-\tau(t)}^{t-\tau_1} \dot{x}(\alpha)^T Z_i \dot{x}(\alpha) d\alpha \\ &\leq -\frac{\tau_{12}}{\tau(t)-\tau_1} \begin{bmatrix} x(t-\tau_1) - x(t-\tau(t)) \\ x(t-\tau_1) + x(t-\tau(t)) - \frac{2}{(\tau(t)-\tau_1)} \int_{t-\tau(t)}^{t-\tau_1} x(\alpha) d\alpha \end{bmatrix}^T \begin{bmatrix} Z_i & 0 \\ 0 & 3Z_i \end{bmatrix} \\ &\quad \times \begin{bmatrix} x(t-\tau_1) - x(t-\tau(t)) \\ x(t-\tau_1) + x(t-\tau(t)) - \frac{2}{(\tau(t)-\tau_1)} \int_{t-\tau(t)}^{t-\tau_1} x(\alpha) d\alpha \end{bmatrix} \end{aligned} \quad (33)$$

$$\begin{aligned} &= -\frac{\tau_{12}}{\tau(t)-\tau_1} \eta(t)^T \begin{bmatrix} e_3 - e_4 \\ e_3 + e_4 - 2e_7 \end{bmatrix}^T \tilde{Z}_i \begin{bmatrix} e_3 - e_4 \\ e_3 + e_4 - 2e_7 \end{bmatrix} \eta(t) \\ &= -\frac{\tau_{12}}{\tau(t)-\tau_1} \eta(t)^T W_4^T \tilde{Z}_i W_4 \eta(t), \\ &-\tau_{12} \int_{t-\tau_2}^{t-\tau(t)} \dot{x}(\alpha)^T Z_i \dot{x}(\alpha) d\alpha \\ &\leq -\frac{\tau_{12}}{\tau_2-\tau(t)} \begin{bmatrix} x(t-\tau(t)) - x(t-\tau_2) \\ x(t-\tau(t)) + x(t-\tau_2) - \frac{2}{(\tau_2-\tau(t))} \int_{t-\tau_2}^{t-\tau(t)} x(\alpha) d\alpha \end{bmatrix}^T \begin{bmatrix} Z_i & 0 \\ 0 & 3Z_i \end{bmatrix} \\ &\quad \times \begin{bmatrix} x(t-\tau(t)) - x(t-\tau_2) \\ x(t-\tau(t)) + x(t-\tau_2) - \frac{2}{(\tau_2-\tau(t))} \int_{t-\tau_2}^{t-\tau(t)} x(\alpha) d\alpha \end{bmatrix} \\ &= -\frac{\tau_{12}}{\tau_2-\tau(t)} \eta(t)^T \begin{bmatrix} e_4 - e_5 \\ e_4 + e_5 - 2e_8 \end{bmatrix}^T \tilde{Z}_i \begin{bmatrix} e_4 - e_5 \\ e_4 + e_5 - 2e_8 \end{bmatrix} \eta(t) \\ &= -\frac{\tau_{12}}{\tau_2-\tau(t)} \eta(t)^T W_5^T \tilde{Z}_i W_5 \eta(t). \end{aligned} \quad (34)$$

In light of (33)-(34) and Lemma 2, there exist matrices Y_1 and Y_2 , such that

$$\begin{aligned}
 & -\tau_{12} \int_{t-\tau_2}^{t-\tau_1} \dot{x}(\alpha)^T Z_i \dot{x}(\alpha) d\alpha \\
 \leq & -\frac{\tau_{12}}{\tau(t) - \tau_1} \eta(t)^T W_4^T \tilde{Z}_i W_4 \eta(t) \\
 & -\frac{\tau_{12}}{\tau_2 - \tau(t)} \eta(t)^T W_5^T \tilde{Z}_i W_5 \eta(t) \\
 \leq & -\eta(t)^T \Psi^T \text{sym}\{Y_1 [I_{2n} \ 0_{2n}] + Y_2 [0_{2n} \ I_{2n}]\} \Psi \eta(t) \\
 & + \frac{\tau(t) - \tau_1}{\tau_{12}} \eta(t)^T \Psi^T Y_1 \tilde{Z}_i^{-1} Y_1^T \Psi \eta(t) \\
 & + \frac{\tau_2 - \tau(t)}{\tau_{12}} \eta(t)^T \Psi^T Y_2 \tilde{Z}_i^{-1} Y_2^T \Psi \eta(t).
 \end{aligned} \tag{35}$$

Then, it follows from (31)-(35) together with (19) that

$$\begin{aligned}
 \mathcal{L}V_4(t) \leq & \dot{x}(t)^T \left[\tau_{12}^2 Z_i + \frac{1}{2} (\tau_2^2 - \tau_1^2) V \right] \dot{x}(t) \\
 & - \tau_{12} \int_{t-\tau_2}^{t-\tau_1} \dot{x}(\alpha)^T Z_i \dot{x}(\alpha) d\alpha \\
 \leq & \eta(t)^T \left[W_{si}^T \left(\tau_{12}^2 Z_i + \frac{1}{2} (\tau_2^2 - \tau_1^2) V \right) W_{si} \right] \eta(t) \\
 & - \eta(t)^T \Psi^T \text{sym}\{Y_1 [I_{2n} \ 0_{2n}] + Y_2 [0_{2n} \ I_{2n}]\} \Psi \eta(t) \\
 & + \frac{\tau(t) - \tau_1}{\tau_{12}} \eta(t)^T \Psi^T Y_1 \tilde{Z}_i^{-1} Y_1^T \Psi \eta(t) \\
 & + \frac{\tau_2 - \tau(t)}{\tau_{12}} \eta(t)^T \Psi^T Y_2 \tilde{Z}_i^{-1} Y_2^T \Psi \eta(t).
 \end{aligned} \tag{36}$$

Then, by (24)-(27), (30), and (36), we arrive at

$$\begin{aligned}
 \mathcal{L}V(t) - J(t) + \rho_i y(t)^T Y_i y(t) - \delta(t)^T Y_i \delta(t) \\
 \leq \eta(t)^T \tilde{\Phi}_i(\tau(t)) \eta(t),
 \end{aligned} \tag{37}$$

where

$$\begin{aligned}
 \tilde{\Phi}_i(\tau(t)) = & \Phi_i(\tau(t)) + \frac{\tau(t) - \tau_1}{\tau_{12}} \Psi^T Y_1 \tilde{Z}_i^{-1} Y_1^T \Psi \\
 & + \frac{\tau_2 - \tau(t)}{\tau_{12}} \Psi^T Y_2 \tilde{Z}_i^{-1} Y_2^T \Psi,
 \end{aligned} \tag{38}$$

and $\Phi_i(\tau(t))$ is defined in Theorem 1.

By convex combination theory, it can be deduced that $\tilde{\Phi}_i(\tau(t)) < 0$ holds if $\tilde{\Phi}_i(\tau_1) < 0$ and $\tilde{\Phi}_i(\tau_2) < 0$. These two inequalities can be obtained by applying Schur complement to (20) and (21), respectively.

Therefore, inequalities (7) and (37) lead to

$$\mathcal{L}V(t) - J(t) < 0. \tag{39}$$

Using the well-known Dykin's formula to (39), we obtain that, for any $\mathcal{T} \geq 0$,

$$\begin{aligned}
 \mathcal{E} \left\{ \int_0^{\mathcal{T}} J(t) dt \right\} & > \mathcal{E} \{ V(\xi_t, \lambda_t, t) \} \\
 & - V(\xi_0, \lambda_0, 0) > 0.
 \end{aligned} \tag{40}$$

Let $\mathcal{T} \rightarrow \infty$. It follows from (40) that there exists $\theta > 0$, such that inequality (12) holds. Thus, we conclude that the filtering error system (9) is dissipative in the sense of Definition 2.

Next, we will prove that the filtering error system (9) is stochastically stable. Let $\omega(t) = 0$, then (39) leads to

$$\mathcal{L}V(t) < e(t)^T \mathcal{L}e(t). \tag{41}$$

Noticing that $\mathcal{L} \leq 0$, we obtain that there exists a sufficient small $\varsigma > 0$, such that $\mathcal{L}V(t) < -\varsigma |x(t)|^2$. Then, along a similar line as in the proof of [41], we derive that the filtering error system (9) is stochastically stable. The proof is complete here. \square

Remark 3. Theorem 1 provides a sufficient condition such that the filtering error system (9) is stochastically stable and dissipative. It is noted that this condition is dependent not only on system modes and time delays but also on event-triggered parameters.

In order to compare with previous results, we consider the case when system (1) with $\omega(t) = 0$:

$$\dot{x}(t) = A_i x(t) + A_{di} x(t - \tau(t)). \tag{42}$$

The stochastic stability condition of system (42) can be derived from Theorem 1 immediately. We give the result in the following corollary.

Corollary 1. For given scalars τ_1 , τ_2 , and μ , system (42) is stochastically stable if there exist matrices $P_i > 0$, $Q_{1i} > 0$, $Q_{2i} > 0$, $Q_{3i} > 0$, $S > 0$, $R_i > 0$, $U > 0$, $Z_i > 0$, $V > 0$, and any matrices Y_1 and Y_2 , such that inequalities (15)-(19) and the following inequalities hold, for each $i \in \mathcal{S}$:

$$\begin{aligned} & \begin{bmatrix} \Xi_i(\tau_1) & \tilde{\Psi}^T Y_2 \\ * & -\tilde{Z}_i \end{bmatrix} < 0, \\ & \begin{bmatrix} \Xi_i(\tau_2) & \tilde{\Psi}^T Y_1 \\ * & -\tilde{Z}_i \end{bmatrix} < 0, \end{aligned} \quad \text{where} \quad (43)$$

$$\begin{aligned} E_i(\tau(t)) = & E_{2i}^T P_i E_1(\tau(t)) + E_1(\tau(t))^T P_i E_{2i} + E_1(\tau(t))^T \left(\sum_{j=1}^s \phi_{ij}(h) P_j \right) E_1(\tau(t)) \\ & + \tilde{e}_1^T (Q_{1i} + Q_{2i} + Q_{3i} + \tau_2 S) \tilde{e}_1 - \tilde{e}_2^T Q_{1i} \tilde{e}_2 - (1 - \mu) \tilde{e}_3^T Q_{2i} \tilde{e}_3 - \tilde{e}_4^T Q_{3i} \tilde{e}_4 \\ & + (A_i \tilde{e}_1 + A_{di} \tilde{e}_3)^T \left(\tau_1^2 R_i + \frac{1}{2} \tau_1^2 U + \tau_{12}^2 Z_i + \frac{1}{2} (\tau_2^2 - \tau_1^2) V \right) (A_i \tilde{e}_1 + A_{di} \tilde{e}_3) \\ & - E_3^T \tilde{R}_i E_3 - \tilde{\Psi}^T \text{sym}\{Y_1 [I_{2n} \ 0_{2n}] + Y_2 [0_{2n} \ I_{2n}]\} \tilde{\Psi}, \end{aligned}$$

$$\tilde{e}_i = \begin{pmatrix} 0_{n, (i-1)n} & I_n & 0_{n, (7-i)n} \end{pmatrix}, \quad i = 1, 2, \dots, 7, \quad (44)$$

$$E_1(\tau(t)) = \text{col}\{\tilde{e}_1, \tau_1 \tilde{e}_5, (\tau(t) - \tau_1) \tilde{e}_6 + (\tau_2 - \tau(t)) \tilde{e}_7\},$$

$$E_{2i} = \text{col}\{A_i \tilde{e}_1 + A_{di} \tilde{e}_3, \tilde{e}_1 - \tilde{e}_2, \tilde{e}_2 - \tilde{e}_4\},$$

$$E_3 = \text{col}\{\tilde{e}_1 - \tilde{e}_2, \tilde{e}_1 + \tilde{e}_2 - 2\tilde{e}_5\},$$

$$E_4 = \text{col}\{\tilde{e}_2 - \tilde{e}_3, \tilde{e}_2 + \tilde{e}_3 - 2\tilde{e}_6\},$$

$$E_5 = \text{col}\{\tilde{e}_3 - \tilde{e}_4, \tilde{e}_3 + \tilde{e}_4 - 2\tilde{e}_7\},$$

$$\tilde{\Psi} = \text{col}\{E_4, E_5\}.$$

Remark 4. Corollary 1 provides a new stochastic stability criterion for SMJSs (42). It is important to note that the condition in Corollary 1 is less conservative than some of the existing ones in the literature, which will be shown by a numerical example in Section 4.

Based on Theorem 1, we give the filter parameter design scheme in the following theorem:

Theorem 2. For given scalars τ_1 , τ_2 , and μ , $0 < \rho_i < 1$, the filtering error system (9) is stochastically stable with dissipativity, if there exist matrices $Q_{1i} > 0$, $Q_{2i} > 0$, $Q_{3i} > 0$, $S > 0$, $R_i > 0$, $U > 0$, $Z_i > 0$, $V > 0$, $M_i > 0$, $X_i > 0$, $P_{2i} > 0$, $P_{3i} > 0$, and $Y_i > 0$, any matrices Y_1 , Y_2 , \bar{A}_{fip} , \bar{B}_{fip} , and \bar{C}_{fip} such that inequalities (15)–(19) and the following inequalities hold, for each $i \in \mathcal{S}$:

$$\begin{bmatrix} M_i & X_i \\ X_i & X_i \end{bmatrix} > 0, \quad (45)$$

$$\begin{bmatrix} \hat{\Phi}_i(\tau_1) & \Psi^T Y_2 & W_0^T \tilde{L}_i^T \tilde{\mathcal{Z}}^T \\ * & -\tilde{Z}_i & 0 \\ * & * & -I \end{bmatrix} < 0, \quad (46)$$

$$\begin{bmatrix} \hat{\Phi}_i(\tau_2) & \Psi^T Y_1 & W_0^T \tilde{L}_i^T \tilde{\mathcal{Z}}^T \\ * & -\tilde{Z}_i & 0 \\ * & * & -I \end{bmatrix} < 0, \quad (47)$$

where

$$\begin{aligned}
 \widehat{\Phi}_i(\tau(t)) &= \widehat{W}_{2i}^T W_1(\tau(t)) + W_1(\tau(t))^T \widehat{W}_{2i} + W_1(\tau(t))^T \left(\sum_{j=1}^s \phi_{ij}(h) P_j \right) W_1(\tau(t)) \\
 &+ e_1^T (Q_{1i} + Q_{2i} + Q_{3i} + \tau_2 S) e_1 - e_3^T Q_{1i} e_3 - (1 - \mu) e_4^T Q_{2i} e_4 \\
 &- e_5^T Q_{3i} e_5 + W_{si}^T \left(\tau_1^2 R_i + \frac{1}{2} \tau_1^2 U + \tau_{12}^2 Z_i + \frac{1}{2} (\tau_2^2 - \tau_1^2) V \right) W_{si} \\
 &- \Psi^T \text{sym} \{ Y_1 [I_{2n} \ 0_{2n}] + Y_2 [0_{2n} \ I_{2n}] \} \Psi - W_3^T \bar{R}_i W_3 \\
 &+ \rho_i (C_i e_1 + C_{di} e_4 + D_i e_{10})^T Y_i (C_i e_1 + C_{di} e_4 + D_i e_{10}) - e_9^T Y_i e_9 \\
 &- 2W_0^T \bar{L}_i^T \mathcal{Y} e_{10} - e_{10}^T \mathcal{X} e_{10}, \\
 \widehat{W}_{2i} &= \text{col} \{ \widehat{W}_{si}, P_{2i} (e_1 - e_3), P_{3i} (e_3 - e_5) \}, \\
 \widehat{W}_{si} &= \text{col} \{ \widehat{A}_i, 0_{2n,n}, \widehat{A}_{di}, 0_{2n,4n}, \widehat{B}_{\delta i}, \widehat{B}_{\omega i} \}, \\
 \widehat{A}_i &= \begin{bmatrix} M_i A_i + \bar{B}_{fi} C_i & \bar{A}_{fi} \\ X_i A_i + \bar{B}_{fi} C_i & \bar{A}_{fi} \end{bmatrix}, \\
 \widehat{A}_{di} &= \begin{bmatrix} M_i A_{di} + \bar{B}_{fi} C_{di} \\ X_i A_{di} + \bar{B}_{fi} C_{di} \end{bmatrix}, \\
 \widehat{B}_{\delta i} &= \begin{bmatrix} -\bar{B}_{fi} \\ -\bar{B}_{fi} \end{bmatrix}, \\
 \widehat{B}_{\omega i} &= \begin{bmatrix} M_i B_i + \bar{B}_{fi} D_i \\ X_i B_i + \bar{B}_{fi} D_i \end{bmatrix}.
 \end{aligned} \tag{48}$$

In this case, a desired filter is obtained with the parameters given by

$$\begin{aligned}
 A_{fi} &= X_i^{-1} \bar{A}_{fi}, \\
 B_{fi} &= X_i^{-1} \bar{B}_{fi}, \\
 C_{fi} &= \bar{C}_{fi}.
 \end{aligned} \tag{49}$$

Proof. Define $P_{li} = \begin{bmatrix} M_i & X_i \\ X_i & X_i \end{bmatrix}$ and $P_i = \text{diag}(P_{1i}, P_{2i}, P_{3i})$. It can be seen from (45) that $P_i > 0$.

Let $\bar{A}_{fi} = X_i A_{fi}$, $\bar{B}_{fi} = X_i B_{fi}$, and $\bar{C}_{fi} = C_{fi}$. Then, substituting those matrices and \bar{P}_i into (46)-(47), we obtain (20) and (21). Therefore, according to Theorem 1, we easily draw the conclusion that the filtering error system (9) is stochastically stable and dissipative; thus, the proof is completed. \square

Remark 5. It should be pointed out that the results given in Theorem 1 are nonconvex inequalities, so it should be further discussed. Motivated by [13, 27], the transition rate $\phi_{ij}(h)$ is usually partly measurable in practice while it satisfies $\underline{\phi}_{ij} \leq \phi_{ij}(h) \leq \bar{\phi}_{ij}$. Then, the $\phi_{ij}(h)$ can be rewritten as the following:

$$\phi_{ij}(h) = \sum_{\alpha=1}^N \psi_{\alpha} \phi_{ij,\alpha}, \quad \sum_{\alpha=1}^N \psi_{\alpha} = 1, \quad \psi_{\alpha} \geq 0,$$

$$\phi_{ij,\alpha} = \begin{cases} \frac{\phi_{ij} + (\alpha - 1) \frac{\bar{\phi}_{ij} - \phi_{ij}}{N - 1}}{\alpha}, & i \neq j, j \in \mathcal{S}, \\ \frac{\bar{\phi}_{ij} - (\alpha - 1) \frac{\bar{\phi}_{ij} - \phi_{ij}}{N - 1}}{\alpha}, & i = j, j \in \mathcal{S}. \end{cases} \quad (50)$$

Now, we give the conditions for the solvability of the present dissipative filter design problem.

Theorem 3. For given scalars τ_1 , τ_2 , μ , and ρ_i , the filtering error system (9) is stochastically stable with dissipativity, if there exist matrices $Q_{1i} > 0$, $Q_{2i} > 0$, $Q_{3i} > 0$, $S > 0$, $R_i > 0$, $U > 0$, $Z_i > 0$, $V > 0$, $M_i > 0$, $X_i > 0$, $P_{2i} > 0$, $P_{3i} > 0$, and $Y_i > 0$, any matrices Y_1 , Y_2 , \bar{A}_{fi} , \bar{B}_{fi} , and \bar{C}_{fi} , such that inequalities (15)–(19) and (45)–(47) hold, for each $i \in \mathcal{S}$, and $\phi_{ij}(h) = \phi_{ij,\alpha}$, $\alpha = 1, 2, \dots, N$. Then, the desired filter parameters can be shown as in (49).

Remark 6. It is noted that the conditions obtained in Theorem 3 are presented in terms of linear matrix inequalities, which can be checked easily by the LMI toolbox of MATLAB.

4. Numerical Examples

Example 1. Consider a semi-Markovian jumping system in (42) with the following parameters [28]:

$$A_1 = \begin{bmatrix} -2 & 0 \\ 0 & -0.9 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -1 & 0.5 \\ 0 & -1 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -1 & 0 \\ -1 & -1 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -1 & 0 \\ -0.1 & -1 \end{bmatrix}. \quad (51)$$

To compare the stability criterion in Corollary 1 with those in [28, 41, 48, 49], we assume that $\tau_1 = 1$, $\mu = 0.5$, $\phi_{11} = -0.1$, and $\phi_{22} = -0.8$. Table 1 lists the maximum allowed τ_2 calculated by the method in [28, 41, 48, 49] and Corollary 1. It can be seen from Table 1 that the result in Corollary 1 is less conservative than those in [28, 41, 48, 49]

TABLE 1: Maximum allowable τ_2 using different methods.

Methods	τ_2
Theorem 1 of [41]	1.471
Theorem 1 of [48]	1.660
Theorem 1 of [49] ($m = 5$)	1.753
Theorem 1 of [28] ($l = 4$)	1.807
Corollary 1	1.982

for this example. It should be noted that the reduced conservatism in Corollary 1 benefits from using the inequalities in Lemma 1 and Lemma 2.

Example 2. Consider the semi-Markovian jumping system in (1) with the following parameters:

$$A_1 = \begin{bmatrix} -3.4888 & 0.8057 \\ -0.6451 & -3.2684 \end{bmatrix},$$

$$A_2 = \begin{bmatrix} -2.4898 & 0.2895 \\ 1.3396 & -0.0211 \end{bmatrix},$$

$$A_{d1} = \begin{bmatrix} -0.8620 & -1.2919 \\ -0.6841 & -2.0729 \end{bmatrix},$$

$$A_{d2} = \begin{bmatrix} -2.8306 & 0.4978 \\ -0.8436 & -1.0115 \end{bmatrix},$$

$$B_1 = \begin{bmatrix} 0.0403 \\ 0.6771 \end{bmatrix},$$

$$B_2 = \begin{bmatrix} 0.5689 \\ 0.2556 \end{bmatrix},$$

$$C_1 = [-0.3775 \quad -0.2959],$$

$$C_2 = [-0.4751 \quad -0.2340],$$

$$C_{d1} = [0.2532 \quad -0.1684],$$

$$C_{d2} = [-0.3010 \quad -0.3564],$$

$$L_1 = [-1.3324 \quad -1.5532],$$

$$L_2 = [-1.4562 \quad -1.2322],$$

$$D_1 = -1.1184,$$

$$D_2 = -0.3148. \quad (52)$$

In this example, the parameters are given by $\tau_1 = 0.1$, $\tau_2 = 0.5$, $\phi_{11}(h) \in (-2.2, -1.8)$, $\phi_{22}(h) \in (-3.4, -2.6)$, $\rho_1 = 0.1$, $\rho_2 = 0.2$, $\mathcal{L} = -4$, $\mathcal{Y} = -1$, and $\mathcal{X} = 2$. Then, by solving the LMIs in Theorem 3, we obtain the dissipative filter parameters as follows:

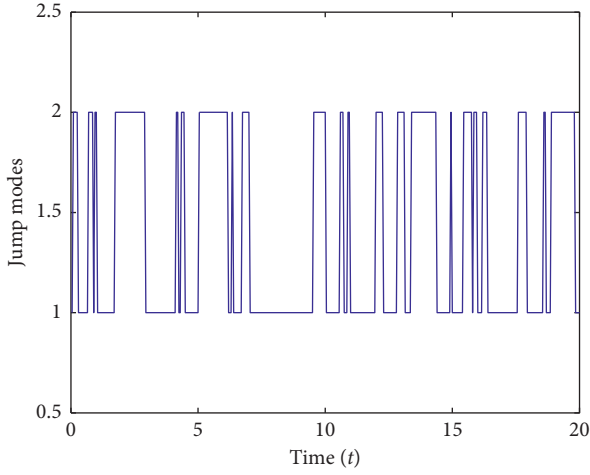
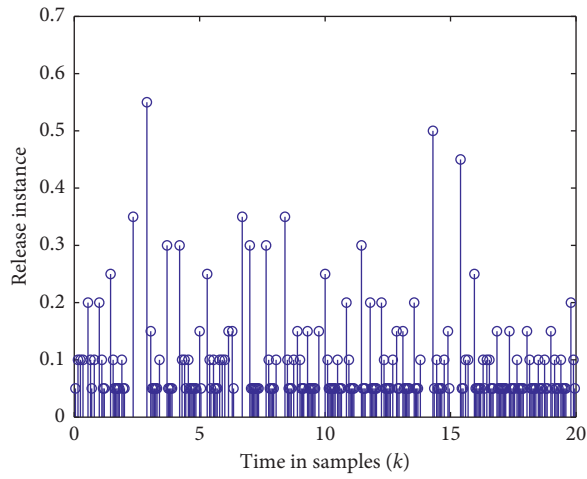
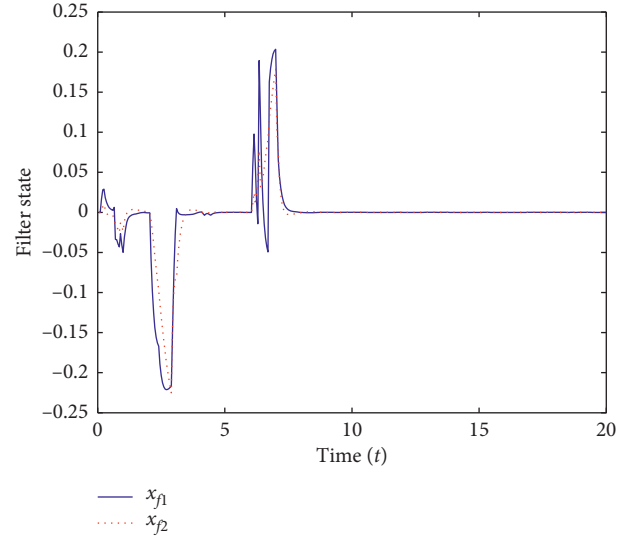
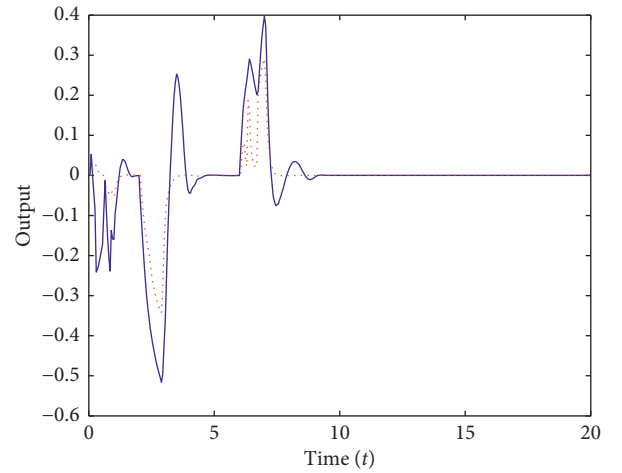

 FIGURE 1: Random jump modes λ_t .


FIGURE 2: Release instants and release interval.

$$\begin{aligned}
 A_{f1} &= \begin{bmatrix} -6.8697 & 0.8567 \\ -3.1332 & -6.4577 \end{bmatrix}, \\
 B_{f1} &= \begin{bmatrix} -0.4229 \\ 0.4993 \end{bmatrix}, \\
 C_{f1} &= \begin{bmatrix} 1.1557 \\ 0.8781 \end{bmatrix}^T, \\
 A_{f2} &= \begin{bmatrix} -5.8309 & -1.4748 \\ 1.3407 & -1.9390 \end{bmatrix}, \\
 B_{f2} &= \begin{bmatrix} 3.4855 \\ 0.6502 \end{bmatrix}, \\
 C_{f1} &= \begin{bmatrix} 0.6011 \\ 0.9456 \end{bmatrix}^T, \\
 Y_1 &= 4.8762, \\
 Y_2 &= 13.9442.
 \end{aligned} \tag{53}$$


 FIGURE 3: Filter states $x_f(t)$.

 FIGURE 4: Estimated signal $z(t)$ and filter output signal $z_f(t)$.

In the simulation, we choose the initial condition $x(0) = [-1 \ 1]^T$ and $x_f(0) = [0 \ 0]^T$, the external disturbance is

$$\omega(t) = \begin{cases} 1, & 2 \leq t \leq 3, \\ -1, & 6 \leq t \leq 7, \\ 0, & \text{otherwise.} \end{cases} \tag{54}$$

Then, the simulation results are provided in Figures 1–4. The possible jump mode λ_t is illustrated in Figure 1. Figure 2 shows the event-triggered release instants and release intervals. Figure 3 plotted the filter state $x_f(t)$. The filter error signal $e(t)$ is presented in Figure 4. These simulation results clearly demonstrate that the filter design scheme developed in this paper is effective.

5. Conclusion

The issue of the dissipative filter for SMJSs with time-varying delays has been investigated in this paper. It should be noted that the event-triggered scheme is employed in order to save the bandwidth resource. The presented scheme provides a sufficient condition to guarantee the resulting filtering error system is stochastically stable and satisfies dissipative. What should be emphasized is that the stability criterion achieved in this paper is less conservative than some existing results. Two numerical examples are given to illustrate the effectiveness and advantages of the proposed filter design scheme. In addition, it is worth mentioning that the scheme developed in this paper can be extended to other dynamic systems, such as neural networks and sampled-data systems with time-varying delays, which will be our future research.

Data Availability

Data used to support the findings of this work are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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