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Research Article

A Combined Upper-Sided Synthetic S^2 Chart for Monitoring the Process Variance

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In order to monitor the process variance, this paper proposes a combined upper-sided synthetic S^2 chart for monitoring the process standard deviation of a normally distributed process. This combined upper-sided synthetic S^2 chart comprises a synthetic chart and an upper-sided S^2 chart. The design and performance of the proposed chart are presented, and the steady-state average run length comparisons show that the combined upper-sided synthetic S^2 chart outperforms the standard synthetic S^2 chart as well as several run rules S^2 charts, especially for larger shifts in the process variance.

1. Introduction

A product that meets the customer's expectations is generally preferred, which means that this product should be produced by a stable process. However, variation is unavoidable in the output of every process. Variation can be attributed to the usual causes of variation and unusual causes of variation. A process working under only usual causes of variation is called statistical in-control (IC) and the output of the process is usually assumed to follow a distribution with nominal mean and variance. A process working under both types of variations is declared out-of-control (OOC) and the assumed distribution of the output of the process deviates from the nominal values (see Montgomery [1]). Statistical process control (SPC) is a powerful collection of tools in achieving process stability through the reduction of variability. Among which, control charts have been shown to be the most effective ones to detect the unusual causes of variation.

Much research has been done on monitoring shifts in the process mean. Among them, the Shewhart-type charts are the most widely used in practice. Collani and Sheil [2] pointed out that some manufacturing circumstances, such as faulty raw material, unskilled operators, and loosening of machine settings, may cause an increase in the process variance without influencing the level of the process mean. Consequently, it is also important to monitor the process variance in practice. The Shewhart S^2 chart is often used for its advantage in detecting larger shifts in the process variance, but becomes less effective in detecting small or moderate shifts.

In order to improve the performance of the classical Shewhart-type charts, many alternative approaches have been proposed, such as the exponentially weighted moving average and cumulative sum control charts. Recently, Wu and Spedding [3] proposed a synthetic chart which comprises an $\overline{X}$ and a CRL (conforming run length) chart. This control chart is known to outperform the classical Shewhart-type charts over the entire range of shifts. Since then, much research has been done on the synthetic type charts. Among them, Davis and Woodall [4] investigated the zero-state and steady-state performances of the synthetic chart using a Markov chain method. Costa and Rahim [5] applied the synthetic chart methodology along with a noncentral chi-square statistic for monitoring the process mean and variance and proved that this approach is more effective and simple than the classical joint $\overline{X}$ and R chart. Costa et al. [6] also proposed a synthetic control chart based on the noncentral chi-square statistic with a two-stage testing, which outperforms the joint $\overline{X}$ and S charts as well as several CUSUM schemes and has a similar performance with the joint $\overline{X}$ and S charts with double sampling (DS). In recent years, Wu et al. [7] proposed a combined synthetic $\overline{X}$ chart for monitoring the process mean. It is shown that the combined synthetic $\overline{X}$ chart always outperforms the individual $\overline{X}$ and the synthetic chart under different conditions. Zhang et al. [8] investigated the effect of process parameter estimation on the performance of the synthetic $\overline{X}$ chart and pointed out that the run length performance of the synthetic chart is quite different in the known and in the estimated parameter cases. Hu et al. [9] investigated the overall performance of the synthetic $\overline{X}$ chart with measurement errors. The above research studies were mainly focused on the univariate synthetic control charts, while for multivariate charts, Pramod and Vikas [10] investigated the effect of some new sampling strategies on the performance of the synthetic T^2 chart. Felipe et al. [11] proposed a synthetic type chart for bivariate autocorrelated processes. The chart was shown to perform better than other competing charts. In order to monitor the multivariate coefficient of variation, Quoc et al. [12] investigated the properties of the one-sided synthetic control charts. For an overview of the synthetic type charts, readers may refer to Athanasios et al. [13].

Unlike the synthetic charts for the process mean or the mean vector, not much research has been done on the synthetic charts for the process variance. Chen and Huang [14] and Huang and Chen [15] developed a synthetic R and S chart for monitoring the process variance, respectively. Being motivated by the above work, we propose a combined upper-sided synthetic S^2 chart for monitoring the process variance and we show that it outperforms the standard upper-sided synthetic S^2 chart and several run rules S^2 charts.

The organization of this paper is as follows. Section 2 discusses the combined upper-sided synthetic S^2 chart in detail. The optimal design of the proposed synthetic chart using the Markov chain method is presented in Section 3. The comparisons between different control schemes are given in Section 4. Finally, conclusions are drawn in Section 5.

2. The Combined Upper-Sided Synthetic S^2 Chart

Assume that at time $i = 1, 2, \ldots$, the quality characteristic X of $n \geq 2$ consecutive items is equal to $\{X_{i,1}, X_{i,2}, \ldots, X_{i,n}\}$. We assume that the $X_{i,j}$'s are independent normal $(\mu_0 + \delta\sigma_0, \rho\sigma_0)$ random variables, where μ_0 and σ_0 are the nominal mean and standard deviation, respectively, both assumed known, while δ and ρ are the standardized mean and standard deviation shifts, respectively. The sample variance for the i^{th} sample $\{X_{i,1}, X_{i,2}, \ldots, X_{i,n}\}$ is given by

$$S_i^2 = \frac{1}{n-1} \sum_{j=1}^{n} \left(X_{i,j} - \overline{X}_i \right), \quad (1)$$

where $\overline{X}_i = (1/n) \sum_{j=1}^{n} X_{i,j}$ is the mean of the i^{th} subgroup. Since we are only interested in monitoring the process variance, we will assume that only the variance is likely to change (i.e., $\delta = 0$). As it is important to find assignable causes that deteriorate the process, in this paper, we will only focus on the increasing variance case (i.e., $\rho > 1$). The combined synthetic upper-sided S^2 chart consists of a synthetic subchart and an upper-sided S^2 subchart. The control flow of the combined upper-sided synthetic S^2 chart is outlined as follows:

(i) Step 1: determine the sample size n, the lower control limit $H \in \{1, 2, \ldots\}$ of the synthetic subchart and, the upper warning limit $\text{WL} = W\sigma_0^2$ and control limit $\text{CL} = K\sigma_0^2$ of the upper-sided S^2 subchart, where $K > W > 0$ are the control and warning limit coefficients, respectively.

(ii) Step 2: at each sample point $i = 1, 2, \ldots$, take a sample of $n (n \geq 2)$ items of the quality characteristic X and compute the sample variance S_i^2 as in equation (1).

(iii) Step 3: if $S_i^2 > \text{CL}$, the process is declared as out-of-control and the control flow goes to step 6.

(iv) Step 4: if $S_i^2 < \text{WL}$, the sample is a conforming one and the control flow goes back to step 2.

(v) Step 5: if $\text{WL} < S_i^2 < \text{CL}$, the sample is considered as nonconforming, and determine the CRL (conforming run length), i.e., the number of conforming samples between two consecutive nonconforming samples plus one. If $\text{CRL} > H$, the process is deemed to be in-control and the control flow goes back to step 2. Otherwise, the process is declared as out-of-control and the control flow goes to step 6.

(vi) Step 6: signal an out-of-control status to indicate an increase in the variance of the process. Find and remove potential assignable causes. Then move back to step 2.

Compared with the standard synthetic S^2 chart, a control limit CL is added to the combined upper-sided synthetic S^2 chart. The basic motivation for this operation is due to the usage of the magnitude of the latest sample and the distance between the two consecutive nonconforming samples.

3. Design of the Combined Upper-Sided Synthetic S^2 Chart

The average run length (ARL) is the expected number of consecutive samples taken until the chart gives an out-of-control signal. In this paper, we compute the steady-state ARL of the combined upper-sided synthetic S^2 chart using the Markov chain method. Other researchers who have considered the steady-state ARL of the synthetic type charts are Wu et al. [7], Khoo et al. [16], Machado and Costa [17], and Khoo et al. [18]. The run length properties of the combined upper-sided synthetic S^2 chart can be determined using a procedure similar to that in Davis and Woodall [4]. To obtain the steady-state ARL of the combined synthetic S^2 chart, we use the Markov chain method, where the $(H + 2) \times (H + 2)$ transition probability matrix $\mathbf{P}$ is equal to

$$\mathbf{P}=\begin{pmatrix}\mathbf{Q} & \mathbf{r}\\ 0^{\mathbf{T}} & 1\end{pmatrix}=\begin{pmatrix} A & B & 0 & \cdots & \cdots & 0 & 1-A-B\\ 0 & 0 & A & \ddots & & 0 & 1-A\\ \vdots & & \ddots & \ddots & \ddots & \vdots & \vdots\\ \vdots & & & & A & 0 & \vdots\\ 0 & \cdots & \cdots & \cdots & 0 & A & 1-A\\ A & 0 & \cdots & \cdots & \cdots & 0 & 1-A\\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & 1\end{pmatrix}, \tag{2}$$

where $0^{\mathbf{T}}=(0,0,\ldots,0)$ is a $1\times(H+1)$ row vector, $\mathbf{Q}$ is a $(H+1)\times(H+1)$ transition probability matrix for the transient states, the $(H+1)\times 1$ column vector $\mathbf{r}$ satisfies $\mathbf{r}=1-\mathbf{Q}1$ with $1=(1,1,\ldots,1)^T$, and the probabilities of a sample in the safe and warning regions on the upper-sided S^2 subchart are $A=P(S_i^2<\mathrm{WL})$ and $B=P(\mathrm{WL}<S_i^2<\mathrm{CL})$, respectively. If the variability in the quality characteristic X shifts from σ_0^2 to $\rho^2\sigma_0^2$, the probabilities of a sample in the safe and warning regions are as follows:

$$\begin{aligned} A &= F_{\chi^2}\left(\frac{W(n-1)}{\rho^2}\mid n-1\right),\\ B &= F_{\chi^2}\left(\frac{K(n-1)}{\rho^2}\mid n-1\right)-F_{\chi^2}\left(\frac{W(n-1)}{\rho^2}\mid n-1\right), \end{aligned} \tag{3}$$

where $F_{\chi^2}(\ldots\mid n-1)$ is the cumulative distribution function (c.d.f.) of the chi-square distribution with $n-1$ degrees of freedom.

Since the number of steps L until the process reaches the absorbing state is known to be a discrete phase-type random variable of parameters $(\mathbf{Q},\mathbf{q})$ (see Neuts [19] and Latouche and Ramaswami [20]), we can easily obtain the probability mass function (p.m.f.) $f_L(l\mid Q,q)$ and the c.d.f. $F_L(l\mid Q,q)$ of the run length distribution of the combined upper-sided synthetic S^2 chart:

$$\begin{aligned} f_L(l\mid\mathbf{Q},\mathbf{q}) &= \mathbf{q}^{\mathbf{T}}\mathbf{Q}^{l-1}\mathbf{r},\\ F_L(l\mid\mathbf{Q},\mathbf{q}) &= 1-\mathbf{q}^{\mathbf{T}}\mathbf{Q}^{l}\mathbf{r}. \end{aligned} \tag{4}$$

The steady-state ARL is given by

$$\mathrm{ARL}=\mathbf{q}^{\mathbf{T}}(\mathbf{I}-\mathbf{Q})^{-1}1, \tag{5}$$

where $\mathbf{q}$ is the vector with the stationary probabilities of being in each nonabsorbing state and $\mathbf{I}$ is an $(H+1)\times(H+1)$ identity matrix, and $\mathbf{Q}$ is the transition probability matrix for the transient states in $\mathbf{P}$. As suggested by Crosier [21] and Khoo et al. [16], we will use the cyclical steady-state probability column vector $\mathbf{q}$ that can be obtained using the simplified procedure proposed by Champ [22]:

(i) Solve $\mathbf{p}=\mathbf{P}^{\mathbf{T}}\mathbf{p}$ for $\mathbf{p}$ subject to $1^T\mathbf{p}=1$

(ii) Compute $\mathbf{q}=(1^{\mathbf{T}}\mathbf{s})^{-1}\mathbf{s}$, where $\mathbf{s}$ is a vector of length $H+1$ obtained from $\mathbf{p}$ by deleting the entry corresponding to the absorbing state

Champ [17] showed that vector $\mathbf{s}$ can directly be obtained using

$$\mathbf{s}=\left(\mathbf{G}-\mathbf{Q}^{\mathbf{T}}\right)^{-1}\boldsymbol{\mu}, \tag{6}$$

$$\mathbf{G}=\begin{pmatrix} 2 & 1 & 1 & \cdots & 1\\ 0 & 1 & 0 & \cdots & 0\\ 0 & 0 & 1 & \cdots & 0\\ \vdots & \vdots & \vdots & \ddots & \vdots\\ 0 & 0 & 0 & \cdots & 1\end{pmatrix}, \tag{7}$$

$$\boldsymbol{\mu}=(1\,0\,0\cdots 0)^T, \tag{8}$$

where $\mathbf{G}$ is a $(H+1)\times(H+1)$ matrix and μ is a column vector of length $H+1$.

The out-of-control ARL (ARL_1) should be small to detect the assignable causes quickly while, at the same time, the in-control ARL (ARL_0) should be large to keep a smaller false alarm rate. In this case, the design of the combined synthetic S^2 chart is based on minimizing the ARL_1 for a desired shift ρ with the constraint $\mathrm{ARL}_0=\tau$, while τ is the smallest allowable in-control ARL, i.e.,

$$\left(H^*,W^*,K^*\right)=\min\mathrm{ARL}(H,W,K,\rho,n), \tag{9}$$

subject to

$$\mathrm{ARL}_0=\tau,\quad K>W>0,\ H\in\{1,2,3,\ldots\}. \tag{10}$$

4. Numerical Results

In this section, we present the performance of the combined upper-sided synthetic S^2 chart, the standard upper-sided synthetic S^2 chart as well as several run rules S^2 chart. The desired ARL_0 is set to be 370.4 and 500.

4.1. Comparison with the Standard Synthetic S^2 Chart. For different combinations of $n\in\{5,10\}$ and $\rho\in\{1.2, 1.4, 1.6, 1.8, 2.0, 2.2, 2.4, 2.6, 2.8, 3.0\}$, Tables 1 and 2 present the optimal parameters (H^*,W^*,K^*) and ARL_1 (denoted as ARL_1^c) of the combined upper-sided synthetic S^2 chart (recorded in columns 3 to 6), as well as the optimal parameters (H^*,W^*) and ARL_1 (denoted as ARL_1^s) of the standard upper-sided synthetic S^2 chart (recorded in columns 7 to 9). As it can be seen from Tables 1 and 2, the optimal parameters (H^*,K^*) of the proposed chart decrease as the shift size increases. For example, when $n=5$ and ρ increases from 1.2 up to 2, H^* decreases from 16 down to 3 and K^* decreases from 5.5 down to 4.5. When the sample size n increases, the ARL_1^C of the proposed chart decreases, which means the chart's performance is getting better than before. For example, in Table 1, when $\rho=1.2$, the ARL_1^C decreases from 28.88 down to 15.50 when n increases from 5 up to 10. Moreover, we can see that the combined upper-sided synthetic S^2 chart always performs better than the standard upper-sided synthetic S^2 chart. For instance, if $\mathrm{ARL}_0=370.4$ and $n=10$, for the combined upper-sided synthetic S^2 chart, we have $\mathrm{ARL}_1^c=15.5$ when $\rho=1.2$ and

Table 1: Comparisons of the combined upper-sided synthetic S^2 chart and the standard upper-sided synthetic S^2 chart when $\mathrm{ARL}_0 = 370.4$.

n	ρ	H^*	W^*	K^*	ARL_1^c	H^*	W^*	ARL_1^s
5	1.2	16	3.1022	5.5	28.88	18	3.1140	29.21
	1.4	8	2.9411	5.2	8.59	10	2.9657	8.87
	1.6	5	2.8463	4.9	4.41	7	2.8726	4.68
	1.8	3	2.7693	4.6	2.94	5	2.7829	3.23
	2.0	3	2.8027	4.5	2.24	4	2.7226	2.56
	2.2	2	2.7417	4.4	1.86	4	2.7226	2.20
	2.4	2	2.8119	4.3	1.62	3	2.6437	1.99
	2.6	1	2.7486	4.2	1.47	3	2.6437	1.86
	2.8	1	2.7486	4.2	1.36	3	2.6437	1.77
	3.0	1	2.7486	4.2	1.28	3	2.6437	1.70
10	1.2	10	2.2221	3.6	15.50	12	2.2393	15.69
	1.4	5	2.1405	3.3	4.29	6	2.1392	4.47
	1.6	3	2.1011	3.1	2.33	4	2.0783	2.56
	1.8	2	2.0799	3.0	1.67	3	2.0340	1.97
	2.0	1	2.0604	2.9	1.37	2	1.9704	1.73
	2.2	1	2.0604	2.9	1.22	2	1.9704	1.62
	2.4	1	2.0604	2.9	1.13	2	1.9704	1.56
	2.6	1	2.0604	2.9	1.08	2	1.9704	1.54
	2.8	1	2.0604	2.9	1.05	2	1.9704	1.52
	3.0	1	2.0604	2.9	1.03	2	1.9704	1.51

Table 2: Comparisons of the combined upper-sided synthetic S^2 chart and the standard upper-sided synthetic S^2 chart when $\mathrm{ARL}_0 = 500$.

n	ρ	H^*	W^*	K^*	ARL_1^c	H^*	W^*	ARL_1^s
5	1.2	18	3.2262	5.8	33.88	21	3.2510	34.22
	1.4	9	3.0659	5.4	9.49	11	3.0868	9.76
	1.6	6	2.9857	5.1	4.72	7	2.9678	4.99
	1.8	4	2.9091	4.9	3.09	6	2.9266	3.37
	2.0	3	2.8856	4.7	2.33	4	2.8166	2.64
	2.2	2	2.8189	4.6	1.92	4	2.8166	2.25
	2.4	2	2.8810	4.5	1.67	3	2.7373	2.03
	2.6	2	2.9800	4.4	1.50	3	2.7373	1.88
	2.8	1	2.7958	4.4	1.38	3	2.7373	1.78
	3.0	1	3.0078	4.3	1.30	3	2.7373	1.72
10	1.2	12	2.3007	3.7	17.78	13	2.3040	17.97
	1.4	5	2.1917	3.4	4.62	6	2.1919	4.80
	1.6	3	2.1501	3.2	2.43	4	2.1308	2.66
	1.8	2	2.1262	3.1	1.72	3	2.0865	2.01
	2.0	1	2.0982	3.0	1.40	3	2.0865	1.75
	2.2	1	2.0982	3.0	1.24	2	2.0229	1.63
	2.4	1	2.0982	3.0	1.14	2	2.0229	1.57
	2.6	1	2.4892	2.9	1.08	2	2.0229	1.54
	2.8	1	2.4892	2.9	1.05	2	2.0229	1.52
	3.0	1	2.4892	2.9	1.03	2	2.0229	1.51

$\mathrm{ARL}_1^c = 1.22$ when $\rho = 2.2$, respectively, while for the same cases of the standard upper-sided synthetic S^2 chart, we have $\mathrm{ARL}_1^s = 15.69$ when $\rho = 1.2$ and $\mathrm{ARL}_1^s = 1.62$ when $\rho = 2.2$, respectively. Denote the absolute and relative advantages of the combined upper-sided synthetic S^2 chart over the standard upper-sided synthetic S^2 chart as $\Delta_1 = \mathrm{ARL}_1^s - \mathrm{ARL}_1^c$ and $\Delta_2 = ((\mathrm{ARL}_1^s - \mathrm{ARL}_1^c)/\mathrm{ARL}_1^s) \times 100\%$, respectively. For the two cases presented above, we have $\Delta_1 = 0.19$ and $\Delta_2 = 1.21\%$ when $\rho = 1.2$, as well as $\Delta_1 = 0.40$ and $\Delta_2 = 24.69\%$ when $\rho = 2.2$. It appears that, with the increase of the shift size ρ, the absolute advantage Δ_1 is always smaller than 1, while increasing the shift size $\rho\,(1 < \rho < 3)$ will cause a significant increase in the relative advantage Δ_2.

4.2. Comparisons with Different Run Rules S^2 Charts. From the work in Section 4.1, it can be seen that the combined synthetic S^2 chart performs better than the standard synthetic S^2 chart for different shift size ρ. However, it is known that adopting different run rules schemes to the Shewhart-type charts can improve the chart's performance (see Castagliola et al. [23], Amdouni et al. [24], Tran [25], Shongwe [26], and so on). Motivated by this fact, the upper-sided run rules S^2 charts with different schemes are presented in this section as a comparison to the proposed chart.

Run rules charts have been studied by many researchers. Among them, Castagliola et al. [23] studied properties of the Shewhart CV (coefficient of variation) charts with 2-of-3, 3-of-4, and 4-of-5 run rules. Tran [25] studied the properties of the 2-of-3, 3-of-4, and 4-of-5 run rules t charts. These works focused on the two-sided Shewhart charts with run rules, while Amdouni et al. [24] investigated the CV chart by means of one-sided 2-of-3 and 3-of-4 Shewhart charts. Since we only focus our attention on the upper-sided chart, the properties of the 2-of-3, 3-of-4, and 4-of-5 run rules upper-sided S^2 charts are investigated and are compared with the proposed scheme. As an example, the transition probability matrix $\mathbf{Q}$ of the 2-of-3 run rules upper-sided S^2 chart is given as follows:

$$\mathbf{Q} = \begin{pmatrix} 0 & 0 & 0 & p_c & p_L & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & p_c \\ p_c & p_L & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_U & p_c & p_L & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & p_U & p_c \\ p_c & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & p_U & p_c & 0 & 0 & 0 \end{pmatrix}. \quad (11)$$

The probabilities in equation (11) are equal to $p_C = P(S_i^2 < \mathrm{CL})$, $p_U = 1 - P(S_i^2 < \mathrm{CL})$, and $p_L = 0$, where $\mathrm{CL} = K_s \sigma_0^2$ is the control limit of the run rules charts. Then using equation (5), we can obtain the ARL of the upper-sided 2-of-3 run rules S^2 chart. Similar method can also be used to obtain the ARL of the upper-sided 3-of-4 and 4-of-5 run rules S^2 charts. Due to the large size of $\mathbf{Q}$, the transition probability matrix of the 3-of-4 and 4-of-5 run rules is not presented here.

For different combinations of n and ρ, Tables 3 and 4 present the ARL_1 values of the combined upper-sided synthetic S^2 chart (recorded in column 3), as well as the ARL_1 values of the run rules S^2 charts (recorded in columns 4 to 6). The optimal parameters K_s of different run rules S^2 charts are obtained with the constraint on the desired ARL_0

Table 3: Comparisons of the combined upper-sided synthetic S^2 chart and several run rules S^2 charts when $ARL_0 = 370.4$.

n	ρ	ARL_1^c	ARL_1 2-of-3	ARL_1 3-of-4	ARL_1 4-of-5
5	1.2	28.88	37.08	37.11	38.33
	1.4	8.59	11.56	12.46	13.65
	1.6	4.41	6.13	7.10	8.18
	1.8	2.94	4.23	5.20	6.22
	2.0	2.24	3.36	4.33	5.33
	2.2	1.86	2.90	3.87	4.85
	2.4	1.62	2.63	3.60	4.58
	2.6	1.47	2.46	3.43	4.41
	2.8	1.36	2.34	3.32	4.30
	3.0	1.28	2.26	3.24	4.23
10	1.2	15.50	19.47	18.95	19.40
	1.4	4.29	5.71	6.37	7.20
	1.6	2.33	3.32	4.16	5.06
	1.8	1.67	2.58	3.49	4.43
	2.0	1.37	2.29	3.23	4.19
	2.2	1.22	2.15	3.12	4.10
	2.4	1.13	2.08	3.06	4.05
	2.6	1.08	2.05	3.03	4.03
	2.8	1.05	2.03	3.02	4.02
	3.0	1.03	2.02	3.01	4.01

Table 4: Comparisons of the combined upper-sided synthetic S^2 chart and several run rules S^2 charts when $ARL_0 = 500$.

n	ρ	ARL_1^c	ARL_1 2-of-3	ARL_1 3-of-4	ARL_1 4-of-5
5	1.2	33.88	44.29	43.83	44.96
	1.4	9.49	12.90	13.72	14.92
	1.6	4.72	6.58	7.54	8.63
	1.8	3.09	4.44	5.41	6.43
	2.0	2.33	3.48	4.45	5.44
	2.2	1.92	2.97	3.94	4.92
	2.4	1.67	2.68	3.64	4.63
	2.6	1.50	2.49	3.46	4.44
	2.8	1.38	2.36	3.34	4.32
	3.0	1.30	2.28	3.26	4.24
10	1.2	17.78	22.60	21.62	21.89
	1.4	4.62	6.15	6.75	7.56
	1.6	2.43	3.46	4.27	5.17
	1.8	1.72	2.64	3.53	4.47
	2.0	1.40	2.31	3.25	4.21
	2.2	1.24	2.16	3.13	4.10
	2.4	1.14	2.09	3.07	4.05
	2.6	1.08	2.05	3.04	4.03
	2.8	1.05	2.03	3.02	4.02
	3.0	1.03	2.02	3.01	4.01

and the ARL_1 values are computed for different shift size ρ. From Tables 3 and 4, it can be noted that the combined upper-sided synthetic S^2 chart performs uniformly better than the run rules S^2 charts, especially for small shifts. For example, when $n = 5$ and $\rho = 1.2$, the $ARL_1^C = 28.88$ is much smaller than the $ARL_1 = 37.08$ of the 2-of-3 run rules S^2 chart. Moreover, we can note that the 2-of-3 run rules S^2 chart is generally preferred to the 3-of-4 or 4-of-5 run rules S^2 charts, especially for large shifts. This may be due to the fact that 3-of-4 or 4-of-5 run rules need to accumulate more samples to make a decision of the process status than the 2-of-3 run rules.

5. Conclusions

In this paper, we proposed the combined upper-sided synthetic S^2 chart for monitoring the process variance. A Markov chain method is used to obtain the run length distribution of the combined upper-sided synthetic S^2 chart. Through the comparisons of two control charts, we can note that the combined upper-sided synthetic S^2 chart performs better than the standard upper-sided synthetic S^2 chart, especially with the shift size increasing. In addition, it is also shown that the proposed chart performs better than several run rules S^2 charts. The combined upper-sided synthetic S^2 chart can be a good alternative in practice for the detection of the process variance. Finally, it could be interesting to further our research on the design of combined synthetic S^2 chart with estimated process variance.

Data Availability

The data used to support the findings of this study are available from the corresponding author upon request.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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