Ingenious Solution for the Rank Reversal Problem of TOPSIS Method

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Abstract

Although the classic TOPSIS method is very practical, there may be a problem of rank reversal in the addition, deletion, or replacement of the candidate set, which makes its credibility greatly compromised. Based on the understanding of the classical TOPSIS method, this paper establishes a new improved TOPSIS method called NR-TOPSIS. Firstly, the historical maximum and minimum values of all attribute indicators from a global perspective during the evaluation process are determined. Secondly, according to whether the attributes belong to the benefit attribute or cost attribute, standardization is carried out. And then, in the case where the historical values of attributes are determined, we fix the positive ideal solution and the negative ideal solution. At the same time, this paper gives the definition of ranking stable and proves that the NR-TOPSIS proposed satisfies ranking stable, which theoretically guarantees that the rank reversal phenomenon does not exist. Finally, in the verification of examples, the results are consistent with the theoretical analysis, which further support the theoretical analysis. The NR-TOPSIS method overcomes rank reversal, which is not only obviously superior to the classical TOPSIS method but also relatively superior to the R-TOPSIS method which has also overcome rank reversal. It is also superior to other reference methods due to its simple calculation.

1. Introduction

Over the last thirty years, the research on multiple attribute (criteria) decision-making (MADM/MCDM) has become a hot issue in different fields of natural science and social science [1–16]. The technique for order preference by similarity to the ideal solution (TOPSIS) is a useful and powerful method for dealing with MADM problems which is proposed by Hwang and Yoon [17, 18]. TOPSIS is a general method for solving MADM problems, which takes into account both positive and negative ideal solutions. Many scholars have combined the TOPSIS method with other intelligent computing methods, resulting in many cross-research results and solving many problems in real life [19–33]. These cross-combination approaches include data envelopment analysis (DEA) [19], hesitant fuzzy correlation coefficient [20], analytic hierarchy process (AHP) [21], fuzzy AHP [22–25], intuitionistic fuzzy number [26, 27], triangular fuzzy number [28], vague sets [29], analytic network process (ANP) [30], weighted grey relational coefficient [31], and neutrosophic sets [32, 33]. These hybrid methods make full use of the advantages of the TOPSIS method, which can quantitatively characterize the difference between the alternatives and the positive and negative ideal solutions. However, they do not discuss the fact that if the TOPSIS method has a reversal of order, the credibility of the hybrid approach will be severely reduced. When the decision-making object changes on the original basis, especially increasing or reducing the evaluation object or replacing a certain evaluation object, the traditional TOPSIS method often has the phenomenon of rank reversal. The quality of evaluation methods often depends on the stability and consistency of evaluation results. That is to say, when the evaluation object is changed, the corresponding results should not be inconsistent. This is what is commonly referred to as rank reversal. In fact, many MADM methods have the problem of rank reversal, such as analytic hierarchy
process (AHP) [34–41], VileKriterijumska Optimizacija I KOmpromisno Resenje (VIKOR) [42], preference ranking organization method for enrichment of evaluations (PROMETHEE) [43], and evaluation based on distance from average solution (EDAS) [44]. Achieving the rank preservation of MADM/MCDM methods has turned into the focus of many scholars in applied study [45–51].

The problem of rank reversal in the TOPSIS method has received great attention, and different scholars have tried to find satisfactory solutions from an experimental or theoretical level [52–57]. By collecting 130 related papers published in international journals from 1980 to 2015, Aires and Ferreira provided an extensive literature review on MCDM methodologies and rank reversals, including the TOPSIS method [52]. Zavadskas et al. [53] also reviewed the development of the TOPSIS method from 2000 to 2015, but did not discuss the solution of rank reversal. Ren et al. [54] proposed a novel M-TOPSIS method which can solve the problems of TOPSIS such as rank reversals and evaluation failure when alternatives are symmetrical, but there is no theoretical proof. Wang and Luo [45] used the counterexample to illustrate the fact that the TOPSIS method has a problem of rank reversal, but did not give a solution. García-Cascales and Lamata [55] considered two aspects to improve the rank reversal problem of the TOPSIS method. On the one hand, a new norm is used for normalization, but it is not sufficient. On the other hand, the absolute mode is used to rewrite the positive ideal solution and the negative ideal solution. The fixed instance verification is feasible, but the indicator attribute is not considered, and the improved method does not have the validity in all cases. Once the cost indicator data are encountered, it will be invalid. Based on the understanding of data dispersion degree, Yang and Wu [56] proposed a new strategy combining improved grey relational analysis and TOPSIS method and verified the controllability of order reversal in the process of case verification. In the process of evaluation, the idea of attribute variable weight was adopted with the change of evaluation knowledge. Aires and Ferreira [57] analyzed that most of the literature on rank reversal of TOPSIS was limited to case studies and then developed an improved TOPSIS method with domain parameters, called R-TOPSIS method, which demonstrated the effectiveness of the method from a statistical point of view. But in theory, there is no reasonable proof and explanation for this method. The key to solving the problem of rank reversal is to analyze the problem from a global perspective and always use the same scale to measure the data conversion. In addition, some scholars have used different integrated MADM methods to focus on the search of the best option and also to determine which method’s best option was better and weakened the attention of the rank reversal problem [46, 58–61]. In doing so, there is no substantive solution to the problem of rank reversal.

In brief, this paper will focus on the following main research: (i) we propose a novel extended TOPSIS method to overcome the rank reversal problem, which is suitable for MADM problems with arbitrary attribute indicators; (ii) we give the definition of ranking stable and ranking unstable and strictly prove in theory that the NR-TOPSIS method proposed in this paper is ranking stable and the R-TOPSIS method is also ranking stable; (iii) we use this method to verify two classic cases, which highlight the effectiveness and consistency of this method, and provide a new simple TOPSIS improvement scheme for solving MADM problems.

The remainder of the paper is organized as follows. Section 2 presents the traditional TOPSIS method, R-TOPSIS method, and a new improvement to solve rank reversal problem in the TOPSIS method which is called the NR-TOPSIS method. Section 3 analyzes the theoretical mechanism of rank reversal problem and proves that the NR-TOPSIS method as well as the R-TOPSIS method is ranking stable. Section 4 describes two numerical cases to verify the validity and consistency of the NR-TOPSIS method. Finally, Section 5 provides some important conclusions.

2. Rank Reversal in TOPSIS Method and Its Improvement

The proposal of the TOPSIS method, in a sense, conforms to the logic of human decision-making and judgment and can reasonably compare the proposed scheme with the positive ideal solution and the negative ideal solution quantitatively, thus giving the order closest to the ideal solution. In this section, we will show the classical TOPSIS method and the improved TOPSIS method named R-TOPSIS that has overcome the rank reversal. Furthermore, a new improved method of TOPSIS to overcome rank reversal based on statistical laws will be proposed.

Formally, consider an MADM problem with $m$ alternatives and $n$ attributes, and the weights of the attributes are known which are denoted by $w_1, w_2, \ldots, w_n$, with $\sum_{j=1}^{n} w_j = 1$, $w_j > 0$, $j = 1, 2, \ldots, n$. Use the symbol $X$ to represent the original decision matrix, which is expressed as the following form:

$$
X = \begin{bmatrix}
A_1 & A_2 & \cdots & A_m \\
\begin{bmatrix}
C_1 & C_2 & \cdots & C_n \\
\begin{bmatrix}
\begin{bmatrix}
x_{i1} & x_{i2} & \cdots & x_{in} \\
\begin{bmatrix}
x_{j1} & x_{j2} & \cdots & x_{jn} \\
\vdots & \vdots & \ddots & \vdots \\
x_{m1} & x_{m2} & \cdots & x_{mn}
\end{bmatrix}
\end{bmatrix}
\end{bmatrix}
\end{bmatrix}
\end{bmatrix}
$$

(1)

where $A_1, A_2, \ldots, A_m$ are alternatives, $C_1, C_2, \ldots, C_n$ are attributes, and $x_{ij}$ are original attribute values.

2.1. The Traditional TOPSIS Method. We briefly explain the implementation of the classic TOPSIS method as follows:

**Step 1.** Compute the normalized decision-making matrix $Y = (y_{ij})_{m \times n}$, where $y_{ij}$ are normalized attribute values expressed as

$$
y_{ij} = \frac{x_{ij}}{\sqrt{\sum_{i=1}^{m} x_{ij}^2}}, \quad i = 1, 2, \ldots, m; \quad j = 1, 2, \ldots, n.
$$

(2)

**Step 2.** Calculate the weighted normalized decision-making matrix $R = (r_{ij})_{m \times n}$. The weighted normalized attribute value $r_{ij}$ is computed by
$$r_{ij} = w_j y_{ij}, \quad i = 1, 2, \ldots, m; j = 1, 2, \ldots, n$$  \hfill (3)

**Step 3.** Determine the positive ideal solution $r^*_j$ and the negative ideal solution $r^-_j$, respectively. If $C_j$ is a benefit attribute, then

$$\begin{align*}
  r^*_j &= \max_i r_{ij}, \\
  r^-_j &= \min_i r_{ij},
\end{align*}$$

$$i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \hfill (4)$$

If $C_j$ is a cost attribute, then

$$\begin{align*}
  r^*_j &= \min_i r_{ij}, \\
  r^-_j &= \max_i r_{ij},
\end{align*}$$

$$i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \hfill (5)$$

**Step 4.** Calculate the Euclidean distances of each alternative with the positive ideal solution and the negative ideal solution, respectively:

$$S^+_i = \sqrt{\sum_{j=1}^{n} (r_{ij} - r^*_j)^2}, \quad i = 1, 2, \ldots, m, \hfill (6)$$

$$S^-_i = \sqrt{\sum_{j=1}^{n} (r_{ij} - r^-_j)^2}, \quad i = 1, 2, \ldots, m. \hfill (7)$$

**Step 5.** Compute the evaluation result $S_i$ for alternative $A_i$ defined as follows:

$$S_i = \frac{S^+_i}{S^+_i + S^-_i}, \quad i = 1, 2, \ldots, m. \hfill (8)$$

**Step 6.** Rank alternatives according to their evaluation results in descending order. The larger the $S_i$, the better the alternative $A_i$.

### 2.2. R-TOPSIS Method

Although the TOPSIS method is very popular and practical, it has a fatal defect, that is, it is prone to the problem of rank reversal when the evaluation alternative changes, which leads to the untrustworthy evaluation results. Aires and Ferreira [57] put forward a new approach named R-TOPSIS and proposed the use of an additional input parameter to the TOPSIS method called “domain,” which proved to be robust in experiments, but did not give a theoretical proof that avoids rank reversal. The R-TOPSIS method is composed mainly of the following seven steps:

**Step 1.** Determine a domain matrix $D = (d_{kj})_{2\times n}$ according to decision makers, experts, or interviewees [57]; $d_{kj} (j = 1, 2, \ldots, n)$ and $d_{k2} (j = 1, 2, \ldots, n)$ are, respectively, the minimum value and the maximum value of $D_j (j = 1, 2, \ldots, n)$, where $D_j (j = 1, 2, \ldots, n)$ is the domain of attribute $C_j (j = 1, 2, \ldots, n)$.

**Step 2.** Calculate the normalized decision matrix $Y = (y_{ij})_{m\times n}$ based on Max or Max-Min way.

Max way:

$$y_{ij} = \frac{x_{ij}}{d^{+}_{2j}}, \quad i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \hfill (9)$$

Max-Min way:

$$y_{ij} = \frac{x_{ij} - d^{-}_{1j}}{d^{+}_{2j} - d^{-}_{1j}}, \quad i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \hfill (10)$$

**Step 3.** Calculate the weighted normalized decision-making matrix $R = (r_{ij})_{m\times n}$ where the weighted normalized attribute value $r_{ij}$ is also computed by equation (3).

**Step 4.** Determine the positive ideal solution $r^*_j$ and the negative ideal solution $r^-_j$, respectively. If $C_j$ is a benefit attribute, then

$$\begin{align*}
  r^*_j &= w_j, \\
  r^-_j &= \frac{d^{-}_{1j}}{d^{+}_{2j}},
\end{align*}$$

$$i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \hfill (11)$$

If $C_j$ is a cost attribute, then

$$\begin{align*}
  r^*_j &= \frac{d^{-}_{1j}}{d^{+}_{2j}}, \\
  r^-_j &= w_j,
\end{align*}$$

$$i = 1, 2, \ldots, m; j = 1, 2, \ldots, n. \hfill (12)$$

Because $d^{-}_{1j} < d^{+}_{2j}$, obviously in this case, if $C_j$ is a benefit attribute, then $r^-_j < r^*_j$; otherwise, $r^-_j > r^*_j$, $j = 1, 2, \ldots, n$.

**Step 5.** Calculate the Euclidean distances of each alternative with the positive ideal solution and the negative ideal solution, respectively.

$$S^+_i = \sqrt{\sum_{j=1}^{n} (r_{ij} - r^*_j)^2}, \quad i = 1, 2, \ldots, m. \hfill (13)$$

$$S^-_i = \sqrt{\sum_{j=1}^{n} (r_{ij} - r^-_j)^2}, \quad i = 1, 2, \ldots, m. \hfill (14)$$

**Step 6.** Calculate the closeness for every alternative $A_i$ to the ideal solution as
\[
R_{Si} = \frac{S_{i}^{-}}{S_{i}^{-} + S_{i}^{+}} \quad i = 1, 2, \ldots, m. \quad (13)
\]

**Step 7.** The alternatives are ranked in descending order according to the scores between each alternative and ideal solution; then, the final evaluation result is obtained.

**Remark 1.** In the process of normalization, the R-TOPSIS method selects Max or Max-Min way and does not consider the difference between the benefit attribute and the cost attribute, but considers it in the fourth step. We will analyze the rationality of the two normalized ways as follows.

### 2.2.1. Max Way

If \( C_j \) is a cost-type criteria, then the less the value of \( x_{ij} \), the better the performance. Because \( y_{ij} = x_{ij}/d_{ij} \) and \( r_{ij} = w_{ij}y_{ij} \) \((i = 1, 2, \ldots, m; j = 1, 2, \ldots, n)\), \( y_{ij} \) and \( r_{ij} \) are also as small as possible. Consider two extreme cases; one is to let \( x_{ij} = d_{ij} \) and the other is to let \( x_{ij} = d_{ij} \). If \( x_{ij} = d_{ij} \), then \( y_{ij} = d_{ij}/d_{ij} \) and \( r_{ij} = (d_{ij}/d_{ij})w_{ij} = r_{ij} \). If \( x_{ij} = d_{ij} \), then \( y_{ij} = 1 \) and \( r_{ij} = w_{ij} = r_{ij} \). For any attribute value \( x_{ij} \), the condition \( d_{ij} \leq x_{ij} \leq d_{ij} \) is satisfied. We can get the following conclusion: \( r_{ij} \leq r_{ij} \leq r_{ij} \), and the smaller the distance between \( r_{ij} \) and \( r_{ij} \), the better the evaluation result, and the larger the distance between \( r_{ij} \) and \( r_{ij} \), the better the evaluation result. The fourth step of the R-TOPSIS method is a clever combination of cost-type criteria and positive and negative ideal solutions, in line with people’s thinking logic.

If \( C_j \) is a benefit attribute, we also consider the next two cases. If \( x_{ij} = d_{ij} \), then \( y_{ij} = d_{ij}/d_{ij} \) and \( r_{ij} = (d_{ij}/d_{ij})w_{ij} = r_{ij} \). If \( x_{ij} = d_{ij} \), then \( y_{ij} = 1 \) and \( r_{ij} = w_{ij} = r_{ij} \). We can easily get the result that \( r_{ij} \leq r_{ij} \leq r_{ij} \).

### 2.2.2. Max-Min Way

If \( C_j \) is a benefit attribute, then the attribute value \( x_{ij} \) is as big as possible and \( y_{ij} \) and \( r_{ij} \) are also as big as possible. Also, calculate the two extremes; one is to let \( x_{ij} = d_{ij} \) and the other is to let \( x_{ij} = d_{ij} \). If \( x_{ij} = d_{ij} \), then \( y_{ij} = d_{ij}/d_{ij} \) and \( r_{ij} = (d_{ij}/d_{ij})w_{ij} = r_{ij} \). If \( x_{ij} = d_{ij} \), then \( y_{ij} = 1 \) and \( r_{ij} = w_{ij} = r_{ij} \). If \( C_j \) is a cost-type criteria, we can also find some flaws. For example, if \( x_{ij} = d_{ij} \), then \( y_{ij} = 0 \) by equation (9) and \( r_{ij} = 0 \). Similarly, \( r_{ij} = 0 \neq r_{ij} \). This is also inconsistent with our logical understanding. If \( x_{ij} = d_{ij} \), then \( y_{ij} = 1 \) and \( r_{ij} = w_{ij} = r_{ij} \).

In summary, for the normalization of the second step of the R-TOPSIS method, we recommend using Max way instead of Max-Min way. We can be sure that, in Max-Min way, for the case of \( x_{ij} = d_{ij} \), the R-TOPSIS method encounters unexplained contradictions, while Max way has no problems.

### 2.3. A New Improved Method of TOPSIS

Aires and Ferreira [57] prove that the R-TOPSIS method can overcome rank reversal through various experimental tests and is an improved TOPSIS scheme with high credibility. In fact, we can also consider the standardized method and the determination of positive and negative ideal solutions and establish a new improved TOPSIS method to overcome rank reversals. A new improved TOPSIS method based on this knowledge will be given below with seven steps. Since the new improved TOPSIS method can solve rank reversal problem in this paper based on the historical maximum value of indicator data, it is abbreviated as the NR-TOPSIS method. The establishment of the following method belongs to an absolute mode [55], which is an evaluation with comprehensive and accumulated historical knowledge of the evaluation problem.

**Step 1.** Determine the minimum value \( m_j \) and maximum value \( M_j \) of each attribute \( C_j \) according to the statistical law of attribute values. That is to say, for any attribute value \( x_{ij} \), the following condition \( m_j \leq x_{ij} \leq M_j \) is satisfied. At the same time, the condition \( m_j \leq x_{ij} \leq M_j \) is still satisfied when the scheme is increased, decreased, or replaced.

**Step 2.** In order to eliminate the influence of dimension on data decision-making, the original decision-making matrix \( X = (x_{ij})_{mn} \) is standardized and transformed to generate standardized decision-making matrix \( Y = (y_{ij})_{mn} \), where \( y_{ij} \) are normalized attribute values. If \( C_j \) is a benefit attribute, then

\[
y_{ij} = \frac{x_{ij} - m_j}{M_j - m_j}. \quad (14)
\]

If \( C_j \) is a cost attribute, then

\[
y_{ij} = \frac{M_j - x_{ij}}{M_j - m_j}. \quad (15)
\]

**Step 3.** Calculate the weighted normalized decision-making matrix \( R = (r_{ij})_{mn} \). Compared with the traditional TOPSIS method, the weighted normalized attribute value \( r_{ij} \) has the same calculation, equation (3), which is omitted here.

**Step 4.** Determine the positive ideal solution \( r_{ij}^+ \) and the negative ideal solution \( r_{ij}^- \), respectively:

\[
\begin{cases}
    r_{ij}^+ = w_{ij}, \\
    r_{ij}^- = 0,
\end{cases} \quad j = 1, 2, \ldots, n. \quad (16)
\]

**Step 5.** Compute the Euclidean distances \( S_i^+ \) and \( S_i^- \) for every alternative \( A_i \) between the positive ideal solution and the negative ideal solution, respectively:

\[
RS_i = \frac{S_{i}^+}{S_{i}^+ + S_{i}^-} \quad i = 1, 2, \ldots, m. \quad (13)
\]
time, we also try to give the explanation that the traditional TOPSIS method is prone to rank reversal. The final step in the three TOPSIS methods mentioned above is to evaluate the value ranking. For the evaluation results, if there is no rank reversal under any circumstances, the ranking of the algorithm is stable; otherwise, the ranking is unstable.

Definition 1. Let \( A = \{A_1, A_2, \ldots, A_m\} \) be an alternative set, and the ranking after evaluation by some evaluation methods is \( A_{(1)} > A_{(2)} > \cdots > A_{(m)} \), where \( A_{(i)}, i = 1, 2, \ldots, m, \) is an alternative in \( A \). In the case of adding, deleting, or replacing alternatives under the original alternative set \( A \), a new alternative set \( B = \{B_1, B_2, \ldots, B_l\} \) is obtained. After ranking by the evaluation method, the ranking result is \( B_{(1)} > B_{(2)} > \cdots > B_{(l)}, \) where \( B_{(k)}, k = 1, 2, \ldots, l, \) is an alternative in \( B \). For any two alternatives \( B_p, B_q \in A \cap B \), if there is no rank reversal, then the evaluation method is ranking stable; otherwise, the ranking is unstable.

Theorem 1. Let \( A = \{A_1, A_2, \ldots, A_m\} \) be an alternative set; the R-TOPSIS method is ranking stable.

Proof. Let \( A = \{A_1, A_2, \ldots, A_m\} \) be an alternative set, and the ranking after evaluation by the R-TOPSIS method is \( A_{(1)} > A_{(2)} > \cdots > A_{(m)} \), where \( A_{(i)}, i = 1, 2, \ldots, m, \) is an alternative in \( A \). In the case of adding, deleting, or replacing alternatives under the original alternative set \( A \), a new alternative set \( B = \{B_1, B_2, \ldots, B_l\} \) is obtained. After ranking by the evaluation method, the ranking result is \( B_{(1)} > B_{(2)} > \cdots > B_{(l)}, \) where \( B_{(k)}, k = 1, 2, \ldots, l, \) is an alternative in \( B \). For any two alternatives \( B_p, B_q \in A \cap B \), if there is no rank reversal, then the evaluation method is ranking stable; otherwise, the ranking is unstable.

Remark 2. The extended NR-TOPSIS method established in this paper is based on the global perspective of evaluation problems and the normalization of two types of attribute indicators: benefit type and cost type. It completely avoids the defects of R-TOPSIS in Max-Min way. Equations (14) and (15) are commonly used linear normalization transformation measures for benefit-type and cost-type indicators, respectively. We will analyze the rationality of the two cases in which the indicator is a benefit type or cost type as follows:

1. If \( C_j \) is a benefit attribute, then the attribute value \( x_{ij} \) is as big as possible and \( y_{ij} \) and \( r_{ij} \) are also as big as possible. Consider two extreme cases; one is to let \( x_{ij} = m_j \) and the other is to let \( x_{ij} = M_j \). If \( x_{ij} = m_j \), then \( y_{ij} = (m_j - m_j)/(M_j - m_j) = 0 \) and \( r_{ij} = 0 \). In this case, \( r_j^+ = 0 \), so \( r_{ij} = r_j^+ \), which is consistent with people’s understanding. If \( x_{ij} = M_j \), then \( y_{ij} = 1 \) and \( r_{ij} = w_j = r_j^− \). For any attribute value \( x_{ij} \), we can easily find that \( r_j^− \leq r_{ij} \leq r_j^+ \).

2. If \( C_j \) is a cost-type criteria, then the attribute value \( x_{ij} \) is as small as possible and \( y_{ij} \) and \( r_{ij} \) are also as small as possible. If \( x_{ij} = d_{ij} \), then \( y_{ij} = (M_j - m_j)/(M_j - m_j) = 1 \) and \( r_{ij} = w_j \). In this case, \( r_{ij} = w_j = r_j^+ \). If \( x_{ij} = M_j \), then \( y_{ij} = 0 \) and \( r_{ij} = 0 \).

In conclusion, the NR-TOPSIS method overcomes the possible deficiencies of the R-TOPSIS method in the selection of normalization. The following section will focus on the ability to maintain order theoretically.

3. Theoretical Discussion on Improving the TOPSIS Method to Overcome Rank Reversal Problem

This section tries to give the theoretical basis of R-TOPSIS and NR-TOPSIS to overcome the rank reversal. At the same
alternative in A. In the case of adding, deleting, or replacing alternatives under the original alternative set A, a new alternative set \( B = \{B_1, B_2, \ldots, B_l\} \) is obtained. After ranking by evaluation method, the ranking result is \( B_{(1)} > B_{(2)} > \cdots > B_{(l)} \), where \( B_{(k)} \), \( k = 1, 2, \ldots, l \), is an alternative in \( B \).

For any two alternatives \( B_p, B_q \in A \cap B \), there are matching alternatives \( A_x, A_y \in A \), subject to \( A_x = B_p \) and \( A_y = B_q \), with \( A_x = \{x_{e1}, x_{e2}, \ldots, x_{ea}\} \) and \( A_y = \{y_{f1}, y_{f2}, \ldots, y_{fn}\} \).

Because the minimum value \( m_j \) and maximum value \( M_j \) of each attribute \( C_j \) are determinate, then \( \{y_{e1}, y_{e2}, \ldots, y_{ea}\} \) and \( \{y_{f1}, y_{f2}, \ldots, y_{fn}\} \) are also invariant by equations (14) and (15).

Then, it is clear that the final evaluation results for \( A_x \) and \( A_y \) obtained through Steps 3–6 are also determined to be constant. Therefore, under the action of the evaluation method for the alternatives that exist before and after the change, the evaluation results are unchanged and the ranking naturally remains unchanged. This completes the proof. \( \square \)

Remark 3. The proof process of Theorems 1 and 2 is straightforward. In particular, the maximum value of the indicator used in NR-TOPSIS is the historical maximum value and it is also the global maximum value, which is a constant, not limited to the maximum value of the indicator in each evaluation. What we can theoretically determine is that the R-TOPSIS and NR-TOPSIS methods ensure the consistency of the evaluation results and are therefore credible. In contrast, the traditional TOPSIS method may change both the positive ideal solution and the negative ideal solution after the change of the alternative set, resulting in the change of the evaluation result of the alternative. In this way, it is easy to cause ranking unstable and rank reversal.

4. Numerical Results

In this section, two classical examples will be used to verify the effectiveness of the NR-TOPSIS method presented above. The first simple example comes from the literature [45], which confirms with the theory to verify that the method can avoid the phenomenon of rank reversal. The second example is used to illustrate the comprehensive effectiveness of the NR-TOPSIS method. A classical weapon performance evaluation problem is selected. Considering that the problem includes both the benefit attribute and cost attribute, the test of the evaluation method is more convincing. According to the analysis of the theoretical part, in the following comparative experiment, the R-TOPSIS method adopts the Max way.

Example 1. Table 1 will show an MADM example in which four alternatives with respect to four benefit attributes are given. Consider the four attribute weights denoted by \( W = \{1/6, 1/3, 1/3, 1/6\} \). It is not difficult for us to determine the historical maximum of each attribute. We use \( m \) to represent the historical minimum value vector of each attribute, and \( M \) to represent the historical maximum value vector of each attribute, where \( m = \{24, 40, 43, 70\} \) and \( M = \{36, 50, 50, 100\} \) in the example. When the scheme is increased, decreased, or replaced, the condition \( m_j \leq x_{ij} \leq M_j \) \( (j = 1, 2, 3, 4) \) must be met.

The weighted normalized decision matrix can be calculated as

\[
X = \begin{bmatrix}
R_1 & 0.1667 & 0.0667 & 0 & 0 \\
R_2 & 0.0139 & 0.3333 & 0.0952 & 0.0556 \\
R_3 & 0.0556 & 0.1667 & 0.3333 & 0.0278 \\
R_4 & 0 & 0 & 0.1905 & 0.1667 \\
\end{bmatrix}.
\]

Also, the positive ideal solution vector and the negative ideal solution vector can be determined by equation (16) as

\[
R^+ = \begin{bmatrix} 1 & 1 & 1 & 1 \end{bmatrix},
\]

\[
R^- = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}.
\]

Figure 1 illustrates the comparison curves of the attributes of each alternative with the positive ideal solution and negative ideal solution after weight normalization. From Figure 1, it can be found that \( A_5 \) of weight normalization has the smallest synthetic distance with the positive ideal solution and the largest synthetic distance with the negative ideal solution, so it is the best among the four alternatives, followed by \( A_3 \) and \( A_1 \) is at the end. The ranking is different from the traditional TOPSIS method and R-TOPSIS method.

If \( A_4 \) is removed from the original alternative set, it is easy to find that \( A_3 \) is also the best, which is consistent with previous ranking results. However, at this time, the traditional TOPSIS method has already undergone rank reversal. Then, we add a new alternative \( A_5 = \{36, 50, 45, 70\} \) to the original alternative set; we can also find that \( A_5 \) is also the best and the ranking results can be shown as

\[ A_3 > A_5 > A_2 > A_4 > A_1. \]
The weighted normalized attribute values and the ideal solutions.

\[
\begin{align*}
R_1 & = \begin{bmatrix} 0.1667 & 0.0667 & 0 & 0 \\ 0.0139 & 0.3333 & 0.0952 & 0.0556 \\ 0.0556 & 0.1667 & 0.3333 & 0.0278 \\ 0.1667 & 0.3333 & 0.0952 & 0 \\
\end{bmatrix} \\
R_2 & = \begin{bmatrix} 0.0667 & 0.3333 & 0 & 0 \\ 0.1667 & 0.3333 & 0.0952 & 0.0556 \\ 0 & 0 & 0.1905 & 0.1667 \\ 0.1667 & 0.3333 & 0.0952 & 0 \\
\end{bmatrix} \\
R_3 & = \begin{bmatrix} 0.1667 & 0.0667 & 0 & 0 \\ 0.0139 & 0.3333 & 0.0952 & 0.0556 \\ 0.0556 & 0.1667 & 0.3333 & 0.0278 \\ 0.1667 & 0.3333 & 0.0952 & 0 \\
\end{bmatrix} \\
R_4 & = \begin{bmatrix} 0.0667 & 0.3333 & 0 & 0 \\ 0.1667 & 0.3333 & 0.0952 & 0.0556 \\ 0 & 0 & 0.1905 & 0.1667 \\ 0.1667 & 0.3333 & 0.0952 & 0 \\
\end{bmatrix}
\end{align*}
\]

Figure 1: The weighted normalized attribute values and the ideal solutions.

Moreover, we make a comparison between the weighted normalized alternatives curve and the positive and negative ideal solution under the new alternative \( A_3 \), and from Figure 2, it is easy to observe that \( A_1 \) is the worst. Therefore, we believe that the NR-TOPSIS method based on global understanding constructed in this paper is better than the R-TOPSIS method, although the R-TOPSIS method also overcomes the problem of rank reversal.

The specific ranking results of the three methods are shown in Table 2. Table 2 gives the ranking results of three TOPSIS methods under the original alternative set, deletion alternative \( A_4 \) and addition alternative \( A_5 \), and the normalized decision data obtained by NR-TOPSIS method. In a sense, the TOPSIS method has positive meaning and value, but it also has the problem of rank reversal. Both the NR-TOPSIS and R-TOPSIS methods inherit the advantages of TOPSIS while overcoming the shortcomings of rank reversal. In all three cases, the NR-TOPSIS method and TOPSIS method have the same ranking in one case. According to the previous analysis, we believe that \( A_3 \) is the best, and NR-TOPSIS keeps the ranking consistency before and after change. Therefore, the NR-TOPSIS method is a better improvement of the TOPSIS method.

**Example 2.** In this example, seven surface-to-air missile weapon system alternatives \( X_1, X_2, \ldots, X_7 \) with eleven attributes \( C_1, C_2, \ldots, C_{11} \) [62] will be evaluated by the NR-TOPSIS method. These attributes include the maximum speed of missile \( C_1 \), the maximum speed of target \( C_2 \), the maximum overload of target \( C_3 \), the highest boundary of killing range \( C_4 \), the number of targets that can simultaneously be shot \( C_5 \), the single-shot kill probability of missiles \( C_6 \), the reaction time of missile weapon system \( C_7 \), the lowest boundary of killing range \( C_8 \), the launching weight of missiles \( C_9 \), and the nearest boundary of killing range \( C_{10} \) and \( C_{11} \). Assuming that according to the statistical law and expert knowledge, we have determined the historical lower limit value of each index and thus determined the historical maximum value \( M_j \) and historical minimum value \( m_j \) of each attribute \( C_j \) as follows:

\[
\begin{align*}
M & = \{ 2, 400, 1, 3, 8, 1, 0.7, 10, 0.025, 85, 0.5 \}, \\
m & = \{ 6, 2300, 6, 27, 100, 8, 0.8, 40, 1, 2375, 8 \}.
\end{align*}
\]

Then, the normalized decision matrix data by equations (14) and (15) are listed in Table 3. According to the references, it is assumed that the weights of 11 attributes are the same, that is, \( w_j = 1/11 \), \( j = 1, 2, \ldots, 11 \).

Determine the weighted normalized multiattribute decision matrix data by (3) as listed in Table 4.

The evaluation score obtained by the NR-TOPSIS method is as follows:

\[
\begin{align*}
R_1 & = 0.6887, \\
R_2 & = 0.7872, \\
R_3 & = 0.6315, \\
R_4 & = 0.2735, \\
R_5 & = 0.4424, \\
R_6 & = 0.4965, \\
R_7 & = 0.4509.
\end{align*}
\]
where $S_i$ is the evaluation score of $X_i$, $i = 1, 2, \ldots, 7$. Thus, the ranking is

$$X_2 > X_1 > X_3 > X_6 > X_7 > X_5 > X_4.$$  \hspace{1cm} (25)

Table 5 shows the evaluation results under four TOPSIS methods, including R-TOPSIS method, the reference method [56], the classical TOPSIS method, and NR-TOPSIS method established in this paper. Figure 3 also shows the distribution of evaluation results under the four TOPSIS methods. It is not difficult to find that the NR-TOPSIS method proposed in this paper is consistent with the reference method [56] and the R-TOPSIS method is consistent with the classical TOPSIS method; there is only a small difference between the two. The ranking results of the R-TOPSIS method and the classical TOPSIS method are

$$X_2 > X_1 > X_3 > X_7 > X_6 > X_5 > X_4.$$  \hspace{1cm} (26)

The difference between the two kinds of methods is that the order of alternatives $X_6$ and $X_7$ is different. Carefully compare the weighted normalized data of alternative $X_6$ and $X_7$ in Table 4; seven attributes of the two alternatives are the same and four attributes are different. According to the principle that the larger the value of weighted normalized data is, the closer it is to the positive ideal solution and the

\begin{table}[h]
\centering
\caption{Normalized decision matrix by NR-TOPSIS and comparison of evaluation results of three TOPSIS methods.}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|}
\hline
\textbf{Alternative} & \textbf{Normalized decision matrix} & \textbf{NR-TOPSIS evaluation results} & \textbf{NR-TOPSIS rank} & \textbf{R-TOPSIS evaluation results} & \textbf{R-TOPSIS rank} & \textbf{TOPSIS evaluation results} & \textbf{TOPSIS rank} \\
\hline
\multicolumn{8}{|c|}{The original alternative set} \\
\hline
$A_1$ & 1 & 0.6 & 0.3 & 0 & 0.2815 & 4 & 0.3971 & 3 & 0.4184 & 3 \\
$A_2$ & 0.0833 & 1 & 0.5 & 0.3333 & 0.5362 & 2 & 0.5027 & 1 & 0.4858 & 1 \\
$A_3$ & 0.3333 & 0.75 & 1 & 0.1667 & 0.6078 & 1 & 0.4836 & 2 & 0.4634 & 2 \\
$A_4$ & 0 & 0.5 & 0.7 & 1 & 0.3881 & 3 & 0.3889 & 4 & 0.3915 & 4 \\
\hline
\multicolumn{8}{|c|}{Drop $A_4$ out of alternative set} \\
\hline
$A_1$ & 1 & 0.6 & 0.3 & 0 & 0.2815 & 3 & 0.3971 & 3 & 0.4319 & 3 \\
$A_2$ & 0.0833 & 1 & 0.5 & 0.3333 & 0.5362 & 2 & 0.5027 & 1 & 0.4742 & 2 \\
$A_3$ & 0.3333 & 0.75 & 1 & 0.1667 & 0.6078 & 1 & 0.4836 & 2 & 0.5007 & 1 \\
\hline
\multicolumn{8}{|c|}{Add a new alternative $A_5$ to the original alternative set} \\
\hline
$A_1$ & 1 & 0.6 & 0.3 & 0 & 0.2815 & 5 & 0.3971 & 4 & 0.4047 & 5 \\
$A_2$ & 0.0833 & 1 & 0.5 & 0.3333 & 0.5362 & 3 & 0.5027 & 2 & 0.4862 & 2 \\
$A_3$ & 0.3333 & 0.75 & 1 & 0.1667 & 0.6078 & 1 & 0.4836 & 3 & 0.4639 & 3 \\
$A_4$ & 0 & 0.5 & 0.7 & 1 & 0.3881 & 4 & 0.3889 & 5 & 0.4061 & 4 \\
$A_5$ & 0.5 & 0.65 & 0.5 & 0.2857 & 0.5696 & 2 & 0.5937 & 1 & 0.5793 & 1 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{The normalized multiattribute decision matrix data.}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
$C_1$ & $C_2$ & $C_3$ & $C_4$ & $C_5$ & $C_6$ & $C_7$ & $C_8$ & $C_9$ & $C_{10}$ & $C_{11}$ \\
\hline
$X_1$ & 1 & 0.1842 & 1 & 0.875 & 1 & 1 & 0.5 & 0.6667 & 0.7179 & 0.6004 & 0.6667 \\
$X_2$ & 0.875 & 1 & 0.8 & 1 & 0.8913 & 1 & 1 & 0.8333 & 1 & 0.6004 & 0.4 \\
$X_3$ & 0.6 & 0.4211 & 0.8 & 1 & 0.7283 & 0.7143 & 0.6 & 0.6667 & 1 & 0.3105 & 0.4 \\
$X_4$ & 0.25 & 0.0105 & 0 & 0.8958 & 0.2609 & 0 & 0.5 & 0 & 0 & 0 & 0 \\
$X_5$ & 0 & 0 & 0 & 0.0833 & 0.0435 & 0.2857 & 0.5 & 1 & 0.5128 & 0.941 & 0.9333 \\
$X_6$ & 0.05 & 0 & 0.2 & 0 & 0 & 0.2857 & 0 & 0.9867 & 0.9744 & 1 & 0.9333 \\
$X_7$ & 0.05 & 0.0053 & 0.2 & 0 & 0 & 0.2857 & 0 & 0.9744 & 1 & 1 & 1 \\
\hline
\end{tabular}
\end{table}

\begin{table}[h]
\centering
\caption{The weighted normalized multiattribute decision matrix data.}
\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|c|c|}
\hline
$C_1$ & $C_2$ & $C_3$ & $C_4$ & $C_5$ & $C_6$ & $C_7$ & $C_8$ & $C_9$ & $C_{10}$ & $C_{11}$ \\
\hline
$X_1$ & 0.0909 & 0.0167 & 0.0909 & 0.0795 & 0.0909 & 0.0909 & 0.0455 & 0.0606 & 0.0653 & 0.0546 & 0.0606 \\
$X_2$ & 0.0795 & 0.0909 & 0.0727 & 0.0909 & 0.081 & 0.0909 & 0.0909 & 0.0758 & 0.0909 & 0.0546 & 0.0364 \\
$X_3$ & 0.0545 & 0.0383 & 0.0727 & 0.0909 & 0.0662 & 0.0649 & 0.0545 & 0.0606 & 0.0909 & 0.0282 & 0.0364 \\
$X_4$ & 0.0227 & 0.001 & 0 & 0.8958 & 0.2609 & 0 & 0.5 & 0 & 0 & 0 & 0 \\
$X_5$ & 0 & 0 & 0.0182 & 0.0076 & 0.004 & 0.026 & 0.0455 & 0.0909 & 0.0466 & 0.0855 & 0.0848 \\
$X_6$ & 0.0045 & 0 & 0.0182 & 0 & 0 & 0.026 & 0.0909 & 0.0897 & 0.0886 & 0.0909 & 0.0848 \\
$X_7$ & 0.0045 & 0.0005 & 0.0182 & 0 & 0 & 0.026 & 0 & 0.0909 & 0.0886 & 0.0909 & 0.0909 \\
\hline
\end{tabular}
\end{table}
farther it is from the negative ideal solution, it can be judged that the alternative $X_6$ is better than the alternative $X_7$, so the evaluation result of NR-TOPSIS is better.

For the above original multiattribute missile selection problem, we will delete or add alternative to evaluate whether there is a rank reversal phenomenon. Generally, an alternative is often deleted to test the rank reversal, which results in the limitation of the discussion. Next, we consider deleting three alternatives $X_4$, $X_5$, and $X_6$ from the original alternative set. In this case, we will compare the results of the NR-TOPSIS method proposed in this paper with other TOPSIS methods, especially to check whether the rank reversal phenomenon occurs. The main results are shown in Table 6.

In this case, we can easily get the following results with dropping $X_4$, $X_5$, and $X_6$ out of the original alternative set by the NR-TOPSIS method:

$$R_1 = 0.6887,$$
$$R_2 = 0.7872,$$
$$R_3 = 0.6315,$$
$$R_7 = 0.4509. \tag{27}$$

Thus, the ranking of the case with dropping $X_4$, $X_5$, and $X_6$ out of the original alternative set is

$$X_2 > X_1 > X_3 > X_7. \tag{28}$$

Table 5: The comparison between the NR-TOPSIS method and reference method.

<table>
<thead>
<tr>
<th>Evaluation results</th>
<th>$R_1$</th>
<th>$R_2$</th>
<th>$R_3$</th>
<th>$R_4$</th>
<th>$R_5$</th>
<th>$R_6$</th>
<th>$R_7$</th>
</tr>
</thead>
<tbody>
<tr>
<td>NR-TOPSIS method</td>
<td>0.6887</td>
<td>0.7872</td>
<td>0.6315</td>
<td>0.2735</td>
<td>0.4424</td>
<td>0.4965</td>
<td>0.4509</td>
</tr>
<tr>
<td>The reference method [56]</td>
<td>0.6242</td>
<td>0.6842</td>
<td>0.5564</td>
<td>0.2604</td>
<td>0.4213</td>
<td>0.4894</td>
<td>0.4342</td>
</tr>
<tr>
<td>R-TOPSIS method</td>
<td>0.7043</td>
<td>0.7644</td>
<td>0.6329</td>
<td>0.2550</td>
<td>0.4534</td>
<td>0.4887</td>
<td>0.4928</td>
</tr>
<tr>
<td>Classical TOPSIS</td>
<td>0.6640</td>
<td>0.7589</td>
<td>0.6207</td>
<td>0.2026</td>
<td>0.4833</td>
<td>0.5347</td>
<td>0.5392</td>
</tr>
</tbody>
</table>

Table 6: The evaluation ranking results of four TOPSIS.

<table>
<thead>
<tr>
<th>Drop $X_4$, $X_5$, and $X_6$ out of the original alternative set</th>
<th>Rank</th>
<th>NR-TOPSIS</th>
<th>Rank</th>
<th>R-TOPSIS</th>
<th>Rank</th>
<th>Classical TOPSIS</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$ 0.4916</td>
<td>2</td>
<td>0.6887</td>
<td>2</td>
<td>0.7043</td>
<td>2</td>
<td>0.4745</td>
<td>4</td>
</tr>
<tr>
<td>$X_2$ 0.5892</td>
<td>1</td>
<td>0.7872</td>
<td>1</td>
<td>0.7644</td>
<td>1</td>
<td>0.6694</td>
<td>1</td>
</tr>
<tr>
<td>$X_3$ 0.4230</td>
<td>3</td>
<td>0.6315</td>
<td>3</td>
<td>0.6329</td>
<td>3</td>
<td>0.5316</td>
<td>2</td>
</tr>
<tr>
<td>$X_7$ 0.3757</td>
<td>4</td>
<td>0.4509</td>
<td>4</td>
<td>0.4928</td>
<td>4</td>
<td>0.5046</td>
<td>3</td>
</tr>
</tbody>
</table>

Add a new alternative $X_8$ to the changed alternative set discussed above:

<table>
<thead>
<tr>
<th>Add a new alternative $X_8$ to the changed alternative set</th>
<th>Rank</th>
<th>NR-TOPSIS</th>
<th>Rank</th>
<th>R-TOPSIS</th>
<th>Rank</th>
<th>Classical TOPSIS</th>
<th>Rank</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_1$ 0.5158</td>
<td>3</td>
<td>0.6887</td>
<td>2</td>
<td>0.7043</td>
<td>2</td>
<td>0.4826</td>
<td>5</td>
</tr>
<tr>
<td>$X_2$ 0.6125</td>
<td>1</td>
<td>0.7872</td>
<td>1</td>
<td>0.7644</td>
<td>1</td>
<td>0.7049</td>
<td>1</td>
</tr>
<tr>
<td>$X_3$ 0.4597</td>
<td>4</td>
<td>0.6315</td>
<td>4</td>
<td>0.6329</td>
<td>4</td>
<td>0.5677</td>
<td>3</td>
</tr>
<tr>
<td>$X_7$ 0.3927</td>
<td>5</td>
<td>0.4509</td>
<td>5</td>
<td>0.4928</td>
<td>5</td>
<td>0.5465</td>
<td>4</td>
</tr>
<tr>
<td>$X_8$ 0.5526</td>
<td>2</td>
<td>0.6463</td>
<td>3</td>
<td>0.6512</td>
<td>3</td>
<td>0.5937</td>
<td>2</td>
</tr>
</tbody>
</table>

Figure 3: Evaluation results and trend.
It can be determined quickly and the evaluation result and ranking of the scheme to be selected are unchanged, and no rank reversal occurs.

At last, a new alternative $X_8 = \{2, 2300, 6, 27, 100, 8, 0.8, 25, 0.025, 1000, 7.5\}$ will be added to the changed alternative set discussed above. The new comparison results are also shown in Table 6.

At the last case, we can obtain the following ranking for the NR-TOPSIS method:

$$X_3 > X_1 > X_8 > X_3 > X_7.$$  \hfill (29)

The example further verifies the theory, showing that the NR-TOPSIS method established in this paper provides an invariable measurement scale, which can ensure that the evaluation results of the alternatives remain unchanged. Therefore, whether deleting the original alternatives or adding new alternatives, the rank stability will be guaranteed and the problem of rank reversal will not arise. Correspondingly, the traditional TOPSIS method has an order reversal phenomenon when deleting and adding new alternatives, such as $X_1$ and $X_3$. In addition, there is no rank reversal between the reference method [56] and the R-TOPSIS method, which are also superior to the classical TOPSIS method, but the reference method is too complicated. The ranking of R-TOPSIS and NR-TOPSIS methods is consistent in Example 2, while the ranking of the reference method [56] is slightly different from that of R-TOPSIS and NR-TOPSIS. In view of the analysis of various situations under the above two examples, we believe that the NR-TOPSIS method constructed in this paper is considered from a global perspective, taking into account the differences of attributes in normalization, overcoming the problem of rank reversal, and is more in line with human thinking logic.

5. Conclusions

Faced with the complicated evaluation problems in real life, the validity and credibility of the evaluation methods are reflected in the consistency of the evaluation results and the stability of the evaluation ranking. However, the classical TOPSIS method is not credible because of the possible phenomenon of rank reversal, so it is necessary to carry out theoretical research and process improvement of TOPSIS to resist rank reversal. In order to make the assessment results fair, reasonable, and consistent, we need to consider and evaluate the problem from a global perspective and ensure that the scale of measurement is always the same. Based on this understanding, this paper constructs a new improved TOPSIS method, which not only inherits the advantages of the TOPSIS method but also makes reasonable improvements, taking into account the attribute characteristics of indicators, making the improved TOPSIS method more in line with human cognition and logic. This paper establishes the definition of ranking stability and theoretically states that the NR-TOPSIS method constructed in this paper and the R-TOPSIS method established in the reference literature are ranking stable, while the classical TOPSIS method is ranking unstable. At the same time, when using the R-TOPSIS method for normalization, we analyze that Max way is more reasonable and Max-Min way has certain flaws. In the example verification process, we used two classic cases and discussed various situations such as deleting and increasing the alternatives. The verification results are completely consistent with the theory. Compared with other types of TOPSIS methods, the NR-TOPSIS method is more reasonable and effective. For a given evaluation problem, the NR-TOPSIS method can ensure that the evaluation results of the alternative remain unchanged, thus ensuring that the ranking is stable. In the future, we will continue to study the use of NR-TOPSIS to solve more complex fuzzy problems involving semantic uncertainty, looking for a new method to solve uncertain MADM problems.

Data Availability

Data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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