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Research Article

Moving Horizon Estimation for Uncertain Networked Control Systems with Packet Loss

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This paper studies a moving horizon estimation approach to solve the constrained state estimation problem for uncertain networked systems with random packet loss. The system model error range is known, and the packet loss phenomena are modeled by a binary switching random sequence. Taking the model error, the packet loss, the system constraints, and the network transmission noise into account, a time-varying weight matrix is obtained by solving a least-square problem. Then, a robust moving horizon estimator is designed to estimate the system state by minimizing an optimization problem with an arrival cost function. The proposed estimator ensures that the optimal estimated state can be obtained in the worst case. Furthermore, the asymptotic convergence of the estimator is analyzed and some sufficient conditions for convergence are given. Finally, the validity of the proposed approach can be demonstrated by numerical simulations.

1. Introduction

Most networked control systems (NCSs) implement control strategies based on the state feedback (see, e.g., [1, 2]). In practice, the system states are usually not directly measurable. Therefore, it is necessary to accurately estimate the states based on the measures before carrying out the performance analysis, control, and optimization of the control system. However, in many control systems, especially in network environments, perfect communication is not always possible. Owing to the limitation of the network bandwidth, the failure of system information in the form of data packets could occur, such as time delay, timing error, packet loss, and quantization distortion (see, e.g., [3–5]), which can affect the accuracy of the state estimates.

The estimation problem of network systems with packet loss has been widely considered by many researchers [6–8]. In [6], the Kalman filter for networked systems with a bounded Markov dropout is studied, and a sufficient condition is obtained for the stability of the peak covariance of the probability transition matrix of the system dynamics and Markov chains. In [7], the problem of optimal H_∞ filtering for network systems with multiple packet loss is studied by modeling the phenomena of packet loss as a Bernoulli distribution and transforming the probability of random packet loss into the stochastic parameters of the system. And then, the H_∞ filtering problem for linear discrete-time systems with time-varying norm-bounded parameter uncertainties is studied in [8], and an estimator is designed to guarantee the estimation error to be quadratic stable. In addition, many other methods have been proposed to solve the problem of packet loss in the process of estimation (see, e.g., [9–11]).

In addition, the state estimation usually depends on the accurate system model, and the accuracy of estimation will be reduced when the model has errors. Because the online recursive filter needs continuous judgment of the system condition in the estimation, the result of the minimized certain indefinite quadratic forms will be inaccurate when special conditions occur in the iterative process, and the estimation error will be biased. In order to solve this problem, reference [12] for the linear time-invariant model, through modifying the robust parameters, the filter can test the stability of the algorithm in any length ahead of time. By using the set-valued estimation approach and considering the noise disturbance, an ellipsoid is constructed around the state estimation problem in [13]. Garulli et al. [14] propose that integral quadratic constraints can be used to describe the uncertain model. It can be seen that the accurate model plays an important role in estimating the effect\enleadertwodots.

Moving horizon estimation (MHE) originated from the model predictive control (MPC) is a state estimation strategy based on moving window and limited information. MHE was applied to improve the effectiveness of the estimator because of its constraint handling ability (see, e.g., [15–23]). In [16], a new unconstrained estimator is designed for network systems with an uncertain model, and the estimation error is proved to be bounded. Xue et al. [17] proposed a new moving time domain estimator to deal with the packet loss problem and gave the convergence condition of the average estimation error norm to guarantee the performance of the estimator. Based on the linear and nonlinear MHE principle, the authors in references [18–21] extend MHE to distributed systems and propose a new distributed estimation algorithm. In order to ensure the estimation error in the moving window is stable, the stability of MHE with linear and nonlinear constraints is studied in [22, 23].

However, in the classical MHE algorithm, the weight matrix is a fixed value, which cannot meet the stability requirements of the system in the presence of multiple uncertainties. Most of the existing RMHE algorithms, such as in [24], transform the optimization problem into a min-max problem, but the performance of this algorithm is not ideal when solving the state estimation problem with physical constraints. Based on the above analysis, it is assumed that the packet loss phenomena are modeled by a binary switching random sequence, and the system model error range is known. This paper designs a new MHE estimator to solve the problem of state estimation for network systems with packet loss and system uncertainty. The proposed method retains the quadratic optimization part of the MHE problem, and the weight matrix of the optimization objective is set as a time-varying matrix and obtained by solving a least-square function. Compared with the existing methods, this design not only ensures to solve the physical constraint problem effectively, but also improves the robustness of the algorithm.

The emphasis of this paper mainly includes the following points: (1) a new robust MHE problem is studied in a unified framework including random packet loss sum, model error, transmission noise, and system constraints. (2) By solving a least-square problem with uncertain parameters, a time-varying weight matrix is obtained. (3) By choosing a random cost function, the optimal estimated state of the system can be obtained by solving a quadratic programming problem. (4) The stability of the estimator is proved. Finally, the validity and rationality of the method are verified by simulation.

Notation. For a column vector x, the notation $\|x\|_Y^2$ stands for $x^T Y x$, where Y is a symmetric positive-semidefinite matrix. The notation A^+ denotes the pseudoinverse of matrix A. The matrix $\mathrm{diag}\{s_1, \ldots, s_n\}$ is a block diagonal with blocks s_n.

2. Problem Statement

Consider the following discrete-time uncertain networked control system with random packet loss

$$\begin{cases} x_{k+1} = (A + \Delta A_k)x_k + Bw_k, \\ y_k = \gamma_k C x_k + v_k, \end{cases} \tag{1}$$

where $x_k \in R^n$ denotes the system state, $y_k \in R^m$ denotes the measurement, $w_k \in R^n$ is the system noise, and $v_k \in R^m$ characterizes network transmission noise derived from the communication network. The two noises have the following statistical characteristics:

$$\begin{cases} E\{w_k\} = 0; \\ E\{v_k\} = 0, \\ E\{w_k v_k'\} = 0; \\ E\{w_i w_k'\} = Q\delta_{jk}, \\ E\{v_i v_k'\} = R_k \delta_{ik}, \\ i, k = 0, 1, 2, \ldots, \end{cases} \tag{2}$$

where δ_{ik} is a sign function and matrices Q and R_k, respectively, satisfying $Q \geq 0$ and $R_k > 0$ reflect the system's confidence in models and measurements. If $Q > R_k$, it means that the influence of the system model on the estimator is greater than measurements, and vice versa.

Furthermore, A, B, and C are known matrices of the appropriate dimensions. ΔA_k representing the model error is a time-varying uncertain matrix expressed as

$$\Delta A_k = M\Lambda_k E, \quad \|\Lambda_k\| \leq 1, \tag{3}$$

where M and E are also the known real constant matrices of the appropriate dimensions and Λ_k is an arbitrary contraction. Perturbation models of the form (3) are common in optimization problems. It can transform the problem with uncertain parameters into a region minimization problem with known norm range.

As shown in (1), the stochastic parameter γ_k taking values of $\{0, 1\}$ indicates whether the measurement is lost at time k, and the packet loss satisfies the Bernoulli distribution. $\gamma_k = 1$, meaning that packet loss does not happen and the measurement data are received; otherwise, $\gamma_k = 0$ indicates that packet loss happens and the only measurement noise v_k is received. Therefore, the stochastic parameter γ_k has the following statistical characteristics:

$$\begin{cases} p(\gamma_k = 1) = \lambda, \\ p(\gamma_k = 0) = 1 - \lambda. \end{cases} \tag{4}$$

Assume that the estimator knows the value of γ_k at time k. The assumption is reasonable because the estimator can get the value of γ_k by comparing the value of y_k with y_{k-1}. Based on the value of the random parameter γ_k indicating whether the measurement is lost, the variance R_k of the system's confidence in measurements can take different values as follows:

$$R_k = \begin{cases} R, & \gamma_k = 1, \\ \sigma^2 I, & \gamma_k = 0, \end{cases} \tag{5}$$

where σ is a scalar parameter and R is the known real constant matrices. As for (5), the variance R_k takes the value in two cases. When the measurement is received at time k, let $R_k = R$ denote the system's confidence in the measurement; otherwise, $R_k = \sigma^2 I$. In the calculation process, the parameter σ is considered to be trending toward ∞ when $\gamma_k = 0$ because the measurement is not received at this moment. In other words, the effect of packet loss on the state estimation is greater than the system uncertainty, and the value of R_k is much greater than Q.

To estimate the system state for the uncertain networked control system (1), a new robust estimator will be designed in the next section.

3. Robust Moving Horizon Estimation

3.1. Description of RMHE. In order to eliminate the adverse effects of the above uncertainties (2)–(4) on the state estimation and improve the estimation performance, a robust MHE method relying on the full information estimation will be discussed and some conclusions will be obtained.

At first, the optimization problem of the full information estimation can be described as

$$J_t^* = \min_{\tilde{x}_{0|t-1}, \{\tilde{w}_{k|t-1}\}_{k=0}^{t-1}} J_t, \tag{6}$$

where the objective function is defined by

$$J_t = \sum_{k=0}^{t-1} \left\| \tilde{v}_{k|t-1} \right\|_{R_k^{-1}}^2 + \left\| \tilde{w}_{k|t-1} \right\|_{Q^{-1}}^2 + \left\| \tilde{x}_{0|t-1} - \hat{x}_{0|0} \right\|_{P_{0|0}^{-1}}^2, \tag{7}$$

s.t. $\tilde{v}_{k|t-1} = y_k - \gamma_k C \tilde{x}_{k|t-1}$ and (1)–(5), where $\hat{x}_{0|0}$ is the prior estimated state of system (1), and $P_{0|0}$ represents the prior weight matrix. At time t, the measurement information $\{y_k\}_{k=0}^{t-1}$ is derived, and then the solutions $\tilde{x}_{0|t-1}^*$ and $\{\tilde{w}_{k|t-1}^*\}_{k=0}^{t-1}$ can be obtained by minimizing the optimization problem (6). In addition, the estimated state at other times can be found by the following equation:

$$\tilde{x}_{k|t-1}^* = A^k \tilde{x}_{0|t-1}^* + \sum_{j=0}^{k-1} A^{k-1-j} B \tilde{w}_{k|t-1}^*, \quad k = 1, 2, \ldots, t-1. \tag{8}$$

However, the full information estimation (6) needs to use all measurements at the previous time for each calculation, so the calculation load will increase continuously. In order to ensure the efficiency and the real-time property of the state estimation, the moving horizon estimation with a fixed window is proposed in the paper, instead of the full moving horizon estimation. Based on the above analysis, the proposed moving horizon state estimation algorithm can be divided into two parts: the full information estimation in the time period $\{0 \le k \le t-n-1\}$ and the moving horizon estimation with a fixed window in the time period $\{t-n \le k \le t-1\}$. Therefore, the optimization problem (6) of full information estimation can be substituted into the following optimization problem of MHE:

$$J_t^* = \min_{\tilde{x}_{t-n|t-1}, \{\tilde{w}_{k|t-1}\}_{k=t-n}^{t-1}} J_t, \tag{9}$$

where

$$J_t = \sum_{k=t-n}^{t-1} \left\| \tilde{v}_{k|t-1} \right\|_{R_k^{-1}}^2 + \left\| \tilde{w}_{k|t-1} \right\|_{Q^{-1}}^2 + J_{t-n}, \tag{10}$$

s.t. (1)–(5) and n is the length of the fixed window.

In the optimization problem (9), the choice of the arrival cost function J_{t-n} which reflects the effect of the full information estimation in time period $\{0 \le k \le t-n-1\}$ is important because of its influence on the estimation performance of MHE (9). For the computational convenience, a usual form of the arrival cost function is adopted as follows:

$$J_{t-n} = \left\| \tilde{x}_{t-n|t-1} - \hat{x}_{t-n|t-1} \right\|_{P_{t-n|t-1}^{-1}}^2. \tag{11}$$

If the random parameter γ_k and the three weight matrices in (9) and (11) are determined, the optimized variables $\tilde{x}_{t-n|t-1}^*$ and $\{\tilde{w}_{k|t-1}^*\}_{k=t-n}^{t-1}$ can be obtained by minimizing the optimization problem (9). Similar to (8), the estimated state in the time period $\{t-n \le k \le t-1\}$ can be calculated from

$$\tilde{x}_{t-n+k|t-1}^* = A^k \tilde{x}_{t-n|t-1}^* + \sum_{j=0}^{k-1} A^{k-1-j} B \tilde{w}_{t-n+k|t-1}^*, \quad k = 1, 2, \ldots, n-1. \tag{12}$$

Remark 1. To simplify the calculation, the uncertain matrix ΔA_k does not appear in (6)–(12), but it will change the form of the arrival cost function by affecting the weight matrix, which will be discussed in the following part.

3.2. Solution of Time-Varying Weight Matrix. It is worth noting that Q, R_k, $\hat{x}_{t-n|t-1}$, and $P_{t-n|t-1}$ must be known when calculating (9). In this paper, Q and R_k are designed as fixed matrices and $\tilde{x}_{t-n+1|t-1}^*$ is chosen as the prior estimated state for the next time. In many studies, the weight matrix $P_{t-n|t-1}$ is usually also designed as a fixed matrix, but in order to get more accurate estimation, the next step is to design a time-varying weight matrix calculation method. Assume that the priori estimated state $\hat{x}_{k-1|k-1}$ and the weight matrix $P_{k-1|k-1}$ are given at time k, along with the measurement y_k. Using this initial information, we can seek to update the estimated state from $\hat{x}_{k-1|k-1}$ to $\hat{x}_{k|k}$ by solving

$$\min_{x_{k-1|k}, \{w_{k-1|k}\}} \max_{\Delta A_{k-1}} \left(\left\| \hat{v}_k \right\|_{R_k^{-1}}^2 + \left\| \hat{w}_{k-1|k} \right\|_{Q^{-1}}^2 + \left\| \hat{x}_{k-1|k} - \hat{x}_{k-1|k-1} \right\|_{P_{k-1|k-1}^{-1}}^2 \right), \tag{13}$$

s.t. $\hat{v}_k = y_k - \gamma_k C (A + \Delta A_{k-1}) \hat{x}_{k-1|k-1}$ and (1)–(5), where $x_{k-1|k}$ and $w_{k-1|k}$, respectively, represent the optimal estimated state and system disturbance of the time $k-1$ at time k, which satisfy

$$\hat{x}_{k|k} = (A + \Delta A_{k-1}) \hat{x}_{k-1|k} + B w_{k-1|k}. \tag{14}$$

To make the calculation simple, we define

$$\begin{aligned}
\widehat{x} &= \begin{bmatrix} \widehat{x}_{k-1|k} - \widehat{x}_{k-1|k-1} \\ w_{k-1|k} \end{bmatrix}, \\
b &= y_k - \gamma_k CA\widehat{x}_{k-1|k-1}, \\
\Delta b &= -\gamma_k CM\Lambda E\widehat{x}_{k-1|k-1}, \\
\Delta F &= CM\Lambda_{k-1}\begin{bmatrix} E & 0 \end{bmatrix}, \\
F &= C\begin{bmatrix} A & B \end{bmatrix}, \\
\widetilde{Q} &= \begin{bmatrix} P_{k-1|k-1}^{-1} & \\ & Q^{-1} \end{bmatrix}, \\
W &= R_k,
\end{aligned} \tag{15}$$

then, (13) can be rewritten as

$$\min_{\widehat{x}} \max_{\Delta F} \left[\|\widehat{x}\|_{\widetilde{Q}}^2 + \|(F+\Delta F)\widehat{x} - (b+\Delta b)\|_W^2 \right]. \tag{16}$$

According to Theorem 1 proposed in [25], the solution of (16) can be expressed as

$$\widehat{x} = \left(\widehat{Q} + F^T\widehat{W}F\right)^{-1}\left(F^T\widehat{W}b + \widehat{\eta}E_a^TE_b\right), \tag{17}$$

where

$$\begin{aligned}
\widehat{W} &= W + WC\left(\widehat{\eta}I - C^TWC\right)^{+}C^TW, \\
\widehat{Q} &= \widetilde{Q} + \widehat{\eta}E_a^TE_a, \\
E_b &= -E\widehat{x}_{k-1|k-1}, \\
E_a &= \begin{bmatrix} E & 0 \end{bmatrix},
\end{aligned} \tag{18}$$

$$\widehat{\eta} = \arg\min_{\eta \geq \|C^TWC\|} \|x(\eta)\|_Q^2 + \eta\left\|E_a x(\eta) - E_b\right\|^2 + \|Fx(\eta) - b\|_{W(\eta)}^2, \tag{19}$$

where

$$\begin{aligned}
W(\eta) &= W + WC\left(\eta I - C^TWC\right)^{+}C^TW, \\
Q(\eta) &= \widetilde{Q} + \eta E_a^TE_a, \\
x(\eta) &= \left(Q(\eta) + F^TW(\eta)F\right)^{-1}\left(F^TW(\eta)b + \eta E_a^TE_b\right).
\end{aligned} \tag{20}$$

By solving (13)–(19), the solution of the optimization problem (12) can be summed up as the following form:

$$\begin{cases}
\widehat{\eta} = \gamma_k\left\|M^TC^TR^{-1}CM\right\| + \alpha, \\
\widehat{R} = R - \widehat{\eta}^{-1}CMM^TC^T, \\
P_{k-1|k} = A\left(P_{k-1|k-1}^{-1} + \widehat{\eta}E^TE\right)^{-1}A^T + BQB^T, \\
P_{k|k} = P_{k-1|k}, -\gamma_k P_{k-1|k}C^T\left(\widehat{R} + CP_{k-1|k}C^T\right)^{-1}CP_{k-1|k},
\end{cases} \tag{21}$$

where α is an adjustable parameter, which satisfies $\alpha > 0$.

Remark 2. In the result of (13), not only the equation of the time-varying weight matrix $P_{k|k}$ can be obtained, but also an equation of a priori estimated state from $\widehat{x}_{k-1|k-1}$ to $\widehat{x}_{k|k}$ can be obtained. But in this paper, we use $\widetilde{x}_{t-n+1|t-1}^*$ to represent the priori estimation of the next time, so in result (21), we only list the equations about $P_{k|k}$ (for more detailed calculations, see [25]).

In conclusion, the algorithm of the RMHE method proposed in this paper can be summarized into the following steps:

Step 1. Initialize and determine $\widehat{x}_{0|0}$, $P_{0|0}$, Q, R, and window length n

Step 2. When $t \leq n$, get the optimal solutions $x_{0|t-1}^*$ and $\left\{w_{k|t-1}^*\right\}_{k=0}^{t-1}$ by solving (6), and then get the estimated state by solving (8)

Step 3. When $t > n$, get $P_{t-n|t-1}$ by solving (21)

Step 4. Based on the measurement data $\{y_k\}_{k=t-n}^{k=t-1}$, get $x_{t-n|t-1}^*$ and $\left\{w_{k|t-1}^*\right\}_{k=t-n}^{t-1}$ by solving (9), then get the estimated state by solving (12)

Step 5. At time $t+1$, return to Step 3

In this section, a state estimator is designed to solve the problem of packet loss and model error in NCSs. In the first part, the state estimation problem is transformed into a quadratic programming problem (9) by selecting a random cost function. Moreover, in order to make the result of (9) more accurate, equation (21) of the weight matrix in (9) is obtained by solving a regularized least-square problem with uncertain parameters. This estimator guarantees that the estimated state is optimal in the worst case and increases its accuracy and practicability.

4. Estimation Properties for Robust MHE

In order to verify the stability of the RMHE designed in this paper, the following sufficient conditions of the asymptotic convergence are deduced. Before the convergence analysis of the estimator, let $\overline{C}^T = \begin{bmatrix} C^T\widehat{R}^{-T/2} & \sqrt{\widehat{\lambda}}E^T \end{bmatrix}$. The recursive formula of the error covariance $P_{k|k}$ in (21) can be rewritten into the following form:

$$\begin{aligned}
P_{k|k} &= AP_{k-1|k-1}A^T + BQB^T - \gamma_k AP_{k-1|k-1}\overline{C}^T \\
&\quad \times \left(I + \overline{C}P_{k-1|k-1}\overline{C}^T\right)^{-1}\overline{C}P_{k-1|k-1}A^T.
\end{aligned} \tag{22}$$

To prove the following theorem, firstly two lemma are introduced as follows.

Lemma 1. *When λ is within a certain range and the following conditions are satisfied, then the recursive state estimator in (21) is asymptotically stable for any $P_{0|0} > 0$. Furthermore, as*

$t \longrightarrow \infty$, $P_{k|k}$ approaches a unique positive definite matrix (see, e.g., [26]).

(a) (A, C) is detectable and A is unstable

(b) $(A, Q^{1/2})$ is controllable

Lemma 2. *Assume $Q \geq 0$, $R_k > 0$, $P_{0|0} > 0$, and (A, C) is detectable; as $t \longrightarrow \infty$, there exist $\tilde{x}^*_{0|t-1}$, $\{\tilde{w}^*_{k\,|\,t-1}\}_{k=0}^{t-1}$, and $\Gamma > 0$ such that*

$$\begin{aligned}&\sum_{k=0}^{t-1}\left\|\tilde{v}^*_{k\,|\,t-1}\right\|^2_{R_k^{-1}} + \left\|\tilde{w}^*_{k\,|\,t-1}\right\|^2_{Q^{-1}} + \left\|\tilde{x}^*_{0\,|\,t-1} - \hat{x}_{0\,|\,0}\right\|^2_{P^{-1}_{0|0}}\\ &\leq \Gamma\left\|\tilde{x}^*_{0\,|\,t-1} - \hat{x}_{0\,|\,0}\right\|.\end{aligned} \tag{23}$$

Furthermore, as $t \longrightarrow \infty$, the conclusion

$$\sum_{k=t-N}^{t-1}\left\|\tilde{v}^*_{k\,|\,t-1}\right\|^2_{R_k^{-1}} + \left\|\tilde{w}^*_{k\,|\,t-1}\right\|^2_{Q^{-1}} \longrightarrow 0, \tag{24}$$

is satisfied (see, e.g., [15]).

According to Lemma 1, it is easy to analyze the related stability of (22) because its analysis process is the same as the proof of Theorem 2 in [26]. Moreover, based on Lemmas 1 and 2, the stability of the RMHE proposed in this paper can be discussed, where the influences of measurement noise, packet loss probability, and network transmission noise are all included in the next proof.

To simplify notation, let $x_{i|t-1} = x_i$ in this section, and so are the others.

Theorem 1. *Suppose (A, C) is detectable and A is unstable and $(A, Q^{1/2})$ is controllable. If the optimization problem (9) is solvable and P_{t-n} can be derived from (21), the expectation of the estimation error will eventually tend to 0 over time.*

Proof. According to Lemma 1, the solutions of problem (9) are assumed to be $\tilde{x}^*_{t-n}$ and $\{\tilde{w}^*_k\}_{k=t-n}^{t-1}$, then the optimal cost function (25) is bounded

$$J^*_t = \sum_{k=t-n}^{t-1}\left\{\left\|\tilde{v}^*_k\right\|^2_{R_k^{-1}} + \left\|\tilde{w}^*_k\right\|^2_{Q^{-1}} + \left\|\tilde{x}^*_{t-n} - \hat{x}_{t-n}\right\|^2_{P^{-1}_{t-n}}\right\} < \infty. \tag{25}$$

In addition, the following formula is satisfied:

$$\begin{aligned}\left\|e_{t-1}\right\| &= \left\|\tilde{x}^*_{t-1} - x_{t-1}\right\|\\ &= \left\|A^{n-1}\tilde{x}^*_{t-n} - \left(A + \Delta A_k\right)^{n-1}x_{t-n} + \sum_{j=0}^{n-2}A^{n-1-j}B\left(\tilde{w}^*_{t-n+j} - w_{t-n+j}\right)\right\|\\ &\leq \left\|A^{n-1}\tilde{x}^*_{t-n} - \left(A + \Delta A_k\right)^{t-1}x_{t-n}\right\| + \sum_{j=0}^{n-2}\|A\|^{n-1-j}\|B\|\left\|\tilde{w}^*_{t-n+j} - w_{t-n+j}\right\|.\end{aligned} \tag{26}$$

For the convenience of the later proof, the expectation of both sides of (26) can be written as

$$\begin{aligned}&E\left\{\left\|e_{t-1}\right\|\right\}\\ &\leq E\left\{\left\|A^{n-1}\tilde{x}^*_{t-n} - \left(A + \Delta A_k\right)^{n-1}x_{t-n}\right\| + \sum_{j=0}^{n-2}\|A\|^{n-1-j}\|B\|\left\|\tilde{w}^*_{t-n+j} - w_{t-n+j}\right\|\right\}\\ &= E\left\{\left\|A^{n-1}\tilde{x}^*_{t-n} - \left(A + \Delta A_k\right)^{n-1}x_{t-n}\right\|\right\} + E\left\{\sum_{j=0}^{n-2}\|A\|^{n-1-j}\|B\|\left\|\tilde{w}^*_{t-n+j} - w_{t-n+j}\right\|\right\}.\end{aligned} \tag{27}$$

By defining

$$\rho=\begin{bmatrix} C & & \\ & \ddots & \\ & & C \end{bmatrix},$$

$$Z_{t-1}=\begin{bmatrix} \gamma_{t-n} & & \\ & \ddots & \\ & & \gamma_{t-1} \end{bmatrix}, \tag{28}$$

$$\Omega=\begin{bmatrix} (A+\Delta A_k)^0 x_{t-n}-A^0\tilde{x}^*_{t-n} \\ \vdots \\ (A+\Delta A_k)^{n-1}x_{t-n}-A^{n-1}\tilde{x}^*_{t-n} \end{bmatrix},$$

then we can get the following formula:

$$\sum_{j=0}^{n-1}\left\|\gamma_{t-n+j}\left\{C(A+\Delta A_k)^j x_{t-n}-CA^j\tilde{x}^*_{t-n}\right\}\right\|^2 = (\Omega Z_{t-1})^T\rho^T\rho(\Omega Z_{t-1}), \tag{29a}$$

$$\sum_{j=0}^{n-1}\left\|\gamma_{t-n+j}\left\{C(A+\Delta A_k)^j x_{t-n}-CA^j\tilde{x}^*_{t-n}\right\}\right\| \geq \sqrt{\xi_{\min}(\rho^T\rho)}\left\|\Omega Z_{t-1}\right\|. \tag{29b}$$

Because (A,C) is detectable, $\rho^T\rho$ is a positive definite matrix. Let $\psi=(1/\sqrt{\xi_{\min}(\rho^T\rho)})$, then equation (29b) can be further reduced to

$$\begin{aligned}\left\|\Omega Z_{t-1}\right\| &\leq \psi\sum_{j=0}^{n-1}\left\|\gamma_{t-n+j}\left\{C(A+\Delta A_k)^j x_{t-n}-CA^j\tilde{x}^*_{t-n}\right\}\right\| \\ &= \psi\sum_{j=0}^{n-1}\left\|y_{t-n+j}-\gamma_{t-n+j}C\tilde{x}^*_{t-n+j}-v_{t-n+j}\right\| \\ &= \psi\sum_{j=0}^{n-1}\left\|\tilde{v}^*_{t-n+j}-v_{t-n+j}\right\|,\end{aligned} \tag{30}$$

and it is obvious that

$$\left\|\gamma_{t-n}\left\{A^{n-1}\tilde{x}^*_{t-n}-(A+\Delta A_k)^{n-1}x_{t-n}\right\}\right\| \leq \left\|\Omega Z_{t-1}\right\|, \tag{31}$$

by combining (30) and (31), it has

$$\left\|\gamma_{t-n}\left\{A^{n-1}\tilde{x}^*_{t-n}-(A+\Delta A_k)^{n-1}x_{t-n}\right\}\right\| \leq \psi\sum_{j=0}^{n-1}\left\|\tilde{v}^*_{t-n+j}-v_{t-n+j}\right\|. \tag{32}$$

To eliminate the random variable γ_{t-n} in (32), we take the expectation of both sides of (32) and obtain

$$E\{\gamma_{t-n}\}E\left\{\left\|A^{n-1}\tilde{x}^*_{t-n}-(A+\Delta A_k)^{n-1}x_{t-n}\right\|\right\} \leq E\left\{\psi\sum_{j=0}^{n-1}\left\|\tilde{v}^*_{t-n+j}-v_{t-n+j}\right\|\right\}. \tag{33}$$

Because the probability λ is given, (33) can be reduced to

$$\lambda E\left\{\left\|A^{n-1}\tilde{x}^*_{t-n}-(A+\Delta A_k)^{n-1}x_{t-n}\right\|\right\} \leq E\left\{\psi\sum_{j=0}^{n-1}\left\|\tilde{v}^*_{t-n+j}-v_{t-n+j}\right\|\right\}, \tag{34}$$

then, combining (27) and (34), it has

$$\begin{aligned}E\left\{\left\|e_{t-1}\right\|\right\} &\leq \frac{1}{\lambda}E\left\{\psi\sum_{j=0}^{n-1}\left\|\tilde{v}^*_{t-n+j}-v_{t-n+j}\right\|\right\}+E\left\{\sum_{j=0}^{n-2}\|A\|^{n-1-j}\|B\|\left\|\tilde{w}^*_{t-n+j}-w_{t-n+j}\right\|\right\} \\ &\leq \frac{1}{\lambda}E\left\{\psi\sum_{j=0}^{n-1}\left\|\tilde{v}^*_{t-n+j}\right\|\right\}+\frac{1}{\lambda}E\left\{\psi\sum_{j=0}^{n-1}\left\|v_{t-n+j}\right\|\right\}+E\left\{\sum_{j=0}^{n-2}\|A\|^{n-1-j}\|B\|\left\|\tilde{w}^*_{t-n+j}\right\|\right\}+E\left\{\sum_{j=0}^{n-2}\|A\|^{n-1-j}\|B\|\left\|w_{t-n+j}\right\|\right\}.\end{aligned} \tag{35}$$

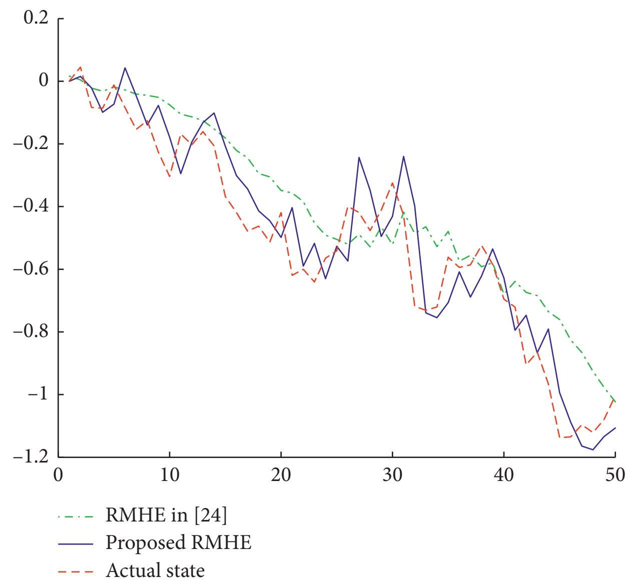

Figure 1: The trajectories of the state x_1.

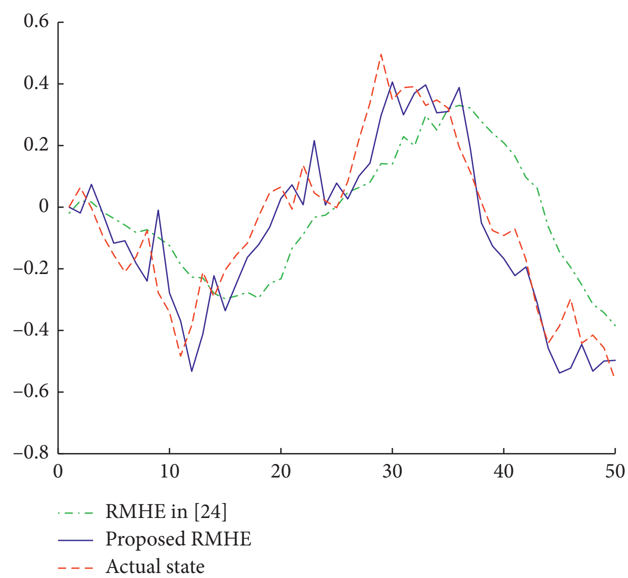

Figure 2: The trajectories of the state x_2.

In view of the expectation of v_k and w_k in (2), (35) can be rewritten as

$$E\left\{\left\|e_{t-1}\right\|\right\} \leq \frac{1}{\lambda} E\left\{\psi \sum_{j=0}^{n-1}\left\|\widetilde{v}_{t-n+j}^{*}\right\|\right\} + E\left\{\sum_{j=0}^{n-2}\|A\|^{n-1-j}\|B\|\left\|\widetilde{w}_{t-n+j}^{*}\right\|\right\}. \tag{36}$$

Because (25) is bounded, $E(\|e_{t-1}\|)$ has an upper bound. Furthermore, if the conditions of Lemmas 1 and 2 are satisfied, that is, when $t \longrightarrow \infty$, it is easy to get $E(\|e_{t-1}\|) \longrightarrow 0$. Therefore, it is concluded that the RMHE method designed in this paper is asymptotically stable.

In Theorem 1, the norm of the estimation error is affected by many factors such as system matrices, system noise, network transmission noise, random packet loss, and system uncertainty. That is, it is not only related to the coefficient matrix A, B, and C of the system, but also affected by the matrices Q and R, window length n, and uncertain matrices Λ and λ. Then, it is very difficult to qualitatively analyze the convergence of the error norm. In order to better compensate the estimation error, it is necessary to appropriately adjust Q and R based on the influence of packet loss and system uncertainties. Different from other methods, such as H_∞ filtering and Kalman filtering, the proposed RMHE method not only guarantees that the error covariance matrix is bounded, but also guarantees that the expectation of the error norm converges to zero if system noise and network noise trend to zero.

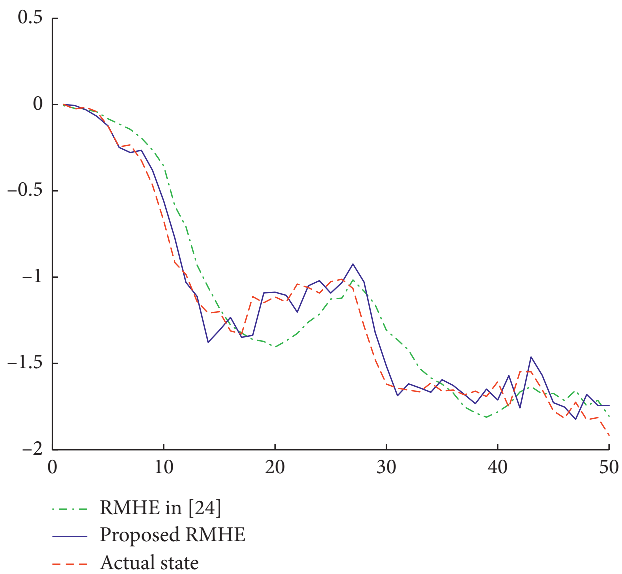

Figure 3: The trajectories of the state x_3.

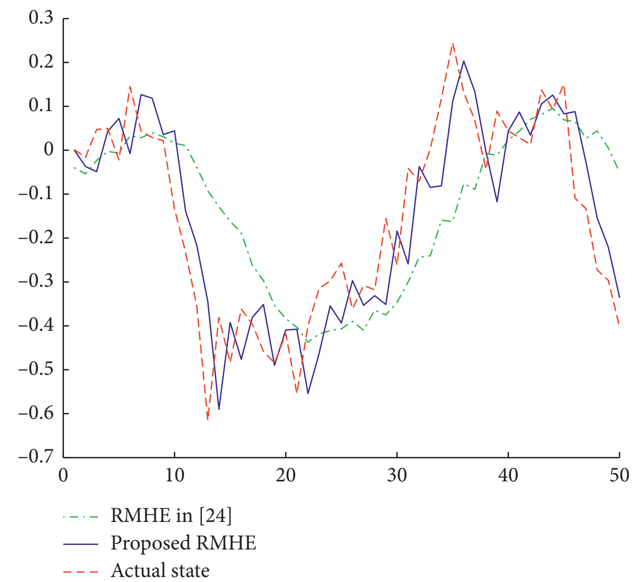

Figure 4: The trajectories of the state x_4.

Remark 3. In this paper, the P in the estimation algorithm is changeable according to the system coefficient at each time, so in the proof process, when the system satisfies some conditions, we guarantee that P is asymptotically stable according to the theorem in [26]. Then, the approach is proved to be asymptotically stable according to the theorem in [15] (for more detailed calculations, see [15, 26]).

5. Verification Simulation

In order to verify the effectiveness of the proposed RMHE, the following unstable state-space model is considered in the simulation analysis:

$$x_{k+1} = \left(\begin{bmatrix} 0.9942 & 0.0537 & 0 & 0 \\ 0.3565 & -0.9156 & 0 & 0.0663 \\ 0.0940 & -0.0047 & 1 & 0.1538 \\ 0.0184 & -0.0956 & 0 & 0.9966 \end{bmatrix} + \Delta A_k \right) x_k + w_k,$$
$$y_k = \gamma_k \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \end{bmatrix} x_k + v_k. \tag{37}$$

It is assumed that $\Delta A_k = M\Lambda_k N$, $\|\Lambda_k\| \le 1$, $M = \begin{bmatrix} 0.01 & 0.01 & 0 & 0 \end{bmatrix}^T$, and $E = \begin{bmatrix} 0.5 & 0.5 & 0 & 0 \end{bmatrix}$. The parameters $R = \mathrm{diag}\{1,1\}$ and $Q = 20$ are given. Set $\lambda = 0.8$ and the constraints of the system noise as $w_k > 0$. Since the system uncertainty has a much greater impact on the state estimation than packet loss in this simulation, the choice of Q is taken larger than R. Furthermore, assume that $P_{0|0} = \mathrm{diag}\{1,1,1,1\}$ and $\hat{x}_{0|0} = \begin{bmatrix} 0 & 0 & 0 & 0 \end{bmatrix}^T$. So far, there is no good conclusion about how the fixed window length N can make the estimator performance better. Generally, it is twice the order of the system, so the fixed window length $N = 10$ is chosen in this paper.

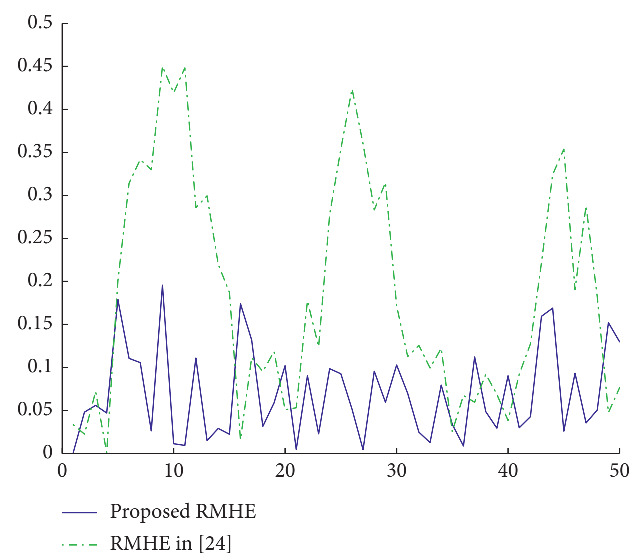

Figure 5: Comparison of the error norm.

Table 1: The mean error norm (MEN) of different methods.

Algorithm	λ			
	0.6	0.7	0.8	0.9
Proposed RMHE	0.1186	0.1044	0.0912	0.0714
RMHE in [24]	0.2055	0.1564	0.1163	0.9250

Table 2: Time(s) for one simulation run for different m.

Algorithm	m			
	50	100	200	400
Proposed RMHE	0.7584	1.3905	2.7265	5.5402
RMHE in [24]	0.7752	1.1055	2.6532	5.5024

To evaluate the estimation performances of such a proposed estimator, the mean error norm (MEN) is adopted as follows:

$$\mathrm{MEN}_t = \left(\sum_{t=1}^{m} \frac{\left\| \tilde{x}_{t|t}^{*} - x_{t|t} \right\|^2}{m} \right)^{1/2}, \tag{38}$$

where $\tilde{x}_{t|t}^{*} - x_{t|t}$ is the estimation error at time t and m is the number of estimated states.

The proposed MHE method is compared with the existing RMHE in [24]. In [24], the objective function of MHE is transformed into the min-max problem, and the optimal estimated state in the worst case is obtained by solving the inequality. However, this method is limited in solving the problems with physical constraints, which cannot get good results or require a lot of recalculation. In this paper, the quadratic optimization is retained and the min-max strategy is used to solve the weight matrix, so the constraint problem is solved with robustness. The system state trajectories are given in Figures 1–4, and Figure 5 gives a comparison of the mean error norm over time. We can know from these results that both methods can estimate the state of the system well in the presence of model error and packet loss, but when the system has constraints, the proposed method can get a smaller estimation error. Table 1 gives a comparison of MEN for different packet loss probabilities; Table 2 shows the time taken to calculate the different data. The results show that the proposed method performs better when the time used is basically the same.

6. Conclusion

Based on the MHE principle, this paper presents a robust moving horizon estimation approach to solve the constrained state estimation problem for uncertain networked systems with random packet loss. The proposed state estimation can be transformed into a quadratic optimization problem with a fixed window, where the initial state and disturbance of the system are estimated by minimizing the performance index. A new unconstrained estimator is given based on the robust least-square principle, which ensures that the optimal weight matrix in the worst case can be obtained. The asymptotically convergent conditions of the estimator are discussed. In the future work, this proposed estimation approach can be expanded to distributed control systems and complex systems.

Data Availability

The raw/processed data required to reproduce these findings cannot be shared at this time as the data also form part of an ongoing study.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

Acknowledgments

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