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Research Article

Transverse Response of an Axially Moving Beam with Intermediate Viscoelastic Support

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This study presented the transverse vibration of an axially moving beam with an intermediate nonlinear viscoelastic foundation. Hamilton's principle was used to derive the nonlinear equations of motion. The finite difference and state-space methods transform the partial differential equations into a system of coupled first-order regular differential equations. The numerical modeling procedures are utilized for evaluating the effects of parameters, such as axial translation velocity, flexure rigidities of the beam, damping, and stiffness of the support on the transverse response amplitude and frequencies. It is observed that the dimensionless fundamental frequency and magnitude of axial speed had an inverse correlation. Furthermore, increasing the flexure rigidity of the beam reduced the transverse displacement, but at the same instant, fundamental frequency rises. Vibration amplitude is found to be significantly reduced with higher damping of support. It is also observed that an increase in the foundation damping leads to lower fundamental frequencies, whereas increasing the foundation stiffness results in higher frequencies.

1. Introduction

Axially moving beams have numerous applications in various branches of engineering, such as manufacturing, civil, mechanical, chemical, and aerospace engineering [1–4]. Typical applications explored in previous studies include conveyor belts, medical nanorobots, power saw, band saw, and magnetic tape [5–9]. At the nano/micron level, the motion of the substrate is significant, and small fluctuations can result in a low-quality product [10–13]. Transverse vibrations triggered by external disturbances during a particular process may be lowered by providing end supports beneath the moving beam in the form of rotating rolls [14]. Though end supports are beneficial, they are not sufficient to stabilize the motion of moving substrate. End support controller was used to stabilize the transverse vibration of an axially moving Kirchhoff beam [15]. Additional intermediate support in the moving span is often required [16]. Intermediate support between the rolls offers superior web processing quality in comparison to the traditional end support. Vibration of an axially moving beam with an intermediate foundation has become a burning research topic since the last decade. Several studies have attempted to analyze axially moving beams with elastic foundations. Dynamic analysis of coupled moving beams through a Winkler elastic support was investigated by Gaith and Müftü [17]. Yang et al. [18] studied the free vibration of axially moving elastic beams resting on an elastic foundation. In their study, they utilized the Galerkin method to discretize the equation of motion. Fundamental frequencies of transverse fluctuations in a moving beam were studied by utilizing different numerical methods [19]. Liu et al. [20] explored the dynamics and instability of the Euler–Bernoulli beam as a function of several variables such as axially moving speed, position, and weight of the lumped mass. The study results revealed that any increase in lumped mass is inversely associated with resulting systems' fundamental frequencies. Dynamics of axially moving beam attached with energy sink and a piezoelectric device was studied in [21, 22]. A significant effect of the attachment on the response was observed. Norouzi and Younesian [23] investigated the chaotic behavior of an axially moving beam supported by a stiff foundation. Cosine-cosine function was used to obtain the approximate solution of the equation of motion. Chaos indication was obtained by setting Lyapunov exponent as a criterion. Mohammadzadeh and Mosayebi [24] investigated the vibration in rail supports and joints by considering it as an axially moving beam resting on an elastic foundation. The effect of rail support and joints in the locality of a bridge in railway track was investigated.

Kural and Ozkaya [25] investigated the performance of a microbeam fluid-carrying media by utilizing the modified couple stress theory. In another study, Yan et al. [26] employed the theoretical model to examine the stability and dynamics of a thin cantilever beam. The beam was attached with a moving station submerged in a fluid. Calim [27] explored the patterns of free and forced fluctuation in a functionally graded beam assembly supported with an intermediate elastic foundation. Tang et al. [28] explored the transverse vibration of an axially accelerating beam having time-dependent axial tension. Analysis revealed that variation in axial speed and axial tension has a direct influence on the resonance. Mirzabeigy and Madoliat [29] studied the large amplitude-free vibration of beams resting on the supporting foundation. The Winkler model and Euler–Bernoulli method were used to study the induced vibrations in elastic foundation and beam, respectively. Ghannadiasl [30] proposed the application of dynamic Green function (DGF) to investigate the dynamic response of railways under varying loading conditions. The load was varying in nature with different speeds and accelerations. Variation in moving load, as well as the elastic parameters of the foundation, was investigated. It was shown by the modeling results that the maximum deflection depends on the increasing or decreasing acceleration of the moving load.

Similarly, Zhang et al. [31] utilized the complex modal analysis to investigate the dynamic response of a moving beam resting on intermediate support. Ding et al. [32] investigated the axially moving beam traveling with supercritical speed. The impact of rotary inertia and shear deformation on the transverse fluctuations was investigated. Zhao et al. [33] analytically investigated an axially moving microbeam. Coupled thermoelastic vibration and heat transfer process was presented. Mohamed et al. [34] presented a novel numerical procedure to forecast the forced steady-state and nonlinear free response of curved beam under clamped-clamped boundary conditions. An and Su [35] presented the generalized integral transform technique as a numerical approach for the dynamic analysis of Timoshenko beam. Esen [36] considered functionally graded Timoshenko beam to investigate the dynamic response using the modified finite element method (FEM). The beam under consideration was resting on elastic support and was exposed to a mass moving with a variable speed. For the first time, Ding et al. [37] proposed a generalized boundary conditions approach for axially moving beams supported by vertical and torsional springs at both ends. Shao et al. [38] investigated the nonlinear dynamic behavior of a moving membrane with fluctuating speed. It was concluded that the nonlinear vibration characteristics of a membrane are sensitive to the initial motion conditions.

A thorough review of the literature reveals a significant lack of comprehensive investigations on the problem of an axially moving beam with nonlinear viscoelastic intermediate support. To the best of author's knowledge, problem of transverse vibration in an axially moving beam is tackled by providing supports at the ends of the moving beam. In authors' opinion, end supports are likely to result in lower quality process. Therefore, intermediate viscoelastic supports are presented here as a possible solution to mitigate the vibration in moving beams. Also, in view of the nature of involved equations of motion, authors also feel the need to investigate the application of the finite difference method (FDM) on the nonlinear dynamics of beams under consideration. In this study, transverse vibrations in axially traveling beams with a roll-to-roll configuration supported by the viscoelastic intermediate foundation were studied. Nonlinear equations of motion for transverse vibrations were derived using Hamilton's principle. A model based on the finite difference method coupled with the state-space approach was presented. The effect of parameters, such as axial translation velocity, flexure rigidities of the beam, damping, and stiffness of the support on the transverse response amplitude and frequencies of the system in the subcritical region, were investigated.

2. Problem Formulation

An axially moving beam on a viscoelastic foundation for analyzing the vibrations characteristics is presented in Figure 1. This model mainly includes stiffness, damping, length, and end supports. The beam has flexural rigidity EI and axial tension T and moves with an axial velocity (v). Transverse displacement of the beam is $w(x, t)$.

Free body diagram of an infinitesimal element of the considered beam is shown in Figure 2. Here, S is the shear force and M_b is the bending moment. Viscous damping force, spring force, and inertial force are represented by F_d, F_s, and F_I, respectively. First, the total kinetic energy and potential energy of the moving beam were calculated by

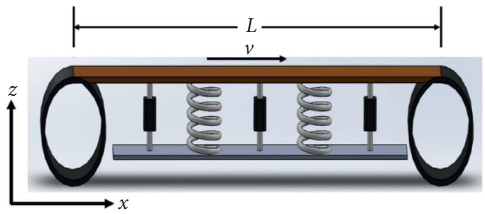


Figure 1: Schematic of an axially moving beam with intermediate support.

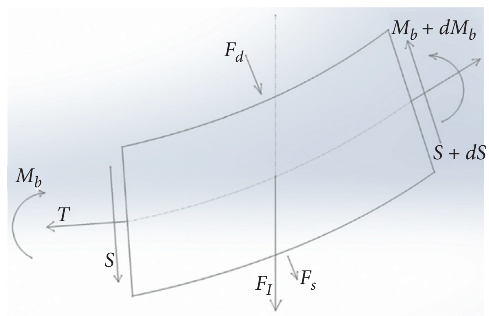


Figure 2: Free body diagram of the infinitesimal beam element.

considering ρ as the density and A as the cross-sectional area of the beam. Hamilton's principle was then applied to derive the equation of motion. Transverse vibration of the beam is described by

$$\rho A\left[\frac{\partial^2 w}{\partial t^2} + 2v\frac{\partial^2 w}{\partial t\,\partial x} + \frac{\partial v}{\partial t}\frac{\partial w}{\partial x} + v^2\frac{\partial^2 w}{\partial x^2}\right] - \left[T\frac{\partial^2 w}{\partial x^2} + EI\frac{\partial^4 w}{\partial x^4}\right] + k_1 w + k_2 w^3 + +\xi\left[\frac{\partial w}{\partial t} + v\frac{\partial w}{\partial x}\right] = 0, \quad (1)$$

where E is the modulus off elasticity, ρ is the density of the beam material, A is the cross-sectional area, and I is the moment inertia of the beam. Axial speed and axial tension in the beam are represented by v and T, respectively. The linear stiffness, nonlinear stiffness, and damping of the foundation are represented by k_1, k_2, and ξ, respectively. Introducing the following dimensionless variables transforms the equation of motion into a normalized equation:

$$x_* = \frac{x}{L},$$

$$w_* = \frac{w}{L},$$

$$t_* = t\sqrt{\frac{T}{\rho A L^2}},$$

$$v_* = v\sqrt{\frac{\rho A}{T}},$$

$$\alpha = \frac{EI}{TL^2},$$

$$k_{1*} = \sqrt{\frac{kL}{T}},$$

$$k_{2*} = \sqrt{\frac{kL^3}{TA}},$$

$$\xi_* = \xi\sqrt{\frac{L}{\rho AT}},$$

$$\frac{\partial^2 w_*}{\partial t_*^2} + 2v_*\frac{\partial^2 w_*}{\partial t_*\partial x_*} + \left(v_*^2 - 1\right)\frac{\partial^2 w_*}{\partial x_*^2} + v_*\frac{\partial w_*}{\partial x_*} + \alpha\frac{\partial^4 w_*}{\partial x_*^4} + k_{1*}^2 w_* + k_{2*}^2 w_*^3 + \xi_*\left(\frac{\partial w_*}{\partial t_*} + v_*\frac{\partial w_*}{\partial x_*}\right) = 0, \quad (2)$$

where v_* represents the dimensionless transport speed (a ratio between physical and wave velocity in the beam), while parameter α is the dimensionless flexural rigidity.

2.1. Finite Difference Formulation. Transverse vibrations of the moving beam on a nonlinear viscoelastic foundation are numerically solved, and essential mathematical formulation is presented in equation (2). An analysis approach based on the finite difference method is used to obtain second-order ordinary differential equations (ODEs) by transforming the fourth-order HPDE. Discretization of the equation is then achieved by the central difference scheme given by the following equations:

$$\left.\frac{\partial w_*}{\partial x_*}\right|_{x_{*i}} = \frac{w_{*(i+1)} - w_{*(i-1)}}{2 * dx_*}, \tag{3}$$

$$\left.\frac{\partial^2 w_*}{\partial x_*^2}\right|_{x_{*i}} = \frac{w_{*(i+1)} - 2 * w_{*i} + w_{*(i-1)}}{dx_*^2}, \tag{4}$$

$$\left.\frac{\partial^4 w_*}{\partial x_*^4}\right|_{x_{*i}} = \frac{w_{*(i+2)} - 4 * w_{*(i+1)} + 6 * w_{*i} - 4 * w_{*(i-1)} + w_{*(i-2)}}{dx_*^4}. \tag{5}$$

The following second-order ordinary differential equations are produced:

$$\left.\begin{array}{l}
\frac{d^2 w_{*1}}{dt_*^0} + \frac{v}{dx_*}\frac{dw_{*2}}{dt_*} + \frac{\left(v_*^2 - 1\right)}{dx_*^2}\left(-2 * w_{*1} + w_{*2}\right) + \frac{\dot{v_*}}{2 * dx_*} * w_{*2} + \cdots \\
\cdots \frac{\alpha}{dx_*^4}\left(6 * w_{*1} - 4 * w_{*2} + w_{*3}\right) + k_{1*}^2 w_{*1} + k_{1*}^2 w_{*1}^3 + \xi_* \frac{dw_{*1}}{dt_*} + \frac{\xi_* v_*}{2 * dx_*} * w_{*2} = 0 \\
\frac{d^2 w_{*2}}{dt_*^2} + \frac{v_*}{dx_*}\left(-\frac{dw_{*1}}{dt_*} + \frac{dw_{*3}}{dt_*}\right) + \frac{\left(v_*^2 - 1\right)}{dx_*^2}\left(w_{*1} - 2 * w_{*2} + w_{*3}\right) + \frac{\dot{v_*}}{2 * dx_*}\left(-w_{*1} + w_{*3}\right) + \cdots \\
\cdots + \frac{\alpha}{dx_*^4}\left(-4 * w_{*1} + 6 * w_{*2} - 4 * w_{*3} + w_{*4}\right) + k_{1*}^2 w_{*2} + k_{2*}^2 w_{*2}^3 + \cdots \\
\cdots + \xi_* \frac{dw_{*2}}{dt_*} + \frac{\xi_* v_*}{2 * dx_*} * \left(-w_{*1} + w_{*3}\right) = 0 \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\vdots \\
\frac{d^2 w_{*(n-1)}}{dt_*^2} + \frac{v_*}{dx_*}\frac{dw_{*(n-2)}}{dt_*} + \frac{\left(v_*^2 - 1\right)}{dx_*^2}\left(w_{*(n-2)} - 2 * w_{*(n-1)}\right) + \frac{\dot{v_*}}{2 * dx_*} * w_{*(n-2)} + \cdots \\
\cdots + \frac{\alpha}{dx_*^4}\left(w_{*(n-3)} - 4 * w_{*(n-2)} + 6 * w_{*(n-1)}\right) + k_{1*}^2 w_{*(n-1)} + k_{2*}^2 w_{*(n-1)}^3 + \cdots \\
\cdots \xi_* \frac{dw_{*(n-1)}}{dt_*} + \frac{\xi_* v_*}{2 * dx_*} * w_{*(n-2)} = 0
\end{array}\right\}, \tag{6}$$

where the total number of spatial points is represented by n and step size is represented by $\mathrm{d}x_*$. The subsequent matrix form of equation (6) is given by

$$M\frac{d^2 w_*}{dt_*^2} + [2 v_* G + \xi_* M] \frac{dw_*}{dt_*} + \left[\left(v_*^2 - 1\right)K_1 + \dot{v_*} * G + \alpha K_2 + \xi_* v_* G + k_{1*}^2 M\right]w_* + k_{2*}^2 M w_*^3 = 0, \tag{7}$$

where M, G, K_1, and K_2 are $(n-1) * (n-1)$ matrices, which have the following structure:

$$M = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & 0 & \vdots \\ 0 & 0 & \ddots & \ddots & 0 \\ \vdots & 0 & \ddots & 1 & 0 \\ 0 & \dots & 0 & 0 & 1 \end{bmatrix}, \tag{7a}$$

$$G = \frac{1}{2 * dx_*}\begin{bmatrix} 0 & 1 & 0 & \dots & 0 \\ -1 & 0 & 1 & 0 & \vdots \\ 0 & -1 & \ddots & \ddots & 0 \\ \vdots & 0 & \ddots & \ddots & 1 \\ 0 & \dots & 0 & -1 & 0 \end{bmatrix}, \tag{7b}$$

$$K_1 = \frac{1}{dx_*^2}\begin{bmatrix} -2 & 1 & 0 & \dots & 0 \\ -1 & -2 & 1 & 0 & \vdots \\ 0 & -1 & \ddots & \ddots & 0 \\ \vdots & 0 & \ddots & \ddots & 1 \\ 0 & \dots & 0 & -1 & -2 \end{bmatrix}, \tag{7c}$$

$$K_2 = \frac{1}{dx_*^4}\begin{bmatrix} 6 & -4 & 1 & 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ -4 & 6 & -4 & 1 & 0 & \cdots & \cdots & \cdots & 0 \\ 1 & -4 & 6 & -4 & 1 & 0 & \cdots & \cdots & 0 \\ 0 & 1 & -4 & 6 & -4 & 1 & 0 & \cdots & 0 \\ \vdots & 0 & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \vdots & \ddots & \ddots & \ddots & \ddots & \ddots & \ddots & 0 \\ \vdots & \vdots & \vdots & \ddots & 1 & -4 & 6 & -4 & 1 \\ \vdots & \vdots & \vdots & \vdots & \ddots & 1 & -4 & 6 & -4 \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 & 1 & -4 & 6 \end{bmatrix}. \tag{7d}$$

System of second-order ordinary differential equations was then transformed into a system of first-order ordinary differential equations by applying state-space approach:

$$\left.\begin{aligned}
&\frac{dy_1}{dt_*} = y_2\\
&\frac{dy_2}{dt_*} = \frac{d^2 w_{*1}}{dt_*^2} = -\frac{v_*}{dx_*} y_4 + \frac{\left(v_*^2 - 1\right)}{dx_*^2}\left(2 * y_1 - y_3\right) - \frac{\dot{v_*}}{2 * dx_*} * y_4 + \cdots\\
&\cdots + \frac{\alpha}{dx_*^4}\left(-6 * y_1 + 4 * y_3 - y_5\right) - k_{1*}^2 y_1 - k_2^2 y_1^3 - \xi_* * y_2 - \frac{\xi_* v_*}{2 * dx_*} * y_4 + f\\
&\frac{dy_3}{dt_*} = y_4\\
&\frac{dy_4}{dt_*} = \frac{d^2 w_{*2}}{dt_*^2} = -\frac{v_*}{dx_*}\left(y_2 - y_6\right) + \frac{\left(v_*^2 - 1\right)}{dx_*^2}\left(-y_1 + 2 * y_3 - y_5\right) - \frac{\dot{v_*}}{2 * dx_*}\left(y_2 - y_6\right) - \cdots\\
&\cdots - \frac{\dot{v_*}}{2 * dx_*} * \left(y_2 - y_6\right) + \frac{\alpha}{dx_*^4}\left(4 * y_1 - 6 * y_3 + 4 * y_5 - y_7\right) - \cdots\\
&\cdots - k_{1*}^2 * y_3 - k_{2*}^2 y_3^3 - \xi_* * y_4 - \frac{\xi_* v_*}{2 * dx_*} * \left(y_2 - y_6\right) + f\\
&\vdots\\
&\vdots\\
&\vdots\\
&\vdots\\
&\vdots\\
&\vdots\\
&\vdots\\
&\frac{dy_{2*(n-1)-1}}{dt_*} = y_{2*(n-1)}\\
&\frac{dy_{2*(n-1)}}{dt_*} = -\frac{v_*}{dx_*} y_{2*(n-1)-2} + \frac{\left(v_*^2 - 1\right)}{dx_*^2}\left(-y_{2*(n-1)-3} + 2 * y_{2*(n-1)-1}\right) - \frac{\dot{v_*}}{2 * dx_*} y_{2*(n-1)-2} - \cdots\\
&\cdots - \frac{\dot{v_*}}{2 * dx_*} * y_{2*(n-1)-2} + \frac{\alpha}{dx_*^4}\left(-y_{2*(n-1)-5} + 4 * y_{2*(n-1)-3} - 6 * y_{2*(n-1)-1}\right) - \cdots\\
&\cdots - k_{1*}^2 y_{2*(n-1)-1} - k_{2*}^2 * y_{2*(n-1)-1}^3 - \xi_* * y_{2*(n-1)} - \frac{\xi_* v_*}{2 * dx_*} * y_{2(n-1)-2} + f
\end{aligned}\right\}, \qquad (8)$$

where

$$w_{*1} = y_1, w_{*2} = y_3, w_{*3} = y_5, \ldots\ldots\ldots, w_{*(n-1)} = y_{2*(n-1)-1}. \quad (9)$$

The initial conditions used in the system of ODE's (8) are

$$y_1(0) = y_3(0) = y_5(0) = \cdots = y_{2^*(n-1)-1}(0) = a(x_*), \quad (10)$$

$$y_2(0) = y_4(0) = y_4(0) = \cdots = y_{2^*(n-1)}(0) = b(x_*), \quad (11)$$

where $a(x_*)$ is the initial transverse deflection and $b(x_*)$ represents the initial transverse speed.

3. Numerical Illustrations and Discussion

Numerical simulations were performed to investigate the effect of different factors such as flexural rigidity of the beam, axial speed of the beam, damping of the foundation, and stiffness of the foundation on the vibration characteristics of the axially traveling beam. Values of the parameters used in the simulations are given in Table 1. Simulations were performed for the initial transverse displacement of "$a(x_*) = 0.01 \sin(x_*)$" and initial transverse speed condition of "$b(x_*) = 0$". An in-house matlab code was used for the simulations.

Table 1: Values of parameters used in simulations.

S. No.	Parameters	Value
1	Length of the beam (L)	1 m
2	Density (ρ)	2710 kg/m^3
3	Axial tension (T)	2.5 kN
4	Width of the beam	0.1 m
5	Thickness of the beam	0.01 m
6	Modulus of elasticity (E)	6.9 * 10^{10} Pa
7	Axial speed of the beam (v)	3 m/s
8	Foundation stiffness (linear, k_1)	800 N/m
9	Foundation stiffness (non-linear, k_2)	200 N/m
10	Foundation damping (ξ)	4 Ns/m^2

First of all, the solution method used in the current analysis (Method 1) was verified by comparing it with the method (Method 2) presented by An and Su [35]. Comparison of the midspan transverse displacement for the case of an axially moving beam having nondimensional speed of "$v_* = 0.1$" and nondimensional flexure rigidity of "$\alpha = 0.2$" is shown in Figure 3.

Response obtained from both solution methods was found to be in good agreement. It was also observed that the solution method used in this study is an efficient approach having long-term stability and excellent convergence.

Figures 4 and 5 show the comparison of transverse vibration as a function of axial speed in an axially moving beam. Transverse displacement at different locations ($x_* = 0.3, 0.5,$ and 0.7) of the beam at different axial speeds, $v_* = 0.1$, 0.3, and 0.5, (equals to 3 m/s, 9 m/s, and 15 m/s, respectively) is presented in Figure 4. Figure 5 plots the trend of the fundamental frequency at different axial speeds. It may be noted from Figure 5 that an increased value of axial speed is accompanied by an exponential decrease in dimensionless fundamental frequency. For the stationary beam, it has a positive value of 7.3, which ultimately reaches zero for an axial speed approaching a value of 1.18 (equals to 35 m/s). The critical speed of the beam can also be interpreted as the speed at which the frequency of the vibration becomes zero; therefore, the critical speed of an axially moving beam under the abovementioned conditions is 35 m/s. Although the fundamental frequency of the vibration is experiencing a downward trend, the amplitude of vibration is unaffected, as evident from Figure 4. Therefore, it is concluded that the critical speed at flexure rigidity of "0.2" is 1.18. Similarly, Figures 6 and 7 show the impact of flexure rigidity on the transverse fluctuations of moving beams resting on the viscoelastic foundation.

Figure 6 shows the effect of flexural rigidity, α, on the midspan transverse displacement of axially moving beam. Based on Figure 6, increasing the flexure rigidity of the beam reduces the transverse displacement (24% decrease in displacement magnitude by increasing α from 0.2 to 0.3, and 47% decrease in magnitude by increasing α from 0.3 to 0.5) but at the same time will increase the nondimensional fundamental frequency from 7.3 to 9.8, as shown in Figure 7. Flexure rigidity also increases the critical speed of the axially moving beam (from 1.18 to 1.65). The higher value of flexure rigidity will somehow increase the internal damping (in terms of lower vibration amplitude) but at the cost of higher fundamental frequency.

Figures 8 and 9 represent the effect of damping of viscoelastic support on the vibration of the beam. Transverse displacement of beam at different locations ($x_* = 0.3, 0.5,$ and 0.5) for different values of the damping factor is given in Figure 9. A sharp decline in the response is observed throughout the beam length with an increase in the damping of the viscoelastic support.

Transverse displacement of midpoint ($x_* = 0.5$) of the beam is plotted for different axial speeds ($v_* = 0.1, 0.3,$ and 0.5) at increasing values of damping factor ($\xi_* = 0.005, 2.0,$ and 3.5) (Figure 9). As shown in Figure 9, with an increase in damping of the support, there is a quite visible reduction in the vibration amplitude. Interestingly, the vibration amplitude reduction is more visible when beam is moving with higher axial speed. A rapid reduction in the vibration magnitude for the case of $\xi_* = 3.5$ is observed when the axial speed is $v_* = 0.5$.

The effect of the support damping on the nondimensional fundamental frequency of the beam is presented in Figure 10. Nondimensional frequency and the critical speed of the beam decrease with an increase in the damping. Nondimensional frequency decreases from 7.3 to 6.4, and critical speed decreases from 1.18 to 1.15 when the damping is increased from 0.05 to 3.5.

Figure 11 presents the variation of nondimensional fundamental frequency as a function of nondimensional axial speed at various magnitudes of support stiffness. An

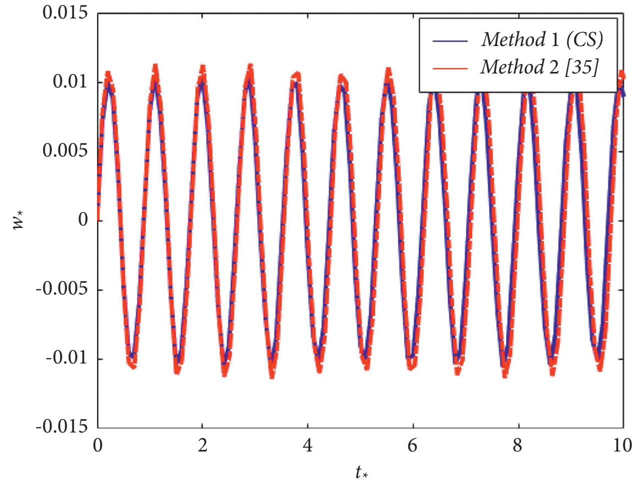


Figure 3: Transverse displacement comparison (current solution and solution from [35]).

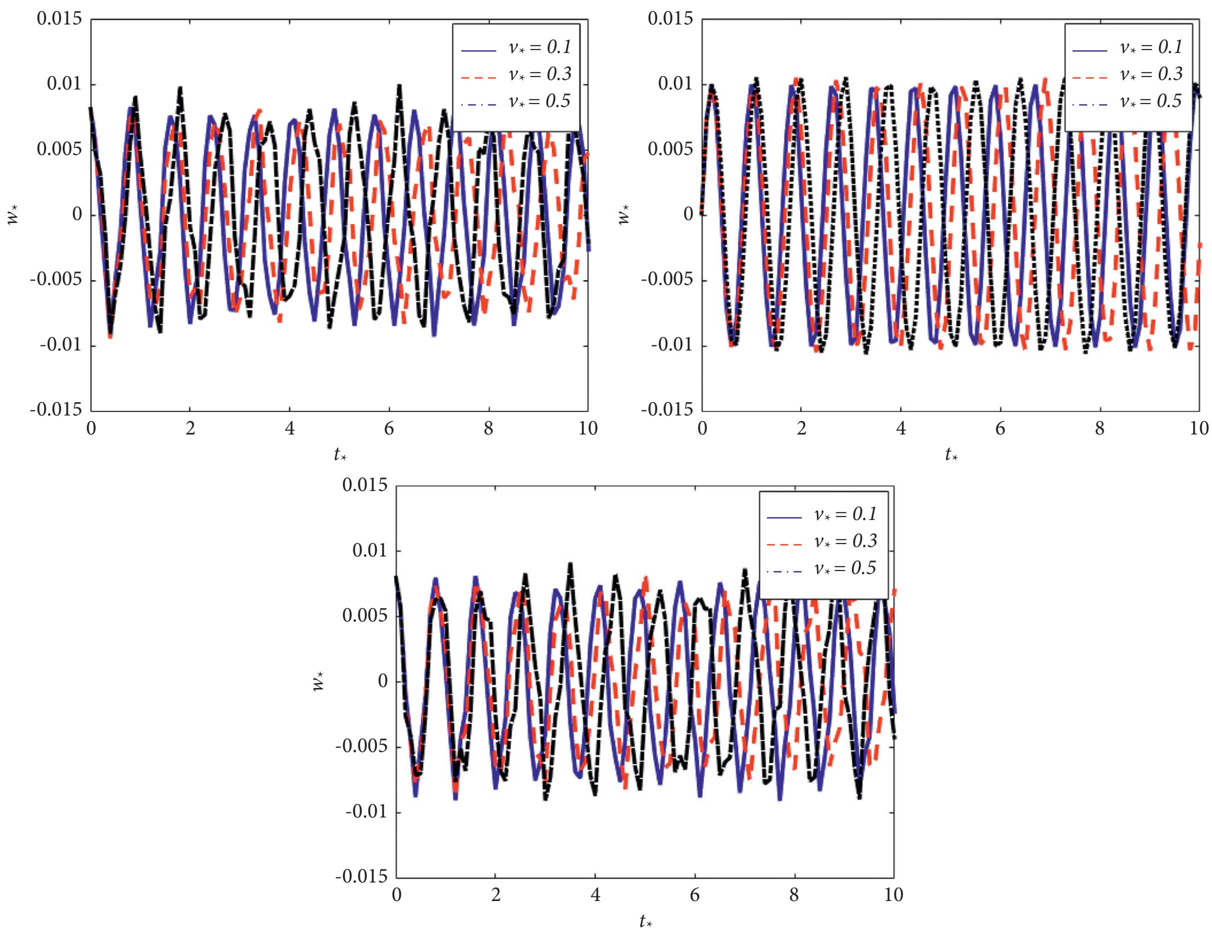


Figure 4: Transverse displacement comparison at different values of axial speed, (a) $x_* = 0.3$ and $\alpha = 0.2$, (b) $x_* = 0.5$ and $\alpha = 0.2$, and (c) $x_* = 0.7$ and $\alpha = 0.2$.

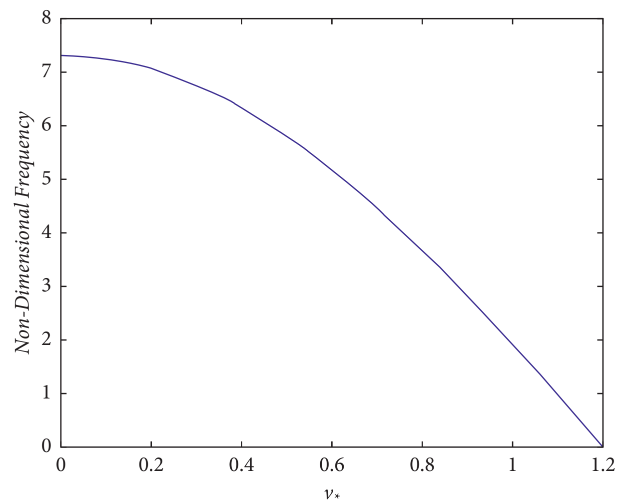


FIGURE 5: Nondimensional frequency vs. nondimensional axial speed.

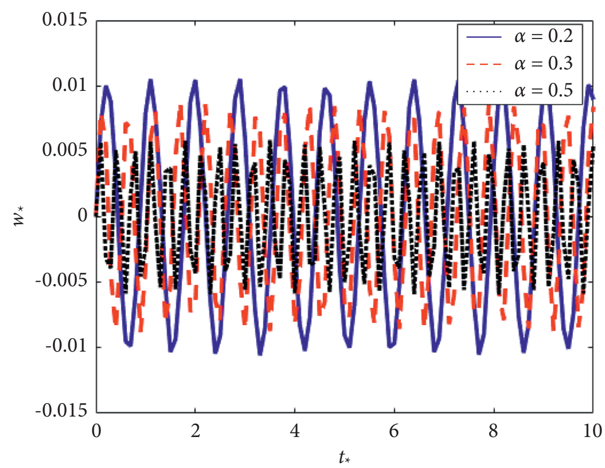


FIGURE 6: Transverse displacement comparison at different values of flexural rigidity ($x_* = 0.5$ and $v_* = 0.1$).

opposite effect to damping on the nondimensional frequency and critical speed is observed in the case of stiffness of the foundation. The frequency of the beam increases from 7.3 to 8.8, and the critical speed increases from 1.18 to 1.43 when the linear stiffness of the beam increases from 0.6 to 1.5 (corresponding to 800 N/m and 6000 N/m, respectively).

A case of a forced vibration was also considered with external excitation force $f(x_*, t_*) = F \sin(\Omega t_*)$. The magnitude of the excitation force is "F," and the frequency of the excitation is Ω. Frequency and displacement of the axially moving beam were measured under various excitation frequencies and are presented in Figure 12.

Figures 12(a)–12(c) show the frequency domain response of the axially moving beam. Starting with lower excitation frequency ($\Omega = 1.0$), frequency domain response, Figure 12(a) indicates that the fundamental frequency of the beam is higher than the excitation frequency ($\omega_2 > \omega_1$). As the excitation frequency is increased, fundamental frequency of the beam and the external excitation frequency match at "$\Omega = 7.4$," and as a result, the response increases exponentially (Figure 12(d)).

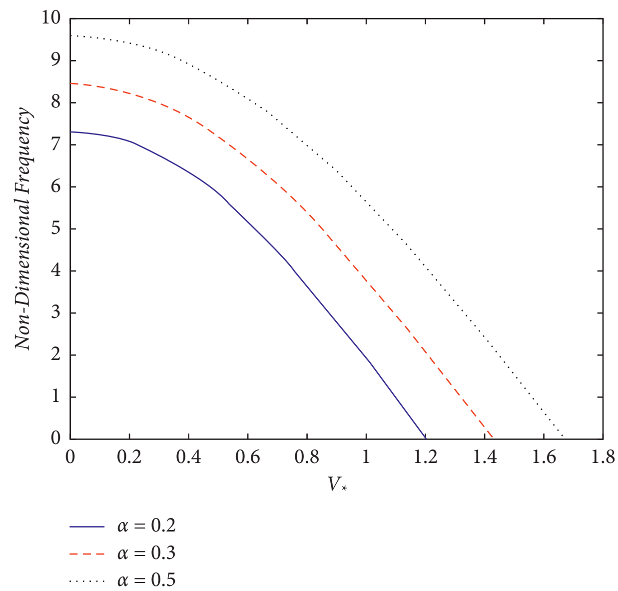


FIGURE 7: Comparison of nondimensional frequencies at different flexural rigidities.

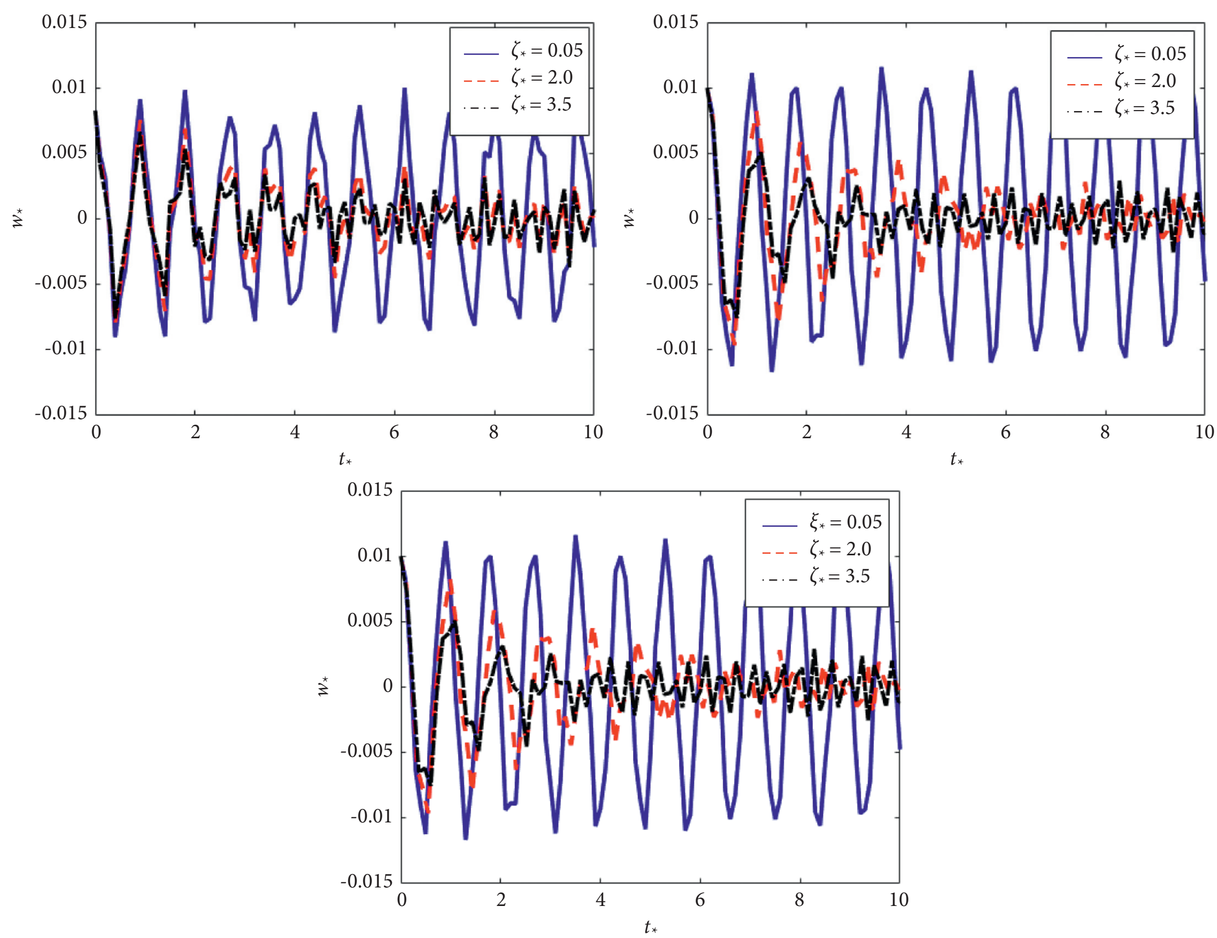


FIGURE 8: Transverse displacement comparison of the beam at different values of the damping factor "ξ_*." (a) $x_* = 0.3$, (b) $x_* = 0.5$, and (c) $x_* = 0.7$.

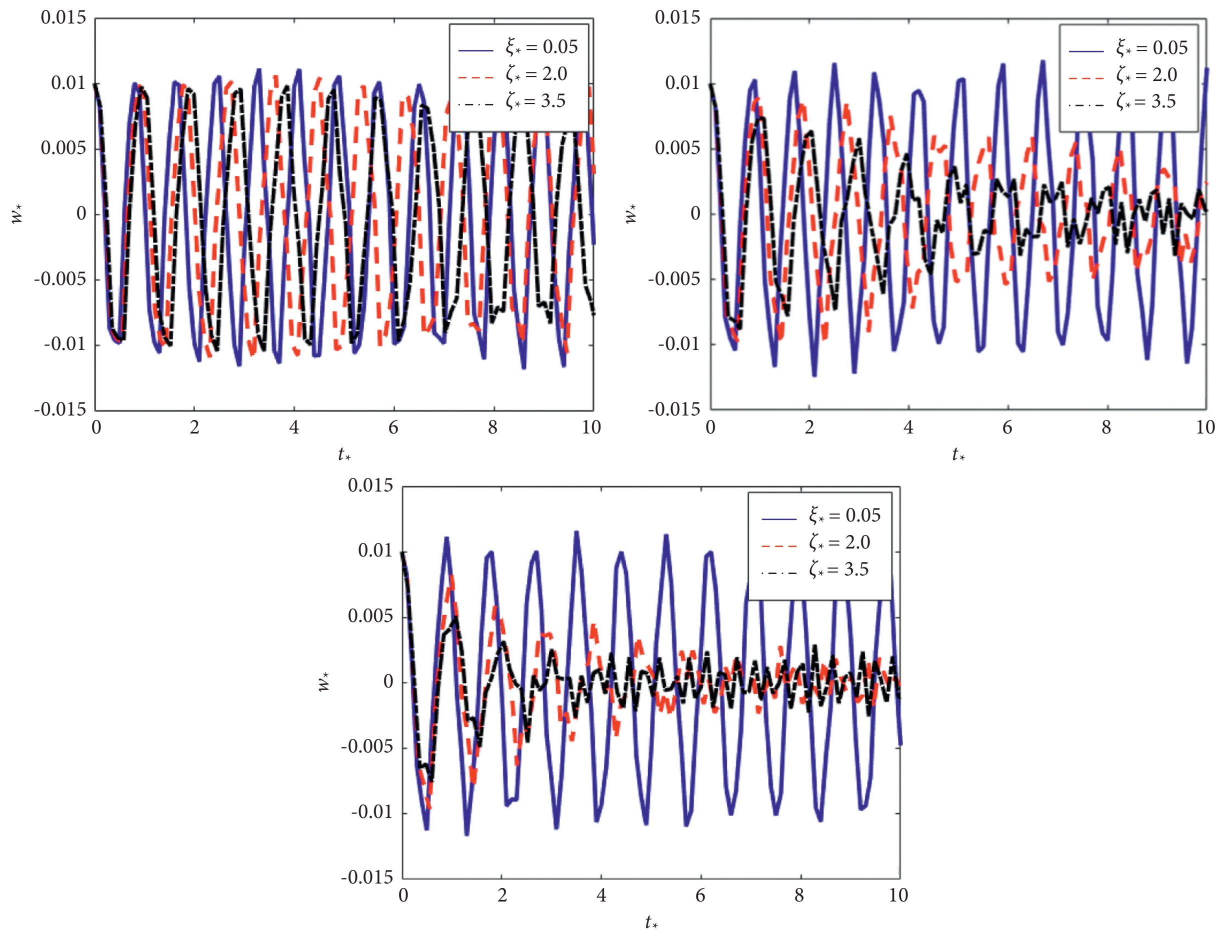


FIGURE 9: Transverse displacement comparison of the beam at different values of the damping factor "ξ_*" and at $x_* = 0.5$: (a) $v_* = 0.1$, (b) $v_* = 0.3$, and (c) $v_* = 0.5$.

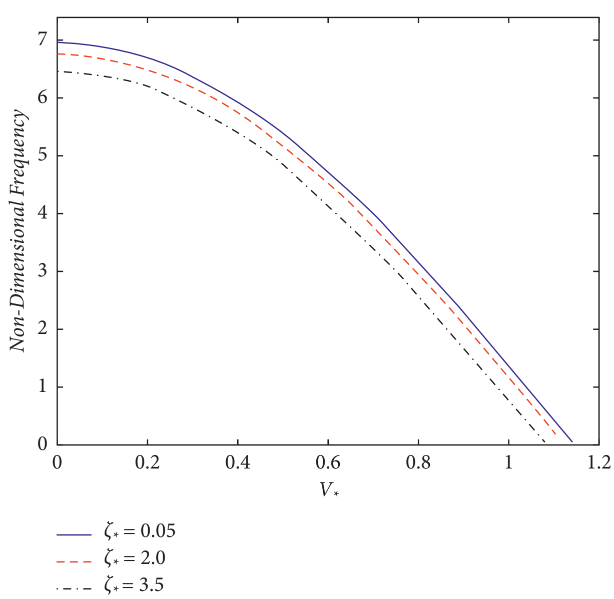


FIGURE 10: Comparison of frequencies at different damping factors '$\xi_* = 0.05, 2.0, 3.5$.'.

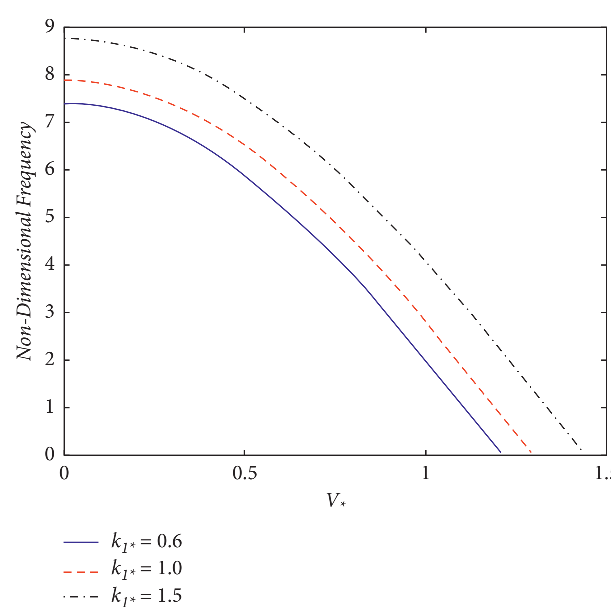
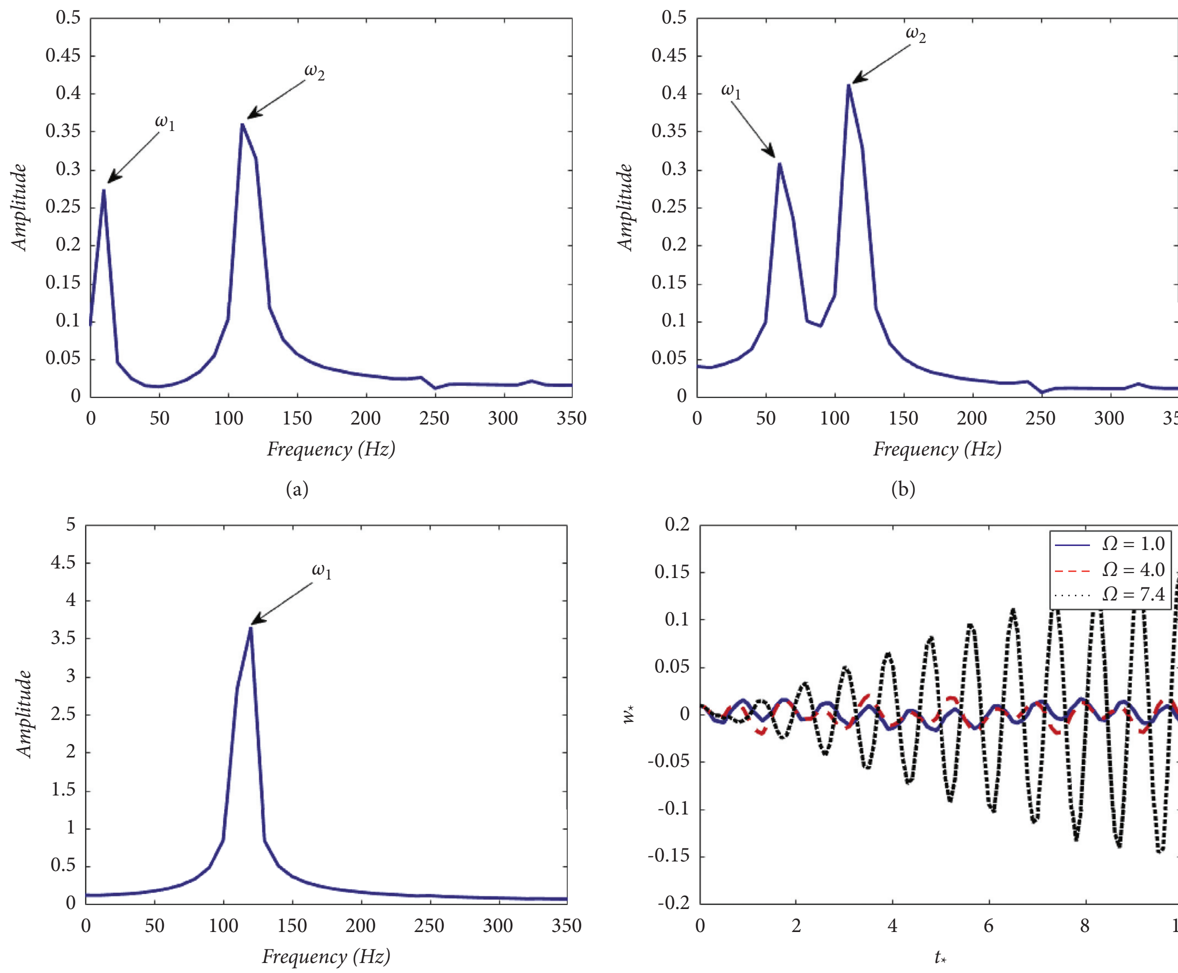


FIGURE 11: Comparison of frequencies at different stiffnesses '$k_{1*} = 0.6, 1.0, 1.5$.'.



(a) (b) (c) (d)

Figure 12: Frequency domain response at (a) $\Omega = 1.0$, (b) $\Omega = 4.0$, (c) $\Omega = 7.4$, and (d) midspan displacement comparison at different values of nondimensional frequency "Ω.".

4. Conclusions

In this study, transverse vibrations in axially traveling beam, supported by the viscoelastic intermediate foundation, was studied. Nonlinear equations of motion were derived using Hamilton's principle. The finite difference method coupled with the state-space approach was presented as an efficient solution technique. The effect of parameters, such as axial translation velocity, flexure rigidities of the beam, damping, and stiffness of the support on the transverse response amplitude and frequencies of the system in subcritical region, was investigated. The major findings of this study may be summarized as follows:

(i) In the absence of effective damping, the dimensionless fundamental frequency was observed to decrease with the increasing value of axial speed. Having the positive value of 7.3 for the case of stationary beam ultimately reaches to zero for an axial speed approaching a value of 1.18. Although, the fundamental frequency of the vibration is decreasing, the amplitude of vibration is unaffected.

(ii) Higher value of flexure rigidity increases the internal damping (in terms of lower vibration amplitude) but at the cost of higher fundamental frequency. A decrease of 24% in the peak magnitude of transverse displacement is observed with increasing α from 0.2 to 0.3 and 47% decrease in the peak magnitude by increasing α from 0.3 to 0.5. In contrast, increasing the flexural rigidity, nondimensional frequency, and critical speed of the beam increases from 7.3 to 9.8 and 1.18 to 1.65, respectively.

(iii) With an increase in the damping of support, there is a visible reduction in the vibration amplitude. Interestingly, the vibration amplitude reduction is found to be more significant when the beam is moving with a higher axial speed. A significant reduction in the peak vibration magnitude is observed for the case of beam having $\xi_* = 3.5$ and axial speed of $v_* = 0.5$. Nondimensional frequency decreases from 7.3 to 6.4, and critical speed decreases from 1.18 to 1.15 when the damping is increased from 0.05 to 3.5.

(iv) Higher value of the foundation stiffness results in the increase of both nondimensional frequencies as well as the critical speed of the beam. Critical speed increases from 1.18 to 1.43, whereas the nondimensional frequency of the beam increases from 7.3 to 8.8 when the linear stiffness of the beam is increased from 0.6 to 1.5.

Abbreviations

A: Cross-sectional area of the beam
v_*: Dimensionless axial speed
E: Modulus of elasticity of beam
$b(x_*)$: Initial transverse speed
T: Tension in beam
ρ: Density of the beam
v: Axial speed of beam
L: Beam length
ξ: Foundation damping
w: Transverse displacement
k_1: Linear stiffness of foundation
α: Dimensionless flexural rigidity
k_2: Nonlinear stiffness of foundation
$a(x_*)$: Initial transverse deflection.

Data Availability

The underlying data supporting the results of this work are found within the article.

Conflicts of Interest

The authors have no conflicts of interest to disclose.

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