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Research Article

Measuring Performance of Ratio-Exponential-Log Type General Class of Estimators Using Two Auxiliary Variables

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In this paper, a ratio-exponential-log type general class of estimators is proposed in estimating the finite population mean using two auxiliary variables when population parameters of the auxiliary variables are known. From the proposed estimator, some special estimators are identified as members of the proposed general class of estimators. The mean square error (MSE) expressions are obtained up to the first order of approximation. This study finds that the proposed general class of estimators outperforms as compared to the conventional mean estimator, usual ratio estimators, exponential-ratio estimators, log-ratio type estimators, and many other competitor regression type estimators. Four real-life applications are used for efficiency comparison.

1. Introduction

In survey sampling, ratio, product, exponential-ratio, log-ratio, and regression type estimators are modified or constructed by many researchers to enhance the precision of the estimators under different sampling designs by using the auxiliary variables. These estimators are commonly used by taking the advantage of correlation coefficient between the study variable and the auxiliary variable(s). Some notable work by the authors includes Olkin [1]; Mohanty [2]; Abu-Dayyeh et al. [3]; Koyuncu and Kadilar [4]; Swain [5]; Lu and Yan [6]; Lu et al. [7]; Sanaullah et al. [8]; Lu [9]; Muneer et al. [10]; Shabbir and Gupta [11]; Akingbade and Okafor [12]; Shabbir et al. [13]; Shabbir et al. [14]; Bhushan et al. [15]; Lone et al. [16]; and Kumari and Thaur [17].

Consider a finite population $\Gamma = \{\Gamma_1, \Gamma_2, \ldots, \Gamma_N\}$ of N units. A sample of size n units is drawn from a population by using simple random sampling without replacement (SRSWOR). Let y_i and (x_i, z_i) be the characteristics of the study variable (Y) and the auxiliary variables (X, Z), respectively. Let $\overline{y} = n^{-1}\sum_{i=1}^{n} y_i$, and $(\overline{x} = n^{-1}\sum_{i=1}^{n} x_i, \overline{z} = n^{-1}\sum_{i=1}^{n} z_i)$, respectively, be the sample means corresponding to the population means $\overline{Y} = N^{-1}\sum_{i=1}^{N} y_i$, and $(\overline{X} = N^{-1}\sum_{i=1}^{N} x_i, \overline{Z} = N^{-1}\sum_{i=1}^{N} z_i)$. To obtain the bias and MSE expressions, we define the following error terms: $\Xi_0 = (\overline{y}/\overline{Y}) - 1$, and $\Xi_1 = (\overline{x}/\overline{X}) - 1$, $\Xi_2 = (\overline{z}/\overline{Z}) - 1$, such that $E(\Xi_i) = 0, (i = 0, 1, 2)$, $E(\Xi_0^2) = \Theta C_y^2$, $E(\Xi_1^2) = \Theta C_x^2$, $E(\Xi_2^2) = \Theta C_z^2$, $E(\Xi_0\Xi_1) = \Theta C_{yx}$, $E(\Xi_0\Xi_2) = \Theta C_{yz}$, and $E(\Xi_1\Xi_2) = \Theta C_{xz}$ where $\Theta = (n^{-1} - N^{-1})$, $C_y = \overline{Y}^{-1} S_y$, $C_x = \overline{X}^{-1} S_x$, $C_z = \overline{Z}^{-1} S_z$, $C_{yx} = \rho_{yx} C_y C_x$, $C_{yz} = \rho_{yz} C_y C_z$, $C_{xz} = \rho_{xz} C_x C_z$, $\rho_{yx} = (S_y S_x)^{-1} S_{yx}$, $\rho_{yz} = (S_y S_z)^{-1} S_{yz}$, $\rho_{xz} = (S_x S_z)^{-1} S_{xz} S_y = \sqrt{(N-1)^{-1}\sum_{i=1}^{N}(y_i - \overline{Y})^2}$, $S_x = \sqrt{(N-1)^{-1}\sum_{i=1}^{N}(x_i - \overline{X})^2}$, $S_z = \sqrt{(N-1)^{-1}\sum_{i=1}^{N}(z_i - \overline{Z})^2}$, $S_{yx} = (N-1)^{-1}\sum_{i=1}^{N}(y_i - \overline{Y})(x_i - \overline{X})$, $S_{yz} = (N-1)^{-1}\sum_{i=1}^{N}(y_i - \overline{Y})(z_i - \overline{Z})$, and $S_{xz} = (N-1)^{-1}\sum_{i=1}^{N}(x_i - \overline{X})(z_i - \overline{Z})$.

2. Some Existing Estimators

Some existing estimators available in the literature are essential to be discussed here.

2.1. Sample Mean Estimator. The usual sample mean estimator and its variance are given as

$$\hat{\bar{Y}}_{(0)} = \bar{y}, \quad (1)$$

$$\operatorname{Var}\left(\hat{\bar{Y}}_{(0)}\right) = \Theta \bar{Y}^2 C_y^2. \quad (2)$$

2.2. Ratio Estimators. The usual ratio estimators when using single and two auxiliary variables are given by

$$\text{(i) } \hat{\bar{Y}}^{(1)}_{(R)} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right),$$

$$\text{(ii) } \hat{\bar{Y}}^{(2)}_{(R)} = \bar{y}\left(\frac{\bar{Z}}{\bar{x}}\right), \quad (3)$$

$$\text{(iii) } \hat{\bar{Y}}^{(3)}_{(R)} = \bar{y}\left(\frac{\bar{X}}{\bar{x}}\right)\left(\frac{\bar{Z}}{\bar{z}}\right).$$

The MSEs of ratio estimators $\hat{\bar{Y}}^{(i)}_{(R)} (i = 1, 2, 3)$ to first order of approximation are given by

$$\text{(i) } \operatorname{MSE}\left(\hat{\bar{Y}}^{(1)}_{(R)}\right) \cong \Theta \bar{Y}^2 \left[C_y^2 + C_x^2 - 2C_{yx}\right], \quad (4)$$

$$\text{(ii) } \operatorname{MSE}\left(\hat{\bar{Y}}^{(2)}_{(R)}\right) \cong \Theta \bar{Y}^2 \left[C_y^2 + C_z^2 - 2C_{yz}\right], \quad (5)$$

$$\text{(iii) } \operatorname{MSE}\left(\hat{\bar{Y}}^{(3)}_{(R)}\right) \cong \Theta \bar{Y}^2 \left[C_y^2 + C_x^2 + C_z^2 - 2\left(C_{yx} + C_{yz}\right) + 2C_{xz}\right]. \quad (6)$$

The ratio estimators $\hat{\bar{Y}}^{(i)}_{(R)} (i = 1, 2, 3)$ are performing better than $\hat{\bar{Y}}_{(0)}$under certain conditions.

2.3. Exponential-Ratio Estimators. The usual exponential-ratio estimators when using single and two auxiliary variables are given by

$$\text{(i) } \hat{\bar{Y}}^{(1)}_{(E)} = \bar{y}\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right),$$

$$\text{(ii) } \hat{\bar{Y}}^{(2)}_{(E)} = \bar{y}\exp\left(\frac{\bar{Z} - \bar{z}}{\bar{Z} + \bar{z}}\right), \quad (7)$$

$$\text{(iii) } \hat{\bar{Y}}^{(3)}_{(E)} = \bar{y}\exp\left(\frac{\bar{X} - \bar{x}}{\bar{X} + \bar{x}}\right)\exp\left(\frac{\bar{Z} - \bar{z}}{\bar{Z} + \bar{z}}\right).$$

The MSEs of exponential-ratio estimators $\hat{\bar{Y}}^{(i)}_{(E)}$ $(i = 1, 2, 3)$ to first order of approximation are given by

$$\text{(i) } \operatorname{MSE}\left(\hat{\bar{Y}}^{(1)}_{(E)}\right) \cong \Theta \bar{Y}^2 \left[C_y^2 + \frac{1}{4}C_x^2 - C_{yx}\right], \quad (8)$$

$$\text{(ii) } \operatorname{MSE}\left(\hat{\bar{Y}}^{(2)}_{(E)}\right) \cong \Theta \bar{Y}^2 \left[C_y^2 + \frac{1}{4}C_z^2 - C_{yz}\right], \quad (9)$$

$$\text{(iii) } \operatorname{MSE}\left(\hat{\bar{Y}}^{(3)}_{(E)}\right) \cong \Theta \bar{Y}^2 \left[C_y^2 + \frac{1}{4}\left(C_x^2 + C_z^2\right) - \left(C_{yx} + C_{yz}\right) + \frac{1}{2}C_{xz}\right]. \quad (10)$$

The exponential-ratio estimators $\hat{\bar{Y}}^{(i)}_{(E)} (i = 1, 2, 3)$ are performing better than $\hat{\bar{Y}}_{(0)}$ and $\hat{\bar{Y}}^{(i)}_{(R)}$ $(i = 1, 2, 3)$under certain conditions.

2.4. Log-Ratio Estimators. Recently many log-type estimators have appeared in the literature in various forms when the logarithmic relationship between the study variable and the auxiliary variables exists.

The usual log-ratio estimators when using single and two auxiliary variables are given by

$$\text{(i) } \hat{\bar{Y}}^{(1)}_{(\log)} = \bar{y}\left[1 + \log\left(\frac{\bar{X}}{\bar{x}}\right)\right],$$

$$\text{(ii) } \hat{\bar{Y}}^{(2)}_{(\log)} = \bar{y}\left[1 + \log\left(\frac{\bar{Z}}{\bar{z}}\right)\right], \quad (11)$$

$$\text{(iii) } \hat{\bar{Y}}^{(3)}_{(\log)} = \bar{y}\left[1 + \log\left(\frac{\bar{X}}{\bar{x}}\right)\right]\left[1 + \log\left(\frac{\bar{Z}}{\bar{z}}\right)\right].$$

The MSEs of log-ratio estimators $\hat{\bar{Y}}^{(i)}_{(\log)}$ $(i = 1, 2, 3)$ to first order of approximation are given by

$$\text{(i) } \operatorname{MSE}\left(\hat{\bar{Y}}^{(1)}_{(\log)}\right) \cong \Theta \bar{Y}^2 \left[C_y^2 + C_x^2 - 2C_{yx}\right], \quad (12)$$

$$\text{(ii) } \operatorname{MSE}\left(\hat{\bar{Y}}^{(2)}_{(\log)}\right) \cong \Theta \bar{Y}^2 \left[C_y^2 + C_z^2 - 2C_{yz}\right], \quad (13)$$

$$\text{(iii) } \operatorname{MSE}\left(\hat{\bar{Y}}^{(3)}_{(\log)}\right) \cong \Theta \bar{Y}^2 \left[C_y^2 + C_x^2 + C_z^2 - 2\left(C_{yx} + C_{yx}\right) + 2C_{xz}\right]. \quad (14)$$

The MSEs of log-ratio estimators $\hat{\bar{Y}}^{(i)}_{(\log)} (i = 1, 2, 3)$ are exactly equal to the MSEs of ratio estimators $\hat{\bar{Y}}^{(i)}_{(R)} (i = 1, 2, 3)$ but their biases are different (not shown here).

2.5. Regression Estimators. The usual regression estimators when using single and two auxiliary variables are given by

$$\text{(i) } \hat{\bar{Y}}^{(1)}_{(\mathrm{Reg})} = \bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right),$$

$$\text{(ii) } \hat{\bar{Y}}^{(2)}_{(\mathrm{Reg})} = \bar{y} + b_{yz}\left(\bar{Z} - \bar{z}\right), \quad (15)$$

$$\text{(iii) } \hat{\bar{Y}}^{(3)}_{(\mathrm{Reg})} = \bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right) + b_{yz}\left(\bar{Z} - \bar{z}\right),$$

where $b_{yx} = (s_{yx}/s_x^2)$ and $b_{yz} = (s_{yz}/s_z^2)$ are the sample regression coefficients.

The MSEs of regression estimators $\hat{\bar{Y}}^{(i)}_{(\text{Reg})}$ $(i = 1, 2, 3)$ are given by

$$\text{(i) MSE}\left(\hat{\bar{Y}}^{(1)}_{(\text{Reg})}\right) \cong \Theta\bar{Y}^2 C_y^2\left(1 - \rho_{yx}^2\right), \quad (16)$$

$$\text{(ii) MSE}\left(\hat{\bar{Y}}^{(2)}_{(\text{Reg})}\right) \cong \Theta\bar{Y}^2 C_y^2\left(1 - \rho_{yz}^2\right), \quad (17)$$

$$\text{(iii) MSE}\left(\hat{\bar{Y}}^{(3)}_{(\text{Reg})}\right) \cong \Theta\bar{Y}^2 C_y^2\left(1 - \rho_{yx}^2 - \rho_{yz}^2 + 2\rho_{yx}\rho_{yz}\rho_{xz}\right). \quad (18)$$

These regression estimators $\hat{\bar{Y}}^{(i)}_{(\text{Reg})}$ $(i = 1, 2, 3)$ are performing better than $\hat{\bar{Y}}_{(0)}$and$\hat{\bar{Y}}^{(i)}_{(j)}$ $(i = 1, 2, 3;\ j = R, E,$ log)under certain conditions.

2.6. Some More Regression Type Estimators. Mohanty [2] suggested the following regression-type estimator:

$$\hat{\bar{Y}}^{(M)}_{(\text{Reg})} = \bar{y} + b_{yx}\left(\bar{X} - \bar{x}\right)\left(\frac{\bar{Z}}{\bar{z}}\right). \quad (19)$$

The MSE of $\hat{\bar{Y}}^{(M)}_{(\text{Reg})}$ is given by

$$\text{MSE}\left(\hat{\bar{Y}}^{(M)}_{(\text{Reg})}\right) \cong \Theta\bar{Y}^2\left[C_y^2\left(1 - \rho_{yx}^2\right) + C_z^2 - 2C_{yz} + 2C_y C_z \rho_{yx}\rho_{xz}\right]. \quad (20)$$

Swain [5] introduced the following regression-type estimator

$$\hat{\bar{Y}}^{(S)}_{(\text{Reg})} = \left[\bar{y} + d_0\left(\bar{X} - \bar{x}\right)\right]\left(\frac{\bar{Z}}{\bar{z}}\right), \quad (21)$$

where $d_0$is constant.

The minimum MSE of $\hat{\bar{Y}}^{(S)}_{(\text{Reg})}$ at optimum value of $d_{0\,(\text{opt})} = (\bar{Y}(C_y\rho_{yx} - C_z\rho_{xz})/\bar{X}C_x)$ is given by

$$\text{MSE}\left(\hat{\bar{Y}}^{(S)}_{(\text{Reg})}\right)_{\min} \cong \Theta\bar{Y}^2 C_y^2\left[\left(C_y^2 + C_z^2 - 2C_{yz}\right) - \left(C_y\rho_{yx} - C_z\rho_{xz}\right)^2\right]. \quad (22)$$

The unbiased regression estimator when using two auxiliary variables is given by

$$\hat{\bar{Y}}^{(U)}_{(\text{Reg})} = \bar{y} + d_1\left(\bar{X} - \bar{x}\right) + d_2\left(\bar{Z} - \bar{z}\right), \quad (23)$$

where $d_1$and $d_2$are constants.

The minimum MSE of $\hat{\bar{Y}}^{(U)}_{(\text{Reg})}$ at optimum values of $d_{1\,(\text{opt})} = (S_y(\rho_{yx} - \rho_{yz}\rho_{xz})/S_x(1 - \rho_{xz}^2))$ and $d_{2\,(\text{opt})} = (S_y(\rho_{yz} - \rho_{yx}\rho_{xz})/S_z(1 - \rho_{xz}^2))$ is given by

$$\text{MSE}\left(\hat{\bar{Y}}^{(U)}_{(\text{Reg})}\right)_{\min} \cong \Theta\bar{Y}^2 C_y^2\left(1 - R_{y.xz}^2\right), \quad (24)$$

where $R_{y\cdot xz}^2 = ((\rho_{yx}^2 + \rho_{yz}^2 - 2\rho_{yx}\rho_{yz}\rho_{xz})/(1 - \rho_{xz}^2))$ is the multiple correlation coefficient.

3. Proposed General Class of Estimators

We propose a ratio-exponential-log type general class of estimators in estimating the finite population mean using two auxiliary variables when some parameters of the auxiliary variables are known. We also obtain different special estimators as members of the general class of estimators which are useful in different real-life situations. The proposed estimator is the combination of three special estimators including ratio, exponential-ratio, and log-ratio by using the linear transformation as

$$\hat{\bar{Y}}^{(\alpha_1,\alpha_2,\alpha_3,\gamma_1,\gamma_2,\gamma_3)}_{(M)} = \bar{y}\left[M_1\left\{\frac{\bar{X}^*}{\bar{x}^*}\right\}^{\alpha_1}\left\{\exp\left(\frac{\bar{X}^* - \bar{x}^*}{\bar{X}^* + \bar{x}^*}\right)\right\}^{\alpha_2}\left\{1 + \log\left(\frac{\bar{X}^*}{\bar{x}^*}\right)\right\}^{\alpha_3} + M_2\left\{\frac{\bar{Z}^*}{\bar{z}^*}\right\}^{\gamma_1}\left\{\exp\left(\frac{\bar{Z}^* - \bar{z}^*}{\bar{Z}^* + \bar{z}^*}\right)\right\}^{\gamma_2}\left\{1 + \log\left(\frac{\bar{Z}^*}{\bar{z}^*}\right)\right\}^{\gamma_3}\right], \quad (25)$$

where $M_i\,(i = 1, 2)$ are constants, whose values are to be determined; α_i $(i = 1, 2, 3)$ and γ_i $(i = 1, 2, 3)$ are scaler quantities; and $\bar{x}^* = a\bar{x} + b$, $\bar{X}^* = a\bar{X} + b$, $\bar{z}^* = c\bar{z} + d$, and $\bar{Z}^* = c\bar{Z} + d$. Here a, b, c, d are the known population parameters of the auxiliary variables which may be coefficients of variation (C_x, C_z), coefficients of kurtosis (β_{2x}, β_{2z}) and correlation coefficients (ρ_{yx}, ρ_{yz}).

Solving (25) in terms of errors to the first order of approximation, we have

$$\hat{\bar{Y}}^{(\alpha_1,\alpha_2,\alpha_3,\gamma_1,\gamma_2,\gamma_3)}_{(M)} - \bar{Y} \cong \bar{Y}\left[M_1\left\{1 + \Xi_0 - \Delta_1 g_1 \Xi_1 - \Delta_1 g_1 \Xi_0 \Xi_1 + \Delta_2 g_1^2 \Xi_1^2\right\} + M_2\left\{1 + \Xi_0 - \Omega_1 g_2 \Xi_2 - \Omega_1 g_2 \Xi_0 \Xi_2 + \Delta_2 g_2^2 \Xi_2^2\right\} - 1\right], \quad (26)$$

where

$$g_1 = \frac{a\overline{X} + b}{a\overline{x} + b},$$
$$g_2 = \frac{c\overline{Z} + d}{c\overline{z} + d},$$
$$\Delta_1 = \alpha_1 + \frac{1}{2}\alpha_2 + \alpha_3,$$
$$\Delta_2 = \alpha_1\alpha_3 + \frac{1}{2}\alpha_2\alpha_3 + \frac{1}{2}\alpha_1(\alpha_1 + 1) + \frac{1}{8}\alpha_2(\alpha_2 + 2) + \frac{1}{2}\alpha_1\alpha_2 + \frac{1}{2}\alpha_3^2,$$
$$\Omega_1 = \gamma_1 + \frac{1}{2}\gamma_2 + \gamma_3,$$
$$\Omega_2 = \gamma_1\gamma_3 + \frac{1}{2}\gamma_2\gamma_3 + \frac{1}{2}\gamma_1(\gamma_1 + 1) + \frac{1}{8}\gamma_2(\gamma_2 + 2) + \frac{1}{2}\gamma_1\gamma_2 + \frac{1}{2}\gamma_3^2. \quad (27)$$

The bias of $\widehat{\overline{Y}}_{(M)}^{(\alpha_1,\alpha_2,\alpha_3,\gamma_1,\gamma_2,\gamma_3)}$ to the first order of approximation is given by

$$\begin{aligned}\text{Bias}\left(\widehat{\overline{Y}}_{(M)}^{(\alpha_1,\alpha_2,\alpha_3,\gamma_1,\gamma_2,\gamma_3)}\right) \cong \overline{Y}\big[&M_1\big\{1 + \Theta\big(\Delta_2 g_1^2 C_x^2 - \Delta_1 g_1 C_{yx}\big)\big\}\\ &+M_2\big\{1 + \Theta\big(\Omega_2 g_2^2 C_z^2 - \Omega_1 g_2 C_{yz}\big)\big\} - 1\big].\end{aligned} \quad (28)$$

The MSE of $\widehat{\overline{Y}}_{(M)}^{(\alpha_1,\alpha_2,\alpha_3,\gamma_1,\gamma_2,\gamma_3)}$ to the first order of approximation is given by

$$\begin{aligned}\text{MSE}\left(\widehat{\overline{Y}}_{(M)}^{(\alpha_1,\alpha_2,\alpha_3,\gamma_1,\gamma_2,\gamma_3)}\right) \cong \overline{Y}^2 E\big[&M_1\big\{1 + \Xi_0 - \Delta_1 g_1 \Xi_1 - \Delta_1 g_1 \Xi_0 \Xi_1 + \Delta_2 g_1^2 \Xi_1^2\big\}\\ &+M_2\big\{1 + \Xi_0 - \Omega_1 g_2 \Xi_2 - \Omega_1 g_2 \Xi_0 \Xi_2 + \Delta_2 g_2^2 \Xi_2^2\big\} - 1\big]^2.\end{aligned} \quad (29)$$

Solving (29), we get

$$\text{MSE}\left(\widehat{\overline{Y}}_{(M)}^{(\alpha_1,\alpha_2,\alpha_3,\gamma_1,\gamma_2,\gamma_3)}\right) \cong \overline{Y}^2\left[1 + M_1^2 A_m + M_2^2 B_m - 2M_1 C_m - 2M_2 D_m + 2M_1 M_2 E_m\right], \quad (30)$$

where

$$\begin{aligned}A_m &= 1 + \Theta\big\{C_y^2\big(\Delta_1^2 + 2\Delta_2\big) g_1^2 C_x^2 - 4\Delta_1 g_1 C_{yx}\big\},\\ B_m &= 1 + \Theta\big\{C_y^2\big(\Omega_1^2 + 2\Omega_2\big) g_2^2 C_z^2 - 4\Omega_1 g_2 C_{yz}\big\},\\ C_m &= 1 + \Theta\big\{\Delta_2 g_1^2 C_x^2 - \Delta_1 g_1 C_{yx}\big\},\\ D_m &= 1 + \Theta\big\{\Omega_2 g_2^2 C_z^2 - \Omega_1 g_2 C_{yz}\big\},\\ E_m &= 1 + \Theta\big\{C_y^2 + \Delta_2 g_1^2 C_x^2 + \Omega_2 g_2^2 C_z^2 - 2\Delta_1 g_1 C_{yx} - 2\Omega_1 g_2 C_{yz} + \Delta_1 \Omega_1 g_1 g_2 C_{xz}\big\}.\end{aligned} \quad (31)$$

Solving (30), the optimum values $M_i\,(i = 1, 2)$ are given as

$$M_{1\,(\text{opt})} = \frac{B_m C_m - D_m E_m}{A_m B_m - E_m^2},$$
$$M_{2\,(\text{opt})} = \frac{A_m D_m - C_m E_m}{A_m B_m - E_m^2}. \tag{32}$$

The minimum MSE of $\widehat{\overline{Y}}_{(M)}^{(\alpha_1,\alpha_2,\alpha_3,\gamma_1,\gamma_2,\gamma_3)}$ to the first order of approximation is given by

$$\text{MSE}\left(\widehat{\overline{Y}}_{(M)}^{(\alpha_1,\alpha_2,\alpha_3,\gamma_1,\gamma_2,\gamma_3)}\right)_{\min} \cong \overline{Y}^2\left[1 - \frac{A_m D_m^2 + B_m C_m^2 - 2C_m D_m E_m}{A_m B_m - E_m^2}\right]. \tag{33}$$

Some special estimators as members of the proposed general class of estimators are given by

(i) Putting $\alpha_1 = \gamma_1 = 1, \alpha_2 = \alpha_3 = \gamma_2 = \gamma_3 = 0$ in (25), we get

$$\widehat{\overline{Y}}_{(M)}^{(1,0,0,1,0,0)} = \overline{y}\left[M_1\left\{\frac{\overline{X}^*}{\overline{x}^*}\right\} + M_2\left\{\frac{\overline{Z}^*}{\overline{z}^*}\right\}\right]. \tag{34}$$

(ii) Putting $\alpha_2 = \gamma_2 = 1, \alpha_1 = \alpha_3 = \gamma_1 = \gamma_3 = 0$ in (25), we get

$$\widehat{\overline{Y}}_{(M)}^{(0,1,0,0,1,0)} = \overline{y}\left[M_1\left\{\exp\left(\frac{\overline{X}^* - \overline{x}^*}{\overline{X}^* + \overline{x}^*}\right)\right\} + M_2\left\{\exp\left(\frac{\overline{Z}^* - \overline{z}^*}{\overline{Z}^* + \overline{z}^*}\right)\right\}\right]. \tag{35}$$

(iii) Putting $\alpha_3 = \gamma_3 = 1, \alpha_1 = \alpha_2 = \gamma_1 = \gamma_2 = 0$ in (25), we get

$$\widehat{\overline{Y}}_{(M)}^{(0,0,1,0,0,1)} = \overline{y}\left[M_1\left\{1 + \log\left(\frac{\overline{X}^*}{\overline{x}^*}\right)\right\} + M_2\left\{1 + \log\left(\frac{\overline{Z}^*}{\overline{z}^*}\right)\right\}\right]. \tag{36}$$

(iv) Putting $\alpha_1 = \alpha_2 = \gamma_2 = 1, \alpha_3 = \gamma_1 = \gamma_3 = 0$ in (25), we get

$$\widehat{\overline{Y}}_{(M)}^{(1,1,0,0,1,0)} = \overline{y}\left[M_1\left\{\frac{\overline{X}^*}{\overline{x}^*}\right\}\left\{\exp\left(\frac{\overline{X}^* - \overline{x}^*}{\overline{X}^* + \overline{x}^*}\right)\right\} + M_2\left\{\exp\left(\frac{\overline{Z}^* - \overline{z}^*}{\overline{Z}^* + \overline{z}^*}\right)\right\}\right]. \tag{37}$$

(v) Putting $\alpha_1 = \alpha_3 = \gamma_2 = 1, \alpha_2 = \gamma_1 = \gamma_3 = 0$ in (25), we get

$$\widehat{\overline{Y}}_{(M)}^{(1,0,1,0,1,0)} = \overline{y}\left[M_1\left\{\frac{\overline{X}^*}{\overline{x}^*}\right\}\left\{1 + \log\left(\frac{\overline{X}^*}{\overline{x}^*}\right)\right\} + M_2\left\{\exp\left(\frac{\overline{Z}^* - \overline{z}^*}{\overline{Z}^* + \overline{z}^*}\right)\right\}\right]. \tag{38}$$

(vi) Putting $\alpha_2 = \alpha_3 = \gamma_2 = 1, \alpha_1 = \gamma_1 = \gamma_3 = 0$ in (25), we get

$$\hat{\hat{Y}}_{(M)}^{(0,1,1,0,1,0)} = \overline{y}\left[M_1\left\{\exp\left(\frac{\overline{X}^* - \overline{x}^*}{\overline{X}^* + \overline{x}^*}\right)\right\}\left\{1 + \log\left(\frac{\overline{X}^*}{\overline{x}^*}\right)\right\} + M_2\left\{\exp\left(\frac{\overline{Z}^* - \overline{z}^*}{\overline{Z}^* + \overline{z}^*}\right)\right\}\right]. \tag{39}$$

(vii) Putting $\alpha_1 = \alpha_2 = \alpha_3 = \gamma_2 = 1, \gamma_1 = \gamma_3 = 0$ in (25), we get

$$\hat{\hat{Y}}_{(M)}^{(1,1,1,0,1,0)} = \overline{y}\left[M_1\left\{\frac{\overline{X}^*}{\overline{x}^*}\right\}\left\{\exp\left(\frac{\overline{X}^* - \overline{x}^*}{\overline{X}^* + \overline{x}^*}\right)\right\}\left\{1 + \log\left(\frac{\overline{X}^*}{\overline{x}^*}\right)\right\} + M_2\left\{\exp\left(\frac{\overline{Z}^* - \overline{z}^*}{\overline{Z}^* + \overline{z}^*}\right)\right\}\right]. \tag{40}$$

(viii) Putting $\alpha_1 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_2 = \alpha_3 = 0$ in (25), we get

$$\hat{\hat{Y}}_{(M)}^{(1,0,0,1,1,1)} = \overline{y}\left[M_1\left\{\frac{\overline{X}^*}{\overline{x}^*}\right\} + M_2\left\{\frac{\overline{Z}^*}{\overline{z}^*}\right\}\left\{\exp\left(\frac{\overline{Z}^* - \overline{z}^*}{\overline{Z}^* + \overline{z}^*}\right)\right\}\left\{1 + \log\left(\frac{\overline{Z}^*}{\overline{z}^*}\right)\right\}\right]. \tag{41}$$

(i) Putting $\alpha_2 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_1 = \alpha_3 = 0$ in (25), we get

$$\hat{\hat{Y}}_{(M)}^{(0,1,0,1,1,1)} = \overline{y}\left[M_1\left\{\exp\left(\frac{\overline{X}^* - \overline{x}^*}{\overline{X}^* + \overline{x}^*}\right)\right\} + M_2\left\{\frac{\overline{Z}^*}{\overline{z}^*}\right\}\left\{\exp\left(\frac{\overline{Z}^* - \overline{z}^*}{\overline{Z}^* + \overline{z}^*}\right)\right\}\left\{1 + \log\left(\frac{\overline{Z}^*}{\overline{z}^*}\right)\right\}\right]. \tag{42}$$

(x) Putting $\alpha_3 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_1 = \alpha_2 = 0$ in (25), we get

$$\hat{\hat{Y}}_{(M)}^{(0,0,1,1,1,1)} = \overline{y}\left[M_1\left\{1 + \log\left(\frac{\overline{X}^*}{\overline{x}^*}\right)\right\} + M_2\left\{\frac{\overline{Z}^*}{\overline{z}^*}\right\}\left\{\exp\left(\frac{\overline{Z}^* - \overline{z}^*}{\overline{Z}^* + \overline{z}^*}\right)\right\}\left\{1 + \log\left(\frac{\overline{Z}^*}{\overline{z}^*}\right)\right\}\right]. \tag{43}$$

Note: We can generate more sub-classes of the proposed general class of estimators by using different combinations.

4. Numerical Example

We use the following four real data sets for a numerical study.

Population 1 (see [19]):

$$\begin{aligned} Y &= \text{Area under wheat in acres in 1974},\\ X &= \text{Area under wheat in acres in 1971},\\ Z &= \text{Area under wheat in acres in 1973},\\ N &= 34,\\ n &= 20,\\ \overline{Y} &= 856.412,\\ \overline{X} &= 208.882,\\ \overline{Z} &= 199.441,\\ C_y &= 0.8561,\\ C_x &= 0.721,\\ C_z &= 0.753,\\ \rho_{yx} &= 0.449,\\ \rho_{yz} &= 0.443,\\ \rho_{xz} &= 0.980,\\ \beta_{2x} &= 2.910,\\ \beta_{2z} &= 3.732. \end{aligned} \tag{44}$$

Population 2 (see [18]):

$$\begin{aligned} Y &= \text{Output of the factory},\\ X &= \text{Number of workers},\\ Z &= \text{Fixed capital},\\ N &= 80,\\ n &= 20,\\ \overline{Y} &= 5182.637,\\ \overline{X} &= 285.125,\\ \overline{Z} &= 1126.463,\\ C_y &= 0.354,\\ C_x &= 0.948,\\ C_z &= 0.751,\\ \rho_{yx} &= 0.915,\\ \rho_{yz} &= 0.941,\\ \rho_{xz} &= 0.988,\\ \beta_{2x} &= 0.698,\\ \beta_{2z} &= 1.050. \end{aligned} \tag{45}$$

Population 3 (Punjab Development Statistics (2019)): This data is taken from Punjab development of statistics of 36 districts of Punjab, Pakistan during 2018.

$$\begin{aligned} Y &= \text{Number of reported crimes by hurt},\\ X &= \text{Number of reported crimes by murdered},\\ Z &= \text{Number of reported crimes by kidnapped},\\ N &= 36,\\ n &= 10,\\ \overline{Y} &= 421.9722,\\ \overline{X} &= 112.3889,\\ \overline{Z} &= 412.2222,\\ C_y &= 0.5718,\\ C_x &= 0.8336,\\ C_z &= 1.4229,\\ \rho_{yx} &= 0.7786,\\ \rho_{yz} &= 0.6900,\\ \rho_{xz} &= 0.8483,\\ \beta_{2x} &= 7.5508,\\ \beta_{2z} &= 27.5415. \end{aligned} \tag{46}$$

Population 4 (see [19]):

This data are based on 69 villages of Doraha development bloc of Punjab, India.

$$\begin{aligned} Y &= \text{Number of tube wells},\\ X &= \text{Number of tractors},\\ Z &= \text{Net irrigated area in hectares},\\ N &= 69,\\ n &= 10,\\ \overline{Y} &= 135.2609,\\ \overline{X} &= 21.2319,\\ \overline{Z} &= 345.7536,\\ C_y &= 0.8422,\\ C_x &= 0.7969,\\ C_z &= 0.8478,\\ \rho_{yx} &= 0.9118,\\ \rho_{yz} &= 0.9224,\\ \rho_{xz} &= 0.9007,\\ \beta_{2x} &= 3.7653,\\ \beta_{2z} &= 7.2159. \end{aligned} \tag{47}$$

The results based on Populations 1–4 are given in Tables 1–11. We use the following expression to obtain the percent relative efficiency (PRE) as

$$\mathrm{PRE} = \frac{\mathrm{Var}\left(\hat{\bar{\bar{Y}}}_{(0)}\right)}{\mathrm{MSE}\,(.)} \times 100, \tag{48}$$

where $(.) = \hat{\bar{\bar{Y}}}_{(0)}, \hat{\bar{\bar{Y}}}^{(i)}_{(R)}, \hat{\bar{\bar{Y}}}^{(i)}_{(E)}, \hat{\bar{\bar{Y}}}^{(i)}_{(\log)}, \hat{\bar{\bar{Y}}}^{(i)}_{(\mathrm{reg})}, \hat{\bar{\bar{Y}}}^{(j)}_{(\mathrm{reg})}$ $(i = 1, 2, 3;$ $j = M, S, U)$.

In Table 1, we observed the following:

(i) The ratio and log-ratio estimators $(\hat{\bar{\bar{Y}}}^{(1)}_{(R)}, \hat{\bar{\bar{Y}}}^{(1)}_{(\log)})$ in population 2, $(\hat{\bar{\bar{Y}}}^{(2)}_{(R)}, \hat{\bar{\bar{Y}}}^{(2)}_{(\log)})$ in populations 2 and 3, and $(\hat{\bar{\bar{Y}}}^{(3)}_{(R)}, \hat{\bar{\bar{Y}}}^{(3)}_{(\log)})$ in all four populations are performing poorly as compared to $\hat{\bar{\bar{Y}}}_{(0)}$.

(ii) The exponential-ratio estimator $\hat{\bar{\bar{Y}}}^{(3)}_{(E)}$ in Populations 2 and 3 is not performing good.

(iii) Mohanty [2] regression estimator $\hat{\bar{\bar{Y}}}^{(M)}_{(\mathrm{Reg})}$ in populations 1–3 and Swain [5] estimator $\hat{\bar{\bar{Y}}}^{(S)}_{(\mathrm{Reg})}$ in Population 3 are not efficient as compared to $\hat{\bar{\bar{Y}}}_{(0)}$.

(iv) Among all the estimators above discussed, the performance of $\hat{\bar{\bar{Y}}}^{(U)}_{(\mathrm{Reg})}$ is the best.

5. Comparison of Estimators

Now we compare the proposed class of estimators with other existing estimators.

Condition 1. By (2) and (33), $\mathrm{MSE}(\hat{\bar{\bar{Y}}}^{(\alpha_1,\alpha_2,\alpha_3,\gamma_1,\gamma_2,\gamma_3)}_{(M)})_{\min} <$ $\mathrm{Var}(\hat{\bar{\bar{Y}}}_{(0)})$ if

$$\left[\Theta C_y^2 + \frac{O_1}{O_2} - 1\right] > 0, \tag{49}$$

where $O_1 = A_m D_m^2 + B_m C_m^2 - 2C_m D_m E_m$ and $O_2 = A_m B_m - E_m^2$.

Condition 2. By ((4)–(6)) or ((12)–(14)) and (33), $\mathrm{MSE}(\hat{\bar{\bar{Y}}}^{(\alpha_1,\alpha_2,\alpha_3,\gamma_1,\gamma_2,\gamma_3)}_{(M)})_{\min} < [\mathrm{MSE}(\hat{\bar{\bar{Y}}}^{(i)}_{(R)}), \mathrm{MSE}(\hat{\bar{\bar{Y}}}^{(i)}_{(\log)})]$ $(i = 1, 2, 3)$ if

$$\left[\Theta\left\{C_y^2 + C_x^2 - 2C_{yx}\right\} + \frac{O_1}{O_2} - 1\right] > 0,$$

$$\left[\Theta\left\{C_y^2 + C_z^2 - 2C_{yz}\right\} + \frac{O_1}{O_2} - 1\right] > 0,$$

$$\left[\Theta\left\{C_y^2 + C_x^2 + C_z^2 - 2\left(C_{yx} + C_{yz}\right) + 2C_{xz}\right\} + \frac{O_1}{O_2} - 1\right] > 0. \tag{50}$$

Condition 3. By ((8)–(10)) and (33), $\mathrm{MSE}(\hat{\bar{\bar{Y}}}^{(\alpha_1,\alpha_2,\alpha_3,\gamma_1,\gamma_2,\gamma_3)}_{(M)})_{\min} < \mathrm{MSE}(\hat{\bar{\bar{Y}}}^{(i)}_{(E)})$ $(i = 1, 2, 3)$ if

Table 1: PRE of different estimators with respect $\hat{\bar{\bar{Y}}}_{(0)}$ when $(a = c = 1, b = d = 0)$.

Estimator	Pop.1	Pop. 2	Pop. 3	Pop. 4
$\hat{\bar{\bar{Y}}}_{(0)}$	100.00	100.00	100.00	100.00
$\hat{\bar{\bar{Y}}}^{(1)}_{(R)}, \hat{\bar{\bar{Y}}}^{(1)}_{(\log)}$	104.932	*	116.936	588.911
$\hat{\bar{\bar{Y}}}^{(2)}_{(R)}, \hat{\bar{\bar{Y}}}^{(2)}_{(\log)}$	100.926	*	*	639.893
$\hat{\bar{\bar{Y}}}^{(3)}_{(R)}, \hat{\bar{\bar{Y}}}^{(3)}_{(\log)}$	*	*	*	*
$\hat{\bar{\bar{Y}}}^{(1)}_{(E)}$	125.129	291.940	252.366	276.952
$\hat{\bar{\bar{Y}}}^{(2)}_{(E)}$	124.688	776.064	120.327	307.879
$\hat{\bar{\bar{Y}}}^{(3)}_{(E)}$	103.718	*	*	870.810
$\hat{\bar{\bar{Y}}}^{(1)}_{(\mathrm{Reg})}$	125.251	614.345	253.948	593.047
$\hat{\bar{\bar{Y}}}^{(2)}_{(\mathrm{Reg})}$	124.692	873.218	190.876	670.339
$\hat{\bar{\bar{Y}}}^{(3)}_{(\mathrm{Reg})}$	100.807	102.181	120.605	120.069
$\hat{\bar{\bar{Y}}}^{(M)}_{(\mathrm{Reg})}$	*	*	*	102.214
$\hat{\bar{\bar{Y}}}^{(S)}_{(\mathrm{Reg})}$	121.910	883.094	*	640.000
$\hat{\bar{\bar{Y}}}^{(U)}_{(\mathrm{Reg})}$	125.349	948.307	255.867	873.966

"*" indicates that the estimators are not efficient as compared to the usual mean estimator.

Table 2: PRE of proposed estimator when $(\alpha_1 = \gamma_1 = 1, \alpha_2 = \alpha_3 = \gamma_2 = \gamma_3 = 0)$.

a	b	c	d	Pop. 1	Pop. 2	Pop. 3	Pop. 4
1	0	1	0	107.697	186.585	132.429	886.815
1	C_x	1	C_z	107.934	184.610	134.467	885.501
β_{2x}	C_x	β_{2z}	C_z	107.780	183.288	132.707	886.865
C_x	β_{2x}	C_z	β_{2z}	107.780	183.288	132.707	886.865
1	ρ_{yx}	1	ρ_{yz}	107.780	183.288	132.707	886.865
1	β_{2x}	1	β_{2z}	108.628	185.787	149.748	852.370
C_x	ρ_{yx}	C_z	ρ_{yz}	107.903	185.219	134.781	883.828
ρ_{yx}	C_x	ρ_{yz}	C_z	108.220	184.398	135.032	885.176
β_{2x}	ρ_{yz}	β_{2z}	ρ_{yz}	107.749	183.672	132.690	886.850
ρ_{yx}	β_{2x}	ρ_{yz}	β_{2z}	109.745	185.671	154.632	846.972

$$\left[\Theta\left\{C_y^2 + \frac{1}{4}C_x^2 - C_{yx}\right\} + \frac{O_1}{O_2} - 1\right] > 0,$$

$$\left[\Theta\left\{C_y^2 + \frac{1}{4}C_z^2 - C_{yz}\right\} + \frac{O_1}{O_2} - 1\right] > 0,$$

$$\left[\Theta\left\{C_y^2 + \frac{1}{4}\left(C_x^2 + C_z^2\right) - \left(C_{yx} + C_{yz}\right) + \frac{1}{2}C_{xz}\right\} + \frac{O_1}{O_2} - 1\right] > 0. \tag{51}$$

Condition 4. By ((16)–(18)) and (33), $\mathrm{MSE}(\hat{\bar{\bar{Y}}}^{(\alpha_1,\alpha_2,\alpha_3,\gamma_1,\gamma_2,\gamma_3)}_{(M)})_{\min} < \mathrm{MSE}(\hat{\bar{\bar{Y}}}^{(i)}_{(\mathrm{Reg})})$ $(i = 1, 2, 3)$ if

TABLE 3: PRE of proposed estimator when $(\alpha_2 = \gamma_2 = 1, \alpha_1 = \alpha_3 = \gamma_1 = \gamma_3 = 0)$.

a	b	c	d	Pop. 1	Pop. 2	Pop. 3	Pop. 4
1	0	1	0	127.090	956.748	264.876	311.868
1	C_x	1	C_z	127.073	956.817	264.643	310.282
β_{2x}	C_x	β_{2z}	C_z	127.084	956.854	264.849	311.339
C_x	β_{2x}	C_z	β_{2z}	126.980	956.775	261.317	319.839
1	ρ_{yx}	1	ρ_{yz}	127.079	956.808	264.675	310.291
1	β_{2x}	1	β_{2z}	127.011	956.783	261.624	314.469
C_x	ρ_{yx}	C_z	ρ_{yz}	127.075	956.803	264.642	310.422
ρ_{yx}	C_x	ρ_{yz}	C_z	127.051	956.824	264.568	310.257
β_{2x}	ρ_{yz}	β_{2z}	ρ_{yz}	127.086	956.846	264.851	311.285
ρ_{yx}	β_{2x}	ρ_{yz}	β_{2z}	126.900	956.788	260.260	316.213

TABLE 4: PRE of proposed estimator when $(\alpha_3 = \gamma_3 = 1, \alpha_1 = \alpha_2 = \gamma_1 = \gamma_2 = 0)$.

a	b	c	d	Pop. 1	Pop. 2	Pop. 3	Pop. 4
1	0	1	0	106.402	186.926	127.151	919.696
1	C_x	1	C_z	106.646	184.941	129.254	921.254
β_{2x}	C_x	β_{2z}	C_z	106.488	183.610	127.443	920.619
C_x	β_{2x}	C_z	β_{2z}	107.724	186.467	149.441	873.745
1	ρ_{yx}	1	ρ_{yz}	106.555	185.296	129.179	921.052
1	β_{2x}	1	β_{2z}	107.357	186.124	144.723	889.198
C_x	ρ_{yx}	C_z	ρ_{yz}	106.615	185.553	129.618	920.360
ρ_{yx}	C_x	ρ_{yz}	C_z	106.941	184.727	129.828	921.127
β_{2x}	ρ_{yz}	β_{2z}	ρ_{yz}	106.456	183.997	127.426	920.719
ρ_{yx}	β_{2x}	ρ_{yz}	β_{2z}	108.505	186.007	149.286	883.271

TABLE 5: PRE of proposed estimator when $(\alpha_1 = \alpha_2 = \gamma_2 = 1, \alpha_3 = \gamma_1 = \gamma_3 = 0)$.

a	b	c	d	Pop. 1	Pop. 2	Pop. 3	Pop. 4
1	0	1	0	126.769	914.223	141.181	807.850
1	C_x	1	C_z	126.777	914.152	141.896	794.946
β_{2x}	C_x	β_{2z}	C_z	126.771	914.197	141.216	804.456
C_x	β_{2x}	C_z	β_{2z}	126.818	913.997	150.724	732.960
1	ρ_{yx}	1	ρ_{yz}	126.774	914.109	141.569	793.158
1	β_{2x}	1	β_{2z}	126.807	914.066	153.920	747.374
C_x	ρ_{yx}	C_z	ρ_{yz}	126.775	914.048	141.497	789.553
ρ_{yx}	C_x	ρ_{yz}	C_z	126.786	914.150	142.202	793.730
β_{2x}	ρ_{yz}	β_{2z}	ρ_{yz}	126.770	914.155	141.204	803.975
ρ_{yx}	β_{2x}	ρ_{yz}	β_{2z}	126.849	914.058	159.313	741.821

TABLE 6: PRE of proposed estimator when $(\alpha_1 = \alpha_3 = \gamma_2 = 1, \alpha_2 = \gamma_1 = \gamma_3 = 0)$.

a	b	c	d	Pop. 1	Pop. 2	Pop. 3	Pop. 4
1	0	1	0	126.737	912.457	139.553	845.147
1	C_x	1	C_z	126.743	912.383	140.132	836.605
β_{2x}	C_x	β_{2z}	C_z	126.739	912.423	139.573	842.896
C_x	β_{2x}	C_z	β_{2z}	126.779	912.237	147.425	797.474
1	ρ_{yx}	1	ρ_{yz}	126.741	912.342	139.830	835.452
1	β_{2x}	1	β_{2z}	126.769	912.303	150.655	806.104
C_x	ρ_{yx}	C_z	ρ_{yz}	126.742	914.284	139.744	833.140
ρ_{yx}	C_x	ρ_{yz}	C_z	126.752	912.380	140.393	835.815
β_{2x}	ρ_{yz}	β_{2z}	ρ_{yz}	126.738	914.383	139.562	842.580
ρ_{yx}	β_{2x}	ρ_{yz}	β_{2z}	126.805	912.295	155.475	802.724

Table 7: PRE of proposed estimator when $(\alpha_2 = \alpha_3 = \gamma_2 = 1, \alpha_1 = \gamma_1 = \gamma_3 = 0)$.

a	b	c	d	Pop. 1	Pop. 2	Pop. 3	Pop. 4
1	0	1	0	126.710	920.174	140.021	784.443
1	C_x	1	C_z	126.715	920.076	140.691	775.612
β_{2x}	C_x	β_{2z}	C_z	126.711	920.126	140.050	782.185
C_x	β_{2x}	C_z	β_{2z}	126.743	919.891	149.088	727.261
1	ρ_{yx}	1	ρ_{yz}	126.713	920.025	140.371	774.362
1	β_{2x}	1	β_{2z}	126.735	919.976	152.300	739.153
C_x	ρ_{yx}	C_z	ρ_{yz}	126.714	919.950	140.293	771.816
ρ_{yx}	C_x	ρ_{yz}	C_z	126.721	920.073	140.983	774.759
β_{2x}	ρ_{yz}	β_{2z}	ρ_{yz}	126.710	920.075	140.038	781.863
ρ_{yx}	β_{2x}	ρ_{yz}	β_{2z}	126.763	919.965	157.553	734.610

Table 8: PRE of proposed estimator when $(\alpha_1 = \alpha_2 = \alpha_3 = \gamma_2 = 1, \gamma_1 = \gamma_3 = 0)$.

a	b	c	d	Pop. 1	Pop. 2	Pop. 3	Pop. 4
1	0	1	0	126.768	907.889	139.833	902.503
1	C_x	1	C_z	126.776	907.822	140.396	890.351
β_{2x}	C_x	β_{2z}	C_z	126.770	907.856	139.852	899.195
C_x	β_{2x}	C_z	β_{2z}	126.818	907.695	147.416	843.179
1	ρ_{yx}	1	ρ_{yz}	126.772	907.787	140.103	888.744
1	β_{2x}	1	β_{2z}	126.806	907.753	150.567	852.694
C_x	ρ_{yx}	C_z	ρ_{yz}	126.774	907.736	140.019	885.566
ρ_{yx}	C_x	ρ_{yz}	C_z	126.785	907.820	140.649	889.257
β_{2x}	ρ_{yz}	β_{2z}	ρ_{yz}	126.769	907.821	139.842	898.731
ρ_{yx}	β_{2x}	ρ_{yz}	β_{2z}	126.850	907.746	155.212	848.913

Table 9: PRE of proposed estimator when $(\alpha_1 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_2 = \alpha_3 = 0)$.

a	b	c	d	Pop. 1	Pop. 2	Pop. 3	Pop. 4
1	0	1	0	123.139	145.452	159.789	669.527
1	C_x	1	C_z	123.185	146.707	161.602	702.708
β_{2x}	C_x	β_{2z}	C_z	123.155	147.296	160.040	678.643
C_x	β_{2x}	C_z	β_{2z}	123.384	146.324	178.354	820.621
1	ρ_{yx}	1	ρ_{yz}	123.168	146.636	161.528	707.218
1	β_{2x}	1	β_{2z}	123.317	146.313	174.823	798.447
C_x	ρ_{yx}	C_z	ρ_{yz}	123.179	146.668	161.898	716.133
ρ_{yx}	C_x	ρ_{yz}	C_z	123.240	146.826	162.099	705.743
β_{2x}	ρ_{yz}	β_{2z}	ρ_{yz}	123.149	147.206	160.025	679.938
ρ_{yx}	β_{2x}	ρ_{yz}	β_{2z}	123.523	146.397	178.726	807.318

Table 10: PRE of proposed estimator when $(\alpha_2 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_1 = \alpha_3 = 0)$.

a	b	c	d	Pop. 1	Pop. 2	Pop. 3	Pop. 4
1	0	1	0	127.304	483.947	272.583	956.165
1	C_x	1	C_z	127.306	485.394	272.843	952.519
β_{2x}	C_x	β_{2z}	C_z	127.304	486.023	272.627	955.357
C_x	β_{2x}	C_z	β_{2z}	127.314	485.060	274.208	928.019
1	ρ_{yx}	1	ρ_{yz}	127.305	485.341	272.869	952.032
1	β_{2x}	1	β_{2z}	127.311	485.006	273.258	933.369
C_x	ρ_{yx}	C_z	ρ_{yz}	127.306	485.413	272.947	952.016
ρ_{yx}	C_x	ρ_{yz}	C_z	127.308	485.528	272.898	952.166
β_{2x}	ρ_{yz}	β_{2z}	ρ_{yz}	127.304	485.948	272.626	955.247
ρ_{yx}	β_{2x}	ρ_{yz}	β_{2z}	127.318	485.105	272.896	931.187

Table 11: PRE of proposed estimator when $(\alpha_3 = \gamma_1 = \gamma_2 = \gamma_3 = 1, \alpha_1 = \alpha_2 = 0)$.

a	b	c	d	Pop. 1	Pop. 2	Pop. 3	Pop. 4
1	0	1	0	122.150	178.926	180.747	649.616
1	C_x	1	C_z	122.196	180.406	181.991	682.237
β_{2x}	C_x	β_{2z}	C_z	122.167	181.159	180.961	658.386
C_x	β_{2x}	C_z	β_{2z}	122.392	179.818	192.910	797.503
1	ρ_{yx}	1	ρ_{yz}	122.179	180.286	182.139	686.675
1	β_{2x}	1	β_{2z}	122.324	179.859	188.365	776.097
C_x	ρ_{yx}	C_z	ρ_{yz}	122.191	180.278	182.524	695.431
ρ_{yx}	C_x	ρ_{yz}	C_z	122.251	180.550	182.263	685.229
β_{2x}	ρ_{yz}	β_{2z}	ρ_{yz}	122.161	181.016	180.955	659.670
ρ_{yx}	β_{2x}	ρ_{yz}	β_{2z}	122.529	179.955	190.082	784.693

$$\left[\Theta C_y^2\left(1-\rho_{yx}^2\right)+\frac{O_1}{O_2}-1\right]>0,$$

$$\left[\Theta C_y^2\left(1-\rho_{yz}^2\right)+\frac{O_1}{O_2}-1\right]>0, \quad (52)$$

$$\left[\Theta C_y^2\left(1-\rho_{yx}^2-\rho_{yz}^2-2\rho_{yx}\rho_{yz}\rho_{xz}\right)+\frac{O_1}{O_2}-1\right]>0.$$

Condition 5. By (20) and (38), MSE $(\widehat{\overline{Y}}_{(M)}^{(\alpha_1,\alpha_2,\alpha_3,\gamma_1,\gamma_2,\gamma_3)})_{\min} < \text{MSE}\,(\widehat{\overline{Y}}_{(\text{Reg})}^{(M)})$ if

$$\left[\Theta\left\{C_y^2\left(1-\rho_{yx}^2\right)+C_z^2-2C_{yz}+2C_yC_z\rho_{yx}\rho_{xz}\right\}+\frac{O_1}{O_2}-1\right]>0. \quad (53)$$

Condition 6. By (22) and (38), MSE $(\widehat{\overline{Y}}_{(M)}^{(\alpha_1,\alpha_2,\alpha_3,\gamma_1,\gamma_2,\gamma_3)})_{\min} < \text{MSE}\,(\widehat{\overline{Y}}_{(\text{Reg})}^{(S)})_{\min}$ if

$$\left[\Theta C_y^2\left\{\left(C_y^2+C_z^2-2C_{yz}\right)-\left(C_y\rho_{yx}-C_z\rho_{xz}\right)^2\right\}+\frac{O_1}{O_2}-1\right]>0. \quad (54)$$

Condition 7. By (32) and (38), MSE $(\widehat{\overline{Y}}_{(M)}^{(\alpha_1,\alpha_2,\alpha_3,\gamma_1,\gamma_2,\gamma_3)})_{\min} < \text{MSE}\,(\widehat{\overline{Y}}_{(\text{Reg})}^{(U)})_{\min}$ if

$$\left[\Theta C_y^2\left(1-R_{y.xz}^2\right)+\frac{O_1}{O_2}-1\right]>0. \quad (55)$$

The proposed class of estimators will perform better when conditions 1–7 are satisfied.

6. Conclusion

In this study, we have proposed ratio-exponential-log type generalized class of estimators by combing a ratio, exponential-ratio, and log-ratio type estimators by using the linear transformation for finite population mean in simple random sampling. Expressions for the bias and MSE of proposed general class of estimators are obtained up to the first order of approximation. Four data sets are used for numerical study. Based on Tables 1–11, we observe that the proposed sub-classes of general estimators are performing well as compared to their competitor estimators. We have generated 10 sub-classes from the proposed general estimators with different combinations which all are efficient in different situation as compared to SRS. So, the proposed general class of estimators is preferable in further study.

Data Availability

The data used to support the findings of this study are included within the article.

Conflicts of Interest

The authors declare that they have no conflicts of interest.

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