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Research Article

Analysis on Altruism-Based Coopetition Game of Oligarchic Enterprises: A Validation of Generation Companies

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For the long-term sustainable development, the modern enterprises should consider both competition and cooperation. In the current studies of corporate competition strategies and games, the quantification of cooperation-competition (coopetition) between enterprises is not deeply investigated. In this paper, we establish a coopetition game model of oligarchic enterprises in the industry by using the quantitative altruistic factor and nonlinear cost function, analyze the influence of altruistic factor on equilibrium variables, and then validate it in the generation side market. The following conclusions are drawn: (1) the coopetition of any form will increase the market equilibrium price and the total equilibrium profit of the industry, which induces the motivation and intention of cooperation between oligarchic enterprises. (2) The increased unilateral altruism is instable and unsustainable because it will produce an altruistic threshold that makes the total equilibrium profit of the industry increase and then decrease. The unilateral altruism of high-cost generation companies is more beneficial for increasing the total equilibrium profit of the industry, but it is realized in a difficult way. Due to a higher initial altruism level, there is lack of motivation for the increased unilateral altruism. (3) The mutually altruistic coopetition is the most effective way for improving the total equilibrium profit of the industry, but it is hard to finally achieve the complete monopoly because of cost differentiation. (4) The established game model of generation market is more universal and provides a certain quantitative interpretation for electricity crisis.

1. Introduction

In the market economy, the enterprises intend to present a competitive or confrontational relationship and pay more attention to their maximum profits. With the deepening of economic globalization, however, the defects of pure competition become more and more nonignorable. The long-term excessive competition will lead to the increased costs and declined risk-withstanding performance of enterprises in the industry, a deteriorating market environment, and a sharp decrease in the overall profit rate of the industry, which has a great negative impact on the industrial healthy and sustainable development. In the information technology era, the competition between enterprises is expanded to multiregional and multi-industrial comprehensive performance under a complex economic environment from the simple improvement of production efficiency. And there is a dual challenge of cooperation and competition for enterprises [1]. After the "Darwinian" competition era is ended, the consideration of both cooperation and competition is a developmental strategy of enterprises in the future, and the cooperation-competition (coopetition) is a potential strategy to seek for win-win cooperation and long-term sustainable development [2]. However, coopetition is not a well-known concept; thus its interpretation and research need to be further deepened [3].

Altruism will promote cooperation, and it is also the connotation and important representation of the later [1, 4]. In a long time, the hypothesis of "economic man" is widely accepted by mainstream economics focusing on the research of competition strategies due to its simplicity. It is not equal to egoism, but most standard economic models are based on the hypothesis that the utility function guiding the individual behaviors is completely determined by pure egoism, without considering the utility of others [5]. However, utility is a function of preference. With the development of experimental economics, the complexity of human behaviors and preference attracts more and more concerns [6]. Many phenomena in the real economy cannot be explained simply by "rationality" and "egoism," and the decision-making often follows the principle of satisfaction, other than the principle of optimality [7]. Therefore, utility is not completely egoistic, yet it is not popularly considered in the related studies.

The enterprises often face a difficulty in directly achieving their strategic objectives because of the restrictions from the internal and external factors (e.g., assets, resources, scale, and policies) and have to realize these objectives indirectly by moderate cooperation (i.e., altruistic behaviors to some extent) [8]. The long-term moderately mutual competition and altruistic cooperation have a significantly positive impact on the industrial innovation and corporate sustainable development, even though the enterprises may not maximize their own profits in the short term [9]. Particularly in the oligarchic industries (e.g., electric power industry, petrochemical industry, steel industry, telecommunication industry, and other scale-economy industries), the strong relation and interdependence between internal enterprises inevitably result in the alternative (or simultaneous) presence of altruistic cooperation and mutual competition of enterprises at different stages and levels. The enterprises can share resources with their competitors in certain fields to enhance the industrial performance and act independently in other fields to improve their own performance [10].

The existing corporate game strategy study methods involve two types of game: cooperative game and noncooperative game. Cooperative game focuses on the profit distribution of various players under the binding contractual conditions for cooperation, while noncooperative game emphasizes the selection of profit maximization strategies under the competition of mutual interest influence. In the real world, various enterprises not only emphasize their self-profit and market share competition, but also pay more attention to the mutual long-term relationship and the industrial sustainable development strength; thus their game relations are often the mixed ones of the above-mentioned two games. In fact, cooperative game and noncooperative game are mutually compatible and have a necessary association, so it is of importantly practical significance to study their organic combination, and the key of such studies is to integrate competition and cooperation. In most current study models, complete cooperation or complete self-interesting is the major objective, and the fact that the decision makers of limited rationality may present slightly altruistic preference and a moderate cooperation tendency is neglected. And the overall research perspective has certain limitations, because the possible nonlinear cost function of oligarchic enterprises under scale economy is not mentioned to facilitate the analysis.

This study aimed to establish a coopetition utility function considering both self-profit and others' profit by introducing slightly altruistic factor (a quantitative index of moderate cooperation) into the noncooperative game model and regarding oligarchic enterprises in the industry as a coopetition system. Firstly, an altruism-based coopetition game model was established for scale-economy oligarchic enterprises with a nonlinear cost function. Secondly, the impact of altruistic factor on the equilibrium output and profit and its sustainability were quantitatively analyzed. Finally, the coopetition relationship between the duopoly generation companies of different costs was further validated.

This paper consists of the following parts: Section 1 introduces study background and significance; Section 2 shows literature review; Section 3 establishes a coopetition model for oligarchic enterprises in the industry based on the altruistic preference and the nonlinear cost function; Section 4 discusses the impact of altruistic factor on the equilibrium variables and its sustainability under the model equilibrium; Section 5 provides validation and analysis on the coopetition relationship between oligarchic generation companies of different costs in the power market; Section 6 carries conclusions and prospect.

2. Review of Literature Studies

The research on the coopetition relationship between enterprises can be traced back to the 1980s [11]. In 1996, Brandenburger and Nalebuff [12] formally proposed the concept of coopetition and pointed out that enterprises were facing a market of cooperation and competition and the coopetition was generally seen in various industries, and then they analyzed the causes of coopetition using a game model. In the real economic market, most enterprises adopt the coopetition strategy because of huge benefits that can be obtained from a coopetition relation [3, 13]. Luo [10, 14] believed that the coopetition was a win-win solution for enterprises to increase the shareable surplus and could improve the customer and financial performance of companies. In addition, coopetition was more helpful for enhancing the innovation capability of enterprises and their competitors, as compared with pure competition or complete cooperation [15, 16]. Kafi et al. [17] thought that the cooperation between competitors would cause changes in the relationship structure of supply chain; thus the profitability of coopetition strategy would be better than that of pure competition strategy. In dual-channel or multilevel supply chains [18, 19], an appropriate coopetition strategy could be beneficial for the better coordination of interests among various parties. The study of Limoubpratum et al. [20] showed that coopetition might help enterprises to achieve the long-term sustainable development and improve the economic, social, and environmental benefits. In the view of Christ et al. [2], coopetition was a potential strategy for the long-term sustainable development of enterprises. Cygler et al. [3] further pointed out that coopetition was a feasible strategy, and its duration was related to different interest types of partners.

Altruism is essentially a kind of cooperation [1, 21], and its emergence is attributed to another possible encounter in the future. The achievement of cooperation depends on the altruistic preference and credibility of different participants, rather than their own benefits [4]. The classical economics experiment of Axelrod [6] revealed that, in multiple games with unknown rounds, the altruistic (cooperative) strategies were more beneficial for individuals than the egoistic (noncooperative) strategies, and they were evolutionarily stable; and Tit-for-Tat strategy was a superior one in the altruistic strategies. According to the social welfare and altruistic preference model established by Charness and Rabin [22], altruism was reciprocal and fair. Marco and Morgan [23] defined the slightly altruistic utility function and mathematically proved the existence and alternative stability properties of slightly altruistic equilibrium. In the subsequent studies, altruistic preference was more often considered in the analysis of supply-chain-related issues. Loch et al. [24] pointed out that the altruistic preference could promote the cooperation among all enterprises in the supply chain and improve the overall systematic efficiency and sustainability. By introducing altruistic preference into the supply chain network, Ge et al. [25] found that the performance of both suppliers and supply chain systems was improved. Xiao et al. [26] proposed a security system that applied the indirect reciprocity principle to combat attacks in wireless networks, which could significantly reduce the attacker population for a wide range of attacks and outperform the existing direct reciprocity-based systems. Wang et al. [27] proposed that, in the Stackelberg game model, the manufacturer's altruistic preference would increase the utility of both the manufacturer and the retailer when some special conditions were met, but the retailer's altruistic preference would only promote the manufacturer's utility. The study of Sun et al. [28] showed that altruistic preference had a significant impact on the optimal decision of suppliers and distributors and on the utility of various members in the supply chain and could markedly improve the efficiency of supply chain. Xu et al. [29] analyzed the relationship between online and offline retail channels by introducing altruistic preference into the coopetition model of dual-channel supply chain and proved the existence and stability of related models. Liu et al. [30] obtained the feasible condition that both logistics service integrator (LSI) and functional logistics service provider (FLSP) had altruistic preferences by using Stackelberg game models, found that supply chain coordination could not be achieved when both LSI and FLSP had altruistic preferences, and then proposed the solution. Lin [31] investigated a supply chain model of one manufacturer and two altruistic preference retailers and found that altruistic preference played a key role in the equilibrium outcome and could increase the corporate profits under certain conditions. Considering the green preference attributes of consumers, Huang et al. [32] addressed the optimal greenness and pricing decisions of supply chain members by introducing the altruistic preference into the supply chain and analyzed the impact of altruistic preferences on supply chain decision-making and profits. Under the limited distribution of demand, Zhai et al. [33] studied the supply chain coordination problem with reciprocal altruistic preference by using robust optimization method. A robust coordination strategy based on buyback contract was presented, and the numerical analysis was carried out.

The electric power industry is a typical oligopolistic industry owing to a high industrial barrier. In the reform process of gradually introducing competition, numerous theories and methods have been developed which involve the selection of superior strategies by the generation companies in the competitive environment to maximize their profits. Benefiting from the oligopolistic characteristics of the industry, the game theory becomes the most suitable method for investigating such issues. The optimal power generation strategy of generation companies can be obtained by solving the equilibrium solution of the game model [34, 35]. If the constraints of power grid and generating units were further considered in the model, it would be closer to the reality, but the process of solution would also be more difficult [36, 37]. Based on the Cournot model of oligopolistic electricity markets, Chen et al. [38] researched the oligopolistic electricity markets by the method of experimental economics. A set of experiments were conducted on the three generating companies, and the experimental results were analyzed with the strict statistical approaches. In the overlapping sealed areas, Li et al. [39] considered a demand response management model for multiple microgrids and multiple users. The Stackelberg game model of microgrids and users was constructed, and the relevant equilibrium strategy was analyzed systematically. Banaei et al. [40] established a comprehensive supply function equilibrium (SFE) model to explore the mutual interactions between forward contracts and the associated day-ahead electricity market, proposed a new risk management method, and then compared it with CVaR method. Liang et al. [41] proposed a novel risk-based uncertainty set optimization method for the energy management of typical hybrid AC/DC microgrids and demonstrated its effectiveness, good applicability, and robustness in comparison to standard robust optimization methods. Yin et al. [42] described the evolution process of four groups and more multiple groups in the open power market by replication dynamics and studied the multiple game strategies of generation enterprises, with the corresponding asymptotic stability conditions given as Lyapunov stability criteria. However, most existing models have been established under the precondition of mutual competition, without considering the moderate cooperation and slightly altruistic preference. So far, there are few studies of the power market to analyze the issues from the perspective of coopetition. By using the coopetition game approach, Kim [43] presented a novel smart grid management scheme which achieved higher energy efficiency and better system performance than other existing schemes. Jiang et al. [7] explored the influence of the variation between slightly altruistic factor and discount rate on the long-term equilibrium of generation enterprises by building coopetition repeated game model. Amin et al. [44] proposed a framework including both noncooperative game and cooperative game, to facilitate the P2P electricity trading when maintaining stability of the contract. The test results have shown that both prosumers and consumers could get benefits from P2P electricity trading under a precondition of maintaining market stability.

The differences between the above studies and our study are described as follows:

(1) In the current studies to investigate the coopetition relationship between enterprises, the empirical analysis method was more frequently used. Meanwhile, most of studies using the game analysis method focused on the classification discussion with the assumption that the decision makers had two completely opposite preferences of "complete competition" or "complete cooperation," in which the quantification of their intermediate state—"partial competition and partial cooperation" (an important feature of coopetition)—was not deeply explored. In this paper, altruistic factor was regarded as a continuous quantitative index to represent "partial cooperation" and applied in the equilibrium analysis on the coopetition game of oligarchic enterprises.

(2) Most of the currently completed studies focused on the relationship between upstream and downstream enterprises in the supply chain and were based on the assumption that the enterprise cost was a linear function. For research supplementation, this paper was conducted to quantitatively analyze the coopetition equilibrium of enterprises in the oligopolistic industry with a nonlinear cost function and its sustainability.

(3) Most studies of the electric power industry emphasized the competition-dominant strategies and equilibrium analysis of generation companies, without investigating the collusion or alliance characteristics of oligarchic generation companies. In this paper, we validated the coopetition equilibrium results and quantitative coordination methods between old and new generation companies in an altruism-based coopetition game model and provided a certain quantitative interpretation for the related electricity crisis.

3. Altruism-Based Coopetition Game Model

3.1. Issues and Assumptions. Cooperation and competition always exist due to the mutual interaction of market power between enterprises in the oligopolistic industry. Some investigators defined the slightly altruistic utility function in mathematics and rigorously proved the existence of equilibrium point in the slightly altruistic game from all aspects [23, 45, 46]. In this paper, with enterprises in the oligopolistic industry as a coopetition system and using the slightly altruistic factor for the quantification of moderate cooperation, the utility function of enterprises was redefined according to the actual situations by introducing the altruistic preference and the profit function of other enterprises. Due to the significant scale-economy characteristics, the short-term and long-term average cost curves of oligarchic enterprises are mostly nonlinear and U-shaped. With the assumption that the corporate cost was a nonlinear function, we established a Cournot coopetition game model for oligarchic enterprises in the industry on the basis of altruistic factor and then quantitatively analyzed the influence of altruistic factor on the equilibrium variables.

In order to facilitate the deduction and mathematical analysis of the model and provide a reasonable application background for the conclusions, we have made the following assumptions:

Assumption 1. N oligarchic enterprises in the regional industry provide nondifferentiated products and are completely informative. Due to regional restrictions, the consumers can only choose and purchase the products from N oligarchic enterprises but do not participate in the game between these enterprises.

Assumption 2. There is the possibility of cooperation between these oligarchic enterprises. For oligarchic enterprises, the profit function is U_i, and the coopetition utility function π_i is reflected by the slightly altruistic function, without the master-slave relationship. The enterprises independently make a decision of competition (egoistic) or moderate cooperation (altruistic).

Assumption 3. During market clearing, the market price of products is determined by the total market volume, and the oligarchic enterprises sell the products at the unified market price. The inverse demand function of the market is a linear function:

$$P = r - s\sum_{i=1}^{N} q_i, \quad (i = 1, 2, \ldots, N), \tag{1}$$

where P is the market price of products, r and s are the price ceiling in the market and the price change rate of products ($r > 0$ and $s > 0$), and q_i is the quantity of products purchased by the users from i^{th} oligarchic enterprise.

Assumption 4. According to the scale-economy characteristics of the oligopolistic industry, the cost function C_i of ith oligarchic enterprise shall be a nonlinear function and thus is assumed to be a quadratic function:

$$C_i = \frac{1}{2}a_i q_i^2 + b_i q_i - d_i, \quad (i = 1, 2, \ldots, N), \tag{2}$$

where $0 < a_i < b_i \ll d_i$ is the production cost coefficient of ith oligarchic enterprise.

3.2. Modeling and Solution. Under the above assumptions, a Cournot altruism-based coopetition game model for N oligarchic enterprises in the industry was established.

The profit function of i^{th} oligarchic enterprise can be expressed as follows:

$$U_i = Pq_i - C_i = \left(r - b_i - s\sum_{i=1}^{N} q_i - \frac{1}{2}a_i q_i\right)q_i - d_i, \quad (i = 1, 2, \ldots, N). \tag{3}$$

Then, the altruistic utility function of i^{th} oligarchic enterprise is obtained [23]:

$$\pi_i = U_i + \varepsilon_i \sum_{j=1, i\neq j}^{N} U_j, \quad (i = 1, 2, \ldots, N), \tag{4}$$

where ε_i is the altruistic factor of i^{th} oligarchic enterprise and indicates the consideration of this oligarchic enterprise on the profits of other competitors; that is, the degree of cooperation is quantitatively reflected by the altruistic factor. However, such consideration is moderate $\varepsilon_i \in [0, 1]$ and does not exceed the concern on the owned profits of enterprises.

In the Cournot altruism-based coopetition game model, the maximization of the altruistic utility function of N oligarchic enterprises is described as follows:

$$\begin{cases} \max\limits_{q_i>0, 0\leq\varepsilon_i\leq 1} [\pi_1, \pi_2, \ldots, \pi_i, \ldots, \pi_N], \\ \pi_i = U_i + \varepsilon_i \sum\limits_{j=1, i\neq j}^{N} U_j, \quad (i = 1, 2, \ldots, N). \end{cases} \tag{5}$$

The condition for the first-order maximization of (5) can be computed as follows:

$$\frac{\partial \pi_i}{\partial q_i} = r - b_i - s\sum_{i=1}^{N} q_i - sq_i - a_i q_i - \varepsilon_i s \sum_{j=1, i\neq j}^{N} q_j = 0, \quad (i = 1, 2, \ldots, N). \tag{6}$$

Then, the response function of i^{th} oligarchic enterprise is obtained:

$$q_i = \frac{r - b_i - (1 + \varepsilon_i)s\sum_{j=1, i\neq j}^{N} q_j}{2s + a_i}, \quad (i = 1, 2, \ldots, N). \tag{7}$$

The following equation is transformed from (6):

$$sq_i + a_i q_i + \left(\varepsilon_i s \sum_{j=1}^{N} q_j - \varepsilon_i s q_i\right) + s\sum_{i=1}^{N} q_i = r - b_i. \tag{8}$$

Thereafter, an equation set is obtained:

$$q_i + \frac{(1+\varepsilon_i)s}{a_i + (1-\varepsilon_i)s}\sum_{i=1}^{N} q_i = \frac{r - b_i}{a_i + (1-\varepsilon_i)s}, \quad (i = 1, 2, \ldots, N). \tag{9}$$

The sum of N equations in (9) is calculated as follows:

$$\sum_{i=1}^{N} q_i + \sum_{i=1}^{N} \frac{(1+\varepsilon_i)s}{a_i + (1-\varepsilon_i)s}\sum_{i=1}^{N} q_i = \sum_{i=1}^{N} \frac{r - b_i}{a_i + (1-\varepsilon_i)s}, \quad (i = 1, 2, \ldots, N). \tag{10}$$

Then, the total coopetition equilibrium output Q^* in the market is expressed as

$$Q^* = \sum_{i=1}^{N} q_i^* = \frac{\sum_{i=1}^{N} r - b_i/[a_i + (1-\varepsilon_i)s]}{1 + \sum_{i=1}^{N}(1+\varepsilon_i)s/[a_i + (1-\varepsilon_i)s]}, \quad (i = 1, 2, \ldots, N). \tag{11}$$

The coopetition equilibrium price P^* in the market is obtained:

$$P^* = r - s\sum_{i=1}^{N} q_i^* = \frac{r + s\sum_{i=1}^{N} b_i + \varepsilon_i r/[a_i + (1-\varepsilon_i)s]}{1 + \sum_{i=1}^{N}(1+\varepsilon_i)s/[a_i + (1-\varepsilon_i)s]}, \quad (i = 1, 2, \ldots, N). \tag{12}$$

By substituting (11) into (9) and simplifying them, the coopetition equilibrium output of single oligarchic enterprises is calculated as

$$q_1^* \frac{r - b_i + s\sum_{j=1, i\neq j}^{N}\left(1+\varepsilon_j\right)(r - b_i) - (1+\varepsilon_i)\left(r - b_j\right)/\left[a_j + \left(1-\varepsilon_j\right)s\right]}{[a_i + (1-\varepsilon_i)s]\left(1 + \sum_{j=1}^{N}\left(1+\varepsilon_j\right)s/\left[a_j + \left(1-\varepsilon_j\right)s\right]\right)}, \quad (i = 1, 2, \ldots, N). \tag{13}$$

In particular, all oligarchic enterprises have no intention of altruistic cooperation when $\varepsilon_i = 0$ $(i = 1, \ldots, N)$, and the market is under a completely competitive environment. And the equilibrium output of each enterprise can be rewritten as follows:

$$q_i^* = \frac{r - b_i + s\sum_{j=1, i\neq j}^{N} b_j - b_i/a_j + s}{(a_i + s)\left(1 + \sum_{j=1}^{N} s/a_j + s\right)}, \quad (i = 1, 2, \ldots, N). \tag{14}$$

Compared with the study results of [34, 35], it can be found by the coopetition game studies of the power market using the above model that (14) is the degenerate form of (13) and also reflects the Cournot equilibrium output of the completely competitive power market in the above references. Therefore, this coopetition game model is more universal and better conforms to the actual situations of oligarchic enterprises, and it expands the game models in the previous studies (which can be regarded as a special case of this model).

4. Analysis on the Coopetition Equilibrium Variables

4.1. Analysis of Equilibrium Output. In order to further quantitatively analyze the influence of altruistic factor ε_i on equilibrium variables, we selected peer duopoly enterprises with different costs and analyzed the relevant changes using the comparative static method. If there are only two oligarchic enterprises 1 and 2 in the market and their products have a certain volume of transaction, the following equations can be got from (11) and (13):

Coopetition equilibrium output of enterprise 1:

$$q_1^* = \frac{(a_2+2s)(r-b_1)-(1+\varepsilon_1)(r-b_2)s}{[a_1+(1-\varepsilon_1)s][a_2+(1-\varepsilon_2)s]\left(1+\sum_{i=1}^{2}(1+\varepsilon_i)s/[a_i+(1-\varepsilon_i)s]\right)} = \frac{A}{H} > 0. \tag{15}$$

Coopetition equilibrium output of enterprise 2:

$$q_2^* = \frac{(a_1+2s)(r-b_2)-(1+\varepsilon_2)(r-b_1)s}{[a_1+(1-\varepsilon_1)s][a_2+(1-\varepsilon_2)s]\left(1+\sum_{i=1}^{2}(1+\varepsilon_i)s/[a_i+(1-\varepsilon_i)s]\right)} = \frac{B}{H} > 0. \tag{16}$$

Total coopetition equilibrium output in the market:

$$Q^* = \sum_{i=1}^{2} q_i = \frac{[a_2+(1-\varepsilon_2)s](r-b_1)+[a_1+(1-\varepsilon_1)s](r-b_2)}{[a_1+(1-\varepsilon_1)s][a_2+(1-\varepsilon_2)s]\left(1+\sum_{i=1}^{2}(1+\varepsilon_i)s/[a_i+(1-\varepsilon_i)s]\right)} = \frac{A+B}{H} > 0, \tag{17}$$

where

$$A = (a_2+2s)(r-b_1)-(1+\varepsilon_1)(r-b_2)s > 0,$$

$$B = (a_1+2s)(r-b_2)-(1+\varepsilon_2)(r-b_1)s > 0,$$

$$H = [a_1+(1-\varepsilon_1)s][a_2+(1-\varepsilon_2)s]\left(1+\sum_{i=1}^{2}\frac{(1+\varepsilon_i)s}{a_i+(1-\varepsilon_i)s}\right) > 0. \tag{18}$$

The following propositions can be proved:

Proposition 1. *In case that the altruism of the rival enterprise is unchanged, the increased unilateral altruism of an enterprise will result in a decrease of its own equilibrium output but an increase in the equilibrium output of the rival enterprise. As the unilateral altruism continuously increases, the decline rate of the equilibrium output for the enterprise providing unilateral altruism will be slowed down, but the rise rate of that for the rival enterprise will be accelerated.*

Proof 1. Providing that ε_2 is constant, the influence of altruistic factor ε_1 on the equilibrium outputs q_1^*, q_2^* of two enterprises is analyzed. The change rate of q_1^*, q_2^* with respect to ε_2 can be proved similarly based on the symmetry of q_1^*, q_2^*.

① An equation is obtained by logarithm to q_1^*:

$$\ln q_1^* = \ln \frac{A}{H} = \ln A - \ln H. \tag{19}$$

Then, a new equation can be calculated as

$$
\begin{aligned}
&\frac{1}{q_1^*}\frac{\partial q_1^*}{\partial \varepsilon_1}=\frac{-(r-b_2)s}{A}+\frac{s^2(1+\varepsilon_2)}{H}\\
&\Longrightarrow \frac{\partial q_1^*}{\partial \varepsilon_1}=\frac{-H(r-b_2)s+As^2(1+\varepsilon_2))}{H^2}\\
&\quad=\frac{1}{H^2}\left\{\begin{array}{c}-s(r-b_2)[a_1+(1-\varepsilon_1)s][a_2+(1-\varepsilon_2)s]\left(1+\sum_{i=1}^{2}\frac{(1+\varepsilon_i)s}{a_i+(1-\varepsilon_i)s}\right)\\ +s^2(1+\varepsilon_2)[(a_2+2s)(r-b_1)-(1+\varepsilon_1)(r-b_2)s]\end{array}\right\}\\
&\quad=-\frac{(a_2+2s)s}{H^2}[(a_1+2s)(r-b_2)-(1+\varepsilon_2)(r-b_1)s]\\
&\quad=-(a_2+2s)s\frac{B}{H^2}<0.
\end{aligned}
\tag{20}
$$

And the following equation is further obtained:

$$
\begin{aligned}
\frac{\partial^2 q_1^*}{\partial \varepsilon_1^2}&=\left[-(a_2+2s)s\frac{B}{H^2}\right]'\\
&=-(a_2+2s)sB\left(-\frac{2}{H^3}\frac{\partial H}{\partial \varepsilon_1}\right)\\
&=-2(a_2+2s)(1+\varepsilon_2)s^3\frac{B}{H^3}<0.
\end{aligned}
\tag{21}
$$

It indicates that q_1^* decreases as ε_1 increases, but such decrease trends to become slower.

② An equation is obtained by logarithm to q_2^*:

$$\ln q_2^*=\ln\frac{B}{H}=\ln B-\ln H. \tag{22}$$

Then, a new equation can be calculated as

$$
\begin{aligned}
&\frac{1}{q_2^*}\frac{\partial q_2^*}{\partial \varepsilon_1}=\frac{0}{B}+\frac{s^2(1+\varepsilon_2)}{H}\\
&\Longrightarrow \frac{\partial q_2^*}{\partial \varepsilon_1}=\frac{s^2(1+\varepsilon_2)B}{H^2}>0.
\end{aligned}
\tag{23}
$$

And the following equation is further obtained:

$$
\begin{aligned}
\frac{\partial^2 q_2^*}{\partial \varepsilon_1^2}&=\left[s^2(1+\varepsilon_2)\frac{B}{H^2}\right]'\\
&=s^2(1+\varepsilon_2)B\left(-\frac{2}{H^3}\frac{\partial H}{\partial \varepsilon_1}\right)=2(1+\varepsilon_2)^2s^4\frac{B}{H^3}>0.
\end{aligned}
\tag{24}
$$

That is, q_2^* increases as ε_1 increases, and such increase trends to become faster. The proof is complete. □

Proposition 2. *Compared with a completely competitive market, the altruistic coopetition of any extent, regardless of unilateral altruism or mutual altruism, will decrease the total equilibrium output (Q^*) in the market and increase the market equilibrium price (P^*). The deeper the altruistic extent is, the fewer the Q^* is, and the more monopolistic the market will emerge.*

Proof 2

① Only considering changes in ε_1 under unilateral altruism, the following equation is obtained according to the results of(20) and (23):

$$
\begin{aligned}
\frac{\partial Q^*}{\partial \varepsilon_1}&=\frac{\partial q_1^*}{\partial \varepsilon_1}+\frac{\partial q_2^*}{\partial \varepsilon_1}=\frac{-(a_2+2s)sB+s^2(1+\varepsilon_2)B}{H^2}\\
&=-[a_2+(1-\varepsilon_2)s]s\frac{B}{H^2}<0.
\end{aligned}
\tag{25}
$$

As the unilateral altruism is increased, the decline rate of the equilibrium output of altruistic enterprises ($\partial q_1^*/\partial\varepsilon_1$) is higher than the rise rate of the equilibrium output of egoistic enterprises ($\partial q_2^*/\partial\varepsilon_1$), resulting in the change rate of total equilibrium output ($\partial Q^*/\partial\varepsilon_1$) which is still negative.

Similarly, (26) is obtained by the symmetry ofQ^*:

$$\frac{\partial Q^*}{\partial \varepsilon_2}=-[a_1+(1-\varepsilon_1)s]s\frac{A}{H^2}<0. \tag{26}$$

Thus, the total equilibrium output (Q^*) decreases with the increase of unilateral altruism.

② Considering changes in both $\varepsilon_1$and ε_2 under mutual altruism and Q^* is a binary function of ε_1 and ε_2, then its gradient is rewritten as

$$\nabla Q^*=\left\{\frac{\partial Q^*}{\partial \varepsilon_1},\frac{\partial Q^*}{\partial \varepsilon_2}\right\}. \tag{27}$$

For $\varepsilon_1, \varepsilon_2 \in (0,1)$ and their change interval can be expressed as $\Omega = \{(\varepsilon_1, \varepsilon_2) \mid 0 \le \varepsilon_1 \le 1, 0 \le \varepsilon_2 \le 1\}$, the direction cosine of path l from $(0,0)$ to $(\varepsilon_1, \varepsilon_2)$ in the interval is calculated as follows:

$$\left\{ \frac{\varepsilon_1}{\sqrt{\varepsilon_1^2 + \varepsilon_2^2}}, \frac{\varepsilon_2}{\sqrt{\varepsilon_1^2 + \varepsilon_2^2}} \right\} = \{\cos\alpha, \cos\beta\}. \tag{28}$$

Then, the change rate of Q^* with respect to $(\varepsilon_1, \varepsilon_2)$ is a directional derivative:

$$\frac{\partial Q^*}{\partial l} = \nabla Q^* \{\cos\alpha, \cos\beta\} = \frac{\partial Q^*}{\partial \varepsilon_1}\cos\alpha + \frac{\partial Q^*}{\partial \varepsilon_2}\cos\beta. \tag{29}$$

It is easily seen that $\partial Q^*/\partial\varepsilon_1 < 0, \partial Q^*/\partial\varepsilon_2 < 0, \cos\alpha > 0, \cos\beta > 0 \Longrightarrow \partial Q^*/\partial l < 0$.

Thus, the total equilibrium output (Q^*) decreases with the increase of mutual altruism.

③ According to Assumption 3, the inverse demand function of the market is linear, which makes the change trend of P^* is opposite to that of Q^*, and, based on the results of ① and ②, the market equilibrium price (P^*) increases with the decrease of the total equilibrium output (Q^*), and the monopoly degree of the industrial market will be also deepened. Under the market which is completely monopolized by complete cooperation between the two sides $(\varepsilon_1 = \varepsilon_2 = 1)$, the minimum Q^* and maximum P^* of products are obtained. The proof is complete. □

4.2. Analysis of Equilibrium Profit. As the unilateral altruism continuously increases, the influence of altruistic factor (ε_i) on the equilibrium profit of oligarchic enterprises (U_i^*) and the total equilibrium profit of the industry (U^*) is analyzed.

Proposition 3. *In case that the rival enterprise altruism is unchanged, despite pushing up the market equilibrium price (P^*), the increased unilateral altruism of an enterprise will result in a decrease of its own equilibrium profit (U_i^*) but a great increase of the equilibrium profit of the rival enterprise.*

Proof 3. Providing that ε_2 is constant, the influence of ε_1 on the equilibrium profits (U_1^*, U_2^*) of two enterprises is investigated. The change rate of U_1^*, U_2^* with respect to ε_2 can be proved similarly based on the symmetry of U_1^*, U_2^*.

① The equilibrium profit of enterprise 1 is computed by (3) as

$$U_1^* = p^* q_1^* - C_1^* = \left(r - b_1 - s\sum_{i=1}^{2} q_i^* - \frac{1}{2}a_1 q_1^* \right) q_1^* - d_1. \tag{30}$$

Then, an equation can be obtained:

$$\frac{\partial U_1^*}{\partial \varepsilon_1} = \left[r - b_1 - (a_1 + 2s)q_1^* - sq_2^* \right] \frac{\partial q_1^*}{\partial \varepsilon_1} - sq_1^* \frac{\partial q_2^*}{\partial \varepsilon_1}. \tag{31}$$

Furthermore, the following equation is obtained according to (15) and (16):

$$\begin{aligned} r - b_1 - (a_1 + 2s)q_1^* - sq_2^* &= \frac{(r - b_1)H - (a_1 + 2s)A - sB}{H} \\ &= \frac{1}{H}\left\{ \begin{array}{c} -(r - b_1)(a_1 + 2s)(1 + \varepsilon_2)s + (r - b_1)(1 + \varepsilon_2)[a_1 + (2 - \varepsilon_1)s]s \\ +(r - b_2)(a_1 + 2s)\varepsilon_1 s \end{array} \right\} \\ &= \frac{\varepsilon_1 s}{H}\left[(a_1 + 2s)(r - b_2) - (1 + \varepsilon_2)(r - b_1)s \right] = \varepsilon_1 s \frac{B}{H}. \end{aligned} \tag{32}$$

Combining (20) and (23), the following result is obtained:

$$\begin{aligned} \frac{\partial U_1^*}{\partial \varepsilon_1} &= \varepsilon_1 s \frac{B}{H}\left[-(a_2 + 2s)s\frac{B}{H^2} \right] - s\frac{A}{H}\frac{s^2(1 + \varepsilon_2)B}{H^2} \\ &= -\left[B(a_2 + 2s)\varepsilon_1 + As(1 + \varepsilon_2) \right] s^2 \frac{B}{H^3} < 0. \end{aligned} \tag{33}$$

That is, U_1^* decreases as ε_1 increases.

② The equilibrium profit of enterprise 2 is computed by (3) as

$$U_2^* = p^* q_2^* - C_2^* = \left(r - b_2 - s\sum_{i=1}^{2} q_i^* - \frac{1}{2}a_2 q_2^* \right) q_2^* - d_2. \tag{34}$$

Then, an equation can be obtained:

$$\frac{\partial U_2^*}{\partial \varepsilon_1} = \left[r - b_2 - (a_2 + 2s)q_2^* - sq_1^*\right]\frac{\partial q_2^*}{\partial \varepsilon_1} - sq_2^*\frac{\partial q_1^*}{\partial \varepsilon_1}. \tag{35}$$

Based on Proof ①, the following equation is obtained according to the symmetry of expression:

$$r - b_2 - (a_2 + 2s)q_2^* - sq_1^* = \varepsilon_2 s\frac{A}{H}. \tag{36}$$

Combining (20) and (23), the following result is obtained:

$$\begin{aligned}\frac{\partial U_2^*}{\partial \varepsilon_1} &= \varepsilon_2 s\frac{A}{H}\frac{s^2(1+\varepsilon_2)B}{H^2} + s\frac{B}{H}\left[(a_2+2s)s\frac{B}{H^2}\right]\\ &= \left[B(a_2+2s) + As(1+\varepsilon_2)\varepsilon_2\right]s^2\frac{B}{H^3} > 0.\end{aligned} \tag{37}$$

That is, U_2^* increases as ε_1 increases. The proof is complete. □

Proposition 4. *As the unilateral altruism continuously increases, the total equilibrium profit* (U^*) *of the industry trends to increase and then decreases, and there is an optimal threshold for unilateral altruism. Moderate unilateral altruism under certain coopetition is beneficial for increasing the total equilibrium profit* (U^*) *of the industry, but excessive altruism has an opposite effect.*

Proof 4. Providing that ε_2 is constant, the influence of ε_1 on the total equilibrium profit (U^*) of the industry is analyzed. The change rate of U^* with respect to ε_2 is similarly proved according to the symmetry of U^*.

The total equilibrium profit of the industry is expressed as $U^* = U_1^* + U_2^*$.

According to (33) and (37), the following equation is calculated as

$$\frac{\partial U^*}{\partial \varepsilon_1} = \frac{\partial U_1^*}{\partial \varepsilon_1} + \frac{\partial U_2^*}{\partial \varepsilon_1} = \left[B(1-\varepsilon_1)(a_2+2s) - As\left(1-\varepsilon_2^2\right)\right]\frac{s^2 B}{H^3}. \tag{38}$$

Let $\partial U^*/\partial \varepsilon_1 = 0$; that is, $B(1-\varepsilon_1)(a_2+2s) - As(1-\varepsilon_2^2) = 0$.

This can be simplified as follows:

$$\begin{aligned}&B(a_2+2s) - B(a_2+2s)\varepsilon_1 - \left[(a_2+2s)(r-b_1) - (1+\varepsilon_1)\right.\\ &\quad\left.\cdot (r-b_2)s\right]\left(1-\varepsilon_2^2\right)s = 0.\end{aligned} \tag{39}$$

The following solution is obtained:

$$\begin{aligned}\varepsilon_1^* &= \frac{B(a_2+2s) + s^2(r-b_2)\left(1-\varepsilon_2^2\right) - (a_2+2s)(r-b_1)\left(1-\varepsilon_2^2\right)s}{B(a_2+2s) - s^2(r-b_2)\left(1-\varepsilon_2^2\right)}\\ &= 1 - s\left(1-\varepsilon_2^2\right)\frac{r - b_1/r - b_2 - 2s/a_2 + 2s}{B/r - b_2 - s^2\left(1-\varepsilon_2^2\right)/a_2 + 2s}.\end{aligned} \tag{40}$$

There are $\varepsilon_1 < \varepsilon_1^*\ \partial U^*/\partial \varepsilon_1 > 0$ and $\varepsilon_1 > \varepsilon_1^*\ \partial U^*/\partial \varepsilon_1 < 0$. It indicates that the maximum $U^*$is obtained atε_1^*. Similarly, the following solution is obtained:

$$\varepsilon_2^* = 1 - s\left(1-\varepsilon_1^2\right)\frac{r - b_2/r - b_1 - 2s/a_1 + 2s}{A/r - b_1 - s^2\left(1-\varepsilon_1^2\right)/a_1 + 2s}. \tag{41}$$

The proof is complete.

The above analysis shows, that under unilateral altruism, the optimal altruistic threshold is affected by both the cost coefficient of the duopoly enterprises and the initial altruistic intensity of their rival enterprises. However, it is seen that, according to the cost of both sides, the positively unilateral altruism of enterprises can increase the total equilibrium profit of the industry within a certain range $(\varepsilon_1 \le \varepsilon_1^*)$, even if the rival enterprises have no cooperation intention at first (e.g., $\varepsilon_2 = 0$).

Then, considering the increased mutual altruism of the duopoly enterprises, the influence of increased $\varepsilon_1$and $\varepsilon_2$on $U^*$is further analyzed. For the purpose of discussion, it is assumed that the duopoly enterprises have the same altruism $(\varepsilon_1 = \varepsilon_2 = \varepsilon)$. □

Proposition 5. *In the case of the mutual altruism between the duopoly enterprises, the total equilibrium profit of the industry* (U^*) *will keep increasing, when the intensity of mutual altruism increase by the altruistic factors of both sides is changed at the same time. And the maximum* U^* *is obtained at complete monopoly.*

Proof 5. Providing $\varepsilon_1 = \varepsilon_2 = \varepsilon$, an equation is obtained according to (20) and (23):

$$\frac{\partial q_1^*}{\partial \varepsilon} = \frac{\partial q_1^*}{\partial \varepsilon_1} + \frac{\partial q_1^*}{\partial \varepsilon_2} = \frac{As^2(1+\varepsilon) - Bs(a_2+2s)}{H^2}. \tag{42}$$

Based on the symmetry of expression, a new equation is obtained:

$$\frac{\partial q_2^*}{\partial \varepsilon} = \frac{\partial q_2^*}{\partial \varepsilon_1} + \frac{\partial q_2^*}{\partial \varepsilon_2} = \frac{Bs^2(1+\varepsilon) - As(a_1+2s)}{H^2}. \tag{43}$$

Combining (32), the following equation is deduced:

$$\begin{aligned}\frac{\partial U_1^*}{\partial \varepsilon} &= \left[r - b_1 - (a_1+2s)q_1^* - sq_2^*\right]\frac{\partial q_1^*}{\partial \varepsilon} - sq_1^*\frac{\partial q_2^*}{\partial \varepsilon}\\ &= \varepsilon s\frac{B}{H}\frac{As^2(1+\varepsilon) - Bs(a_2+2s)}{H^2}\\ &\quad - s\frac{A}{H}\frac{Bs^2(1+\varepsilon) - As(a_1+2s)}{H^2}\\ &\quad - \frac{1}{H^3}\left[ABs^3(1+\varepsilon)(\varepsilon-1) - B^2s^2\varepsilon(a_2+2s)\right.\\ &\quad \left.+ A^2s^2(a_1+2s)\right].\end{aligned} \tag{44}$$

Similarly, another new equation is obtained:

$$\frac{\partial U_2^*}{\partial \varepsilon} = \left[r - b_2 - (a_2 + 2s)q_2^* - sq_1^*\right]\frac{\partial q_2^*}{\partial \varepsilon} - sq_2^*\frac{\partial q_1^*}{\partial \varepsilon}$$
$$= \frac{1}{H^3}\left[ABs^3(1+\varepsilon)(\varepsilon - 1) - A^2 s^2 \varepsilon (a_1 + 2s) + B^2 s^2 (a_2 + 2s)\right]. \tag{45}$$

The change rate of U^* with respect to ε can be obtained:

$$\frac{\partial U^*}{\partial \varepsilon} = \frac{\partial U_1^*}{\partial \varepsilon} + \frac{\partial U_2^*}{\partial \varepsilon}$$
$$= \frac{s^2(1-\varepsilon)}{H^3}\left[B^2(a_2 + 2s) + A^2(a_1 + 2s) - 2ABs(1+\varepsilon)\right]$$
$$= \frac{s^2(1-\varepsilon)}{H^3}\left\{\begin{array}{c} B^2\left[a_2 + (1-\varepsilon)s\right] + A^2\left[a_1 + (1-\varepsilon)s\right] \\ +(A-B)^2 s(1+\varepsilon) \end{array}\right\} \geq 0. \tag{46}$$

Thus, U^* continuously increases as ε increases, and the maximum U^* is obtained at $\varepsilon = 1$. The proof is complete.

Through the deductive proofs of the above propositions, the following conclusions can be drawn:

(1) Under the certain coopetition, the increased unilateral altruism of oligarchic enterprises in the industry will result in a decrease of their own equilibrium output and profit but an increase in the equilibrium output and profit of rival enterprises. As the unilateral altruism continuously increases, the altruistic intensity of the altruistic enterprises will be declined, and the output of the rival enterprises will be accelerated. Resultantly, the continuously increased unilateral altruism is unsustainable.

(2) Compared with the complete competition pattern, the altruistic coopetition of any form will decrease the total equilibrium output in the market and increase the market equilibrium price. With the deepening of altruistic cooperation, the industry will show the trend of monopoly.

(3) The increased altruistic cooperation between oligarchic enterprises will always help in improving the total equilibrium profit of the industry. Therefore, there is the long-standing motivation and tendency of alliance or collusion between oligarchic enterprises. But the total equilibrium profit of the industry trends to increase and then decreases when the unilateral altruism keeps increasing continuously. Furthermore, the enterprises may not earn excessive profits for themselves if there is the unreasonable internal distribution in the industry. Conclusively, the increased unilateral altruism is instable and unsustainable. However, the total equilibrium profit of the industry will keep increasing with the equal increase of mutual altruism and reach a peak when the market becomes completely monopolized.

In order to validate the abovementioned propositions and further analyze the coopetition equilibrium characteristics of oligarchic enterprises with a nonlinear cost function in the industry, the duopoly generation companies in the power market were selected as the study objects for further investigation. □

5. Validation of the Coopetition between Generation Companies

The electric power industry is a foundational energy industry for the lifeline of national economy. Although the competitive mechanism is introduced into the current power marketization reform, the generation market is still not a fully competitive market because of a large investment scale, power transmission restrictions, grid loss, capacity limitations, etc. The characteristics of oligopoly will exist in a long time, and most generation companies have market power more or less and thus can influence the market through physical or economic withholding [7].

After 2015 new power reform in China, the power grid is not responsible for the purchase and sale business of all electricity quantity and also not the unique buyer any longer; the share of bilaterally direct trading electricity becomes bigger and bigger in the power market, and the electricity price is also determined by many market factors, other than only by the coal price (though it is still the major influential factor). Particularly, under the direct bidding trading environment of electricity sellers and buyers, the collusion between generation companies will greatly affect the trading electricity price. The practices also have shown that the disordered competition and the unregulated market power will inevitably lead to a large fluctuation of power grid and volatile market [47, 48]. When the stable power supply is maintained, how to balance and coordinate the interests of all participants on the generation side and actively promote the deepening of power industry reform so as to achieve the sustainable development of the industry? It is necessary to consider the relationship between altruism and egoism and study the balance between competition and cooperation from every aspect.

5.1. Description of Issues. The generation market has the characteristics of scale economy and oligopoly. The production cost of thermal power generating units conforms to the quadratic function, which meets the characteristics of study objects in the above model. The duopoly generation companies on the generation side were selected to validate the above model conclusions. In order to better match the realities of generation side and facilitate the analysis and discussion, the issues are described as follows:

(1) The duopoly generation companies met all the assumptions in Section 3.1.

(2) The model was established under the scenario of bilateral market and spot transaction; i.e., the power users directly purchased electricity from generation companies in a bilateral mode and made the deals in

a form of spot transaction, without the participation of independent system operators (ISO). The restrictions of generating units and power grid as well as the power transmission congestion were not considered in the model calculations.

(3) The data of IEEE system in [35] were used for the inverse demand function of the power market and the cost coefficients of two generation companies. After analysis on the marginal cost functions of two generation companies, generation company 1 was regarded as a low-cost generation company (or new power plant) and generation company 2 was regarded as a high-cost generation company (or old power plant). The preset parameters are shown in Table 1.

5.2. Analysis and Conclusions

5.2.1. Electricity Price Subsidy. A real-time supply-demand balance is required for the stable running of power grid, while the total power demand in the bilateral market presents little change in a short term. Based on the conclusion of Proposition 2 in Section 4, the mutually altruistic coopetition between oligarchic generation companies can lead to the fluctuation of total power output, which will disturb the power demand and electricity purchasing plan of power users, thus increasing the uncertainty risk of bilateral market and harming the safe running and deployment of power grid. To stabilize the bilateral trading electricity quantity, we analyzed the quantitative relation between the degree of collusion among generation companies and the market clearing electricity price and further worked out the electricity price subsidy index in the coopetition market according to the price transmission mechanism.

From (11), the total power output of duopoly generation companies at coopetition equilibrium was calculated as follows:

$$Q^* = \sum_{i=1}^{2} q_i = \frac{\sum_{i=1}^{2} r - b_i/a_i + (1-\varepsilon_i)s}{1 + \sum_{i=1}^{2}(1+\varepsilon_i)s/a_i + (1-\varepsilon_i)s}. \tag{47}$$

It indicated that when the cost coefficient and altruistic factor were constant, increasing the exogenous variable r would enhance the total power output (Q^*) of altruistic equilibrium. According to Assumption 3, r was the electricity price ceiling in the inverse demand function of the power market. If the total equilibrium power output in a completely competitive market (Q_0^*) (as the benchmark power supply) is kept unchanged, the power users should provide a certain price subsidy. Supposing that the adjustment of electricity price was linear, i.e., the power users could adjust the electricity price ceiling (r) for m times, the following calculation was done:

$$\begin{aligned} Q_0^* &= \frac{\sum_{i=1}^{2} mr - b_i/a_i + (1-\varepsilon_i)s}{1 + \sum_{i=1}^{2}(1+\varepsilon_i)s/a_i + (1-\varepsilon_i)s} \\ &= \frac{\left[a_2 + (1-\varepsilon_2)s\right](mr - b_1) + \left[a_1 + (1-\varepsilon_1)s\right](mr - b_2)}{H}. \end{aligned} \tag{48}$$

And the price subsidy index was obtained:

$$m = \frac{Q_0^* H + \left\{\left[a_2 + (1-\varepsilon_2)s\right]b_1 + \left[a_1 + (1-\varepsilon_1)s\right]b_2\right\}}{r\left\{\left[a_2 + (1-\varepsilon_2)s\right] + \left[a_1 + (1-\varepsilon_1)s\right]\right\}}. \tag{49}$$

When the cost coefficient was constant, m was a binary function of altruistic factor $(\varepsilon_1, \varepsilon_2)$; i.e., m was mainly determined by the cooperation intensity between two generation companies, and m could be calculated at any value of $(\varepsilon_1, \varepsilon_2)$. Under complete competition $(\varepsilon_1 = \varepsilon_2 = 0)$, the power users did not need to pay the price subsidy $(m = 1)$; under complete monopoly $(\varepsilon_1 = \varepsilon_2 = 1)$, the power users have to pay the highest price subsidy. In practice, the benchmark power supply (Q_0^*) could be the total power demand of the power users in the bilateral market, so as to figure out a price subsidy index with more practical significance.

It is concluded that if there is the collusion between oligarchic generation companies or the total power demand (Q_0^*) is huge and inelastic (i.e., the supply cannot meet the demand) in the bilateral market, the electricity price will be continuously increased to maintain the supply and demand balance of electricity, and the electricity price ceiling will be increased accordingly. If the price changes could not be transmitted in time or there is lack of effective supervision and punishment mechanism, it will eventually bring huge risks and harms or even a crash to the operation of power market. California power crisis is a realistic case [47]. It happened in summer: the power supply in California was in shortage, and the generation companies jointly colluded to push up the electricity price for distributors because of no effective supervision; however, the electricity price for users was fixed and thus the high electricity price failed to be transmitted to the final users, but the distribution companies had to continuously purchase electricity from the spot market at an excessive high price, which eventually triggered the power crisis of large-scale blackouts due to the overspending. Our quantitative analysis also has revealed the causes for the nonsustainability of such power market operation. Similarly, the pilot power market in Northeast China was suspended due to an increase of electricity price caused by local market power in 2006. The electricity price subsidy index (m) quantitatively indicates the relation between the degree of collusion on the generation side and the market electricity price; thus it can be used as a quantitative index to measure the degree of coopetition in the market.

Table 1: Technical and economic parameters.

Power user	r($/MWh)		s($/MW^{2}h)
	50		0.015
Generation company	a_i ($/MW^{2}h)	b_i($/MWh)	d_i($)
1	0.02	2	3601
2	0.06	1	3384

5.2.2. Analysis on the Coopetition Equilibrium. According to the parameters in Table 1, the function diagrams between the equilibrium profit U_i^* $(i = 1, 2)$ of duopoly generation companies and the altruistic factors $(\varepsilon_1, \varepsilon_2)$ are obtained.

As shown in Figures 1(a) and 1(b), the equilibrium profit of generation companies is increased with the increase in the altruism of rival companies and decreased with the increase in their own altruism, and its change rate will be slowed down with the increase of altruism. The minimum profit of a generation company is made when it is completely altruistic and the rival company is completely egoistic, rather than under the situation of complete competition $(\varepsilon_1 = \varepsilon_2 = 0)$; the maximum profit of a generation company is reached when it is completely egoistic and the rival company is completely altruistic, rather than under the situation of complete monopoly $(\varepsilon_1 = \varepsilon_2 = 1)$.

In Figure 2, the minimum and maximum total equilibrium profits of generation side are gotten under complete competition $(\varepsilon_1 = \varepsilon_2 = 0)$ and complete monopoly $(\varepsilon_1 = \varepsilon_2 = 1)$, respectively. If the duopoly enterprises have the same altruism $(\varepsilon_1 = \varepsilon_2 = \varepsilon)$, increasing the intensity of mutual altruism will continuously increase the total equilibrium profit of the industry, and the coopetition under such pattern is an effective way for rapidly increasing the abovementioned profit. With the increase in the unilateral altruism of any side at the initial coopetition level, there is an optimal threshold for the altered altruistic factor that can maximize the total equilibrium profit of the industry. The unilateral altruism less than this threshold will increase the total profit of the electric power industry.

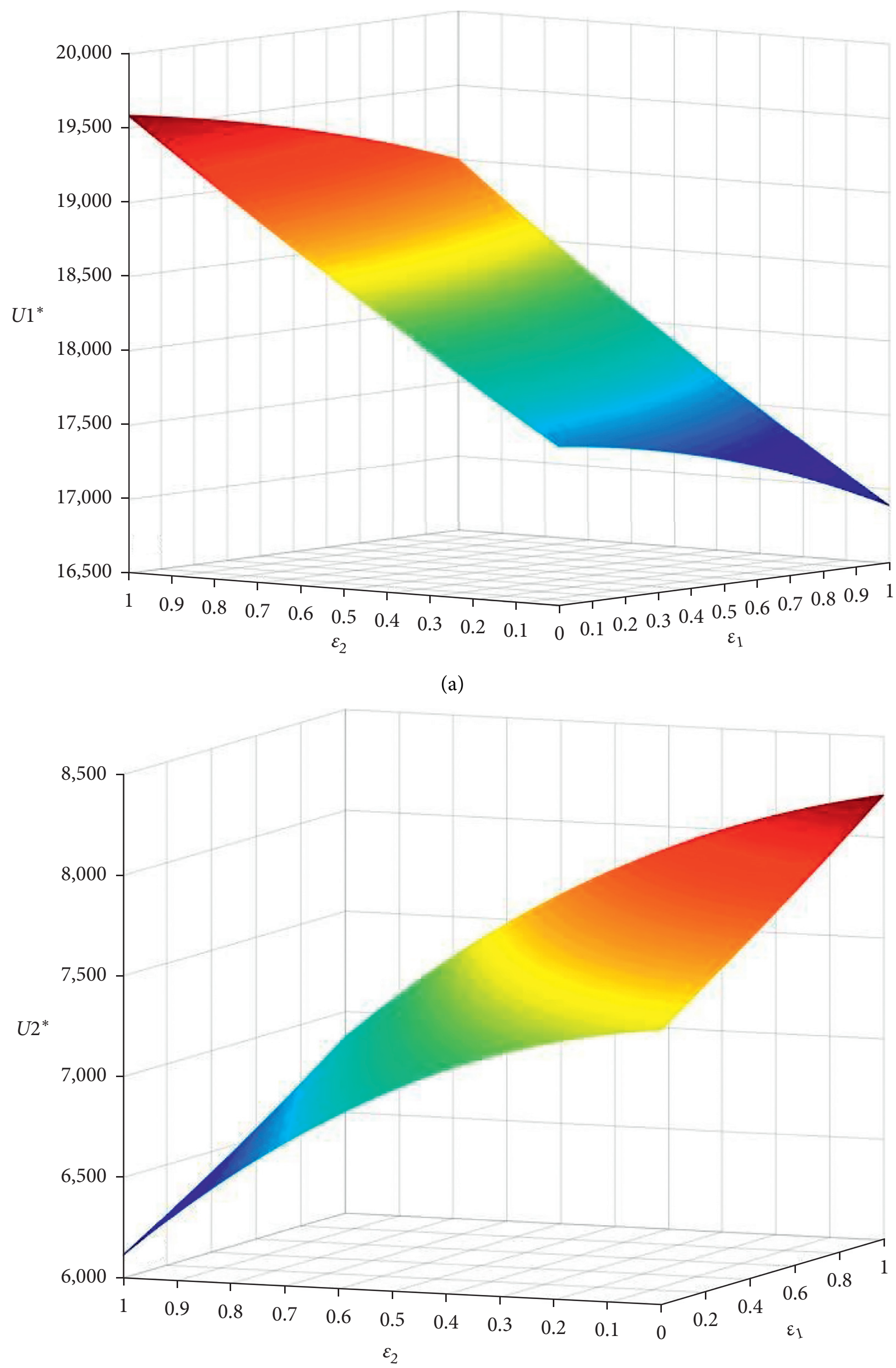
In addition, the relationship between coopetition equilibrium variables (**CEV**) and the altruistic factor (**AF**) under the increased mutual or unilateral altruism was discussed by using the relevant data at several different altruistic levels.

According to Table 2, the total equilibrium profit of the industry (U^*) is continuously increased when the generation companies under mutually altruistic coopetition cut down the equilibrium power output (q_i^*) simultaneously. With the deepening of mutually altruistic cooperation, the equilibrium profit of the low-cost generation company (U_1^*) is continuously increased, while the equilibrium profit of the high-cost generation company (U_2^*) trends to increase and then decreases, and there is a threshold for its maximum equilibrium profit.

These findings show that selecting the proper altruism for moderate coopetition is an effective way for high-cost generation companies to make excessive profits, and it is difficult for generation companies with different costs to achieve the complete monopoly or ensure the intrinsic stability of monopoly in the finite repeated coopetition games only focusing on short-term profits.

Based on Tables 3 and 4 (compared with the initial data in Table 2), the increased unilateral altruism of generation companies will result in a decrease of their own equilibrium power output (q_i^*) and profit (U_i^*) but an increase in the equilibrium power output and profit of rival companies. Meanwhile, the total equilibrium power output of the industry (Q^*) is continuously decreased, and the total equilibrium profit of the industry (U^*) is further increased. As shown in Tables 3 and 4, the increased unilateral altruism of high-cost generation companies at the initial coopetition level will further cut down the total equilibrium power output (Q^*) and better improve the total equilibrium profit of the industry (U^*) than that of low-cost generation companies. However, the exorbitant optimal threshold of altruistic factor (ε_2^*) indicates that the maximized total equilibrium profit of the industry (U^*) is only achieved at the expense of huge benefits from high-cost generation companies, which is almost impossible in the real world. The higher the initial altruism level between the two sides is, the larger the optimal threshold of unilateral altruistic factor (ε_i^*) will be required, and the less likely it will be to increase altruism unilaterally. As a result, both the decline rate of equilibrium power output (q_i^*) and profit (U_i^*) of unilaterally altruistic companies and the rise rate of equilibrium power output (q_i^*) and profit (U_i^*) of rival companies are smaller, the increase in the total equilibrium profit (U^*) of the industry is not obvious, and there is lack of motivation for the increased unilateral altruism.

The duopoly generation companies achieve the coopetition equilibrium by adjusting the power output from both sides, which may lead to the fluctuation of total equilibrium power output (Q^*) in the spot market. As shown in Tables 2–4, the higher the overall cooperation degree of two parties is, the larger the electricity price subsidy index (m) will be required for stabilizing the fluctuation of trading electricity quantity; thus, the above index will reflect the degree of coopetition between two parties. If the high-cost generation companies further enhance the altruism, the market clearing electricity price will be increased more significantly, and a higher electricity price subsidy index (m)



(b)

FIGURE 1: Relations between the equilibrium profit of generation companies and the altruistic factors. (a) The generation company is a low-cost company (new power plant). (b) The generation company is a high-cost company (old power plant).

will be needed. In addition, the mutually altruistic level can be calculated according to the trading power output, so as to determine the degree of coopetition in the market. For example, when the trading electricity quantity of two generation companies in the market is ($q_1^* = 779.49$ MWh, $q_2^* = 349.57$ MWh), the mutually altruistic level is calculated as (0.721, 0.5) using (15) and (16) and then substituted into (49) to figure out the degree of coopetition in the market ($m = 1.1$) (electricity price subsidy index).

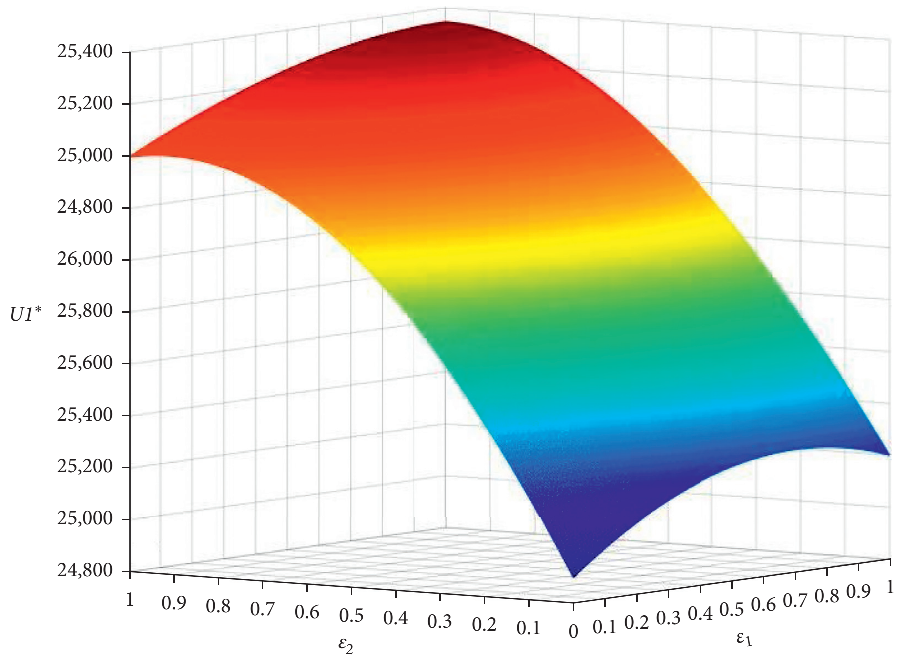


Figure 2: Relation between the total equilibrium profit of generation side and the altruistic factor.

Table 2: Equilibrium variables under the mutual altruism of generation companies.

CEV	AF $(\varepsilon_1, \varepsilon_2)\varepsilon_1 = \varepsilon_2$						
	(0, 0)	(0.1, 0.1)	(0.3, 0.3)	(0.5, 0.5)	(0.6, 0.6)	(0.8, 0.8)	(1, 1)
q_1^* (MWh)	838.60	830.58	816.68	805.63	801.22	794.75	791.67
q_2^* (MWh)	404.68	392.17	367.50	343.04	330.78	306.02	280.56
Q^* (MWh)	1243.27	1222.75	1184.17	1148.67	1132.01	1100.77	1072.22
U_1^* ($)	13980.1	14134.33	14423.55	14697.86	14833.26	15108.17	15399
U_2^* ($)	3985.41	4025.5	4044.04	3984.03	3925.13	3748.67	3489.61
U^* ($)	17965.51	18159.83	18467.59	18681.88	18758.39	18856.84	18888.61
m	1.00	1.02	1.05	1.08	1.09	1.12	1.15

Table 3: Equilibrium variables under the unilateral altruism threshold of the low-cost generation company.

CEV	AF $(\varepsilon_1^*, \varepsilon_2)\varepsilon_1^* \geq \varepsilon_2$						
	(0.705, 0)	(0.712, 0.1)	(0.717, 0.3)	(0.721, 0.5)	(0.751, 0.6)	(0.846, 0.8)	(1, 1)
q_1^* (MWh)	745.10	751.10	764.95	779.49	783.83	789.69	791.67
q_2^* (MWh)	420.26	406.74	378.71	349.57	335.42	307.54	280.56
Q^* (MWh)	1165.36	1157.84	1143.66	1129.06	1119.25	1097.23	1072.22
U_1^* ($)	13587.39	13765.44	14142.54	14537.07	14719.37	15070.99	15399
U_2^* ($)	4563.89	4519.06	4373.4	4158.7	4045.13	3786.36	3489.61
U^* ($)	18151.27	18284.49	18515.93	18695.77	18764.5	18857.35	18888.61
m	1.06	1.07	1.08	1.10	1.11	1.13	1.15

Table 4: Equilibrium variables under the unilateral altruism threshold of the high-cost generation company.

CEV	AF $(\varepsilon_1, \varepsilon_2^*)\varepsilon_1 \le \varepsilon_2^*$						
	(0, 0.910)	(0.1, 0.915)	(0.3, 0.922)	(0.5, 0.926)	(0.6, 0.935)	(0.8, 0.963)	(1, 1)
q_1^* (MWh)	880.78	872.20	854.41	835.69	826.67	808.88	791.67
q_2^* (MWh)	264.05	266.06	270.76	276.24	277.78	279.86	280.56
Q^* (MWh)	1144.84	1138.26	1125.16	1111.93	1104.44	1088.73	1072.22
U_1^* ($)	15793.53	15765.39	15690.22	15589.92	15550.11	15472.51	15399
U_2^* ($)	2928.4	2986.65	3114.11	3255.09	3310.44	3409.03	3489.61
U^* ($)	18721.93	18752.04	18804.33	18845.02	18860.56	18881.54	18888.61
m	1.08	1.09	1.10	1.11	1.12	1.14	1.15

6. Conclusions and Prospect

6.1. Conclusions. This paper focused on the possible altruistic preference and cooperation intention of decision makers with limited rationality, which were ignored by most models only with a consideration on the self-profits in the current studies of corporate competition and game. With the oligarchic industry as a coopetition system and by introducing the altruistic factor to quantify the moderate cooperation, we established a coopetition game model with a nonlinear cost function for oligarchic enterprises in the industry, then quantitatively analyzed the changes of equilibrium variables under different coopetition patterns, and finally completed the related validation by the application of the model in the electric power industry. The major conclusions are described as follows:

(1) Compared with a completely competitive market, the altruism-based coopetition of any form between oligarchic enterprises in the industry will decrease the total equilibrium output in the market, increase the market equilibrium price, and promote the growth in the total equilibrium profit of the industry. Therefore, there is always the motivation and tendency of cooperation or alliance between oligarchic enterprises, and the industry trends to be more monopolized as the altruistic cooperation between enterprises is deepened.

(2) Under the initial coopetition, the increased unilateral altruism of oligarchic enterprises will decrease their own equilibrium output and profit but increase the equilibrium output and profit of rival enterprises. The increased unilateral altruism is instable and unsustainable, because the altruistic intensity of the altruistic enterprises will be declined and the output of the rival enterprises will be accelerated. Meanwhile, the total equilibrium profit of the industry trends to increase and then decreases, there is a unilateral altruistic threshold, and the altruistic enterprises may not earn excessive profits. The validation using the electric power industry as a case has further shown that the unilateral altruism of high-cost oligarchic enterprises is more beneficial for increasing the total equilibrium profit of the industry yet at the expense of huge self-interests; thus, it is hard to achieve. In addition, the higher the initial altruism level is, the less likely it will be to increase altruism unilaterally. The increase in the total equilibrium profit of the industry is not obvious, and there is lack of motivation for the increased unilateral altruism.

(3) The total equilibrium profit of the industry will keep increasing with the equal increase of mutually altruistic coopetition and reach the maximization when the market becomes completely monopolized; thus, such coopetition was an effective way for improving the total equilibrium profit of the industry. As shown by the validation, the equilibrium profit of high-cost enterprises under such coopetition pattern will increase and then decreases, which indirectly shows that it is difficult for oligarchic enterprises with different costs to finally achieve the complete monopoly of the market or ensure the intrinsic stability of monopoly.

(4) Using the established model, the study conclusions have been further validated and improved by analyzing the coopetition relationship between duopoly generation companies. The complete competition models in the existing literature of power market are the special cases of our coopetition model when the mutually altruistic level is (0, 0), and the analytic expression for the equilibrium solution of these original complete competition models is a degenerative form for the equilibrium solution of our coopetition model. The extension of the models and solutions in the previous studies is realized in this paper. Furthermore, the electricity price subsidy index for maintaining the stability of total power output has been proposed. The quantitative analysis has revealed that the mutually altruistic collusion between oligarchic generation companies and the exorbitant power demand in the market will cause an increase in the electricity price ceiling and eventually induce the electricity crisis if there are no effective transmission mechanism and supervision mechanism of electricity price.

6.2. Prospect. There are some deficiencies in this paper that shall be improved, and some issues still need to be further expanded and deeply investigated.

(1) In this paper, the coopetition game model was established under the precondition of complete

information. However, the information in the real market is usually incomplete, and such incompleteness can be embodied by the altruistic uncertainty of market players when the cooperation intention is expressed using the quantitative altruism. In the subsequent studies, with the condition of incomplete information, it may be considered that slightly altruistic factor follows some probability distribution, so that the model is more practical.

(2) The single-stage coopetition game of oligarchic enterprises was analyzed with the comparative static analysis method in this paper. However, the formation of coopetition between oligarchic enterprises is in fact a result of long-term repeated game, which is exactly our ongoing follow-up research work.

(3) In order to facilitate the deduction and analysis of the model, the restrictions in the real power market were not involved in the coopetition game model validated by using the electric power industry as a case, which affects the practicality of the model. The restrictions of power system may be gradually included in the further research to improve the practical value of the model. In addition, the relevant data of generating units (e.g., coal consumption, water consumption, and running hours) in China are sensitive for generation companies, and they are difficult to obtain. Therefore, we completed the numerical validation of the model using the simulative data from the existing literature. If the actual data are available in the future, we will make the subsequent empirical analysis.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that there are no conflicts of interest regarding the publication of this paper.

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